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# Optimal design of sequential excitation for identification of multi-variable systems

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**Abstract:** While designing excitation signals for identification of industrial processes, it is important to obtain desired model accuracies, reduce the experimental time and limit the output amplitudes within the specified bounds to avoid serious disruptions of the nominal process operation. In this work, we design a multi-frequency multi-amplitude square wave (multi-square) input based on a nominal model by minimizing the experiment length and placing constraints on the model accuracy (in the frequency domain) and the output amplitudes. A separate design is carried out for each input where the resulting optimization problem has the same structure as a semi-definite program but with the decision variables restricted to integers corresponding to the number of half-periods of each square-wave. For processes with multiple inputs, the corresponding designs are carried out sequentially. The violations in the output constraints either due to model-plant mismatch or unmeasured disturbances should be mitigated by appropriate closed loop control actions. The efficacy of the proposed design is shown by means of a simulation case study.

*Keywords:* Experiment design, System identification, Multi-square, Sequential excitation, Output constraints

## 1. INTRODUCTION

For a commercial MPC project the most time consuming step is to carry out experiments and subsequently from collected data identify models (Ogunnaike, 1996). For an automation vendor the time spent on site is probably the most costly part of the project. With this motivation, this paper aims at minimizing the length of the identification experiment. Furthermore, if the process plant is in operation it is important, if possible, to still produce sellable product during the experiment, why constraints on the process variables are important.

Optimal input design for dynamical systems has a long history. Drawing from work in statistics, e.g. (Wynn, 1970), the foundations were laid out in the 1970s, essentially for linear time-invariant (LTI) single-input single-output (SISO) systems. Key issues were parametrizations of the input (often a sum of sinusoids were used) enabling desired metrics of Fisher information matrix to be achieved, computational techniques, and the development of different general purpose criteria (Mehra, 1974; Goodwin and Payne, 1977; Zarrop, 1979). A new direction for optimal input design opened up in the 1980s when expressions for the high order asymptotic variance of estimated transfer functions became available, thanks to the work of Ljung and co-workers (Ljung, 1985; Yuan and Ljung, 1984; Ljung and Wahlberg, 1992). These expressions allowed for frequency by frequency design of the input spectrum (Yuan and Ljung, 1985; Ljung, 1999) and through the multi-input multi-output (MIMO) extension of the variance theory (Zhu, 1989), also input design for MIMO systems could be done (Zhu, 2001). Employing these methods, the design criteria were extended to application related criteria (Gevers and Ljung, 1986; Ljung, 1999). With computational tools for semi-definite programming (SDP) becoming widely available around the turn of the century, the focus shifted back to input design based on finite order variance expressions as many such problems could be cast as SDPs in the input spectrum (Cooley et al., 1998; Lindqvist and Hjalmarsson, 2000; Jansson and Hjalmarsson, 2005). These techniques were also amenable to LTI MIMO systems. Building on this framework, least-costly input design was introduced where, instead of optimizing a model quality criterion based on a constrained experimental budget, the objective function and constraint were interchanged, leading to the minimization of the experimental budget subject to quality constraints (Bombois et al., 2006). A general perspective on application oriented input design is given in Hjalmarsson (2009).

During the first decade of this century, the major focus had been on LTI systems with convex optimization techniques employing the relaxation that instead of the input sequence, the input spectrum is designed. The input sequence is subsequently generated by filtering white noise through a filter obtained via spectral factorization of the designed input spectrum. This means that for such methods it is difficult to incorporate amplitude constraints in the input design formulation. A remedy to this is proposed in Hägg

et al. (2013) where MPC is used to generate the input such that both input and output constraints are maintained at the same time as that desired spectral properties are obtained. Direct design of the input sequence has also been considered, where amplitude constraints can be included. Such techniques are greatly simplified by considering that the decision variables are restricted to a finite alphabet, for example in terms of allowed input amplitudes or subsequences of the input (De Cock et al., 2016; Forgione et al., 2014). An early attempt to convexify also amplitude constraints is given in (Manchester, 2010). Similar to the linear case, the input design problem can be relaxed into a convex problem for nonlinear models by considering the probability density of the input sequence, or a linear parametrization thereof, as decision variable (Valenzuela et al., 2015). Computational complexity tends to become very high for this family of methods. An alternative minimization method for the direct design of an input sequence that yields a desired Fisher information matrix is suggested by Parsa and Hjalmarsson (2021). The technique is applicable to FIR models and thus allows direct input sequence design in an application oriented framework. Also particle filter methods have been proposed for input design (Gopaluni et al., 2011). The bulk of methods considering amplitude constraints are restricted to SISO systems.

