Observer-Based Distributed Sensor Fault-Tolerant Model Predictive Control for Intermittent Faults ⋆

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Abstract: For distributed control systems with intermittent sensor faults, a distributed fault and state combined observer is designed, and an observer switching mode strategy is proposed to improve the observer convergence performance, then a distributed fault-tolerant model predictive is designed based on the observer. In the proposed observer, the distributed system observer matrices, and state and fault estimation are computed in a totally distributed way with some transition matrices, which can consider the coupled states. The switching mode observer can be more targeted. The two modes: state only estimation, both state and fault estimation are switched due to the absence or presence of the sensor faults. The approach is shown effective through a simulated example.

Keywords: Model predictive control, distributed control, fault-tolerant control, fault estimation, distributed observer

1. INTRODUCTION

Distributed control can be applied to deal with large-scale systems and geographically separate systems with communications (Christofides, 2001; Xu and Bao, 2009). Distributed control can reduce the computation burden and simplify the modeling work using distributed process structures and models.

Modern chemical processes often work with larger scales and flexible production requirements. Many chemical processes contain several units coupled through some physical connections, such as mass flow and recycles. Centralized controllers may be very complex and with high computation burden. Distributed controllers are widely used in such processes to reduce the control complexity and computation burden, especially the distributed model predictive control (DMPC). DMPC can deal with the process constraints when design the control law by solving an online optimization (Mayne et al., 2000; Christofides et al., 2013). A lot of work about DMPC in chemical processes has been proposed, some focus on subsystems cooperation communication problems (Stewart et al., 2011), some work on plantwide optimization or stability (Venkat et al., 2005).

The previous works are all in the absence of actuator or sensor faults. While in large-scale control systems, there are much more components, resulting in a higher probability of faults. Thus fault-tolerant control (FTC) of the distributed control systems is an important subject. The sensor faults may provide inaccurately or even fault measurements for controllers, leading to unstable controlled processes. Sometimes the system states cannot be measured directly, and the controllers are designed based on state estimation. Some works have been proposed to solve output feedback model predictive control. For example, the moving horizon estimation is combined with MPC in (Ellis et al., 2014). With the help of the estimation-based controller idea, one way to restrain the faults is designing the controllers based on fault estimation (Gao, 2015; Huang et al., 2018; Qin et al., 2017, 2016). With faults estimation, the controller can be designed with the fault-free estimated states.

The distributed observer in this work is different from the fault and state estimation of distributed sensor network systems (Song and Chen, 2014; Liang et al., 2011), this is for distributed control system (Yin and Liu, 2019). Moreover, for distributed control system state estimation, the most existing states observers are usually designed based on decentralized systems (Razavinasab et al., 2017), which will lose all the coupled information, leading to a slower converge performance. And also, the existing fault and state observer-based control always estimates the fault and states together, no matter the current system is with or without faults(Xiao and Liu, 2020). This kind of estimation can work, but the estimation performance can be improved by distinguishing the faulty and unfaulty estimation mode. The switching mode observer proposed in this paper can work under state estimation only mode and both state and fault estimation according to whether there is a sensor fault or not. The switching mode observer will improve the observer convergence performances and

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then controller performances, especially for intermittent faults in distributed control systems.

In this paper, a distributed state observer is designed for a distributed control system with coupled states. The observer matrices are designed in a distributed way offline, and states are also estimated in distributed structure, reducing the computation burden and allowing more online flexibility. Meanwhile, a switching mode observer is designed, different modes of the observer will work under the situation with or without sensor faults. The distributed model predictive controller is designed based on fault-free estimated states. Finally, the distributed sensor fault-tolerant model predictive control is implemented.

2. PRELIMINARIES AND PROBLEM FORMULATION

Notations: $*$ represents the symmetric matrix of the symmetric position in a LMI. $\mathbb{R}^n$ is real numbers with n dimensions. $\mathcal{N}$ is the set of subsystem number index. $\|a\|$ is the standard norm symbol, $\mathcal{L}_2$ norm of a discrete time signal a is defined as $\|a\|_2^2 = \sum_{k=0}^{\infty} a^\top(k)a(k)$, $P \succ 0$ denotes the symmetric matrix P is positive definite.

