An Adaptive Filter for Parameter Estimation of Damped Sinusoidal Signals

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Abstract: In this paper, we introduce a continuous-time adaptive algorithm for online estimation of the parameters of exponentially damped sinusoidal signals. The proposed algorithm can also reconstruct the original signal from its corrupted measurements in real-time and without extra computational cost. For sufficiently slow adaptation gains, we establish the local stability of the filter and local exponential convergence of the estimated parameters to their true values. We also obtain a linear approximation of the adaptation laws which can be utilized for tuning the adaptation gains. Simulation studies are also provided to corroborate our claims.

1. INTRODUCTION

Parameter estimation of the harmonic signals is a fundamental problem in signal processing and systems theory. It has a wide range of applications in various branches of science and engineering, for instance in resolving the location of long-distance stars by radio telescopes, geophysical explorations, underwater surveillance, sonar, radar, communications, speech analysis, power systems, active noise and vibration control, process control, and many other fields. In the signal processing literature, there are several well-established algorithms to estimate the frequency of a sinusoid such as Pisarenko's method, Pisarenko (1973), multiple signal classification (MUSIC), Schmidt (1986), and estimation of signal parameters via rotational invariance technique (ESPRIT), Roy and Kailath (1989). For a detailed review, see the classical paper, Kay and Marple (1981), or for a comprehensive list of basic references, see Stoica (1993).

From a control perspective, the online estimation of the parameters of such signals is crucially important. The challenge of the online frequency estimation problem stems from the nonlinear dependency of the measurable signal on the unknown frequency. Therefore, designing globally convergent frequency estimators is not a straightforward task. In discrete-time, the nonlinear parameterization obstacle does not arise. However, the estimated frequency critically depends on the sampling time, particularly for the frequencies close to 0 or π, Hsu et al. (1999). Hence, the global convergence cannot be acheived in the discrete-time case. Consequently, over the past two decades, several globally convergent continuous-time algorithms were proposed for the frequency estimation of sinusoidal signals—e.g., see Hsu et al. (1999), Marino and Tomei (2002), Xia (2002), Obregon-Pulido et al. (2002).

In this paper, we are interested in the problem of continuous-time online estimation of the parameters of exponentially damped sinusoidal signals. This contains a broader class of signals, including the harmonic signals. The stationarity of the measured signal is an underlying assumption in most of the existing algorithms for online frequency estimation. In contrast to harmonic signals, the exponentially damped sinusoidal signals are non-stationary and vanishing. Therefore, the parameter estimation of such signals is relatively more challenging. As this broader class of signals capture the impulse response of causal linear time-invariant systems, it is evident that estimation of the parameters of such signals plays an important role in many practical applications. For instance, in speech processing, each pitch frame of the speech waveform for vowel sounds can be assumed to be a sum of exponentially damped signals. In radar target identification, the estimated parameters of the exponential model are used to discriminate different targets. In power systems, the exponentially damped sinusoidal signals arise as a result of interaction among different components. Hence, fast and accurate identification of such oscillations is critical in detecting any faults or equipment malfunctions.

The traditional non-iterative algorithms for estimating the parameters of exponentially damped sinusoidal signals includes, Prony's algorithm and its variations, Kumaresan and Tufts (1982), Osborne and Smyth (1995), the matrix-pencil methods, Hua and Sarkar (1990), Badeau et al. (2002), and subspace identification methods, Van Overschee and De Moor (1994). These techniques depend heavily on solving linear algebraic equations, eigenvalue problems, and singular-value decomposition. Another important class of algorithms are algebraic identification methods, Mboup (2009). Several online adaptive algorithms were also reported. For instance, Lu and Brown (2008), Lu and Brown (2010), used the internal-model principle to develop online discrete-time and continuous-time adaptive algorithms. In a recent work, Chen et al. (2019) employed Volterra integral operators with novel kernel-functions to develop a finite-time estimation algorithm for exponentially damped sinusoidal signals. A Dynamic regressor extension and mixing (DREM)-based algorithm has also been reported in Vediakova et al. (2020), where a linear regression model was first obtained in terms of (nonlinear) functions of the unknown parameters of the measured signal. Hence, in the end, the unknown parameters were reconstructed via additional nonlinear computations. This is a common theme among all the proposed globally convergent frequency estimators for harmonic signals, except for Hsu et al. (1999).

In this paper, motivated by the popular technique of adaptive notch filters, Regalia (1991), Hsu et al. (1999), Mojiri et al. (2007), we propose an alternative continuous-time algorithm for online estimation of the parameters of exponentially damped sinusoidal signals. The proposed scheme has a simple structure and provides direct estimates of the parameters of the signal. In contrast to Vediakova et al. (2020), there are no extra nonlinear computations involved for reconstructing the parameters of the signal. For sufficiently slow adaptations, the (local) stability and the convergence of the proposed algorithm has been investigated by time-scale separation and averaging techniques, Riedle and Kokotovic (1986).