Despite the vast literature on optimal input design, there appears to be no work that minimizes the experiment time while having constraints on the process outputs.

## 2. PROBLEM DESCRIPTION

The main goals of this paper can be formulated as:

- Construct excitation signals to minimize the experiment time needed to obtain a dynamic model with a given accuracy.
- Alternatively, construct excitation signals to obtain the most accurate model with a given experiment time.

Ideally, it would be nice to define the excitation in terms of the control performance. Some attempts have been done for SISO systems (Hjalmarsson, 2005; Barenthin and Hjalmarsson, 2008; Hjalmarsson and Ninness, 2006), but this is considered as too difficult a task for the multivariable case here.

For the forthcoming control design and closed loop control it is, from a robustness perspective, important that the transfer function is accurate at frequencies slightly under and up to the phase crossover frequency.

Frequencies higher than the crossover are of less interest for control since it is assumed that the process gain decreases significantly for these frequencies, and is well above the bandwidth of the closed loop system. It can be debated if excitation at high frequencies are meaningful. While it can be argued that information at high frequencies are extrapolated to other frequencies when the true system and the used model both are of low order, with the true system in the model set, these assumptions are rarely met in practice. Instead such excitation may have a negative impact. Therefore excitation and evaluation of accuracy is recommended to take place below the crossover frequency. The required standard deviation of the estimated model is here defined relative to the static gain

$$R(\omega) = r|G(0)|, \qquad \forall \omega$$

Other requirements would also be possible, e.g. relative to frequency varying gain $R(\omega) = r|G(\omega)|$, up to the cross-over frequency. For higher frequencies it is recommended to have a fixed bound because the gain of the frequency response is so low that a relative bound will be unnecessarily tight. Here $r = 0.03$ is used which would give a $3\sigma$ limit of about 9% relative error at low frequencies.

## 3. CHARACTERISTICS OF THE INPUT SIGNAL

As a starting point for the experiment design we assume that a seed model is available. This seed model can be obtained in different ways. If it is a model re-identification in an already running MPC installation, the model used in the MPC will naturally serve as the seed model. If we are dealing with a new MPC project there are at least two ways to estimate a model: Either one can screen historic operating data to find intervals suitable for identification (see Bittencourt et al. (2015) for a SISO model approach), or an initial simple experiment has to be performed.

From the step response of the seed model, it is possible to obtain information related to its frequency characteristics. We will use $t_{90}$ which is defined as the time when a step reaches 90% of its final value. If needed, an adequate sampling interval $T_s$ could also be chosen from the step response of the seed model using recommendations in standard text books.

Based on $t_{90}$ it is possible to approximate the lowest frequency where it is relevant to excite the system:

$$\omega_{lo} = \frac{2\pi}{2t_{90}} \tag{1}$$

which corresponds to a period time $T_p = 2t_{90}$ for a square wave excitation signal. For a delay dominated SISO system, this choice would correspond to a frequency somewhat lower than the phase cross-over frequency, i.e. where it from robustness would be good to have an accurate model.

### *3.1 Choice of Excitation Signal*

Here square wave input signals are used for the excitation. A square wave signal has the Fourier series expansion

$$u(t) = \frac{4A}{\pi} \sum_{m \in I} \frac{1}{m} \sin(\omega_0 m t) \tag{2}$$

where $A$ is the amplitude, $\omega_0$ is the base frequency of the square wave, and $I = \{1, 3, 5, \ldots\}$. Hence, the square wave has its main energy at the frequencies $\omega_0$ and $3\omega_0$, and the energy is then decaying for higher frequencies.