Consider a large-scale discrete time linear system that can be divided into M subsystems, each subsystem can be described as:

$$\begin{aligned} x_i(k+1) &= A_i x_i(k) + \sum_{j\in\mathcal{N}_i, j\neq i} A_{ij} x_{ij}(k) + B_i u_i(k) \\ z_i(k) &= C_i x_i(k) \qquad \forall i \in \{1, 2, \ldots, M\} \end{aligned} \tag{1}$$

where $i \in \{1, 2, \ldots, M\}$ is the number of each subsystem, $\mathcal{N}_i$ are the set of neighbor subsystems numbers of subsystem i, the neighbor subsystems are the subsystems that have shared (coupled) states with subsystem i. $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$ and $z_i \in \mathbb{R}^{p_i}$ are variables of states, control inputs and measured outputs of subsystem i, x_{ij} are coupled states from subsystem j ($j \in \mathcal{N}_i, j \neq i$). A_i, A_{ij}, B_i and C_i are constant matrices, A_{ij} matrix related to the coupled states. Consider the sensor faults and noises of outputs measurements, system (1) with measurements are:

$$\begin{aligned} x_i(k+1) &= A_i x_i(k) + \sum_{j\in\mathcal{N}_i, j\neq i} A_{ij} x_{ij}(k) + B_i u_i(k) \\ y_i(k) &= C_i x_i(k) + F_i f_i(k) + w_i(k) \\ &\quad \forall i \in \{1, 2, \ldots, M\} \end{aligned} \tag{2}$$

where $f_i(k) \in \mathbb{R}^{p_i}$, if the sensor faults are additive faults, F_i is an identity matrix, w_i are measurements noises. f_i and w_i are unknown but bounded by $\|f_i\| \leq \epsilon$ and $\|w_i\| \leq \sigma$, ϵ and σ are known bounds. Then z_i in (1) are the physical outputs, y_i in (2) are actual measured outputs.

Problem Description: We assume that all the states can be estimated from the measured outputs and can be controlled in the proposed systems, i.e. the following assumptions are true.

Assumption 1. Each subsystem i is controllable.

Assumption 2. Each subsystem i is observable.

The estimated states are used for model predictive controller (3) design, and during the process, sensor faults may happen, leading to the wrong states estimations, which will give an unexpected control performance. To overcome the effects from sensor faults, we need to design the sensor faults observer for system (2) in a distributed way, and to improve the observer performance, a switching observer is designed for the faulty mode and unfaulty mode.

Distributed model predictive control problem:

$$\begin{aligned} &\min_{u_i} \sum_{k=1}^{N} l_i(x_i(k), u_i(k)) \\ &\text{s.t.} x_i(k+1) = A_i x_i(k) + \sum_{j\in\mathcal{N}_i, j\neq i} A_{ij} x_{ij}(k) \\ &\quad + B_i u_i(k) \\ &\quad x_i \in \mathcal{X}_i, u_i \in \mathcal{U}_i, \quad i \in \{1, 2, \ldots, M\} \end{aligned} \tag{3}$$

where $l_i(x_i, u_i)$ is the MPC objective function of subsystem i, the convex set $\mathcal{X}_i$ and $\mathcal{U}_i$ are states and control inputs constraints of subsystems.

3. SWITCHING OBSERVER DESIGN FOR DISTRIBUTED CONTROL SYSTEMS BASED ON FAULT SIGNAL

In this section, a states and sensor fault observer is given for distributed control systems, the observer is designed in a distributed way for each subsystem. Furthermore, to improve the convergence performance, a switching rule is proposed according to the fault signal, the observer can be switched between *the states only observer* and *the both states and sensor faults observer.*

3.1 States only observer for distributed control system

When there is no sensor fault in the the subsystem i, actually only the states need to be estimated, the sensor faults are not necessary in the observer, moreover, if they are still considered, it will slow down the convergence. Thus the states observer is only designed for system (1).

Consider the state observer form:

$$\begin{aligned} \hat{x}_i(k+1) &= A_i \hat{x}_i(k) + \sum_{j\in\mathcal{N}_i, j\neq i} A_{ij} \hat{x}_{ij}(k) \\ &\quad + B_i u_i(k) + L_i (y_i(k) - \hat{y}_i(k)) \\ \hat{y}_i &= C_i \hat{x}_i(k) + w_i \end{aligned} \tag{4}$$

where $\hat{x}$ is the estimation variable, L_i is the observer gain for subsystem i.

The L_i is designed according to the following Theorem.