The remainder of the paper is organized as follows. In Section 2, we formulate the problem and present the proposed algorithm. The main results and convergence analysis are provided in Section 3. Finally, Section 4 provides some numerical simulations followed by a brief conclusion in Section 5.

2. PROBLEM STATEMENT AND THE PROPOSED ALGORITHM

Consider the following measurable exponentially damped sinusoidal signal

$$u(t) = A_0 e^{-\sigma_0 t}\sin(\omega_0 t + \varphi_0), \tag{1}$$

where its amplitude, A_0, damping factor, $\sigma_0 \geq 0$, frequency $\omega_0 > 0$, and initial phase φ_0 are all fixed but unknown.

The task is to design continuous-time algorithms for online estimation of the frequency and damping factor of the pure exponentially damped sinusoidal signal (1).

The challenge of this problem is two-folded: (a) The exponentially damped sinusoidal signals are energy signals, i.e., $u(\cdot) \in \mathcal{L}_2$, and hence not persistently exciting (PE). (b) The measurable signal (1) depends on the unknown frequency and damping factor in a nonlinear fashion.

Remark 1. It is important to point out that it is possible to find linear parameterization for the signal (1), in terms of ω_0^2, σ_0 and σ_0^2, however, such parameterizations lead to online estimates of the squared frequency/damping factor and further nonlinear computations are required to produce online estimates of the frequency and damping factor. This could, in turn, lead to sensitivity of the estimates in the presence of noise.

To motivate our proposed algorithm, consider the following filter

$$H(p) = \frac{(p+\sigma)^2 + \omega^2}{p^2 + 2(\zeta\omega + \sigma)p + (\sigma^2 + \omega^2)}, \tag{2}$$

where $p := \frac{d}{dt}$, and $\zeta > 0$ is a design parameter. Throughout this paper, with an abuse of notation, we refer to ζ as the damping ratio.

Feeding the exponentially damped sinusoidal signal (1) to the filter (2) will provide (up to exponentially decaying terms) a zero output when $\sigma = \sigma_0$ and $\omega = \omega_0$. Therefore, it is reasonable to adjust the parameters of the second-order filter (2) online with estimators that exploit the information from the filter output. Notice that the denominator polynomial of the filter (2), can be in general replaced by any arbitrary second-order Hurwitz polynomial. This particular choice, however, could be interpreted as a generalization of Regalia's notch filter, Regalia (1991), Bodson and Douglas (1997). As pointed out earlier, the unknown parameters enter nonlinearly—see the discussion under Remark 1. Therefore, the question is, how to design such adaptation laws to guarantee the stability and asymptotic convergence of the algorithm? To address this question, we will consider the following time-domain representation of the second-order filter (2)

$$\ddot{x} + 2\sigma\dot{x} + (\sigma^2 + \omega^2)x = (2\zeta\omega)(u - \dot{x}). \tag{3}$$

Let $e := u - \dot{x}$ to be the error. It is straightforward to verify that, for fixed values of ω and σ, the transfer function between the input u and output e is exactly the same as the transfer function of the filter (2). Suppose, $\sigma = \sigma_0$ and $\omega = \omega_0$. Then for appropriately chosen initial conditions, we have

$$\ddot{x} + 2\sigma_0\dot{x} + (\sigma_0^2 + \omega_0^2)x = 0, \qquad \forall t \geq 0.$$

The foregoing identity implies that

$$\begin{aligned} \omega_0^2 x &= -\ddot{x} - 2\sigma_0\dot{x} - \sigma_0^2 x, \\ 2\sigma_0\dot{x} &= -\ddot{x} - (-\sigma_0^2 + \omega_0^2)x. \end{aligned}$$

Hence, by using (3), and close to the true parameters, where $\omega \approx \omega_0$ and $\sigma \approx \sigma_0$, we expect to have

$$\begin{aligned} \omega_0^2 x &\approx -\ddot{x} - 2\sigma\dot{x} - \sigma^2 x = \omega^2 x - 2\zeta\omega e, \\ 2\sigma_0\dot{x} &\approx -\ddot{x} - (-\sigma^2 + \omega^2)x = -2\sigma\dot{x} - 2\zeta\omega e. \end{aligned}$$

Equivalently, after multiplying by appropriate factors, we have

$$\begin{aligned} (\omega_0^2 - \omega^2)x^2 &\approx -2\zeta\omega x e, \\ (\sigma_0 - \sigma)\dot{x}^2 &\approx -\zeta\omega\dot{x}e. \end{aligned}$$

Hence, in the vicinity of the true parameter values, it is reasonable to assume that $(\omega_0^2 - \omega^2)x^2 \propto (-xe)$, and $(\sigma_0 - \sigma)\dot{x}^2 \propto (-\dot{x}e)$.

The derivations above suggest the following adaptation laws

$$\dot{\hat{\omega}} = -\gamma_1 \frac{\hat{\omega} x e}{m}, \tag{4}$$

$$\dot{\hat{\sigma}} = -\gamma_2 \frac{\hat{\sigma}\dot{x} e}{m}, \tag{5}$$

where $\gamma_1, \gamma_2 > 0$ determine the speed of adaptations, and m, is an appropriately chosen (positive) normalization signal that will be introduced later.

