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Model Predictive Control for Bottleneck Isolation with Unmeasured Faults

Evren Mert Turan * **Sigurd Skogestad** * **Johannes Jäschke** *

* *Department of Chemical Engineering, Norwegian University of Science and Technology (NTNU), Høgskoleringen 1, Trondheim, 7034, Norway. (e-mail: evren.m.turan@ntnu.no, sigurd.skogestad@ntnu.no, johannes.jaschke@ntnu.no).*

Abstract: We address the task of allocating process inventories to maximise production and bottleneck isolation using a model predictive control (MPC) scheme. This scheme implicitly defines “set-points” for the inventories based on current operating conditions, and automatically adjusts these set-points when the operating conditions change. This problem has previously been identified as a challenge for MPC, and likely to requiring a forecast of disturbances or multi-scenario approach. In contrast, we address this challenge with an appropriate choice of the MPC objective and design of a disturbance model. The combined scheme does not require a forecast of disturbances or involve significant computational expense while allowing for the MPC to automatically correct for misidentified bottlenecks or unmeasured faults.

Keywords: Process control, Inventory control, Model predictive control

1. INTRODUCTION

Despite large variations in the design and operating considerations of chemical process plants, nearly all plants share the task of managing inventories. The inventories typically need to be controlled within given minimum and maximum bounds, with the set-point of these inventories as degrees of freedom. These set-points are important to the process economics because they act as buffers that prevent disturbances from cascading through a process and disrupting throughput (Belanger and Luyben, 1997; Zotică et al., 2022). The task of automatically adjusting these set-points based on operating concerns is a key challenge that has been identified in several works, e.g. Skogestad (2023) and the references therein. This paper considers the development of a model predictive control (MPC) scheme, with disturbance model, that implicitly defines set points for the inventories that are optimal when the process goal is to maximise throughput.

Inventory control has two competing goals (a) mitigate changes/fluctuations in inventories and (b) mitigate the effects of a reduction in the maximum flow allowed through a section or unit of the plant. These goals directly compete with each other as addressing goal (b) may necessitate changing the set-points of the process inventories based on current information.

If changes in process operations lead to a bottleneck that persists over a long enough period, then it becomes necessarily to change the inventory set-points to mitigate the influence of future bottlenecks. Likewise, once the bottleneck is relieved, the set-points have to be changed again. Automatic selection of good set-points of the inventories are key to mitigating the influence of bottlenecks on the process throughput, and is the focus of this paper, i.e. goal (b). There are predominantly two challenges in meeting goal (b) (Zotică et al., 2022; Skogestad, 2023):

- **Challenge 1.** *Use of intermediate storage for bottleneck isolation (containment): How to optimally select the inventory (level) setpoints to maximize the time until a new bottleneck makes it is necessary to decrease the throughput?*
- **Challenge 2.** *Inventory control rearrangement: How to implement a logic that automatically rearranges the inventory loops / setpoints to maintain consistent inventory control when encountering a new bottleneck?*

These challenges have been addressed in a Zotică et al. (2022), in which a system consisting of serially connected inventories is considered, and a decentralised control structure consisting of simple control elements was proposed to address these challenges. In particular a bidirectional inventory control scheme (Shinskey, 1981) is proposed and shown to be optimal for the class of systems under consideration. This control scheme was extended by Bernardino and Skogestad (2023) to consider systems with minimum flow constraints.

The inventory control problem summarised by challenges 1 and 2 was presented as a challenge for MPC as it was supposed that MPC would require either a disturbance measurement or forecast which is unrealistic, or a multi-scenario approach which would greatly increase the required computational complexity (Zotică et al., 2022). Later it was noted that challenges 1 and 2 could be addressed without minimum flow constraints through the use of unreachable set-points, assuming no model mismatch or misidentified bottlenecks (Skogestad, 2023).

In this work we make three contributions: (1) we show how for serially connected inventories model predictive control (MPC) can be developed in two ways to meet the inventory challenges, (2) we demonstrate how with a suitable disturbance model the MPC scheme can still meet the challenges despite inaccurate operating information, and (3) how the first goal of inventory control may also be incorporated in the MPC scheme. Importantly, in all the MPC implementations we do so *without* relying on a forecast of disturbances, a scenario tree or any other significant computational complications to the standard MPC problem. Instead our approach relies on either the selection of an unreachable set-point or a selection of weights for tank levels. The MPC problem is sparse and convex it can be solved rapidly and reliably by modern solvers even for large systems. Furthermore, a dynamic model of the *inventory* alone is required, i.e. a full dynamic model of the plant or process economics is not required. Thus we avoid many of the typical concerns of the complexity of implementing an MPC solution.

The paper is structured as follows: Section 2 briefly reviews the essential background of the paper, including the inventory control problem, and the MPC model, Section 3 introduces the proposed approaches when considering units in series, with Section 4 detailing practical concerns in the implementation of the MPC, including the use of a disturbance model and tuning of the transient response. Lastly we end with a discussion and conclusion in Sections 5 and 6.