One reason for using square waves is that it is essentially a sequence of step changes which is a well known and accepted test signal in process industry. Furthermore, in a situation where output amplitude constraints are essential, a square wave is preferred over, for example, a sinusoid since as noted in (2) the contribution from the fundamental frequency has amplitude $4A/\pi$, despite that the square wave amplitude is only $A$. Then as an extra bonus we get a number of harmonics too. You simply get a lot of excitation compared to the signal amplitude. While PRBS has the

similar property, a multi-square provides a much better handle on exactly which frequencies at what amplitudes that are injected.

Therefore in this paper, an appropriate number of such square waves having optimal length and amplitude (given by input design) are concatenated to form an input sequence for performing the identification experiment.

### *3.2 Choice of Frequency Range and Candidate Square Waves*

It is important that the excitation takes place in a frequency interval where an accurate model is desired, otherwise unnecessary long experiments will occur.

The lowest characteristic frequency of interest is given by (1). In fact for a single-input multiple-output (SIMO) system, the lowest excitation frequency would be chosen as the lowest of the $\omega_{lo}$ among the individual transfer functions in the corresponding input column of the transfer function matrix.

The highest frequency of interest is chosen as

$$\omega_{hi} = 10\,\omega_{lo}$$

In addition the highest frequency for use is limited to $\omega_{hi} \leq \pi/(2T_s)$, i.e. half the Nyquist frequency which means that the square wave with the highest frequency will have at least two samples in each half period.

The following candidate square waves are proposed for the excitation:

$$[\omega_{lo}, 2\omega_{lo}, 3\omega_{lo}, \cdots, \omega_{hi}]$$

Among these, the frequencies to be used will be determined using optimization which decides how many half periods of each candidate frequency that are needed to obtain the desired model accuracy.

The requested covariance profile was defined in Section 2. The evaluation frequencies are recommended to start slightly lower than the base square wave frequency. Here eight frequencies in $[0.6\,\omega_{lo}, 10\,\omega_{lo}]$ were used.

### *3.3 Choice of Input Signal Amplitude*

As stated in the introduction, for the experiment design it is required that the input signal amplitudes are chosen such that the outputs remain within the desired zones during the experiment.

Appropriate amplitudes can be obtained from the seed model by simply simulating the individual transfer functions in one column of the model (corresponding to one input signal). Given a number of square wave candidates, the amplitude for each of these could be determined directly from the simulated outputs, such that

$$|y_i(t)| = |y_i^0(t)A| \leq z_i, \qquad \forall i, \forall t$$

where $y_i^0(t)$ is the output from the seed model for a square wave input with amplitude 1, $A$ is the amplitude for the actual square wave, and $z_i$ are user requested bounds for each output (sometimes in the sequel referred to as zone limits).

If the individual transfer functions in a column of a MIMO system have a large variation in gain it may not be possible to obtain good models for each of them. This is especially pronounced in a zone-limit scenario where it is not possible to increase the amplitude of the excitation signal without exceeding the allowed zones.

## 4. ACCURACY OF ESTIMATED TRANSFER FUNCTIONS

This section discusses ways to predict the accuracy of estimated transfer functions given a certain experiment.

### *4.1 Standard Theory for OE Model*

From, for example, Ljung (1999), the asymptotic covariance of the parameters for an identified transfer function can be estimated by

$$\hat{P}_N = \hat{\lambda}_N \left[ \frac{1}{N} \sum_{t=1}^{N} \psi(t, \hat{\theta}_N) \psi^T(t, \hat{\theta}_N) \right]^{-1}$$

where $\psi$ are derivatives of the prediction with respect to the parameters, and the noise variance is estimated by

$$\hat{\lambda}_N = \frac{1}{N} \sum_{t=1}^{N} \epsilon^2(t, \hat{\theta}_N)$$

For a sequential combination of two square wave sequences it follows that

$$\begin{aligned} \hat{P}_N \;=\; & \hat{\lambda}_N \Big[ \tfrac{1}{N} \Big( \textstyle\sum_{t=1}^{N_1} \psi_1(t, \hat{\theta}_N) \psi_1^T(t, \hat{\theta}_N) + \\ & + \textstyle\sum_{t=N_1+1}^{N} \psi_2(t, \hat{\theta}_N) \psi_2^T(t, \hat{\theta}_N) \Big) \Big]^{-1} = \\ & \Big[ \tfrac{1}{N_1+N_2} \Big( \tfrac{N_1}{\hat{\lambda}_N} \Big( \tfrac{1}{N_1} \textstyle\sum_{t=1}^{N_1} \psi_1(t, \hat{\theta}_N) \psi_1^T(t, \hat{\theta}_N) \Big) \\ & + \tfrac{N_2}{\hat{\lambda}_N} \Big( \tfrac{1}{N_2} \textstyle\sum_{t=N_1+1}^{N_1+N_2} \psi_2(t, \hat{\theta}_N) \psi_2^T(t, \hat{\theta}_N) \Big) \Big) \Big]^{-1} \end{aligned}$$

