

Robust Multi-Scenario Dynamic Real-Time Optimization with Embedded Closed-Loop Model Predictive Control

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Abstract: Economic optimization is a key tool in ensuring competitive chemical plant operation. Traditional steady-state real-time optimization (RTO) is suboptimal in many applications where the plant exhibits frequent transitions or slow dynamics, thus requiring the use of dynamic RTO (DRTO). Additionally, DRTO algorithms exhibit faster response when able to account for the behavior of the underlying model predictive control (MPC) systems. This work seeks to combine closed-loop (CL) prediction of the plant response under the action of MPC with a scenario based robust modeling approach to account for plant uncertainty. The CL prediction is handled by directly modeling the MPC calculations and reformulating the resulting multilevel optimization problem as a single-level mathematical program with complementarity constraints (MPCC). The proposed robust CL DRTO formulation is compared against a single-scenario nominal CL DRTO in terms of maximizing economic performance in a case study involving a nonlinear CSTR. The robust DRTO is shown to outperform the nominal DRTO in this metric on average across the scenarios tested.

Keywords: dynamic real-time optimization, model predictive control, economic optimization, scenario-based robust control, closed-loop prediction, uncertainty

1. INTRODUCTION

Chemical plants typically use a control hierarchy in order to separate tasks of differing scope and time scale. Real-time optimization (RTO) and model predictive control (MPC) are often included as two adjacent layers within the hierarchy. RTO traditionally computes optimal economic operating conditions based on a steady-state plant model (Darby et. al., 2011). This optimal operating point is then communicated to the MPC as the set-point target for the controller. The MPC typically uses a target-tracking objective with a simple dynamic model of the plant to drive the plant conditions to this target (Qin and Badgwell, 2003). By assuming steady-state conditions, the RTO is limited if the process undergoes frequent or slow transitions as it can not predict the dynamics of the plant. Therefore, dynamic RTO (DRTO) has been developed to mitigate this limitation.

Tosukhowong et. al. (2004) incorporate a reduced order dynamic model into an RTO framework and execute the resulting DRTO at a lower frequency than the underlying MPC to reduce computational requirements. Kadam et. al. (2002) use a two-layer DRTO-MPC architecture as well, but computational requirements are kept reasonable by employing a trigger mechanism where the DRTO layer executes only when certain disturbance estimation criteria are reached, rather than at a pre-determined frequency. Ellis and Christofides (2014) integrate the control and economic optimization problems by including control criteria in the DRTO layer (essentially converting it into an economic MPC). Swartz and Kawajiri (2019) provide a review of

dynamic optimization applications for operation and design problems.

The above DRTO strategies all assume an MPC at a lower level of control, but only optimize for the performance of the plant itself. However, the set-points provided to the MPC will not be reached instantaneously. The MPC will instead use its own optimization problem to determine the best path to achieve the set-points provided. Jamaludin and Swartz (2015) directly predict this MPC behavior in the DRTO problem, allowing the DRTO to provide set-point trajectories which are optimal for the entire MPC-plant system, rather than the plant alone. This closed-loop (CL) prediction was shown to improve performance over a corresponding open-loop (OL) DRTO which does not include consideration of the MPC behavior. This work was then extended through use of approximation methods (Jamaludin and Swartz, 2017a) to reduce the computational requirements of the CL DRTO. It was also applied to distributed systems (Jamaludin and Swartz, 2017b; Li and Swartz, 2019) where multiple MPC subsystems were coordinated with a single DRTO.

This paper seeks to extend the CL DRTO framework to problems with uncertain plant models. Plant uncertainty has been widely studied in the context of robust MPC. Bemporad and Morari (1999) provide an excellent review of the major robust MPC techniques. In particular, the method of multi-scenario prediction is of interest here. In this method, the uncertain behavior is handled as a discrete set of possible plant realizations. Each realization generates a scenario of potential plant behavior and by modeling each scenario, the MPC can predict the plant response across a

range of possible uncertainty realizations. Lucia et. al. (2012) use this method in a nonlinear MPC to generate a scenario tree, where each node of the tree branches according to the number of scenarios. This approach accommodates future input action but scales exponentially with the length of the branching horizon. Mastragostino et. al. (2014) use a similar technique for robust decision making of supply chains, where inclusion of approximate CL prediction is shown to improve performance. In the present work, robust multi-scenario prediction is utilized at the DRTO level, such that there are multiple scenario predictions within the DRTO, and incorporated into the CL DRTO method developed previously. This allows the DRTO to predict multiple uncertain plant realizations and predict the MPC behavior for each of these realizations. The DRTO determines the optimal set-point trajectory for the underlying plant MPC system based on the expected performance across multiple uncertain model scenarios.