Theorem 1. If there exists a positive definite matrix $P_{\mathcal{N}_i}$, a matrix Y_i and a constant γ_i, making the following optimization problem solved, then the error dynamic of the observer (4) is stabilized for subsystem i, and for unknown and bounded w_i, the performance $\sum_{k=0}^{T} \|e_i\|^2 \leq \sum_{k=0}^{T} \gamma_i \|w_i\|^2$ is satisfied.

$$\begin{aligned} &\min \gamma_i \\ &\text{s.t.} \begin{bmatrix} -P_{\mathcal{N}_i} + I & * & * \\ 0 & -\gamma_i^2 I & * \\ P_i A_{\mathcal{N}_i} - Y_i C_{\mathcal{N}_i} & P_i I & -P_i \end{bmatrix} \prec 0 \\ &P_{\mathcal{N}_i} \succ 0 \\ &\gamma_i > 0 \end{aligned} \tag{5}$$

where $L_i = P_i^{-1} Y_i$. $P_{\mathcal{N}_i}$ and P_i are with different dimensions, $P_{\mathcal{N}_i} \in \mathbb{R}^{n_{\mathcal{N}_i} \times n_{\mathcal{N}_i}}$ is the dimension extended symmetric matrix of P_i, with the same dimension of all states in subsystem i and its coupled states. P_i is a part of $P_{\mathcal{N}_i}$ with $P_i = U_i P_{\mathcal{N}_i} U_i^\top$, $U_i \in \mathbb{R}^{n_i \times n_{\mathcal{N}_i}}$ is a transition matrix, composed by 0 and 1.

Proof. Define the error of estimation (4):

$$e_i(k) = x_i(k) - \hat{x}_i(k) \tag{6}$$

the error dynamic is

$$\begin{aligned} & e_i(k+1) = x_i(k+1) - \hat{x}_i(k+1) \\ & = A_i x_i(k) + \sum_{j \in \mathcal{N}_i, j \neq i} A_{ij} x_{ij}(k) + B_i u_i(k) \\ & - A_i \hat{x}_i(k) - \sum_{j \in \mathcal{N}_i, j \neq i} A_{ij} \hat{x}_{ij}(k) - B_i u_i(k) \\ & - L_i C_i (x_i(k) - \hat{x}_i(k)) \\ & = A_i e_i(k) + \sum_{j \in \mathcal{N}_i, j \neq i} A_{ij} e_{ij}(k) - L_i C_i e_i(k) \end{aligned} \tag{7}$$

In this form, because of the existence of coupled states x_{ij}, L_i is still need to be calculated in a centralized way, or the coupled will be omitted. The centralized design and computation cannot provide a flexible control framework, when one parameter changes, all the subsystems need to be recomputed, and also, centralized design will increase the computation burden. In the following proposed observer, the transition matrices with $0-1$ binary variables T_i, $U_{\mathcal{N}i}$ are introduced to realize the distributed observer design for coupled subsystems. The following transition descriptions are used:

$$\begin{aligned} x_i(k) &= T_i x_{\mathcal{N}i}(k) \\ e_i(k) &= T_i e_{\mathcal{N}i}(k) \end{aligned} \tag{8}$$

where $x_{\mathcal{N}i} = [x_i^T \quad x_j^T]^T$, $e_{\mathcal{N}i} = [e_i^T \quad e_j^T]^T$, $j \in \mathcal{N}i$, and the extended system matrices are:

$$\begin{aligned} A_{\mathcal{N}i} &= \begin{bmatrix} A_i & \dots & A_{ij} \\ \mathbf{0} & \cdots & \\ \mathbf{0} & \cdots & A_j \end{bmatrix} \\ C_{\mathcal{N}i} &= [C_i \ \ \mathbf{0}_{p_i \times n_{\mathcal{N}i}}] \end{aligned} \tag{9}$$

In this way, the error dynamic can be described as:

$$e_i(k+1) = (T_i A_{\mathcal{N}i} - L_i T_i C_{\mathcal{N}i}) e_{\mathcal{N}i} \tag{10}$$