For our design, the averaged sums in the expression above, which we may denote $\bar{P}_1$ and $\bar{P}_2$, will be pre-calculated by simulating square waves through the seed model for appropriately large values of $N_1$ and $N_2$. Large enough for the sums to converge to resemble expected values. Hence it follows that

$$\hat{P}_N = \left[ \frac{1}{N} \left( \frac{N_1}{\hat{\lambda}_N} \bar{P}_1 + \frac{N_2}{\hat{\lambda}_N} \bar{P}_2 \right) \right]^{-1}$$

Introduce

$$M_j = \frac{N_j}{2} \frac{1}{\hat{\lambda}_N} \bar{P}_j$$

which is the contribution to the covariance expression from a half period of a square wave with the frequency $\omega_j = 2\pi/(N_j T_s)$. Then the covariance of the parameters for an estimated transfer function is

$$\hat{P}_N = \left[ \frac{1}{N} \left( x_1 M_1 + x_2 M_2 \right) \right]^{-1}$$

where $N$ is the length of the experiment and $x_j$ is the number of half periods of the square wave $j$. Note that the above formulas generalize straightforwardly to more than two square waves. Here we used two for simplicity of illustration. Also, a specific $M_j$ is determined for each candidate square wave and for each output in the SIMO system. The square wave amplitudes $A$ are chosen as outlined in Section 3.3.

From Ljung (1999) the parameter covariance over an experiment of $N$ samples length is given as

$$\operatorname{cov}\hat{\theta}_N \approx \frac{1}{N}\hat{P}_N \approx (x_1 M_1 + x_2 M_2)^{-1}$$

To determine the derivatives $\psi(t,\hat{\theta}_N)$, a model structure and also the estimated parameters $\hat{\theta}$ are needed. To illustrate this, consider that the model is a first order discrete time OE model

$$y(k) = \frac{b_1 + b_2 q^{-1}}{1 + a_1 q^{-1}}\, q^{-d}\, u(k) + e(k)$$

with parameters as

$$\theta = [a_1 \quad b_1 \quad b_2]^T$$

and further

$$\varphi(t,\theta) = [-w(t-1) \quad u(t-d) \quad u(t-d-1)]^T$$

where $w(t)$ is the simulated output from the model above with $e(k) = 0$. Also introduce for use below the notation $A(q^{-1})$ for the denominator of the model, i.e.

$$A(q^{-1}) = 1 + a_1 q^{-1}$$

The prediction is given by

$$\hat{y}(t|\theta) = \varphi^T(t,\theta)\theta$$

and the derivatives of the predictor with respect to the parameters

$$\psi(t,\hat{\theta}_N) = \left(\frac{d\hat{y}}{da_1} \quad \frac{d\hat{y}}{db_1} \quad \frac{d\hat{y}}{db_2}\right)^T$$

can be obtained by filtering $\varphi(t,\hat{\theta})$ through the filter $1/A(q^{-1})$, i.e.

$$A(q)\psi(t,\theta) = \varphi(t,\theta)$$

Further, the covariance of the frequency response magnitude is given by

$$\begin{aligned}\operatorname{cov}|G(e^{i\omega T_s})| &= \Gamma^*(e^{i\omega T_s}) \cdot \operatorname{cov}\hat{\theta}_N \cdot \Gamma(e^{i\omega T_s}) \\ &= \Gamma^*(e^{i\omega T_s}) \cdot (x_1 M_1 + x_2 M_2)^{-1} \cdot \Gamma(e^{i\omega T_s})\end{aligned}$$

where $\Gamma(e^{i\omega T_s})$ is the derivative of the magnitude function with respect to the model parameters.

