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Performance-Based Plant-Model-Mismatch Detection in Soft-Sensor Control Loops

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Abstract: The predictive performance of soft sensors deteriorates over time which is called the performance change of a soft sensor. These changes occur due to differences between the current characteristics of the process or plant and the soft sensor model. The deviation is a type of plant-model mismatch (PMM). Initially, this mismatch may be acceptable. However, over time, the PMM can become so large that it affects the prediction quality of the soft sensor and may become unacceptable. This paper develops a new method to evaluate the impact of PMM on closed-loops with soft sensors. Using coprime factorisation and small-gain theory, a performance-change index is developed to characterise the PMM-induced performance degradation. Then, a performance-based online PMM detection method is proposed using this performance-change index. To validate the effectiveness of the proposed algorithm, we use a numerical example and a continuous stirred tank reactor (CSTR). It is shown that that the proposed index can detect the change of the PMM.

Keywords: **Plant-model mismatch, soft sensor, performance-change index, Coprime factorisation**

1. INTRODUCTION

In most industrial environments, failure to measure critical variables quickly enough or accurately enough can lead to financial loss or safety issues. In both cases, soft sensors are designed to estimate unknown variables using more easily available process data (Bosca & Fissore, 2011; Yu, 2012).

Soft sensors can be divided into three categories. First, soft sensors can be developed using first-principle models, Kalman filters, and observers (de Assis & Maciel Fiho, 2000; Heineken, Flockerzi, Steyer, Voigt, & Sundmacher, 2007; Mangold, 2012). As well, inference-based soft sensors (Fortuna, Graziani, Rizzo, & Xibilia, 2007) are subsumed into model-based soft sensors as relevant variables are used to predict the desired result (Shardt & Huang, 2012). Model-based soft sensors require an understanding of the process and considerable effort to develop the model. Another approach involves using data-driven techniques to create soft sensors, where process data and plant knowledge play a crucial role (Dufour, Bhartiya, Dhurjati, & Doyle III, 2005; Facco, Doplicher, Bezzo, & Barolo, 2009). Techniques like principal component regression (PCR) and partial least squares (PLS) are used in constructing linear models. Progress has been made in quality estimation from collinear high dimensional data (Krdlec, Gabrys, & Strandt, 2009). However, even if soft sensors are initially accurate, the predictive performance of the models deteriorates over time due to various factors. These factors include changes in raw materials properties, fluctuations in catalyst activity, changes in the external environment, or even shifts in the operational condition (Urhan & Alakent, 2020). This situation is known as a soft-sensor performance change, which causes the true process (also known as the plant) to differ from the initial process model. This deviation is called plant-model mismatch (PMM). PMM refers to the difference between the dynamics of the initial process model and the actual process behaviour. To tackle this problem, it is necessary to detect PMM and develop effective performance evaluation tools (Shardt, et al., 2012).

Detection of PMM is the first step for performance change detection of a soft sensor. There are many approaches to detect PMM. A partial correlation approach was developed by Badwe *et al.* (2009), which used the correlation between the residuals and the manipulated variable. Three specific indices were proposed (Jing, Li, & Shah, 2007): η_{ABC}, η_{AC}, and η_C, which can be used to detect PMM in the matrices of model predictive control (MPC) systems. This approach is based on discrete-time state-space models. To identify PMM in a closed-loop control system, Ling *et al.* (2017) developed an evaluation indicator for process models. This indicator is based on calculating the ratio between the variance of the disturbance innovations and the variance of the model quality variables. A larger value implies a smaller PMM. An approach based on the subspace method was developed by Wang, Song, and Xie (2012) to detect PMM.

Compared to detecting PMM, there are only a few methods in the literature to assess the impact of PMM. An assessment criterion is proposed based on minimum variance for evaluating the control loop performance change caused by the PMM (Harris, 1989). Although the method extends from univariate to multivariate systems, the main purpose of this index is not to evaluate the PMM, and if there are deviations, it does not mean that the model has changed, it could mean that the controller has been poorly tuned. (Hong, Tore, & Zhihuan, 2012) proposed a metric called the integral absolute error (IAE) index. They demonstrated that the IAE index increases as PMM is increased. However, none of the criteria and decisions are indicated for comparison with the defined index or indicate how the index should be like under normal working environment. A closer relationship between PMM and robustness of soft-sensor control loop has not been suggested elsewhere in the literature. However, robustness should be considered when studying detection of PMM. It is concerned with the stability of controlled systems. In addition to this, PMM can lead to either an increase or decrease in the performance of the soft-sensor control loop which require engineers to respond differently. System robustness can effectively detect these changes. Therefore, when evaluating PMM, it is important to consider the robustness of the soft-sensor control loop.