According to the foregoing derivations, whenever the estimated parameters $\hat{\omega}, \hat{\sigma}$ are close to their nominal values and adaptation is slow, the search in parameter space will go in the right direction (e.g., when $\hat{\omega} > \omega_0 \Rightarrow \dot{\hat{\omega}} \leq 0$, or when $\hat{\sigma} < \sigma_0 \Rightarrow \dot{\hat{\sigma}} \geq 0$, or vice versa).

Remark 2. The term $\hat{\omega}$ (resp. $\hat{\sigma}$) on the right-hand side of (4) (resp. (5)) is introduced to make the set $\{\hat{\omega} \equiv 0\}$ (resp. $\{\hat{\sigma} \equiv 0\}$) invariant, and hence impose the positivity of the estimated frequency (resp. damping factor) without using the projection. The role of normalization signal m will be unveiled in the course of stability and convergence analysis.

Before we analyze the stability of the proposed adaptive filter, we characterize its equilibria.

Fact 3. For the input signal $u(t) = A_0 e^{-\sigma_0 t} \sin(\omega_0 t + \varphi_0)$, the dynamical system (3), (4), (5), has a unique solution $\mathcal{S} := [\bar{x}, \dot{\bar{x}}, \bar{\omega}, \bar{\sigma}]'$, with constant and correct estimated frequency and damping factor given by

$$\mathcal{S} = \begin{bmatrix} \dfrac{-A_0}{\sigma_0^2 + \omega_0^2} e^{-\sigma_0 t} \left(\omega_0 \cos(\omega_0 t + \varphi_0) + \sigma_0 \sin(\omega_0 t + \varphi_0)\right) \\ A_0 e^{-\sigma_0 t} \sin(\omega_0 t + \varphi_0) \\ \omega_0 \\ \sigma_0 \end{bmatrix} \quad (6)$$

Remark 4. It is clear from the second component of $\mathcal{S}$ in (6), i.e., $\dot{\bar{x}}$, that the proposed adaptive filter also reconstructs the signal without any extra computations. This could be of interest in some applications, especially when the measured signal $u(\cdot)$ is corrupted with noise or other harmonics.

Next, we introduce the normalization signal m as follows:

$$m := \left(\frac{\hat{\sigma} x + \dot{x}}{\hat{\omega}}\right)^2 + x^2. \quad (7)$$

Remark 5. Note that it is important to initialize the filter (3) with non-zero states. Indeed, by using the initial guesses for A_0, ω_0 and σ_0, it is clear from (6) that one can easily choose the initial conditions of the adaptive filter $(x(t_0), \dot{x}(t_0), \hat{\omega}(t_0), \hat{\sigma}(t_0))$ to meet this requirement.

Remark 6. It is not hard to verify that for $\hat{\omega} = \omega_0, \hat{\sigma} = \sigma_0$, $x = \bar{x}$ and $\dot{x} = \dot{\bar{x}}$, we have $m = A_0^2 e^{-2\sigma_0 t}/(\sigma_0^2 + \omega_0^2)$.

3. ANALYSIS

In this section, we investigate the stability of the proposed adaptive filter and the asymptotic convergence of the estimated parameters. The main result of this paper is summarized in Theorem 7.

Let $\chi := (x\,, \dot{x})'$, be the states of the filter (3), and define the new variables η_1, η_2 such that $\gamma_1 = \epsilon\eta_1$ and $\gamma_2 = \epsilon\eta_2$. Similar to other adaptive systems, the estimated frequency $\hat{\omega}$ and damping factor $\hat{\sigma}$ are expected to evolve slowly compared to the filter states χ for sufficiently small adaptation gains γ_1, γ_2 and after a transient. By resorting to the classical concept of the integral manifold of slow adaptation, Riedle and Kokotovic (1986), we utilize a time-scale separation, to provide a proof of stability.

Theorem 7. Consider the pure exponentially damped sinusoidal signal (1) applied to the adaptive filter of (3)–(5), with the normalization signal defined in (7). Then, there exists ϵ^*, such that for $\epsilon \in [0, \epsilon^*]$, the adaptive filter of (3)–(5) has a uniquely defined integral manifold $M_\epsilon = \{t, \hat{\omega}, \hat{\sigma}, \chi : \chi = h_\epsilon(t, \hat{\omega}, \hat{\sigma})\}$, i.e., a time-varying two-dimensional surface in $\mathbf{R}^4$. This ϵ-family of integral manifolds arbitrarily approaches the *"frozen-parameter"* manifold M_0 as $\epsilon \to 0$. Furthermore, on the manifold M_ϵ, the adaptation laws are *locally exponentially stable* in the sense that $\hat{\omega} \to \omega_0$, and $\hat{\sigma} \to \sigma_0$ as $t \to \infty$.