Define the error dynamic Lyapunov function as: $V_i(k) = e_i^T(k) P_i e_i(k)$, first consider $\omega_i = 0$, we have:

$$\begin{aligned} & V_i(k+1) - V_i(k) \\ & = e_{\mathcal{N}i}{}^T(k) (T_i A_{\mathcal{N}i} - L_i T_i C_{\mathcal{N}i})^T P_i (T_i A_{\mathcal{N}i} - L_i T_i C_{\mathcal{N}i}) \\ & \times e_{\mathcal{N}i}(k) - (U_i e_{\mathcal{N}i}(k))^T P_i (U_i e_{\mathcal{N}i}(k)) \\ & = e_{\mathcal{N}i}{}^T(k) \Big[(T_i A_{\mathcal{N}i} - L_i T_i C_{\mathcal{N}i})^T P_i (T_i A_{\mathcal{N}i} - L_i T_i C_{\mathcal{N}i}) \\ & - P_{\mathcal{N}i} \Big] e_{\mathcal{N}i}(k) \leq 0 \end{aligned} \tag{11}$$

i.e.

$$(T_i A_{\mathcal{N}i} - L_i T_i C_{\mathcal{N}i})^T P_i (T_i A_{\mathcal{N}i} - L_i T_i C_{\mathcal{N}i}) - P_{\mathcal{N}i} \leq 0 \tag{12}$$

which shows the error dynamic can be stabilized. Then when there are $w_i \neq 0$ and bounded, the time difference of Lyapunov function need satisfies the following to guarantee the performance:

$$\begin{aligned} & V_i(e_i(k+1)) - V_i(e_i(k)) \\ & = [(T_i A_{\mathcal{N}i} - L_i T_i C_{\mathcal{N}i}) e_{\mathcal{N}i}(k) + w_i(k)]^T P_i \\ & \times [(T_i A_{\mathcal{N}i} - L_i T_i C_{\mathcal{N}i}) e_{\mathcal{N}i}(k) + w_i(k)] \\ & - (U_i e_{\mathcal{N}i}(k))^T P_i (U_i e_{\mathcal{N}i}(k)) \\ & = e_{\mathcal{N}i}^T(k) \Big[(T_i A_{\mathcal{N}i} - L_i T_i C_{\mathcal{N}i})^T P_i (T_i A_{\mathcal{N}i} \\ & - L_i T_i C_{\mathcal{N}i}) - P_{\mathcal{N}i} + I \Big] e_{\mathcal{N}i}(k) \\ & + 2 w_i^T(k) P_i (T_i A_{\mathcal{N}i} - L_i T_i C_{\mathcal{N}i}) e_{\mathcal{N}i}(k) \\ & + w_i^T(k) (P_i - \gamma_i^2 I) w_i(k) - e_{\mathcal{N}i}^T(k) e_{\mathcal{N}i}(k) \\ & + \gamma_i^2 w_i^T(k) w_i(k) \end{aligned} \tag{13}$$

Write them in matrix form:

$$\begin{aligned} & \begin{bmatrix} e_{\mathcal{N}i}{}^T(k) & \omega_i{}^T(k) \end{bmatrix} \times \\ & \begin{bmatrix} (T_i A_{\mathcal{N}i} - L_i T_i C_{\mathcal{N}i})^T P_i (T_i A_{\mathcal{N}i} - L_i T_i C_{\mathcal{N}i}) - P_{\mathcal{N}i} + I_i & (T_i A_{\mathcal{N}i} - L_i T_i C_{\mathcal{N}i})^T P_i^T \\ P_i (T_i A_{\mathcal{N}i} - L_i T_i C_{\mathcal{N}i}) & P_i - \gamma_i^2 I \end{bmatrix} \begin{bmatrix} e_{\mathcal{N}i}(k) \\ \omega_i(k) \end{bmatrix} \prec 0 \end{aligned} \tag{14}$$

or simplified as:

$$\begin{bmatrix} (T_i A_{\mathcal{N}i} - L_i T_i C_{\mathcal{N}i})^T P_i (T_i A_{\mathcal{N}i} - L_i T_i C_{\mathcal{N}i}) & (T_i A_{\mathcal{N}i} - L_i T_i C_{\mathcal{N}i})^T P_i^T \\ P_i (T_i A_{\mathcal{N}i} - L_i T_i C_{\mathcal{N}i}) & P_i \end{bmatrix} + \begin{bmatrix} -P_{\mathcal{N}i} + I & \mathbf{0} \\ \mathbf{0} & -\gamma_i^2 I \end{bmatrix} \prec 0 \tag{15}$$

Then apply the Schur complement lemma twice, the LMI in Theorem 1 is obtained. If sum up $V_i(k)$ from $k=0$ to $k=T$, from (13), $\sum_{k=0}^T \|e_i\|^2 \leq \sum_{k=0}^T \gamma_i \|w_i\|^2$ is derived.