Thus, the objectives of the paper are: to develop an online PMM detection algorithm considering the robustness of the soft-sensor control loop and provide the associated design decision logic; to test the PMM detection algorithm on a numerical example; and to test the PMM detection algorithm on an industrial continuous, stirred tank reactor (CSTR).

2. BACKGROUND AND PROBLEM FORMULATION

2.1. Background

Consider the controlled soft-sensor system shown in Figure 1, where $G(z)=(A_G, B_G, C_G, D_G)$ is the plant (or true process), $\hat{G}(z)=(A, B, C, D)$ is the assumed process model, $K(z)$ is the controller, $G_B(z)$ is the bias update term, $v(z) \in R^n$ is the reference signal, $u(z) \in R^m$ is the input to the plant and the soft sensor, $y(z) \in R^n$ is the (true) plant output, $\theta(z) \in R^n$ is the model output, $y_\beta(z) \in R^n$ is the compensation signal from the bias update term, $y_m(z) \in R^n$ is the compensated output, and $d(z) \in R^n$ is the disturbance, which is assumed to be white, Gaussian noise. The bias update term $G_B(z)$ and the model $\hat{G}(z)$ create the soft sensor $S(z)$.

According to the design characteristics of the bias update term (Shardt & Huang, 2012), when there is a difference between $\theta(z)$ and $y(z)$, $G_B(z)$ provides a compensation signal to guarantee that $y_m(z)$ is close to the true output $y(z)$.

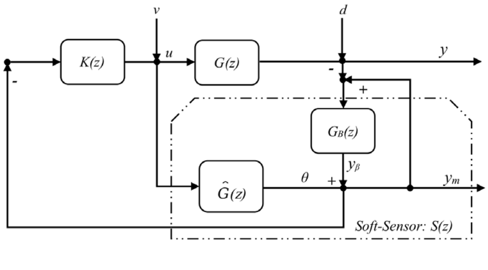

Figure 1: Closed-loop structure with soft sensors (Shardt & Huang, 2012)

In Figure 1, $y_m(z)$ is the actual output of the soft sensor. Therefore, we have

$$v(z) = u(z) - K(z)y_m(z) \tag{1}$$

$$y_m(z) = S(z)u(z) \tag{2}$$

The soft-sensor can be described by the following equation:

$$\begin{bmatrix} u \\ y_m \end{bmatrix} = \begin{bmatrix} I & -K \\ -S & I \end{bmatrix}^{-1} \begin{bmatrix} v \\ 0 \end{bmatrix} \tag{3}$$

Definition 1 (Zhou & Doyle, 1998): In the RH_∞ space, there are two matrices $\hat{M}(z)$ and $\hat{N}(z)$ with an equal number of rows, if there are two other matrices $\hat{X}(z)$ and $\hat{Y}(z)$ in the same space that satisfy the following relationship:

$$\begin{bmatrix} \hat{M}(z) & \hat{N}(z) \end{bmatrix} \begin{bmatrix} \hat{X}(z) \\ \hat{Y}(z) \end{bmatrix} = I \tag{4}$$

Then, it is said that in the RH_∞ space, the matrices $\hat{M}(z)$ and $\hat{N}(z)$ are left coprime, and equivalent to $[\hat{M}(z) \quad \hat{N}(z)]$, which are right invertible in the RH_∞ space.

Similarly, in the RH_∞ space, there are two matrices $M(z)$ and $N(z)$ with an equal number of columns, if there are other matrices $X(z)$ and $Y(z)$ in the same space that satisfy the following relationship:

$$\begin{bmatrix} X(z) & Y(z) \end{bmatrix} \begin{bmatrix} M(z) \\ N(z) \end{bmatrix} = I \tag{5}$$