$$\Gamma(e^{i\omega T_s}) = \left(\frac{d|G(e^{i\omega T_s})|}{da_1} \quad \frac{d|G(e^{i\omega T_s})|}{db_1} \quad \frac{d|G(e^{i\omega T_s})|}{db_2}\right)^T$$

Notice that the derivation above is described for a first order model structure. If the true system is of higher order this will lead to bias which is not accounted for in these expressions. It is, however, straightforward to redo the derivation for higher-order models or other model structures. We have, for example, in subsequent work that will presented in Sigurdsson et al. (2024) used higher order Laguerre models to reduce the bias. This will lead to very similar expressions in the next chapter, but of course lead to slightly different optimal inputs.

## 5. EXPERIMENT DESIGN

The covariance of the magnitude of the frequency response is given by the derivation in the previous section. In the experiment design there are constraints on this covariance, for each individual transfer function, $G_i$, and for each frequency $\omega$ chosen for evaluation. It follows that

$$\begin{aligned}\operatorname{cov} G_i(e^{i\omega T_s}) &= \Gamma_i^*(e^{i\omega T_s}) \cdot \left(\textstyle\sum_j x_j M_{ij}\right)^{-1} \cdot \Gamma_i(e^{i\omega T_s}) \\ &\le R_i^2(\omega) \qquad \forall\omega \quad \forall i\end{aligned}$$

where $M_{ij}$ are parameter covariance contributions for the square wave candidates defined in Section 4.1 and the amplitudes have beforehand been determined as discussed in Section 3.3. Further, $x_j$ are the integer number of half periods for each square wave candidate. The inequality above should be satisfied for all evaluation frequencies $\omega$ and all transfer functions $G_i$ in the actual column in the multivariable transfer function. Using a Schur complement, the constraint can be re-written to a matrix inequality as

$$\begin{pmatrix} R_i^2(\omega) & \Gamma_i^*(e^{i\omega T_s}) \\ \Gamma_i(e^{i\omega T_s}) & \sum x_j M_{ij} \end{pmatrix} \ge 0 \qquad \forall\omega \quad \forall i \tag{3}$$

This would be a linear matrix inequality (LMI) in the decision variables $x_j$ if these were free variables. However, here they are integers representing the number of half-periods of each square-wave. Fortunately, it is still fairly easy to solve the optimization problem with satisfactory precision. Here YALMIP Löfberg (2004) is used with the solver CUTSDP and the lower level MILP solver GLPK (2012).

As discussed above two different optimization problems can now be defined depending on the objective.

### *5.1 Shortest Experiment*

To find the shortest possible experiment that gives the desired accuracy of the estimated transfer functions, the task is to select a number of half periods from a given a set of square wave candidates with the period times $N_i$ and pre-specified amplitudes. The shortest experiment is obtained by minimizing

$$\sum_{j=1}^{n_{sqw}} x_j N_j/2$$

subject to (3), where the free integer variable $x_j$ is the number of half periods of square wave $j$.

Notice that one optimization is made per input signal. Since we here assume identification of one SIMO system per input signal, it does not matter in which order the sequential inputs are applied.

### *5.2 Most Accurate Model in Given Time*

An alternative (dual) formulation of the experiment design problem is to find the most accurate model for a given experiment length. This is obtained by minimizing $z$ subject to a modification of (3) where the covariance is scaled by $z$, i.e

$$\begin{pmatrix} zR_i^2(\omega) & \Gamma_i^*(e^{i\omega T_s}) \\ \Gamma_i(e^{i\omega T_s}) & \sum x_j M_{ij} \end{pmatrix} \ge 0 \qquad \forall\omega \quad \forall i$$

and subject to a constraint on the experiment length

$$\sum_{j=1}^{nsq} x_j N_j/2 \le N_{max}$$

where $N_{max}$ is the maximal experiment length expressed in samples.

Since $z$ is a scaling of the desired covariance profile $R^2(\omega)$, an optimal value of $z > 1$ means that accuracy has to be sacrificed at the required experiment length. An optimal $z < 1$, however, means that a shorter experiment would be able to achieve the originally specified accuracy.