3.2 States and sensor faults observer

In this subsection, distributed observer is designed for both states and sensor faults in the meantime. The observer for system (2) is designed as:

$$\begin{aligned} & \hat{x}_i(k+1) \\ & = A_i \hat{x}_i(k) + \sum_{j \in \mathcal{N}_i, j \neq i} A_{ij} \hat{x}_{ij}(k) + B_i u_i(k) \\ & + L_i [y_i(k) - C_i \hat{x}_i(k) - F_i \hat{v}_i(k)] \\ & m_i(k+1) \\ & = L_{Fi} \Bigg[C_i A_i \hat{x}_i(k) + C_i \sum_{j \in \mathcal{N}_i, j \neq i} A_{ij} \hat{x}_{ij}(k) \\ & + C_i B_i u_i(k) \Bigg] + L_{Xi} [y_i(k) - C_i \hat{x}_i(k) - \hat{v}_i(k)] \\ & \hat{v}_i(k) = m_i(k) - L_{Fi} y_i(k) \end{aligned} \tag{16}$$

where m_i is an auxiliary vector for sensor fault estimation, $v_i(k) = f_i(k) + w_i(k)$ combines the sensor fault and sensor noises (if F_i is identity matrix, these can be combined). L_i and L_{Xi} are designed observer gains.

The states estimations and sensor faults estimations error are:

$$\begin{aligned} e_i(k) &= x_i(k) - \hat{x}_i(k) \\ \epsilon_i(k) &= v_i(k) - \hat{v}_i(k) \end{aligned} \tag{17}$$

Then the corresponding error dynamics can be expanded:

$$
\begin{aligned}
e_i(k+1) &= x_i(k+1) - \hat{x}_i(k+1) \\
&= A_i e_i(k) + \sum_{j\in\mathcal{N}_i, j\neq i} A_{ij} e_{ij}(k) \\
&- L_i\left[y_i(k) - C_i\hat{x}_i(k) - F_i\hat{\upsilon}_i(k)\right]
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
&\varepsilon_i(k+1) = \upsilon_i(k+1) - \hat{\upsilon}_i(k+1) \\
&= \upsilon_i(k+1) - m_i(k+1) + L_{Fi} y_i(k+1) \\
&= \upsilon_i(k+1) + L_{Fi}\Bigg[C_i A_i x_i(k) + C_i \sum_{j\in\mathcal{N}_i, j\neq i} A_{ij} x_{ij}(k) \\
&+ C_i B_i u_i(k) + F_i \upsilon_i(k)\Bigg] - L_{Xi}\left[y_i(k) - C_i\hat{x}_i(k)\right. \\
&\left.- F_i\hat{\upsilon}_i(k)\right] - L_{Fi}\Bigg[C_i A_i \hat{x}_i(k) + C_i \sum_{j\in\mathcal{N}_i, j\neq i} A_{ij}\hat{x}_{ij}(k) \\
&+ C_i B_i u_i(k) + F_i\hat{\upsilon}_i(k)\Bigg] \\
&= \left(L_{Fi}C_iA_i - L_{Xi}C_i\right)e_i(k) + L_{Fi}C_i \sum_{j\in\mathcal{N}_i, j\neq i} A_{ij} e_{ij}(k) \\
&+ \left(L_{Fi}F_i - L_{Xi}F_i\right)\varepsilon_i(k) + w_i(k)
\end{aligned} \tag{19}
$$

Based on the definitions and analysis, the observer gains are designed from the Theorem 2.