2. BACKGROUND

2.1 Inventory control

Level control is a common task in process plants and there is an extensive literature on the topic, see Belanger and Luyben (1997); Zotică et al. (2022); Skogestad (2023) and the references therein. Important concepts from the literature are the throughput manipulator (TPM) and bottleneck. The TPM is defined as the variable (usually a flow rate) used to set the (steady state) throughput rate for the entire process. The production bottleneck is a constraint that limits further increase in the steady state throughput of the system. A bottleneck may thus be a wide range of things, e.g. operating temperature, but can often be written (sometimes implicitly) as a flow rate constraint. Note that this definition presupposes that an increase in the steady state throughput would be economically preferred. When considering process economics (or equivalently the maximisation of production) a good choice is to locate the TPM near the production bottleneck (Downs and Skogestad, 2011). For units in series, to satisfy the "pair-close" rule from inventory control one should follow the radiation rule (Price et al., 1994), that is, inventory control should be be in the direction of flow downstream of the TPM and it should be opposite the direction of flow upstream of the TPM. When a new bottleneck emerges, the TPM should move requiring a rearrangement of the inventory loops. Automatically performing this task is the crux of challenge 2.

Bidirectional inventory control (Shinskey, 1981) has recently been shown to resolve these challenges for units in series (Zotică et al., 2022; Bernardino and Skogestad, 2023; Skogestad, 2023), see section 3. In this work we show that a simple MPC formulation is able to meet these challenges, while also allowing for misrepresentation of process bottlenecks.

2.2 Model predictive control

Model predictive control (MPC) is a popular control strategy for constrained systems with multiple inputs and outputs, especially when explicit implementation of a control policy becomes complex. A key requirement of a successful MPC scheme is the use an adequate model. Although finding a model can generally be an arduous task, for inventory control we are able to only consider the *inventory* dynamics and thus can use a simple first principle model.

We consider a system of N_I inventories or vessels, and N_F flows. For simplicity we use a volumetric basis and assume the inventories are in rectangular tanks. Practically the inventories can be arbitrary units or process sections, as the methodology can easily be used with other appropriate extensive variables. From a volume balance we write the discrete time model:

$$ah(t_{k+1}) = ah(t_k) + MF(t_k) \tag{1a}$$

$$M_{ij} = \begin{cases} 1 \text{ if } F_j \text{ enters vessel } i \\ -1 \text{ if } F_j \text{ exits vessel } i \\ 0 \text{ otherwise} \end{cases} \tag{1b}$$

where $h \in \mathbb{R}^{N_I}$ is a vector of levels, $a \in \mathbb{R}^{N_I}$ is a vector of cross sectional areas, $F \in \mathbb{R}^{N_F}$ is a vector of flows, and $M \in \mathbb{R}^{N_I \times N_F}$ is an incidence matrix that describes the connectivity of the system.

We assume that flow rate F is our control variable, with it acting as the set point for a lower-level controller, which we assume is controlled perfectly. By this assumption we avoid non-linearities that would otherwise be included in the formulation. Additionally, in Section 4 we address how a disturbance model can be used to handle the case where the lower level controller is unable to meet the desired flow specification. We thus consider the MPC problem:

$$\min_{h,F} \; J \tag{2a}$$

$$h(t_{k+1}) = Ah(t_k) + BF(t_k) \tag{2b}$$

$$h_{\min} \leq h(t_k) \leq h_{\max} \tag{2c}$$

$$0 \leq F(t_k) \leq F_{\max} \tag{2d}$$

where J is an objective function (specified later), $h_{\min} \in \mathbb{R}_{+}^{N_I}$ and $h_{\max} \in \mathbb{R}_{+}^{N_I}$ are vectors that define the range of allowable levels, $F_{\max} \in \mathbb{R}_{++}^{N_F}$ defines the range of allowable flow rates, and N_k is the number of time points considered. To put the dynamics into standard form we have defined $B \in \mathbb{R}^{N_I \times N_F}$ as the component-wise division M/a, and $A \in \mathbb{R}^{N_I \times N_I}$ as the identity.

Note that if J is convex, then this is a convex optimisation problem. Additionally, we have assumed that there is no lower limit on the flow rate because specifying a lower limit can lead to an infeasible problem.

In this model a bottleneck occurs due to the entries in $F_{\max}$. As discussed above, a bottleneck may also be a temperature or similar, and thus only implicitly a flow rate. In this case the entries of $F_{\max}$ may be uncertain and/or incorrect, however this can also be addressed by an appropriate disturbance model (Section 4). We note that in the current MPC formulation, $F_{\max}$ is assumed to not vary in time, i.e. we do not have a forecast. If we had a forecast, this can easily be incorporated into the framework.

3. INVENTORY ALLOCATION OF UNITS IN SERIES

Consider a plant consisting of units in series, a simple example of which is shown in Figure 1. To isolate bottlenecks, while aiming to maximise throughput, one can follow the following rule (Zotică et al., 2022), which is motivated by the following example.

Rule for challenge 1. To isolate the effect of a bottleneck, the inventory set-points before the bottleneck should be set *high*, and those after should be set *low*.