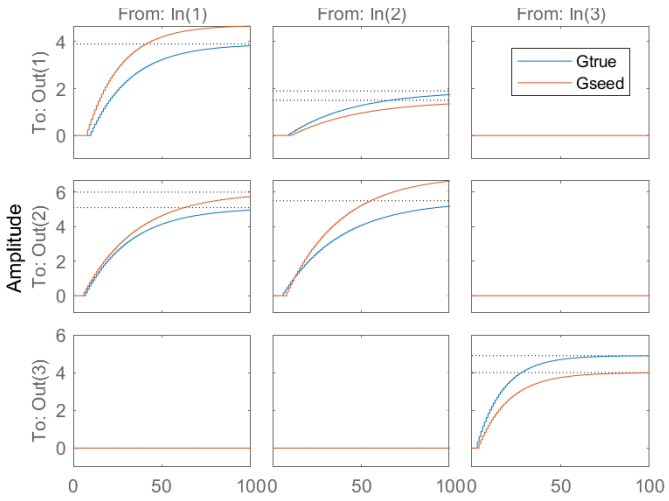


Fig. 1. Step responses for true system and seed model

## 6. EVALUATION

The method presented in this paper has been tested for many processes. Here due to space limitations we focus on one example which has three inputs $u(k)$ and three outputs $y(k)$ and is defined as

$$y(k) = G(q)u(k) + H(q)e(k)$$

where $G(q)$ and $H(q)$ are discrete time transfer functions, and $e(k)$ is a white noise vector with each element having unit variance. Further $G(q)$ is the discrete time counterpart of

$$G_0(s) = \begin{pmatrix} \frac{3.9}{23s+1}e^{-9s} & \frac{1.9}{37s+1}e^{-8s} & 0 \\ \frac{5.1}{26s+1}e^{-6s} & \frac{5.5}{33s+1}e^{-5s} & 0 \\ 0 & 0 & \frac{4.9}{15s+1}e^{-2s} \end{pmatrix}$$

when sampled with $T_s = 1$ s, and the noise filter is given by

$$H(q) = \text{diag}\left( \frac{0.04}{q-0.2}, \frac{0.08}{q-0.17}, \frac{0.04}{q-0.25} \right)$$

For the simulations presented below the following seed model is used

$$G_s(s) = \begin{pmatrix} \frac{4.7}{19s+1}e^{-7s} & \frac{1.5}{39s+1}e^{-10s} & 0 \\ \frac{6.0}{30s+1}e^{-5s} & \frac{7.0}{31s+1}e^{-7s} & 0 \\ 0 & 0 & \frac{4.0}{17s+1}e^{-3s} \end{pmatrix}$$

The step responses for true system as well seed model are presented in Figure 1.

For the experiment design it was assumed that for each output there is an independent white measurement noise with standard deviation

$$\hat{\sigma}_N = (\, 0.05 \;\; 0.1 \;\; 0.05 \,)$$

which is in fact slightly higher than that of the coloured noise. Furthermore for this example, it was assumed that all three outputs have identical design constraints and the same with the three inputs:

$$|y_i(t)| \leq 0.5; |u_j(t)| \leq 1;$$

To determine the shortest excitation sequence, the optimization procedure based on OE model theory is repeated three times, one for each input.

Figure 2 shows the resulting optimal experiment. The bottom three rows show the designed input signals and the three top rows show the three outputs and their respective zone limits. Each of the optimal sequential excitation signals is followed by a section where the input is zero to allow time for the system to respond completely to the previous input (see vertical red line for the start of next part of the experiment). For this particular example, the three optimal excitation signals show slightly different behaviours. The optimal first input is based on only one rather high frequency, while the second is a combination of one low and one high frequency. The optimal third input is simply one step up, and one step down. In a way the proposed method can be viewed as a systematic approach to design a generalized step test to obtain the desired model accuracy.

For outputs 1 and 2, the process outputs mainly stay within the defined output zone constraints, except for some samples where the noise causes the zone violation. The third output, however, significantly exceeds the upper constraint. This is caused by the seed model having much lower gain for $G_{33}$, which creates an overly optimistic amplitude in input 3. If this upper limit is a critical one, the experiment would have to be carried out in closed loop. We have investigated this too, using an MPC without setpoint but only soft constraints for the zone limits. One simulation example of this is presented in Sigurdsson et al. (2024).