Theorem 2. (Xiao and Liu, 2020) If there exists a positive definite matrix $P_{\mathcal{N}_i}$, a matrix Y_i and a constant γ_i, making the following optimization problem solved, then the error dynamic of the observer (16) is stabilized for subsystem i, and for unknown and bounded w_i, the performance $\sum_{k=0}^{T}\|e_i\|^2 \leq \sum_{k=0}^{T}\gamma_i\|w_i\|^2$ is satisfied.

$$
\begin{aligned}
&\min \gamma_i \\
&\text{s.t.} \begin{bmatrix} -P_{\mathcal{N}_i} + I & * & * \\ 0 & -\gamma_i^2 I & * \\ P_i\Theta_i - Y_i\Phi_i & P_i I & -P_i \end{bmatrix} \prec 0 \\
&P_{\mathcal{N}_i} \succ 0 \\
&\gamma_i > 0
\end{aligned} \tag{20}
$$

the matrices are defined as:

$$
\begin{aligned}
\Theta &= \begin{bmatrix} T_i A_{\mathcal{N}i} & \mathbf{0} \\ L_{Fi}C_iT_iA_{\mathcal{N}i} & \mathbf{0} \end{bmatrix} \\
\Phi_i &= \left[T_i C_{\mathcal{N}I} \;\; I\right] \\
\left[L_i \;\; L_{Xi}\right]^T &= P_i^{-1} Y_i
\end{aligned}
$$

The proof of Theorem 2 is a extended of proof of Theorem 1, and the similar proof can be found in our previous work (Xiao and Liu, 2020).

Switching mode: The observer mode is chosen based on the fault detection signal, assume that $\mathcal{F}$ is the fault detection result, when $\mathcal{F} = 1$, there is sensor fault detected, when $\mathcal{F} = 0$, there is no sensor fault detected.

$$
\begin{cases} \mathcal{F} = 0, \text{state observer only} \\ \mathcal{F} = 1, \text{state and fault observer} \end{cases} \tag{21}
$$

4. STATE OBSERVER BASED DISTRIBUTED MODEL PREDICTIVE CONTROL

In this section, distributed model predictive controller is designed. The controller design follows the previous states and sensor fault estimation. Due to the simultaneous estimation, the states information used for controller design is fault-free, thus actually sensor faults tolerant is implemented by the state observer.

For each subsystem, the MPC control law is obtained by solving the following online optimization problem

$$
\begin{aligned}
\min_{u_i} J_i &= x_i(N)^T P_{Ni} x_i(N) \\
&+ \sum_{s=0}^{N-1}\Big(x_i(k+s)^T Q_i x_i(k+s) \\
&+ u_i{}^T(k+s) R_i u_i(k+s)\Big)
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
\text{s.t.} x_i(k+1) &= A_{ii}x_i(k) + \sum_{j\in\mathcal{N}_i, j\neq i} A_{ij}x_j(k) \\
&+ B_i u_i(k)
\end{aligned} \tag{23}
$$

$$
\begin{aligned}
&x_i \in \mathcal{X}_i, u_i \in \mathcal{U}_i \\
&x_i(N) \in \mathcal{X}_{i,T} \\
&\forall i \in \{1, 2, \cdots, M\}
\end{aligned} \tag{24}
$$

where Q_i and R_i are weighting matrices of MPC, P_{Ni} is terminal cost weighting matrix. The cost function for DMPC of the whole system is the summation of all subsystems costs.

$$
J = \sum_{i=1}^{M} J_i \tag{25}
$$

In the MPC design, the vector x are used as estimated states. The controller design details can be found in our previous work (Xiao and Liu, 2020), we omit the design theory here.

Algorithm 1 synthesizes the Section 3 and Section 4, gives the integrated sensor faults and states observer and DMPC controller design algorithm.

5. EXAMPLE

Example 1:
Consider a distributed control system with two coupled subsystems, subsystem 1 shares states with subsystem 2, as shown in (26) and (27).

$$
\begin{aligned}
A_1 &= \begin{bmatrix} 1 & 0 & 0.2 & 0 & 0 \\ 0 & 2 & -2 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -0.6 & 1 \end{bmatrix} \\
B_1 &= \begin{bmatrix} 1 & 0.2 & 0 & 0 \\ -0.2 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -0.3 \\ 0 & 0 & -1 & 0 \end{bmatrix} \\
C_1 &= \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & -1 & 1 & -1 & 1 \end{bmatrix}
\end{aligned} \tag{26}
$$

$$
\begin{aligned}
A_2 &= \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ -1 & 0 & 1 \end{bmatrix} A_{12} = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
B_2 &= \begin{bmatrix} 0 & 1 \\ -1 & 0.3 \\ -2 & 1 \end{bmatrix} C_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\end{aligned} \tag{27}
$$

The distributed MPC controllers are designed for each subsystem, control input u_2 is constrained by $-5 \leq u_2 \leq 5$.