Example 1. *Consider a single tank, with a valve before and after the tank (i.e. the system of F_0, F_1 and unit 1 in Figure 1) Consider that the process has been operating at a steady state of $F_0 = F_0^{max} = F_1^{max} = F_1$, i.e. there is no bottleneck as both valves are fully open. Now consider that a reduction in F_0^{max} occurs i.e. F_0 is now the bottleneck, without any forecast on how long the bottleneck will last.*

To minimise the effect of the bottleneck of F_0 on F_1 one should keep the flow rate of F_1 the same, which can be done as long as the tank level is sufficiently above it's minimum heights. If the bottleneck persists until the tank depletes, then the system should be operated with the level set point of $h_{\min}$, and $F_1 = F_0$. If F_0 becomes further reduced then as there is no buffer inventory available F_1 is similarly reduced (set point remains $h_{\min}$). Now consider that the reduction is lifted, with the new $F_0^{max} \geq F_1^{max}$, i.e. F_1 is the bottleneck. To isolate the effect of the F_1 bottleneck, and *to maximise the isolation time of a future bottleneck, one should operate with $F_0 = F_0^{max}$, i.e. the set point becomes $h_{\max}$. The above argument can be extended for an arbitrary sequence of tanks in series, leading to the rule.*

The bidirectional control structure, shown in Figure 1, implicitly follows this rule, while also resulting in automatic control rearrangement (challenge 2) (Shinskey, 1981; Zotică et al., 2022). The core of the control structure is that each flow is linked through a `min` selector to (1) their upper limit, (2) the high-level control of the downstream vessel, and (3) the low-level control of the upstream vessel. As such unless a stream is the bottleneck (controlled by its upper limit) it will be controlled by the higher level limit of the downstream tank if it is before the bottleneck and the lower level limit of the upstream tank if it is after the bottleneck. Although this scheme explicitly assigns flow rates, it implicitly allocates set points for the inventories in accordance to the proposed rule. Later this scheme was extended to include minimum flow rate constraints (Bernardino and Skogestad, 2023).

3.1 An MPC solution

We now present two simple MPC solutions that are based on the same logic as the rule 1. The key challenge is that one cannot specify set-points for the tank levels, as these set points should be implicitly defined by the controller and automatically adjusted based on the current operation (challenge 2). Note that the dynamics of these schemes (and the previous) depend on their tuning and thus may not be the same. However, if the schemes yield the same steady state then they are consistent. As such, for simplicity of presentation for now we neglect terms that can be included to shape the MPC transients and focus on an objective that will implicitly select the same set-points as the bidirectional inventory scheme.

Unreachable set-points Instead of specifying a set point for the tank levels, we instead consider implementing a standard MPC with an unreachable set point for the flow rates of the system. In addition we require that the objective at future time steps is subject to a discount factor, e.g. for a system of N_I serially connected tank:

$$J = \sum_{k=0}^{N_k} \gamma^k \|F(t_k) - F_{sp}\|_2^2 \tag{3}$$

where $0 < \gamma < 1$ is the discount factor. The discount factor must be used as otherwise there is "time symmetry" and the solution of the MPC problem is non-unique. Using this objective, the MPC will maximise the throughput of the system subject to the system constraints. If one works through Example 1 then it is clear that this MPC scheme is consistent with rule 1 and the control structure proposed by Zotică et al. (2022).

Use of a "Economic" objective The previous MPC scheme meets challenges 1 and 2 for the case of serial tanks, however it immediately suggests that an economic objective based on the throughput may also suffice. The idea is to maximise the sum of the time discounted flow rate out of the system, e.g. $\gamma^k F_3$ in Figure 1, and the weighted heights of the vessels, $\alpha_i h_i$. For the system in Figure 1 this gives an objective of:

$$J = -\sum_{k=0}^{N_k} \gamma^k \left(F_N(t_k) + \sum_{i=1}^{N_I} \alpha_i h_i(t_k) \right) \tag{4}$$

where $0 < \gamma < 1$ is the discount factor and $0 < \alpha_1 < \cdots < \alpha_N < 1$ are the weights of the tank heights. Although we call this an economic objective we note that it is *not* the true economic objective of the integrated profit over infinite time. Instead (4) is designed to yield solutions equivalent to this more complicated objective.

Importantly (4) is a linear objective and thus at any time step there is a priority in the maximisation, namely: the flow rates are to be allocated to maximise F_3, if F_3 is constrained then h_3 should be maximised, if it is constrained then so on. This ordering is achieved by the constraint on α. Note that if the αs were the same this would lead to multiple optima. Similarly to the previous case γ breaks time symmetry. By inspection of the objective it is clear that this allocates inventories consistently with the bidirectional scheme, as levels after the bottleneck will be set low ($F_{N,k}$ is preferred) while levels before the bottleneck will be set high (as it can be done without reducing $F_{N,k}$).

An additional motivation of (4) over (3) is that if there are some units that preferentially should not have high inventory (e.g. they become less efficient) then this can be easily incorporated by changing the ordering of the constraint on the αs.