Proof. For the purpose of analysis, it is convenient to use the state-space realization of the adaptive filter (3)–(5) given by

$$\dot{\chi} = \begin{bmatrix} 0 & 1 \\ -(\hat{\omega}^2 + \hat{\sigma}^2) & -(2\zeta\hat{\omega} + \hat{\sigma}) \end{bmatrix} \chi + \begin{bmatrix} 0 \\ 2\zeta\hat{\omega} \end{bmatrix} u \quad (8)$$

$$\dot{\hat{\omega}} = -\epsilon \frac{\eta_1\, \hat{\omega}\, \chi_1\, (u - \chi_2)}{\left(\frac{\hat{\sigma}\, \chi_1 + \chi_2}{\hat{\omega}}\right)^2 + \chi_1^2}, \quad (9)$$

$$\dot{\hat{\sigma}} = -\epsilon \frac{\eta_2\, \hat{\sigma}\, \chi_2\, (u - \chi_2)}{\left(\frac{\hat{\sigma}\, \chi_1 + \chi_2}{\hat{\omega}}\right)^2 + \chi_1^2}. \quad (10)$$

For the frozen-parameters $\hat{\theta} := [\hat{\omega}, \hat{\sigma}]'$, and the pure exponentially damped sinusoidal signal (1), the steady-state response of (8) denoted by $\chi^0(t, \hat{\theta})$, can be fully characterized as follows:

$$\chi^0(t, \hat{\theta}) := K e^{-\sigma_0 t} \begin{bmatrix} \sin(\omega_0 t + \varphi_0 + \psi) \\ -\sigma_0 \sin(\omega_0 t + \varphi_0 + \psi) + \omega_0 \cos(\omega_0 t + \varphi_0 + \psi) \end{bmatrix},$$

where $K = A_0 \left|T(-\sigma_0 + j\omega_0)\right|$, $\psi = \angle\, T(-\sigma_0 + j\omega_0)$ and

$$T(s) = \frac{2\zeta\hat{\omega}}{s^2 + 2(\zeta\hat{\omega} + \hat{\sigma})s + (\hat{\sigma}^2 + \hat{\omega}^2)}.$$

For the sake of brevity, and throughout the remainder of the paper, we will simply omit the arguments $(t, \hat{\theta})$ and denote the quasi-static response $\chi^0(t, \hat{\theta})$ by χ^0. Also, notice that χ^0 is a smooth function of $\hat{\theta}$.
Let $z := \chi - \chi^0$, and

$$A(\hat{\theta}) := \begin{bmatrix} 0 & 1 \\ -(\hat{\omega}^2 + \hat{\sigma}^2) & -(2\zeta\hat{\omega} + \hat{\sigma}) \end{bmatrix}.$$

Since $\hat{\omega}, \hat{\sigma} > 0$ (recall the discussion under Remark 2), the frozen-parameter matrix $A(\hat{\theta})$ is Hurwitz. Considering this fact, and using the previous observation about the smoothness of χ^0, we can show that the conditions of (Riedle and Kokotovic, 1986, Thm. 3.1) are satisfied. Due to page restriction, however, we omitted the verification of these conditions. Hence, for sufficiently small ϵ, the existence of the ϵ-family of integral manifolds $h_\epsilon(t, \hat{\theta}) := \chi^0(t, \hat{\theta}) + \epsilon h_1(t, \hat{\theta}) + \epsilon^2 h_2(t, \hat{\theta}) + \cdots$, immediately follows from (Riedle and Kokotovic, 1986, Thm. 3.1).

Next, we invoke (Riedle and Kokotovic, 1986, Thm. 4.1), to analyze the stability of the adaptation laws. Note that for $z = 0$, the adaptation laws reduce to

$$\dot{\hat{\omega}} = -\epsilon \frac{\eta_1\, \hat{\omega}\, \chi_1^0\, e^0}{m^0},$$
$$\dot{\hat{\sigma}} = -\epsilon \frac{\eta_2\, \hat{\sigma}\, {\chi_2}^0\, e^0}{m^0},$$

where $e^0 := u - \chi_2^0$, and $m^0 := \left((\hat{\sigma}\, \chi_1^0 + \chi_2^0)/\hat{\omega}\right)^2 + {\chi_1^0}^2$. It is not hard to verify that

$$e^0 = \frac{1}{2\zeta\hat{\omega}} \left[\left((\hat{\omega}^2 - \omega_0^2) + (\hat{\sigma} - \sigma_0^2)\right) \chi_1^0 + 2(\hat{\sigma} - \sigma_0)\chi_2^0\right].$$