Algorithm 1 Observer-based fault-tolerant distributed MPC

Initialization: At time k, receive the measurements $y(k)$ from sensors;

Fault detection: Detect if there is sensor fault, then send the fault detection signal $\mathcal{F}$;
Switching mode state and fault estimation:
For each subsystem i, $i \in \{1, \ldots, M\}$
Receive the fault detection signal $\mathcal{F}$,
Receive the coupled estimated states from neighbor subsystems observer,
if $\mathcal{F} = 0$, estimation turns to states only observer mode as the designed observer (4);
if $\mathcal{F} = 1$, estimation turns to both sensor faults and states observer mode as the designed observer (16), get $\hat{x}(k)$ and $\hat{v}(k)$;

Send the states estimations $\hat{x}_i(k)$ and coupled states estimations $\hat{x}_{ij}(k)$ to each subsystem i controller;

Distributed model predictive control design:
For all subsystem i, $i \in \{1, \ldots, M\}$
Receive the estimated states $\hat{x}_i(k)$ and coupled states $\hat{x}_{ij}(k)$of neighbor subsystems (estimated sensor faults are not used in controller design);
Solve the online optimization problem (22) using the estimated states, and obtain the DMPC control law $u_i(k)$;
Wait for the next sampling time $k+1$

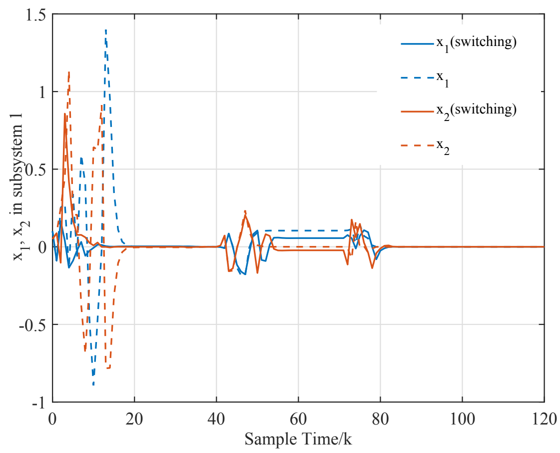

Fig. 1. States x_1, x_2 in subsystem 1

The sensor fault $f_s = 0.05$ arises from $k = 40$ to $k = 80$ in subsystem 1, thus when $k = 40$ to 80, the distributed observer of subsystem 1 is switched to faults and states estimation mode, and at other instants, the distributed observers for two subsystems only estimate the states. Figure 1, 2 and 3 show the controlled states results. In the figures, the solid lines represent the states revolutions under switching mode observers, and the dot lines represent the states under one observer mode (faults and states estimation). The improvements are mainly in two aspects: one is during the tracking stage, the system under switching mode observers can track the reference faster than the one mode observer, and with a flatter adjusting process,

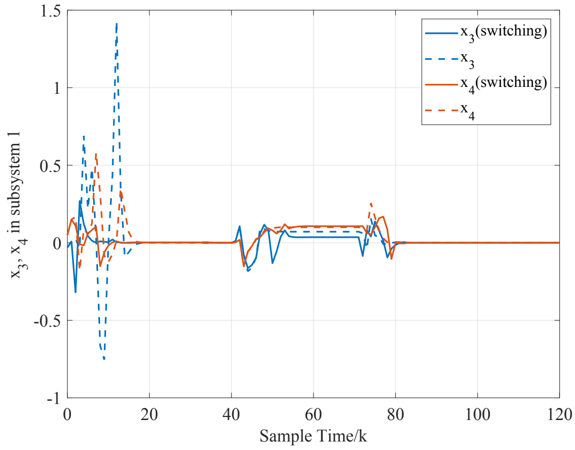

Fig. 2. States x_3, x_4 in subsystem 1

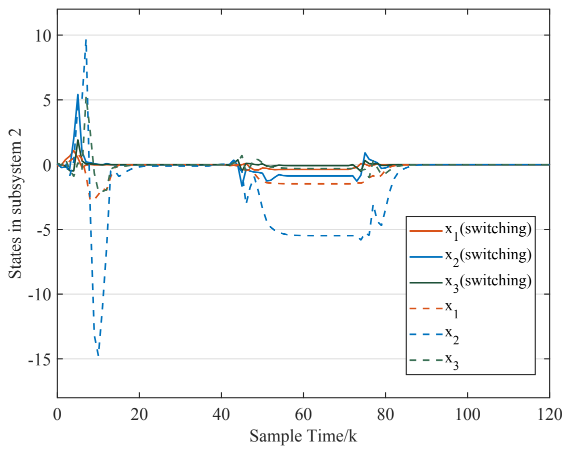

Fig. 3. States in subsystem 2

the other is during the faulty period, the system under switching mode also get a better performance, especially in subsystem 2, because that subsystem 2 does not under the fault and states estimation mode, which will give a more accurate estimation.