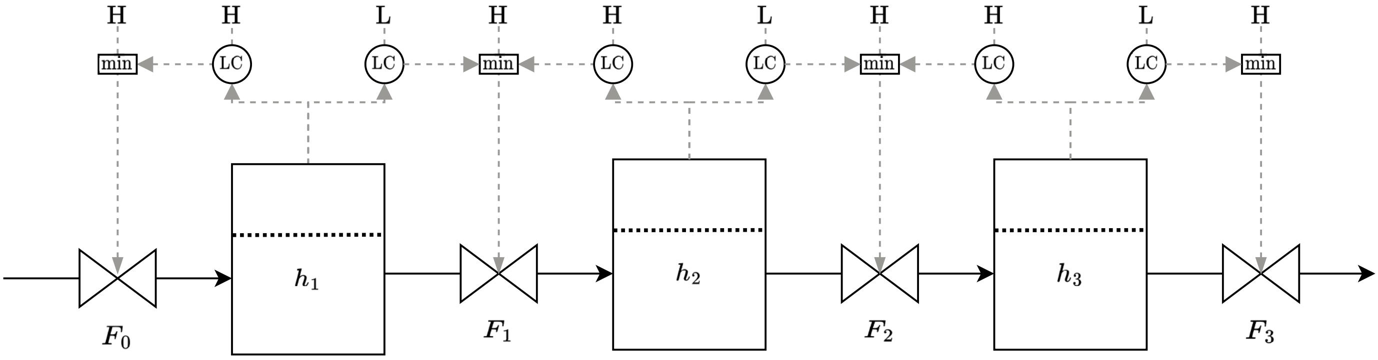


Fig. 1. Three tanks in series, with bidirectional inventory control structure proposed by Zotică et al. (2022) in grey. The grey dashed lines represent control signals, LC represent a level controller, and the min blocks represent min selectors. H and L represent high and low limits, of their corresponding LC or min selector. We neglect a subscript to show their relationship as this is clear from context. The level setpoints vary between the high and low limits automatically to isolate the effects of the current, and future, bottlenecks.

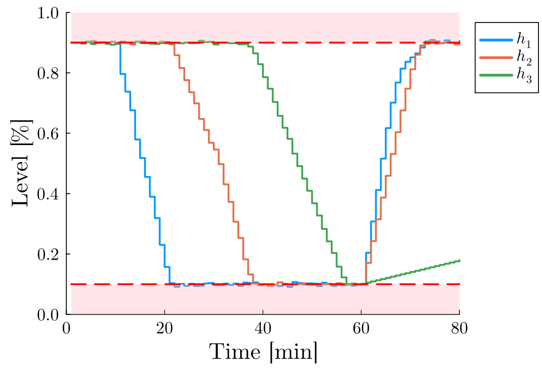
3.2 Simulation of proposed scheme

We consider the closed loop performance of the proposed MPC scheme using (3) on Example 2.

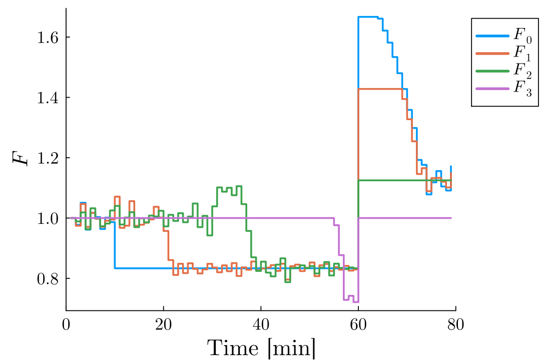
Example 2. *Consider a system of three tanks, as in Figure 1, with* $a = [1.0\ 1.5\ 2.0]\ m^2$ *,* $h_{tanks} = [2.3\ 2.8\ 3.2]\ m$*,* $h_{\min} = 0.1h_{tanks}$*, and* $h_{\max} = 0.9h_{tanks}$*, with all levels at their upper limit at* $t = 0$ *and a time discretisation of 1 minute. We vary the maximum flow rate as follows:* $F_{\max} = [1.667\ 1.428\ 1.125\ 1.0]\ m^3/min$ *for 10 minutes,* $F_{\max} = [0.833\ 1.428\ 1.125\ 1.0]\ m^3/min$ *for 50 minutes, and* $F_{\max} = [1.667\ 1.428\ 1.125\ 1.0]\ m^3/min$ *for the last 20 minutes. This means that the bottleneck is originally at* F_4*, then* F_1 *and then* F_4 *again. Furthermore, after 30 minutes, we introduce an uncontrolled depletion of 0.05* m^3/min *from tank 3 if the level is above the minimum, e.g. due to a leak. In the simulation we add normally distributed noise (mean zero, standard deviation* 0.1*) to the height measurements.*

We consider using MPC with the unreachable set-point objective (3) for inventory control of Example 2. The closed loop simulation of the proposed proposed MPC scheme is shown in Figure 2. Figure 2b shows that production rate of the system is only reduced when all tanks are empty, thus meeting challenge 1 (bottleneck containment). This is done through implicit set-points of the levels, e.g. at time 0-40 minutes the level of tank 3 is implicitly set to be high resulting in the flow rates being adjusted once the leakage begins occuring at $t = 30\ min$, without use of a scenario tree, forecast of the bottlenecks, etc. We note that due to the measurement noise there is minor violation of the level constraints, however without an additional back-off term this is unavoidable.

This scheme has two significant disadvantages (1) it requires the bottlenecks and operating information to be accurately identified and (2) the changes in flow rates is very aggressive leading to rapid changes in tank level and hence going against goal (a) of inventory control. These points are addressed in the following section.



(a) Tank levels of the MPC scheme with measured change in bottleneck. Red areas of the graph correspond to violation of the upper and lower level limits.