From the derivations above, it is clear that (ω_0, σ_0) is an equilibrium point of (9), (10). Let $\tilde{\omega} := \hat{\omega} - \omega_0$, and $\tilde{\sigma} := \hat{\sigma} - \sigma_0$, be the deviations of the parameters, and $\tilde{\theta} := \hat{\theta} - \theta_0$. Note that $\chi_1^0(t, \theta_0) = \bar{x}$, $\chi_2^0(t, \theta_0) = \dot{\bar{x}}$, where $\bar{x}$, and $\dot{\bar{x}}$ are introduced in (6). By linearizing the reduced dynamics about $\theta_0 := (\omega_0, \sigma_0)$, we obtain the linear time-varying (LTV) system

$$\dot{\tilde{\theta}} = \epsilon \begin{bmatrix} -\dfrac{\eta_1\, \omega_0\, \bar{x}^2}{\zeta \bar{m}} & -\dfrac{\eta_1\, (\sigma_0\, \bar{x}^2 + \bar{x}\dot{\bar{x}})}{\zeta \bar{m}} \\ -\dfrac{\eta_2\, \sigma_0\, \bar{x}\, \dot{\bar{x}}}{\zeta \bar{m}} & -\dfrac{\eta_2\, \sigma_0\, (\sigma_0\, \bar{x}\, \dot{\bar{x}} + \dot{\bar{x}}^2)}{\omega_0 \zeta \bar{m}} \end{bmatrix} \tilde{\theta}, \qquad (11)$$

where $\bar{m} := \left((\sigma_0\, \bar{x} + \dot{\bar{x}})/\omega_0\right)^2 + \bar{x}^2$. By Remark 6, it is clear that $\bar{m} = A_0^2 e^{-2\sigma_0 t}/(\sigma_0^2 + \omega_0^2)$. To analyze the stability of the LTV system (11), we invoke (Riedle and Kokotovic, 1986, Thm. 4.1) and the averaging theorem—e.g., see Sastry and Bodson (1989).

A close inspection of the LTV system (11) reveals the importance of the normalization signal m in the dynamics of the proposed adaptation laws (4), (5).

By applying the averaging theorem to the LTV dynamics in (11), and using (6), we obtain the following linear time invariant (LTI) system

$$\dot{\theta}_{av} = \epsilon \begin{bmatrix} -\dfrac{\eta_1\, \omega_0}{2\zeta} & 0 \\ +\dfrac{\eta_2\, \sigma_0^2}{2\zeta} & -\dfrac{\eta_2\, \sigma_0\, \omega_0}{2\zeta} \end{bmatrix} \theta_{av}, \qquad (12)$$

where θ_{av} denotes the averaged state. Clearly, the averaged LTI system (12) is exponentially stable. Therefore, in light of (Khalil, 2002, Thm. 4.15) and (Riedle and Kokotovic, 1986, Thm. 4.1), for sufficiently small ϵ, the adaptive filter (3)–(5) is *locally* stable and the estimated parameters will converge exponentially to their true values. This completes the proof.

Remark 8. It is important to emphasize that the established convergence result is just valid locally.

Remark 9. The preceding analysis shows that the (local) rate of convergence of the proposed algorithm, and hence the tuning parameters, depend on the (unknown) parameters of the signal. It also reveals how to circumvent this issue by appropriate scaling of the adaptation laws and projection on the nonnegative real numbers. Indeed, with a similar analysis, we can show that the eigenvalues of the averaged LTI system associated with the following adaptation laws are independent of the unknown parameters σ_0 and ω_0:

$$\begin{aligned} \dot{\hat{\omega}} &= -\gamma_1 \frac{\hat{\omega}^2\, x\, e}{(\hat{\sigma}x + \dot{x})^2 + (\hat{\omega}x)^2}, \\ \dot{\hat{\sigma}} &= \text{Proj}\left(-\gamma_2 \frac{\hat{\omega}\, \dot{x}\, e}{(\hat{\sigma}x + \dot{x})^2 + (\hat{\omega}x)^2} \right). \end{aligned} \qquad (13)$$

This could enhance the tuning of the design parameters, ζ, γ_1 and γ_2. Furthermore, if an upper-bound is known *a priori* for σ_0, it could be easily incorporated into the Proj operator.

Remark 10. A simple examination of the adaptations laws (4)–(5), reveals that due to the product terms xe or $\dot{x}e$, vanishing double-frequency ripples will appear in the estimated parameters. As the parameters converge to their true values, however, the double-frequency ripples also keep being removed from the estimations. In other words, there will be no double-frequency ripple on the estimated parameters, when the true values are reached. Nevertheless, it is possible to mitigate these double-frequency ripples by passing the cross-terms xe and $\dot{x}e$ through a low-pass filter. Indeed, if a lower bound $\underline{\omega}$ of the nominal frequency ω_0 is known *a priori*, it is possible to employ a first-order low-pass filter $F(p) = \lambda/(p + \lambda)$, with $0 < \lambda \ll 2\underline{\omega}$, to smooth out the estimated parameters.

Remark 11. In practice, the measured signal could have a dc bias, i.e., $u_b(t) = c + u(t)$, where c denotes the constant, yet unknown, offset on the input signal $u(t)$ defined in (1). The performance of the estimated parameters in the proposed structure are prone to error in the presence of such a dc component. To resolve this problem, and by resorting to the internal model principle of control, Francis and Wonham (1976), it is sufficient to augment an extra integrator to the structure of the filter (2). The amended filter is given by:

$$\begin{aligned} \ddot{x} + 2\sigma\dot{x} + (\sigma^2 + \omega^2)x &= (2\zeta\omega)e \\ \dot{z} &= \eta e \end{aligned} \qquad (14)$$

where $e := u_b - \dot{x} - z$, denotes the modified error signal that will be employed in the adaptation laws (4), (5), and $\eta > 0$ determines the speed of adaptation of the augmented state z.