The following sections present the generalized discrete time formulation of the robust CL DRTO problem. First, the overall strategy of the optimization problem is discussed, including the interactions between the DRTO and the CL MPC predictions. The primary DRTO problem is then presented, followed by the embedded MPC subproblems which make up the CL predictions of the DRTO. Next, the interactions between the MPC subproblems and DRTO are expressed in detail. A reformulation approach is then presented to transform the two optimization problems into a single-level problem. The performance of the resulting formulation is compared against a single-scenario nominal CL DRTO implementation on a CSTR case study. The plant model used has a single uncertain parameter and the DRTO models three scenarios: minimum, nominal, and maximum values of the uncertain parameter. The DRTO provides set-point trajectories to maximize the expected profit over three plant scenarios with different values of the uncertain parameter.

2. FORMULATION

2.1 General Purpose and Strategy

The central idea of the proposed robust DRTO formulation is to apply a multi-scenario stochastic programming approach to the CL DRTO strategy of Jamaludin and Swartz (2015). This allows the DRTO to predict the plant response under the action of MPC for different plant scenarios, and take this behavior into account when determining optimal set-point trajectories. This process is illustrated below in Figure 1.

2.2 Primary Dynamic Optimization Problem

The primary optimization problem described here in the general form includes an economic objective function, a dynamic model of the plant behaviour (with different parameter values for each scenario), and any process constraints necessary for plant operation, including path constraints on the outputs and set-points.

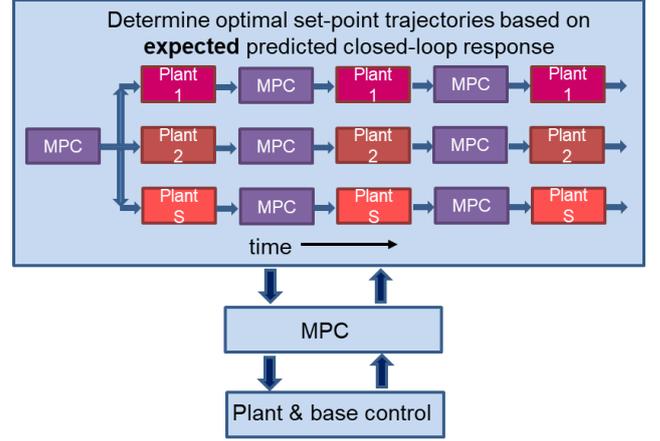


Fig. 1. Illustration of robust CL DRTO architecture.

Equations (2)-(6) are applied for all scenarios $i = 1, \dots, S$.

$$\max_{\mathbf{y}^{Ref}, \mathbf{u}^{Ref}} \phi = \sum_{i=1}^S w(i) * \sum_{j=0}^N Profit_j^i(\mathbf{u}_j^i, \mathbf{x}_j^i, \mathbf{y}_j^i) \quad (1)$$

$$\text{s.t. } \mathbf{x}_{j+1}^i = \mathbf{f}^i(\mathbf{x}_j^i, \mathbf{u}_j^i), \quad j = 0, \dots, N-1 \quad (2)$$

$$\mathbf{y}_j^i = \mathbf{h}^i(\mathbf{x}_j^i, \mathbf{u}_j^i) + \mathbf{d}_j^i, \quad j = 1, \dots, N \quad (3)$$

$$\mathbf{0} \leq \mathbf{g}^i(\mathbf{x}_j^i, \mathbf{u}_j^i), \quad j = 0, \dots, N \quad (4)$$

$$\mathbf{u}_j^i = \mathbf{f}^{MPC}(\tilde{\mathbf{y}}_j^{SP}, \tilde{\mathbf{u}}_j^{SP}, \mathbf{y}_j^i), \quad j = 0, \dots, N-1 \quad (5)$$

$$\mathbf{0} = \mathbf{h}^{Ref}(\mathbf{y}^{Ref}, \mathbf{u}^{Ref}, \tilde{\mathbf{y}}^{SP}, \tilde{\mathbf{u}}^{SP}) \quad (6)$$

$$\mathbf{y}_{min}^{Ref} \leq \mathbf{y}_j^{Ref} \leq \mathbf{y}_{max}^{Ref}, \quad j = 0, \dots, N-1 \quad (7)$$

$$\mathbf{u}_{min}^{Ref} \leq \mathbf{u}_j^{Ref} \leq \mathbf{u}_{max}^{Ref}, \quad j = 0, \dots, N-1 \quad (8)$$