Another observation is that the experiment is significantly longer for the second input. A reason for this is that the gain of $G_{12}$ is lower than the other gains, and therefore the amplitude for output one is lower and results in a lower signal-to-noise-ratio, which leads to a longer experiment to reach the desired accuracy. The fact that for the design we overestimated the gain of $G_{22}$ (which is limiting the amplitude of input 2) as well as the noise standard deviation both contribute in the same direction.

A final remark on the design is that knowing that $G_{33}$ is not coupled to the upper $2 \times 2$ sub-system (which we did actually assume knowledge of in our seed model), one could of course reduce the experiment time by running the $u_3$ experiment simultaneously with one of the other two experiments.

The three excitation signals were then used in an open-loop Monte-Carlo simulation with 50 different noise realizations for each input. Despite that OE was used for design, ARMAX models were identified for the different data sets to allow for a possible colour in the noise. Figure 3 shows Bode magnitude diagrams for the estimated models. The +-marks show the $3\sigma$ deviation from the nominal transfer function. The frequency responses for the identified models for the most part reside well between the markers why, despite significant error in the seed model, the experiment design was able to find excitation signals that lead to model estimates which meet the desired frequency accuracy.

## 7. CONCLUSIONS

This paper describes a framework to design experiments for process identification. Based on an approximate seed model of the process and the required model accuracy of in the frequency domain, together with allowed ranges for inputs and outputs, an optimal experiment is obtained using mathematical optimization.