Remark 12. It is very straightforward to generalize the proposed algorithm to cope with the input signals composed of N exponentially damped sinusoidal signals with distinct frequencies, i.e. $\tilde{u}(t) = \sum_{i=1}^{N} A_i e^{-\sigma_i t} \sin(\omega_i t + \varphi_i)$. With a simple modification in the driving error signal e, a parallel structure composed of N blocks of the proposed adaptive algorithm can be utilized to estimate the parameters of signal $\tilde{u}(t)$. In this manner, each block extracts the information of a different component of the signal $\tilde{u}(t)$. For the i-th block of the parallel structure, the dynamics of the second-order filter is given by

$$\ddot{x}_i + 2\sigma_i\dot{x}_i + (\sigma_i^2 + \omega_i^2)x_i = (2\zeta_i\omega_i)e, \qquad (15)$$

where $e := \tilde{u} - \sum_{i=1}^{N} \dot{x}_i$ denotes the modified error signal that feeds the adaptation laws in each block. If, in addition, the input signal $\tilde{u}(t)$ has a dc component, an extra integrator can be augmented to the parallel structure of N blocks, as discussed in Remark 11, and the error signal should be modified accordingly.

In the presence of additive measurement noise, as time evolves, the signal to noise ratio (SNR) of the measurable signal decreases. Consequently, when the SNR drops below a certain threshold, the suggested normalization factor will adversely magnify the noise, and it could eventually lead to the parameter drift phenomenon. Hence, it is required to modify the normalization signal, e.g. we can use $[1 + (\hat{\sigma}x + \dot{x})^2 + (\hat{\omega}x)^2]$ in (13).

4. SIMULATIONS

In this section, we present some numerical simulations to demonstrate the performance of the proposed algo-

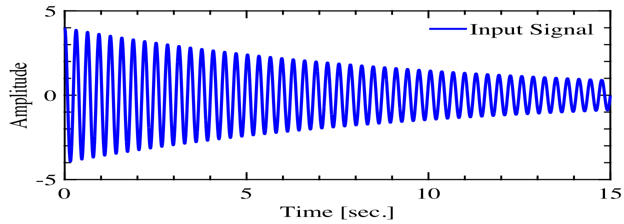

Fig. 1. The pure exp. damped sinusoidal input signal

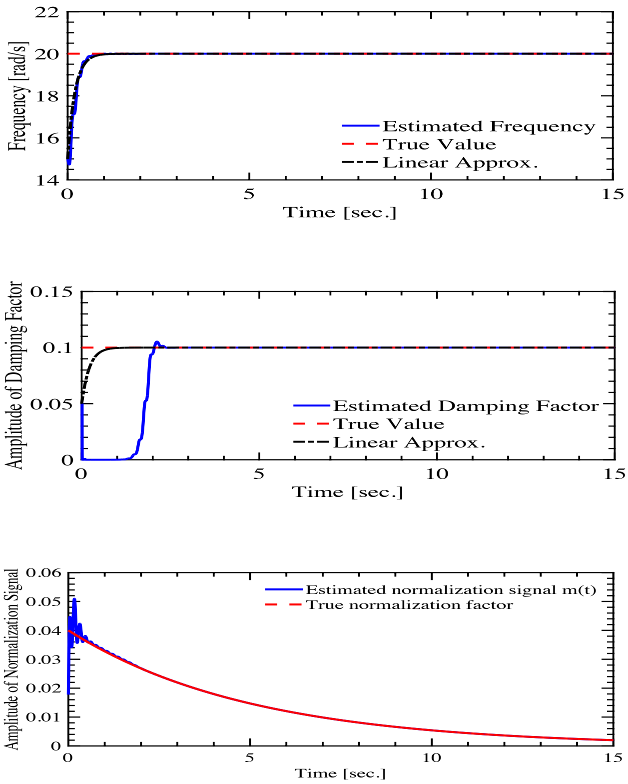

Fig. 2. Performance of the proposed algorithm for a pure exponentially damped sinusoidal signal

rithm. All the simulations are performed in the MATLAB/Simulink environment.

First, we consider a pure exponentially damped sinusoidal signal to assess the performance of the adaptive filter of (3)–(5), with the normalization signal defined in (7). Fig. 2 illustrates the response of the proposed algorithm to $u(t) = 4\,e^{-0.1t}\cos(20t)$. The input signal $u(\cdot)$ is depicted in Fig. 1. The design parameters $\zeta = 1$, $\gamma_1 = 0.5$, and $\gamma_2 = 9$ are chosen for the simulation.

It is evident that the approximate LTI model (12) provides an accurate estimate of the convergence time of the proposed algorithm. The error signal $e(\cdot)$ is also depicted in Fig. 3.