(b) Flow rates of the MPC scheme with measured change in bottleneck

Fig. 2. Performance of the MPC scheme using objective (3) for Example 2.

4. PRACTICAL CONCERNS

Two important concerns for the inventory control scheme are (1) robustness to misidentified bottlenecks and (2) tuning of the controller. The control schemes of Skogestad (2023); Zotică et al. (2022); Bernardino and Skogestad (2023) use PID controllers, and hence due to feedback and the integral term these control schemes can inherantly correct (1). In contrast the proposed MPC scheme needs to be augmented by an integrating state, or the model parameters have to be adapted online to have similar properties. In this section we show how (1) can be achieved by appropriate disturbance modelling, and (2) can be performed effectively by an additional constraint.

4.1 Handling of disturbances

For the inventory problem unmeasured disturbances can cause the bottleneck of the process to shift and if not corrected can result in assignments of flowrates that lead to infeasible operation. These disturbances can have a wide range of causes, e.g. a leak in a tank, or change in the maximum flow rate across a valve, or (temporary) error in the estimation of the maximum production achievable by the lower level controller. Although it is likely that the model can be adjusted if the error disturbance persists, it is important for the MPC to handle such errors without unsafe operation or significant reduction in throughput. In this section we briefly review disturbance models for MPC, and hence initially move away from the inventory control problem. After an introduction to the essential theory we demonstrate that the "standard simple" tunings result in very poor performance for the inventory control problem and showcase the use of a marginally more complex, simple tuning. For further information and theory see the tutorial paper Pannocchia (2015) and the references therein.

A brief introduction to disturbance models To handle disturbances MPC algorithms normally rely on some disturbance model and observer, with a range of different formulations in the literature. In this text we use an augmented model in which the nominal system model is augmented with disturbances, d, which are integrating states estimated from output measurements, y (Muske and Badgwell, 2002; Pannocchia and Rawlings, 2003). Usage of this formulation is not restrictive, as it has been shown that several other formulations are special cases of this formulation (Pannocchia, 2015). The augmented model is:

$$x_{k+1} = Ax_k + Bu_k + B_d d_k \tag{5a}$$
$$d_{k+1} = d_k \tag{5b}$$
$$y_k = Cx_k + C_d d_k \tag{5c}$$
$$x_0 = \hat{x}_{0|0},\ d_0 = \hat{d}_{0|0} \tag{5d}$$

where $x_k \in \mathbb{R}^{n_x}$ is the state, $u_k \in \mathbb{R}^{n_u}$ is the control input, $d_k \in \mathbb{R}^{n_d}$ is the disturbance, y_k is a measurement at time k, $C \in \mathbb{R}^{n_y \times n_x}$ is relates measurements to states in the measurement equation (5c), and $C_d \in \mathbb{R}^{n_y \times n_d}$ and $B_d \in \mathbb{R}^{n_x \times n_d}$ are matrices describing how the disturbances effect the augmented system. For the inventory control system $x_k = h(t_k),\ u_k = F(t_k)$.

At each iteration the initial value of the state and disturbance are set to their *estimated* value at t_0 (5d). These estimates are evolved by the observer:

$$e_k = y_k - C\hat{x}_{k|k-1} - C_d\hat{d}_{k|k-1} \tag{6a}$$
$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_x e_k \tag{6b}$$
$$\hat{d}_{k|k} = \hat{d}_{k|k-1} + K_d e_k \tag{6c}$$

where e_k is the error estimate, and the notation $\hat{x}_{k|k-1}$ refers to the prediction of $\hat{x}_k$ from $\hat{x}_{k-1}$ (using the augmented model).

For the augmented system to be detectable we require that the original system (C,A) is detectable and

$$\texttt{rank}\begin{bmatrix} A-I & B_d \\ C & C_d \end{bmatrix} = n_x + n_d \tag{7}$$

B_d and C_d can be chosen to satisfy this condition if and only if $n_d \le n_y$. Typically one chooses $n_d = n_y$ to ensure integration for all measurements. Lastly, the observer gains should be chosen such that the augmented observer is stable, i.e.:

$$\max|\lambda(A_a - K_a C_a A_a)| \le 1 \tag{8a}$$
$$A_a = \begin{bmatrix} A & B_d \\ 0 & I \end{bmatrix},\ C_a = [C\ C_d],\ K_a = [K_x\ K_d] \tag{8b}$$

where $\max\lambda(\cdot)$ is the largest absolute eigenvalue. Disturbance modelling thus requires the appropriate choice of: $B_d,\ C_d,\ K_x,\ K_d$ to meet these requirements. Unfortunately the choice of these matrices is non-trivial with good MPC performance typically requiring a well chosen disturbance model. The detectability condition offers some guidance – as (A, C) is detectable the submatrix $\begin{bmatrix} A-I \\ C \end{bmatrix}$ has rank n_x, and thus any $n_d \le n_y$ columns that are independent of $\begin{bmatrix} A-I \\ C \end{bmatrix}$ can be chosen for $\begin{bmatrix} B_d \\ C \end{bmatrix}$ to meet the rank condition for detectability.