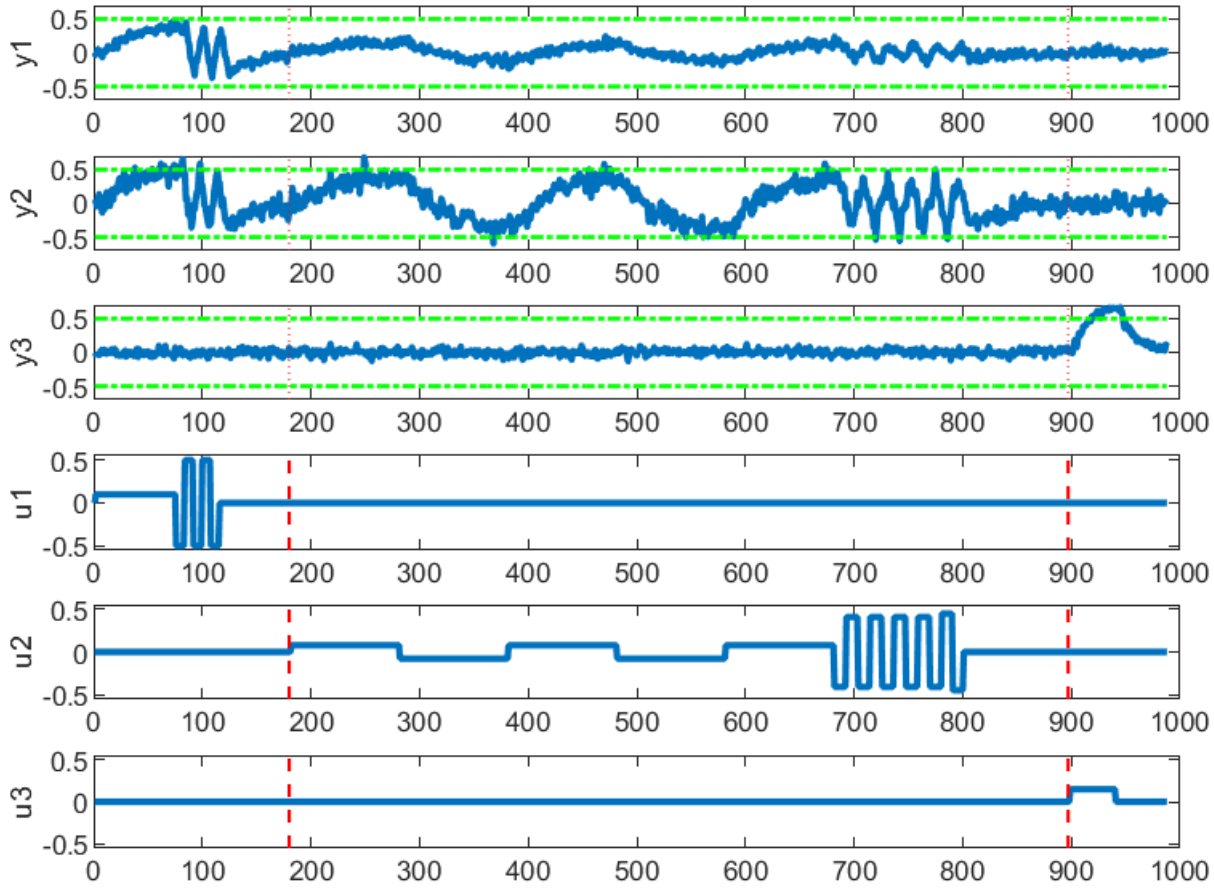


Fig. 2. Optimal input signals and resulting simulated noisy outputs. Horizontal green lines mark the output constraints, while vertical red lines mark the border between sequential experiments

One simulation example was presented here where the seed model was manually selected. However, the method has been extensively tested on a number of test models where also the seed model has first been estimated using, for example, a preliminary step experiment. Although, the procedure is approximate since it combines asymptotic theory with finite experiment lengths, our test results indicate that the method is robust and provides adequate excitation in most cases. As was pointed out in the previous section an experiment in input 3 could have been run in parallel with the sequential one in inputs 1 and 2. This opens up the general question whether simultaneous excitation could potentially lead to a shorter experiment. This, however, gives a much more complex optimization problem since both experiment length as well as all input amplitudes need to be free variables. Such a study is not within the scope of this paper and will be presented in Sigurdsson et al. (2024).

Since the optimal theory builds on knowledge of the true process, the quality of the seed model will inevitably have an impact on the achieved model accuracy. We would propose to use an iterative procedure where the obtained accuracy is evaluated using standard theory, followed by additional experiments in inputs whose transfer functions are still not good enough.

Estimation of the delay in the process is crucial and several approaches have been tested. However, the proposed experiment design considers only the standard deviation of the process gain, not the phase shift. A different experiment design approach would probably provide better excitation for time delay estimation. Notice that processes with integrating dynamics are not covered in this paper, and need special treatment.

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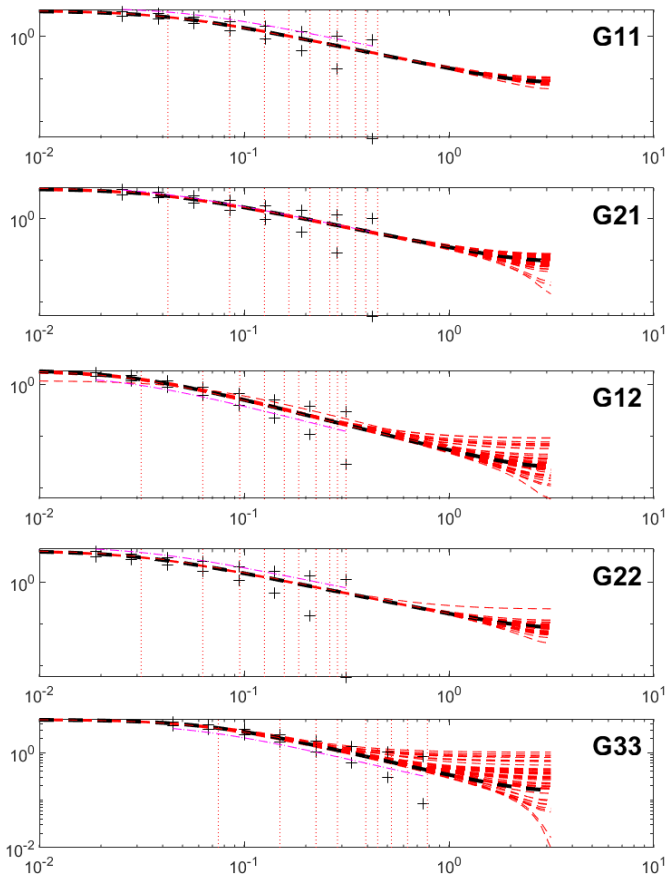


Fig. 3. Frequency responses of identified models from Monte-Carlo simulations.

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