Next, we consider the case where the measured signal has an unknown dc component. For the sake of simulations, we consider the signal $u_b(t) = 0.7 + 4\,e^{-0.1t}\cos(20t)$. According to Remark 11, an extra integrator has been augmented to the proposed adaptive filter. Fig. 4 illustrates the performance of the modified structure. The design parameters $\zeta = 1$, $\eta = 2.5$, $\gamma_1 = 0.25$, and $\gamma_2 = 2.5$ are chosen for the simulation.

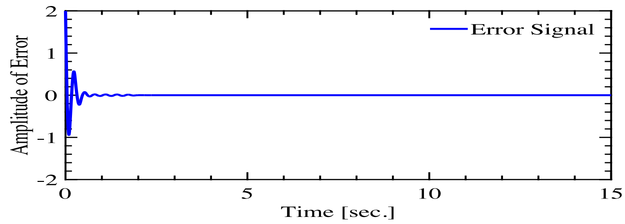

Fig. 3. The error signal of the proposed algorithm

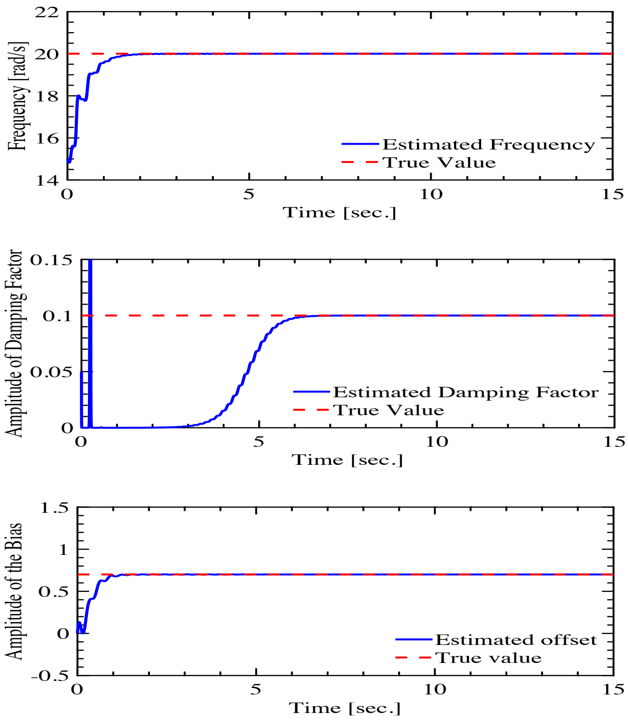

Fig. 4. Performance of the modified algorithm in the presence of dc component

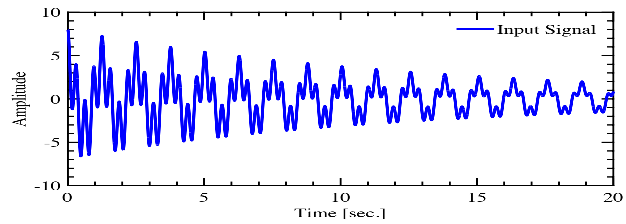

Fig. 5. The multi-component input signal

Finally, we evaluate the capability of the proposed algorithm for extracting multiple components. We consider the input signal $\tilde{u}(t) = 4\,(e^{-0.055t}\cos(5t) + e^{-0.1t}\cos(20t))$. Fig. 5 depicts the signal $\tilde{u}(\cdot)$.

The parallel structure composed of two blocks of the proposed algorithm and the modified error $e = \tilde{u} - \dot{x}_1 - \dot{x}_2$ is fed to each block—refer to Remark 12. The simulation results are depicted in Fig. 6.

5. CONCLUSION

A continuous-time algorithm for online estimation of the frequency and damping factor of exponentially damped sinusoidal signals is proposed in this paper. In contrast

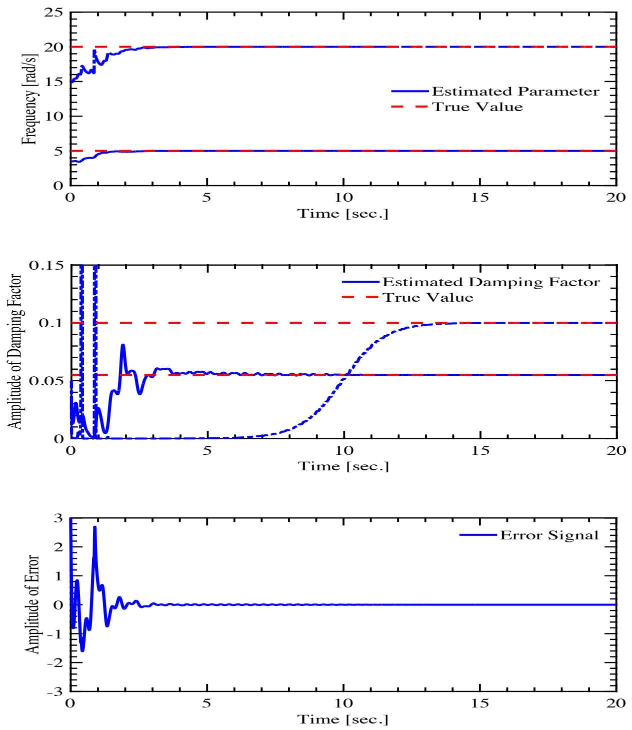

Fig. 6. The performance of parallel structure for extracting different components

with the existing algorithms, the proposed algorithm has a simple structure and directly estimates the parameters of the signal without any further computations. The proposed algorithm can easily be modified to estimate multiple modes in the presence of a dc bias. The main drawback of the method lies in its local (exponential) stability and convergence.