We now consider three simple tuning strategies for the disturbance model applied to the inventory control problem. We assume that the level of each tank is measured and select N_I disturbances i.e. $n_d = n_x = n_y = N_I,\ C = I$.

Deadbeat output disturbance model The "standard" industry practice is to use a dead output disturbance model in which any error is assumed to be due to a step (constant) disturbance in the output. This corresponds to a choice of:

$$B_d = 0,\ C_d = I,\ K_x = 0,\ K_d = I, \tag{9}$$

and is equivalent to designing a deadbeat Kalman filter for the augmented system. However, this cannot be applied to the inventory system as the levels are integrators, and thus h cannot be distinguished from the integrating disturbances (the rank condition is not met).

Deadbeat input disturbance model This simplest alternative to the standard deadbeat output model is to simply move the disturbance to the input instead, corresponding to a choice of:

$$B_d = I,\ C_d = 0,\ K_x = 0,\ K_d = I. \tag{10}$$

In this model any disturbance is assumed to be solely due to a disturbance at the input. By design this avoids the rank issue, and often can give better performance than the output disturbance model. Additionally, if $n_d = n_u$ then one could select $B_d = B$. However, for the inventory control system we cannot do this ($n_d \le n_y < n_u$) and we instead assign an independent input disturbance to each state equation. However, applied to the inventory control problem this leads to the non-augmented system matrix $(A - K_xCA)$ having eigenvalues at 1, and the augmented observer having positive eigenvalues with non-zero imaginary parts. Thus the system will show poor performance.

Youla-Kucera parameterisation of disturbance model Lastly we examine the tuning suggested in Pannocchia (2015, 2023), based on a prior formulation of Tatjewski

(2014), in which the choice of the four disturbance matrices is replaced by the choice of a single matrix $Q \in \mathbb{R}^{n_x \times n_d}$. Q should be selected such that the non-augmented system characteristic matrix $(A - QCA)$ has desired properties, e.g. has eigenvalues in interior of the unit circle. Then the disturbance matrices can be set to:

$$B_d = Q,\ C_d = I - CQ,\ K_x = Q,\ K_d = I. \tag{11}$$

For any such choice of Q the augmented system is detectable, and the observer is asymptotically stable.

Note that as $K_d = I$ this tuning places n_y poles of the observer at the origin (and thus may lead to sensitivity to output noise) and results in the disturbance being set to the difference between the measurement and predicted state value, i.e. the innovation $y_k - C\hat{x}_{k|k-1}$:

$$\hat{d}_{k|k} = \hat{d}_{k|k-1} + K_d e_k = \hat{d}_{k|k-1} + e_k \tag{12a}$$

$$= \hat{d}_{k|k-1} + \left(y_k - C\hat{x}_{k|k-1} + C_d\hat{d}_{k|k-1}\right) \tag{12b}$$

$$= y_k - C\left(A\hat{x}_{k-1|k-1} + Bu_{k-1}\right) \tag{12c}$$

$$= y_k - C\hat{x}_{k|k-1} \tag{12d}$$

This means that the augmented model output is readjusted to exactly match the measured output at each iteration.

4.2 Numerical simulation

Example 3. *We consider the same system in example 2, but the MPC only receives level measurements, i.e. (1) the change $F_{\max}$ is not provided to the MPC, i.e. it uses the $F_{\max} = [1.667\ 1.428\ 1.125\ 1.0]$ m^3/min throughout the horizon and (2) the draining from tank 3 (0.05 m^3/min when above the lower limit) is not in the MPC model.*

To prevent infeasibility of the MPC problem the state constraints are replaced with soft constraints. In the simulation, if the controller allocates a flow-rate higher than the actually achievable flow rate, then the maximum allowable flow rate is used. Similarly, if the tank level exceeds 100 % then the level is set to 100 % and it is assumed that excess is lost.

We consider applying the nominal MPC solution and the discussed disturbance schemes to the inventory control problem, with the results summarised in Figure 3. The nominal MPC solution (Figure 3a), significantly violates the level constraints of the first two tanks, with both of the tanks running dry. However, due to the feedback of the MPC implementation the leakage disturbance is adequately controlled, although it does result in violation of the lower level constraint of tank 3 at $t = 59$ min.

The use of the deadbeat output disturbance model is shown in Figure 3b. As expected this leads to very poor performance as the augmented system is not detectable and hence the state and disturbance estimates are entirely inaccurate leading to violation of all the tank levels, and the inventories not shifting once the bottleneck is lifted.

Although the deadbeat input disturbance model, Figure 3c, correctly accounts for the leakage disturbance (see level 3 before $t = 40$ min) it handles the unobserved reduction in the maximum flow rate very poorly. Good performance of the leakage is expected as this can be entirely captured by an input disturbance. Similarly, the poor performance is expected due to eigenvalues of the observer inducing oscillations (the large imaginary components) while not providing asymptotic stability.

On the contrary, the use of the Youla-Kucera parameterisation with $Q = 1.1I$, is able to avoid extended infeasible operation, see Figure 3d, by correctly handling both disturbances. We note that there is some violation of the level constraints, and that the system is not compensate as effectively for the leakage of tank 3, however the performance is very similar to when the MPC has full information (2a) and so we judge this acceptable. There is a minor backoff of the level of tank 1 from the constraint, due to the combined influence of the disturbance and noise, however there is some room for adjustment of this by tuning of Q.