Lemma 1 (Zhou & Doyle, 1998): Suppose $\hat{G}(z)$ is a real, rational matrix that has a stabilisable and observable realisation. Let $\mathcal{F}$ and $\mathcal{L}$ satisfy, respectively, $\mathcal{A}+\mathcal{BF}$ and $\mathcal{A}-\mathcal{LC}$ stability. Therefore, the above eight transfer matrices can also be expressed as

$$\begin{aligned} M(z) &= \begin{bmatrix} A+BF & B \\ F & I \end{bmatrix}, & N(z) &= \begin{bmatrix} A+BF & B \\ C+DF & D \end{bmatrix} \\ \hat{X}(z) &= \begin{bmatrix} A+BF & B \\ F & I \end{bmatrix}, & \hat{Y}(z) &= \begin{bmatrix} A+BF & -L \\ F & 0 \end{bmatrix} \\ \hat{M}(z) &= \begin{bmatrix} A-LC & L \\ -C & I \end{bmatrix}, & \hat{N} &= \begin{bmatrix} A-LC & B-LD \\ C & D \end{bmatrix} \\ X(z) &= \begin{bmatrix} A-LC & -B+LD \\ L & I \end{bmatrix}, & Y(z) &= \begin{bmatrix} A-LC & -L \\ F & 0 \end{bmatrix} \end{aligned} \tag{6}$$

Then, $\hat{G}(z) = N(z)M(z)^{-1} = \hat{M}^{-1}(z)\hat{N}(z)$ are, respectively, the right coprime factorisation (RCF) and left coprime factorisation (LCF) of $\hat{G}(z)$. Furthermore, Bézout's identity holds:

$$\begin{bmatrix} X(z) & Y(z) \\ -\hat{N}(z) & \hat{M}(z) \end{bmatrix} \begin{bmatrix} M(z) & -\hat{Y}(z) \\ N(z) & \hat{X}(z) \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{7}$$

The LCF and RCF are obtained for the system $\hat{G}(z)$ and the controller $K(z)$ described in Figure 1 to give:

$$\hat{G}(z) = N(z)M(z)^{-1} = \hat{M}^{-1}(z)\hat{N}(z) \tag{8}$$

$$K(z) = U(z)V(z)^{-1} = \hat{V}^{-1}(z)\hat{U}(z) \tag{9}$$

Lemma 2 (Tay & Mareels, 1998): For the real, rational model $\hat{G}(z)$ and controller $K(z)$, $K(z)$ stabilises $\hat{G}(z)$ if and only if there exist coprime factorisations Equation (8) and (9) such that the following Bézout's identity holds:

$$\begin{bmatrix} \hat{V} & \hat{U} \\ -\hat{N} & \hat{M} \end{bmatrix} \begin{bmatrix} M(z) & -U(z) \\ N(z) & V(z) \end{bmatrix} = I \tag{10}$$

2.2 Problem formulation

The PMM in the soft sensor $S(z)$ is equivalent to the PMM in $\hat{G}(z)$, since $G_B(z)$ is not affected by PMM. We suppose PMM exists in the $\hat{G}(z)$ of the soft sensor:

$$S_P = \hat{G}_P = \hat{M}^{-1}\hat{N} = \left(\hat{M}_0 + \Delta_{\hat{M}}\right)^{-1}\left(\hat{N}_0 + \Delta_{\hat{N}}\right) \tag{11}$$

where $\hat{M}_0, \hat{N}_0 \in RH_\infty$ are the LCF when the soft sensor is fault-free and $\Delta_{\hat{M}}, \Delta_{\hat{N}} \in RH_\infty$ are the model uncertainties (Vinnicombe, 2000).

Let us assume that:

- The soft-sensor can be described as a linear and time-invariant system.

- A controller and an observer can be designed to meet the requirement of performance and stability in the soft-sensor control loop.
- The reference signal *v(z)* is persistently excited.

3. PMM DETECTION

3.1 Constructing a performance-change index (PCI

Combining Equations (3)(8)(9) and (11), we get:

$$\begin{bmatrix} u \\ y_m \end{bmatrix} = \begin{bmatrix} I & -K \\ -S_p & I \end{bmatrix}^{-1} \begin{bmatrix} v \\ 0 \end{bmatrix} = \begin{bmatrix} \hat{V} & \hat{U} \\ -\hat{N} & \hat{M} \end{bmatrix}^{-1} \begin{bmatrix} \hat{V} \\ 0 \end{bmatrix} v \quad (12)$$