Example 2: Consider a two-CSTR chemical process, the two linear models are given as:

$$A_1 = \begin{bmatrix} 0.8198 & 0.0065 \\ 0 & 0.8187 \end{bmatrix} \qquad A_{21} = \begin{bmatrix} 0.1587 & 0 \\ 0 & 0.1586 \end{bmatrix} \quad (28)$$

$$A_2 = \begin{bmatrix} 0.8909 & 0.0067 \\ 0 & 0.8899 \end{bmatrix} \qquad A_{12} = \begin{bmatrix} 0.063 & 0 \\ 0 & 0.0629 \end{bmatrix} \quad (29)$$

$$B_1 = \begin{bmatrix} 1.96 \times 10^{-5} & 8.69 \times 10^{-5} \\ 0 & 0.0226 \end{bmatrix} \quad (30)$$

$$B_2 = \begin{bmatrix} 6.81 \times 10^{-6} & 1.75 \times 10^{-4} \\ 0 & 0.0472 \end{bmatrix} \quad (31)$$

The sensor fault in CSTR2 arises from $k = 200$ to $k = 230$ as a sine function of amplitude 6. The Figure 4 and 5 show the controlled $x - x^*$ dynamics of production concentration and reaction temperature in two CSTRs.The blue line is temperature, the red line is concentration. The results tell the proposed sensor fault-tolerant DMPC approach is effective.

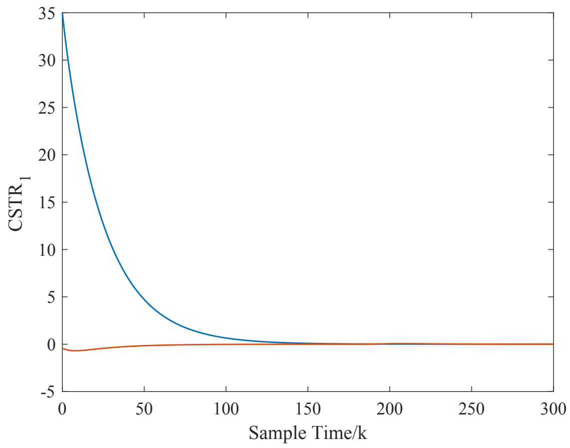

Fig. 4. States in CSTR1.

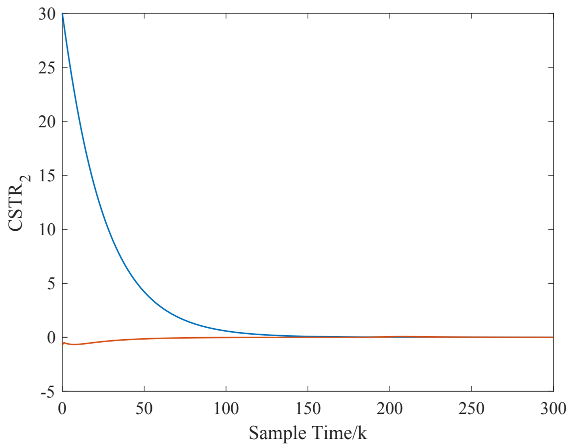

Fig. 5. States in CSTR2.

6. CONCLUSION

In this paper, distributed sensor fault-tolerant model predictive control is considered, the controller is designed based on fault and state estimation for intermittent sensor faults. Two central problems are solved: one is a distributed observer for the distributed control system, the other is an observer switching mode strategy. With a totally distributed design, distributed system observer matrices is designed offline with some transition matrices, and state and fault estimation are computed online with communicated coupled states information. The observer are designed in two modes: state only estimation, both state and fault estimation are switched due to the absence or presence of the sensor faults. The design in this paper provides more flexibility for distributed model predictive control systems, especially in flexible chemical processes.

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