4.3 Tuning of transients

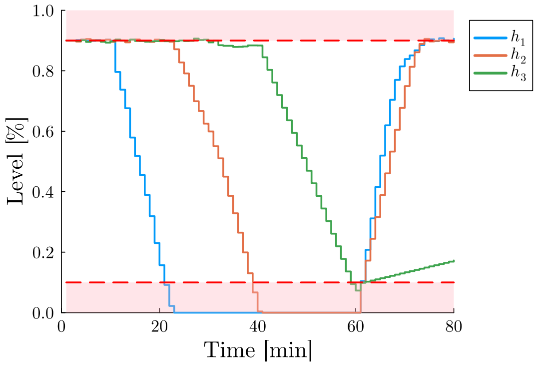
The use of objective (3) or (4) will lead to a controller that maximises the time between interruptions in production due to bottlenecks. Although this is desirable, another aspect of inventory control is to mitigate short-term fluctuations in the inventories. This is clearly at odds with the aggressive goal of maximising throughput. The simplest way to consider these dual goals is to introduce a constraint on the constraint on the change in the levels, i.e.

$$|h_i(t_k) - h_i(t_{k-1})| \leq \Delta_{\max} h_i, \qquad i = 1, ..., N_I \tag{13}$$

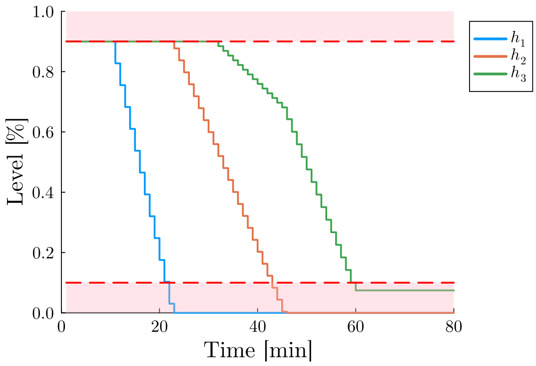
where $\Delta_{\max} h_i$ limits the allowable change of h_i. Although it is more common to penalise or restrict the change in the control variable, this is undesirable for the inventory problem as we are okay with fluctuations in the flow if these don't significantly influence the levels. We also note that this constraint implicitly contains the logic that goal a applies to shorter time scales, and goal (b) to a longer time scale. Additionally, small level variations as this is typically acceptable.

Practically we note that (1) this constraint should be enforced as a soft constraint to prevent infeasibility of the MPC problem and (2) this constraint introduces a new bottleneck source, as sometimes the inflow or outflow from a tank can be limited by this constraint. Practically one may also wish to consider some kind of allowed "acceleration" of the change in height, this can easily be incorporated by allowing $\Delta_{\max} h_i$ to increase (to some upper limit) if this constraint was active at a previous iteration. We note that if $\Delta_{\max} h$ is chosen sufficiently small then this not only restricts not the transients but can induce a bottleneck by making a large flow rate infeasible due to it resulting in a inventory increasing too fast.

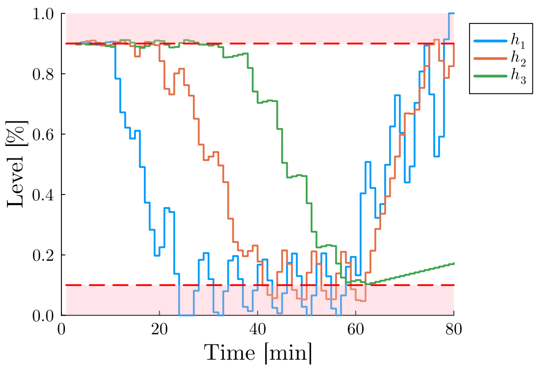
As pathological example of what can result from an excessively small $\Delta_{\max} h$ we consider Example 2 with (13) and $\Delta_{\max} h_i = 0.03 h_{\max,i}$ for all tanks. The results are shown in Figure 4 and should be compared with Figure 2. This choice of $\Delta_{\max} h$ results in a bottleneck when the levels are refilled after the disturbance to F_0 has ended (Figure 4b). In addition, due to the constraint the levels of the first two tanks drop together, and the level of the third tank begins to drop when the *first* tank reaches its lower limit, as if it did not the level of the second tank would decrease too fast. Thus the constraint can induce a non-intuitive coupling between non-adjacent inventories.



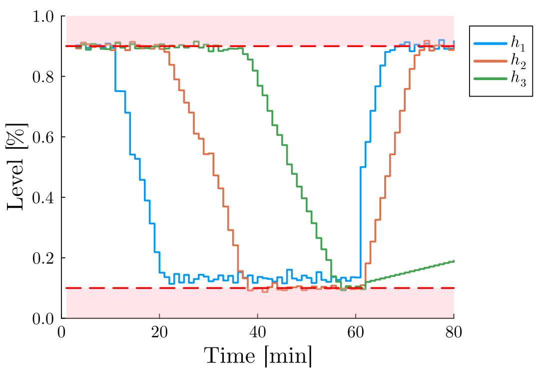
(a) Level profiles of applying the MPC scheme without disturbance model to a system with incorrect F_{max}