Combining Equations (11) and (12):

$$\begin{bmatrix} \hat{V} & \hat{U} \\ -\hat{N} & \hat{M} \end{bmatrix}^{-1} = \left(\begin{bmatrix} \hat{V} & \hat{U} \\ -\hat{N}_0 & \hat{M}_0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ -\Delta_{\hat{N}} & \Delta_{\hat{M}} \end{bmatrix} \right)^{-1} \quad (13)$$

Bézout's identity (7) gives:

$$\begin{bmatrix} \hat{V} & \hat{U} \\ -\hat{N} & \hat{M} \end{bmatrix}^{-1} = \begin{bmatrix} M_0 & -U \\ N_0 & V \end{bmatrix} \left(I + \begin{bmatrix} 0 & 0 \\ -\Delta_{\hat{N}} & \Delta_{\hat{M}} \end{bmatrix} \begin{bmatrix} M_0 & -U \\ N_0 & V \end{bmatrix} \right)^{-1} \quad (14)$$

According to small-gain theory (Ding S. X., 2008), the transfer function is stable if:

$$\left\| \begin{bmatrix} -\Delta_{\hat{N}} & \Delta_{\hat{M}} \end{bmatrix} \begin{bmatrix} M_0 & -U \\ N_0 & V \end{bmatrix} \right\|_\infty < 1 \quad (15)$$

Equation (15) contains uncertainties $\Delta_{\hat{N}}$ and $\Delta_{\hat{M}}$. It is clear that uncertainty has an impact on the stability of the model. If Equation (15) is satisfied, then transfer function (14) is stable. Hence, Equation (15) reflects the performance change of the soft sensor (Tay & Mareels, 1998; Ding S. X., 2021), and can be regarded as the PCI. If PCI is close to 1, it indicates proximity to the stability limit. To detect PCI more accurately, we can design a maximum acceptable limit PCI_{th} for the performance degradation of the soft sensor:

$$PCI_{th} \geq sup \left\| \begin{bmatrix} -\Delta_{\hat{N}} & \Delta_{\hat{M}} \end{bmatrix} \begin{bmatrix} M_0 & -U \\ N_0 & V \end{bmatrix} \right\|_\infty \quad (16)$$

If PCI_{th} <1, then

$$\left\| \begin{bmatrix} -\Delta_{\hat{N}} & \Delta_{\hat{M}} \end{bmatrix} \begin{bmatrix} M_0 & -U \\ N_0 & V \end{bmatrix} \right\|_\infty < PCI_{th} < 1 \quad (17)$$

From the above discussion, it can be seen that the predictive behaviour of the soft sensor is affected by PMM. If there is no PMM in the system, PCI = 0. Once PMM occurs, PCI starts to increase. Once PCI is greater than PCI_{th}, the prediction accuracy will decline to an unacceptable level. From this point of view, PCI can be regarded as the performance change. To monitor changes in the predictive behaviour, we use the following logic:

$$\begin{cases} PCI < PCI_{th} \Rightarrow \text{good performance} \\ PCI \geq PCI_{th} \Rightarrow \text{performance anomaly (alarm)} \end{cases} \quad (18)$$

If $PCI \geq PCI_{th}$, an alarm will be triggered, which alerts the engineer that PMM has occurred that causes the closed-loop performance to degrade to an unacceptable level.

3.2 Online detection of the PCI

Construct the state observer and the observer-based residual generator (Ding S. X., 2008) for the assumed process model $\hat{G}(z)$:

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + L\big(y_m(k) - \hat{y}_m(k)\big) \quad (19)$$

$$\hat{y}_m(k) = C\hat{x}(k) + Du(k), r = y_m(k) - \hat{y}_m(k) \quad (20)$$

where $\mathcal{A}, \mathcal{B}, \mathcal{C}$, and $\mathcal{D}$ are the state-space realisation and the known constant matrices of the soft sensor, and matrix $\mathcal{L}$ stabilises ($\mathcal{A} - \mathcal{L}\mathcal{C}$).