REFERENCES

Badeau, R., Boyer, R., and David, B. (2002). Eds parametric modeling and tracking of audio signals.

Bodson, M. and Douglas, S.C. (1997). Adaptive algorithms for the rejection of sinusoidal disturbances with unknown frequency. *Automatica*, 33(12), 2213–2221.

Chen, B., Li, P., Pin, G., Fedele, G., and Parisini, T. (2019). Finite-time estimation of multiple exponentially-damped sinusoidal signals: A kernel-based approach. *Automatica*, 106, 1–7.

Francis, B.A. and Wonham, W.M. (1976). The internal model principle of control theory. *Automatica*, 12(5), 457–465.

Hsu, L., Ortega, R., and Damm, G. (1999). A globally convergent frequency estimator. *IEEE Transactions on Automatic Control*, 44(4), 698–713.

Hua, Y. and Sarkar, T.K. (1990). Matrix pencil method for estimating parameters of exponentially damped/undamped sinusoids in noise. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, 38(5), 814–824. doi:10.1109/29.56027.

Kay, S.M. and Marple, S.L. (1981). Spectrum analysis—a modern perspective. *Proceedings of the IEEE*, 69(11), 1380–1419.

Khalil, H.K. (2002). *Nonlinear systems*. Prentice hall Upper Saddle River, NJ, 3 edition.

Kumaresan, R. and Tufts, D. (1982). Estimating the parameters of exponentially damped sinusoids and pole-zero modeling in noise. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, 30(6), 833–840. doi:10.1109/TASSP.1982.1163974.

Lu, J. and Brown, L. (2010). Internal model principle-based control of exponentially damped sinusoids. *International Journal of Adaptive Control and Signal Processing*, 24(3), 219–232.

Lu, J. and Brown, L.J. (2008). Identification of exponentially damped sinusoidal signals. *IFAC Proceedings Volumes*, 41(2), 5089–5094.

Marino, R. and Tomei, P. (2002). Global estimation of n unknown frequencies. *IEEE Transactions on Automatic Control*, 47(8), 1324–1328. doi:10.1109/TAC.2002.800761.

Mboup, M. (2009). Parameter estimation for signals described by differential equations. *Applicable Analysis*, 88(1), 29–52.

Mojiri, M., Karimi-Ghartemani, M., and Bakhshai, A. (2007). Time-domain signal analysis using adaptive notch filter. *IEEE Transactions on Signal Processing*, 55(1), 85–93. doi:10.1109/TSP.2006.885686.

Obregon-Pulido, G., Castillo-Toledo, B., and Loukianov, A. (2002). A globally convergent estimator for n-frequencies. *IEEE Transactions on Automatic Control*, 47(5), 857–863.

Osborne, M.R. and Smyth, G.K. (1995). A modified prony algorithm for exponential function fitting. *SIAM Journal on Scientific Computing*, 16(1), 119–138.

Pisarenko, V.F. (1973). The retrieval of harmonics from a covariance function. *Geophysical Journal International*, 33(3), 347–366.

Regalia, P.A. (1991). An improved lattice-based adaptive iir notch filter. *IEEE transactions on signal processing*, 39(9), 2124–2128.

Riedle, B. and Kokotovic, P. (1986). Integral manifolds of slow adaptation. *IEEE Transactions on Automatic Control*, 31(4), 316–324.

Roy, R. and Kailath, T. (1989). Esprit-estimation of signal parameters via rotational invariance techniques. *IEEE Transactions on acoustics, speech, and signal processing*, 37(7), 984–995.

Sastry, S. and Bodson, M. (1989). *Adaptive control: stability, convergence and robustness*. Englewood Cliffs, NJ: Prentice-Hall.

Schmidt, R. (1986). Multiple emitter location and signal parameter estimation. *IEEE Transactions on Antennas and Propagation*, 34(3), 276–280. doi:10.1109/TAP.1986.1143830.

Stoica, P. (1993). List of references on spectral line analysis. *Signal Processing*, 31(3), 329–340.

Van Overschee, P. and De Moor, B. (1994). N4sid: Subspace algorithms for the identification of combined deterministic-stochastic systems. *Automatica*, 30(1), 75–93.

Vediakova, A., Vedyakov, A., Bobtsov, A., and Pyrkin, A. (2020). Drem-based parametric estimation of bias-affected damped sinusoidal signals. In *2020 European Control Conference (ECC)*, 214–219. IEEE.

Xia, X. (2002). Global frequency estimation using adaptive identifiers. *IEEE Transactions on Automatic Control*, 47(7), 1188–1193. doi:10.1109/TAC.2002.800670.