(b) Level profiles of applying the MPC scheme with disturbance model deadbdeat ouput to a system with incorrect F_{max}

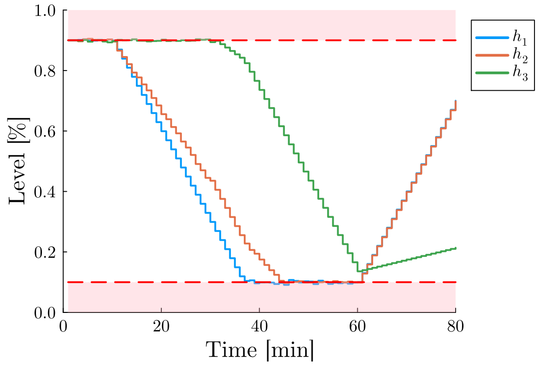


(c) Level profiles of applying the MPC scheme with disturbance model deadbdeat input to a system with incorrect F_{max}

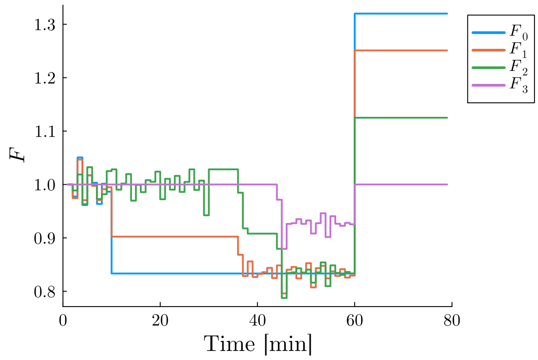


(d) Level profiles of applying the MPC scheme with disturbance model Youla to a system with incorrect F_{max}

Fig. 3. Use of MPC for the inventory control problem with incorrect identification of bottleneck (Example 3).



(a) Tank levels of the MPC scheme with measured disturbances and constraint on Δh.



(b) Flow rates of the MPC scheme with measured change in bottleneck and constraint on Δh. Note that the maximum flow rates are not reached due to the Δh constraint.

Fig. 4. Performance of the MPC scheme using objective (3) for Example 2.

Lastly, in terms of the process throughput the scheme still prioritises keeping F_3 at its maximum, however due to the constraint the effective maximum flow rate when the tank is being depleted is around 0.93 (Figure 4a, around $t = 50\ min$).

5. DISCUSSION

5.1 Comparison to prior work

This paper considers the same inventory control problem as Zotică et al. (2022); Skogestad (2023) in which a a decentralised control structure consisting of simple control elements was proposed. In comparison, this work proposes the use of an MPC scheme. Under identical scenarios both control schemes will find the same steady state operating point, with the transient behaviour of the schemes determined entirely by their tunings. As such, it is not informative to compare these schemes quantitatively. Qualitatively, the decentralised control scheme is computationally simple as it uses simple control elements. However, the MPC scheme does not represent a computational burden as a small, convex optimisation problem is solved which can be done very reliably and efficiently. An important benefit of the MPC scheme is that if future information is available, then this can be directly incorporated in the MPC problem. Similarly, the MPC scheme can be augmented in several ways, with two suggestions outlined below. Based on simplicity, it is likely that unless more complex process topologies and constraints are considered (and the schemes are suitably extended), the decentralised control scheme will be preferred.

5.2 Delay in transportation

In this work we have considered that there is no delay in the transport between units. Because of this the outflow from a unit can be more than amount of substance in that unit. To avoid this one can introduce a constraint of the form:

$$M^{out}F(t_k) \leq a_i h_i(t_k) \tag{14a}$$

$$M_{ij}^{out} = \begin{cases} -1 \text{ if } F_j \text{ exits vessel } i \\ 0 \text{ otherwise} \end{cases} \tag{14b}$$

where $M^{out} \in \mathbb{R}^{N_I \times N_F}$ is an incidence matrix that describes the processes outflows. This constraint requires the total outflow from a unit is less than or equal to the amount of substance in the unit at a point in time.

5.3 Incorporation of economics

The proposed scheme does not make direct use of the process economics. This choice is an intentional choice, as economic considerations of processes can be very complex. Instead based on assumption of increasing throughput being economically preferred, we avoid explicit inclusion of the process economics. In practice this may not be the case e.g. due to increased cost or inefficiencies if some inventories are kept high. If this is the case then one can design tiered soft constraints that promote operating in a predefined "Goldilock" zone, but allows for short transients in the less profitable operating regions if necessitated.

6. CONCLUSION

We address the task of allocating process inventories to maximise production and bottleneck isolation using a model predictive control (MPC) scheme. This approach addresses the two challenges associated with mitigating the effects of bottlenecks, while also allowing incorporation of the goal of minimising short term inventory fluctuation. Unlike the claims of previous works this is done without requiring a computational expensive formulation, bottleneck forecast, multi-scenario approach or similar. Furthermore, we also investigated the use of a Youla-Kucera tuning of a disturbance model for the MPC scheme and showed it to be effective when provided with incorrect operational information, e.g. leaks or misidentified bottlenecks.

Further work can include considering systems that are more complicated, this includes recycles, parallel processing sections, multiple product streams and units that require a minimum flow to safely operate, e.g. a compressor.

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