Substituting Equation (20) into the Equation (19) gives:

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + L\big(y(k) - C\hat{x}(k) - Du(k)\big)$$
$$= (A - LC)\hat{x}(k) + (B - LD)u(k) + Ly(k) \quad (21)$$

Performing the *z*-transformation gives:

$$\hat{x}(z) = [zI - (A - LC)]^{-1}(B - LD)u(z) + [zI - (A - LC)]^{-1}Ly_m(z) \quad (22)$$

Substituting Equation (22) into the *z*-transformation of Equation (20) gives

$$\hat{y}_m(z) = C[zI - (A - LC)]^{-1}(B - LD)u(z) + C[zI - (A - LC)]^{-1}Ly_m(z) + Du(z) \quad (23)$$

and $r(z) = y_m(z) - \hat{y}_m(z)$, we have:

$$r(z) = y_m(z) - C[zI - (A - LC)]^{-1}(B - LD)u(z) - C[zI - (A - LC)]^{-1}Ly_m(z) - Du(z)$$
$$= [I - C[zI - (A - LC)]^{-1}L]y_m(z) - [D + C[zI - (A - LC)]^{-1}(B - LD)]u(z) \quad (24)$$

Form the following matrices

$$\hat{M}(z) = \begin{bmatrix} A - LC & L \\ -C & I \end{bmatrix}, \quad \hat{N} = \begin{bmatrix} A - LC & B - LD \\ C & D \end{bmatrix}$$

Therefore, we get the residual generator:

$$r = \hat{M}(z)y_m(z) - \hat{N}(z)u(z) \quad (25)$$

Equation (25) is the new form of the residual generator, which can be built using data-driven methods (Ding S. X., 2013). When there is no PMM, Equation (25) can be rewritten as

$$\hat{M}_0(z)y_m(z) - \hat{N}_0(z)u(z) = 0 \quad (26)$$

In light of Equations (11), (25) and (26),

$$r = \hat{M}_0 y_m(z) + \Delta_{\hat{M}} y_m(z) - \hat{N}_0 u(z) - \Delta_{\hat{N}} u(z)$$
$$= \begin{bmatrix} -\Delta_{\hat{N}} & \Delta_{\hat{M}} \end{bmatrix} \begin{bmatrix} u \\ y_m \end{bmatrix}$$

Combining Equation (12), the dynamics process of the residual generator is given by:

$$r = \begin{bmatrix} -\Delta_{\hat{N}} & \Delta_{\hat{M}} \end{bmatrix} \begin{bmatrix} \hat{V} & \hat{U} \\ -\hat{N} & \hat{M} \end{bmatrix}^{-1} \begin{bmatrix} \hat{V} \\ 0 \end{bmatrix} v \quad (27)$$

Using Equation (14) and Bézout's identity gives

$$\begin{bmatrix} \hat{V} & \hat{U} \\ -\hat{N} & \hat{M} \end{bmatrix}^{-1} = \begin{bmatrix} M_0 & -U \\ N_0 & V \end{bmatrix} \left(I + \begin{bmatrix} 0 & 0 \\ -\Delta_{\hat{N}} & \Delta_{\hat{M}} \end{bmatrix} \begin{bmatrix} M_0 & -U \\ N_0 & V \end{bmatrix} \right)^{-1} \quad (28)$$

Substituting Equation (28) into Equation (27) gives

$$r = \left(I + \begin{bmatrix} -\Delta_{\hat{N}} & \Delta_{\hat{M}} \end{bmatrix} \begin{bmatrix} -U \\ V \end{bmatrix} \right)^{-1} \times \begin{bmatrix} -\Delta_{\hat{N}} & \Delta_{\hat{M}} \end{bmatrix} \begin{bmatrix} M_0 \\ N_0 \end{bmatrix} \hat{V} v \quad (29)$$

Equation (29) gives a transfer function between the residual and reference signals. The purpose is to estimate the detection logic online. Comparing Equations (29) and (15), we need Lemma 3 to determine the required relationship.

Lemma 3:

Let $\Delta_1, \Delta_2 \in H_\infty$ and $\left\| \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} \right\|_\infty < \gamma < 1$, then

$$\| \Delta_1(I+\Delta_2)^{-1} \|_\infty < \frac{\gamma}{\sqrt{1-\gamma^2}}$$

Proof: See (Georgiou & Smith, 1990).
We have $\| \Delta_1, \Delta_2 \| \leqslant \gamma < 1$, then

$$\| (I+\Delta_2)^{-1}\Delta_1 \|_\infty \leq \frac{\gamma}{\sqrt{1-\gamma^2}} \quad (30)$$

Comparing Equations (17) and (28), we see that:

$$\Delta_1 = \begin{bmatrix} -\Delta_{\hat{N}} & \Delta_{\hat{M}} \end{bmatrix} \begin{bmatrix} M_0 \\ N_0 \end{bmatrix}, \Delta_2 = \begin{bmatrix} -\Delta_{\hat{N}} & \Delta_{\hat{M}} \end{bmatrix} \begin{bmatrix} -U \\ V \end{bmatrix}, \gamma = PCI_{th}$$

If $PCI_{th} < 1$, then

$$\| (I+\Delta_2)^{-1}\Delta_1 \|_\infty \leq \frac{PCI_{th}}{\sqrt{1-PCI_{th}^2}} \quad (31)$$

Hence, we get an evaluation function and a new threshold, then we can specify the logic equivalent to Equation (18):

$$J(k) = \| (I+\Delta_2)^{-1}\Delta_1 \|_\infty, \; J_{th} = \frac{PCI_{th}}{\sqrt{1-PCI_{th}^2}}$$

$$\begin{cases} J(k) < J_{th} \Rightarrow \text{good performance} \\ J(k) \geq J_{th} \Rightarrow \text{performance anomaly (alarm)} \end{cases} \quad (32)$$

$J(k)$ represents the RH_∞ norm of the transfer function from r to v. Following Ding (2008), the evaluation steps for PMM are:

1. Construct the Hankel matrix and calculate the evaluation function:

$$J(k) = \| (I+\Delta_2)^{-1}\Delta_1 \|_\infty = \sigma_{max}(RV^T(VV^T)^{-1}), V = \hat{\mathrm{V}}v$$

$$R = \begin{bmatrix} r_{j-N+1} & \cdots & r_j \\ \vdots & \ddots & \vdots \\ r_{j-N+s} & \cdots & r_{j+s-1} \end{bmatrix},$$

$$V = \begin{bmatrix} \hat{\mathrm{V}}v_{j-N+1} & \cdots & \hat{\mathrm{V}}v_j \\ \vdots & \ddots & \vdots \\ \hat{\mathrm{V}}v_{j-N+s} & \cdots & \hat{\mathrm{V}}v_{j+s-1} \end{bmatrix} \quad (33)$$

2. Determine the threshold:

$$J_{th} = \frac{PCI_{th}}{\sqrt{1-PCI_{th}^2}}, 0 < PCI_{th} < 1 \quad (34)$$

3. Evaluate Equation (32) to determine the condition.

5. SIMULATION RESULTS

5.1 Numerical example

Create a plant $G(z)$, an assumed process model $\hat{G}(z)$, and a controller, which have the following realisations:

$$A = \begin{bmatrix} 0.98 & 0.02 & 0.01 \\ -0.01 & 1.03 & 0.02 \\ 0.02 & -0.01 & 0.98 \end{bmatrix}, B = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \\ 0.1 & -0.2 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, D = 0;$$

$$A_s = \begin{bmatrix} 0.98 & 0.02 & 0.01 \\ -0.01 & 1.03 & 0.02 \\ 0.02 & -0.01 & 0.98 \end{bmatrix}, B_s = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \\ 0.1 & -0.2 \end{bmatrix},$$

$$C_s = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, D_s = 0;$$

$$K = \hat{\mathrm{V}}^{-1}(z)\hat{U}(z);$$

$$\hat{\mathrm{V}} = \begin{bmatrix} A-LC & -(B-LD) \\ F & I \end{bmatrix}, \hat{U} = \begin{bmatrix} A-LC & L \\ F & 0 \end{bmatrix}$$

Let us consider $\mathcal{F}$ and $\mathcal{L}$ such that $\mathcal{A}+\mathcal{B}\mathcal{F}$ and $\mathcal{A}-\mathcal{L}\mathcal{C}$ remain stable. The bias term is (2012):

$$G_B = \frac{-z^{-1}}{1-z^{-1}} \quad (35)$$

The reference input is a random constant ranging between 0 and 1, the disturbance variable follows a Gaussian distribution with mean 0 and a variance 1. The simulation time is 11,000 s. The assumptions given in Section 2 are satisfied. Collect residual signal r and reference signal v and construct the Hankel matrix in Equation (33). Set s = 100 and N = 5000. PCI_{th} is set to 0.6 which implies that J_{th} = 0.75. For simulation purposes, PMM is simulated from the 6000th sample which leads to the change in model $\hat{G}(z)$ matrix as follows

$$A_{s,p} = A_s + \Delta, \Delta = k(i)\bar{\Delta} \;,$$

$$\bar{\Delta} = \begin{bmatrix} -0.02 & 0.005 & 0 \\ 0.02 & -0.08 & 0 \\ -0.008 & 0 & -0.01 \end{bmatrix} \quad (36)$$

$$k(i) = \begin{cases} 0, i \leq 6000 \\ 1, i > 6000 \end{cases}$$

Afterwards the change in the performance of the soft sensor is determined using Equations (33), (34) and (32). Figure 2 shows the simulation results. The real-time evaluation value is represented by the blue line, while the red line shows the threshold based on the Equation (16). It can be noted that PMM results in an increase in the estimated value, which means that the performance is degraded. When the evaluation value exceeds the threshold, an alarm is triggered.

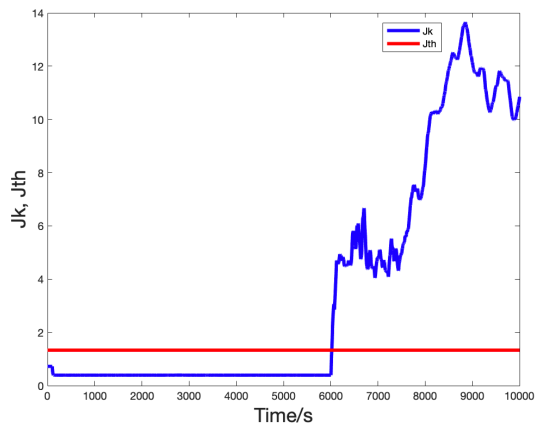

Figure 2: The performance change in a soft sensor

5.2 Industrial example

We will apply the proposed method to a CSTR. It consists of exothermic, first-order, irreversible reactions in a constant-volume reactor with the concentrations of the reactants as output. The plant model *G(z)* and the model $\hat{G}(z)$ and corresponding parameters are given in (Shardt & Huang, 2012), so that model $\hat{G}(z)$ is

$$A_s = \begin{bmatrix} -1.16 & -0.14 \\ 1 & 0 \end{bmatrix}, B_s = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$C_s = [0 \quad 1], D_s = 0 \tag{37}$$

Situation A: To design the bias update term G_B we will follow Equation (35). Simulation time is 10,000 s, where the disturbance variable follows a Gaussian distribution, with mean 0 and a variance 1. PCI_{th} is set to 0.74, which yields J_{th}= 1.1. PMM is simulated from the 6000th sample, which leads to the change in the $S(z)$ matrix:

$$A_{s,p} = A_s + \Delta, \Delta = k(i)\bar{\Delta} \quad and$$

$$\bar{\Delta} = \begin{bmatrix} 0 & -0.03 \\ 0.03 & 0 \end{bmatrix} \text{ with } k(i) = \begin{cases} 0, i \leq 6000 \\ 1, i > 6000 \end{cases} \tag{38}$$

Figure 3 shows the simulation results. The blue line shows the performance evaluation of the soft sensor over time, while the red line is the threshold. It shows that the unacceptable performance degradation caused by PMM was detected at 6800 s.

Situation B: In this case, we will consider the effect of an impulse disturbance on the proposed method. Assume that an impulse disturbance occurs between 6500 s and 7500 s, and other parameters remain the same as in Situation A.

The simulation results are shown in Figure 4. The blue line shows the performance based on a Gaussian distribution, while the black line shows the performance for the impulse disturbance between 6500 and 7500 s. The results show that under the influence of different disturbances, Situation B triggers the alarm about 100 s earlier than Situation A. Therefore, the sensitivity of this method will vary due to different disturbances.

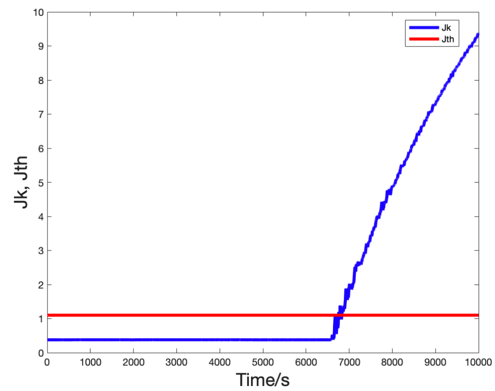

Figure 3: The performance change in the soft sensor for the CSTR (Situation A)

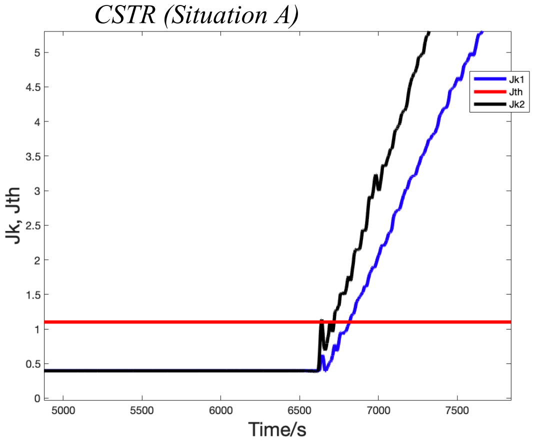

Figure 4: The performance change estimation of soft sensor. in CSTR (Situation B)

Situation C: Now set the bias update term as (Shardt & Huang, 2012)

$$G_{B,p} = \frac{-0.7z^{-1}}{1 - z^{-1}} \tag{39}$$

The simulation time is 10,000 s. The disturbance variable follows a Gaussian distribution with mean 0 and a variance 1. PCI_{th} is set to 0.74, which yields J_{th} = 1.1. PMM is simulated from the 6000th sample, which leads to a change in the $\hat{G}(z)$ matrix:

$$A_{s,p} = A_s + \Delta, \Delta = k(i)\bar{\Delta} \quad \text{and}$$

$$\bar{\Delta} = \begin{bmatrix} 0 & -0.03 \\ 0.03 & 0 \end{bmatrix} \text{ with } k(i) = \begin{cases} 0, i \leq 6000 \\ 1, i > 6000 \end{cases} \tag{40}$$

Compared with Situation A, the only difference between Situations C and A is the bias update term. Figure 5 shows the simulation results. It shows that Situation C does not trigger the performance-degradation alarm at the same time, which shows that the soft sensor in Situation A is more sensitive to the influence of PMM and has better tracking behaviour (Shardt & Huang, 2012).

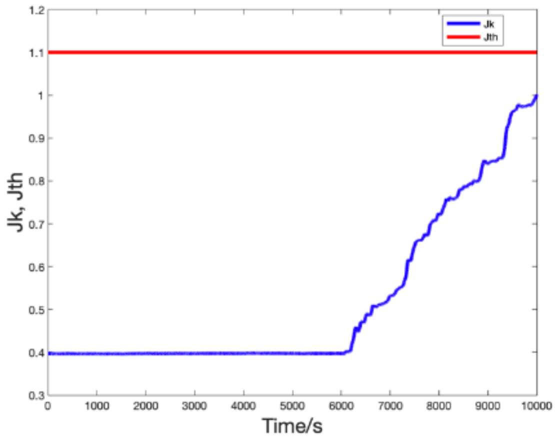

Figure 5: The performance change for soft sensor in the CSTR (Situation C)

The focus of the discussion is on performance changes of the control loop. Although the Harris index (Harris, 1989) can also evaluate control-loop performance, the evaluation is not the primary purpose. If there is a performance deviation in the loop, it may be that the minimum-variance controller has been poorly tuned.

However, the IAE indicator (Hong, Tore, & Zhihuan, 2012) was developed under the same conditions, but they did not show what the indicator should be like without PMM. The method proposed in the paper can remedy the limitations in conventional methods. This method can also be used for controller tuning. When the controller is designed and the soft sensor model and disturbance signal are fixed, the optimal controller can be obtained by comparing different controllers using the PCI.

6. CONCLUSIONS

This paper proposed an algorithm to detect PMM and evaluate the effect of the PMM on the controlled soft-sensor system by considering the robustness of the control loop. Through small-gain theory, PCI is proposed to detect and evaluate the effect of the PMM. As well, decision logic based on the PCI is formulated. Subsequently, the relationship between the PCI and the residual signal is established. Based on this, a PMM online detection algorithm using the process data and the H_∞–norm is proposed. Finally, a numerical example and CSTR are used to verify the accuracy of the proposed method, as well as the effects of different disturbances and parameter variations on the sensitivity of the index. In the future, methods to mitigate the detected performance changes and how to extend the performance-oriented PMM monitoring to nonlinear systems will be studied.

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