In the above, i, j are the scenario and time index, respectively; S, N are the total number of scenarios and optimization horizon, respectively; ϕ is the objective function; $w(i)$ are the scenario weights; $\mathbf{u}_j^i, \mathbf{x}_j^i, \mathbf{y}_j^i$ are the vectors of inputs, states, and outputs, respectively; $\mathbf{f}^i, \mathbf{h}^i$ are the dynamic and algebraic equations for the plant model, respectively; \mathbf{g}^i are the inequality process constraints; \mathbf{f}^{MPC} are the equations defining the MPC subproblems; $\mathbf{y}^{Ref}, \mathbf{u}^{Ref}$ are the composite vectors of output and input reference trajectories, respectively; $\tilde{\mathbf{y}}_j^{SP}, \tilde{\mathbf{u}}_j^{SP}$ are the setpoint trajectories provided to the MPC subproblems; \sim denotes a variable specific to the MPC subproblems; \mathbf{h}^{Ref} are the equations connecting the reference trajectories with the setpoints; and $\mathbf{y}_{min}^{Ref}, \mathbf{y}_{max}^{Ref}, \mathbf{u}_{min}^{Ref}, \mathbf{u}_{max}^{Ref}, \mathbf{y}_{min}, \mathbf{y}_{max}$ are the lower and upper bounds on the reference trajectories and outputs, respectively.

The primary optimization problem predicts the future plant evolution under different realizations of the uncertain plant parameter in (2) and (3). It uses these future plant conditions to determine the profit obtained by the plant in each of these scenarios and uses a weighted sum of the scenario profits as the objective function in (1). Additional inequality constraints for the process are included in (4). The input moves for each scenario are determined by the MPC subproblems, expressed in (5). The MPC subproblems receive subsets of the overall reference trajectories, defined

in (6). Finally, the reference trajectories are constrained in (7) and (8). The optimization degrees of freedom for this problem are the single unified reference trajectories from which set-point trajectories are extracted and provided to the plant MPC.

2.3 Embedded MPC Subproblems

Embedded within the DRTO problem are MPC subproblems which predict the behavior of the MPC given the set-point trajectories determined by the primary problem. In this study we consider linear MPC. The MPC subproblems are executed for each scenario $i = 1, \dots, S$ and for each step of the optimization horizon $j = 0, \dots, N - 1$.

$$\min_{\tilde{\mathbf{y}}_{j,k}^i, \tilde{\mathbf{u}}_{j,k}^i} \tilde{\phi}_j^i = \left(\sum_{k=1}^p (\tilde{\mathbf{y}}_{j,k}^i - \tilde{\mathbf{y}}_{j,k}^{SP})^T Q (\tilde{\mathbf{y}}_{j,k}^i - \tilde{\mathbf{y}}_{j,k}^{SP}) + \sum_{k=0}^{m-1} (\Delta \tilde{\mathbf{u}}_{j,k}^i)^T R (\Delta \tilde{\mathbf{u}}_{j,k}^i) + \sum_{k=0}^{m-1} (\tilde{\mathbf{u}}_{j,k}^i - \tilde{\mathbf{u}}_{j,k}^{SP})^T S (\tilde{\mathbf{u}}_{j,k}^i - \tilde{\mathbf{u}}_{j,k}^{SP}) \right) \quad (9)$$

$$\text{s.t. } \tilde{\mathbf{x}}_{j,k+1}^i = A \tilde{\mathbf{x}}_{j,k}^i + B \tilde{\mathbf{u}}_{j,k}^i \quad k = 0, \dots, m - 1 \quad (10)$$

$$\tilde{\mathbf{x}}_{j,k+1}^i = A \tilde{\mathbf{x}}_{j,k}^i + B \tilde{\mathbf{u}}_{j,m-1}^i \quad k = m, \dots, p - 1 \quad (11)$$

$$\tilde{\mathbf{y}}_{j,k}^i = C \tilde{\mathbf{x}}_{j,k}^i + \tilde{\mathbf{d}}_{j,k}^i \quad k = 0, \dots, p \quad (12)$$

$$\Delta \tilde{\mathbf{u}}_{j,k}^i = \tilde{\mathbf{u}}_{j,k}^i - \tilde{\mathbf{u}}_{j,k-1}^i \quad k = 0, \dots, m - 1 \quad (13)$$

$$\mathbf{u}_{min} \leq \tilde{\mathbf{u}}_{j,k}^i \leq \mathbf{u}_{max} \quad k = 0, \dots, m - 1 \quad (14)$$

In the above, p, m are the prediction and control horizons, respectively; Q, R, S are the output deviation, input move penalty, and input deviation weighting matrices, respectively; A, B, C are the state-space model matrices; $\tilde{\mathbf{d}}_{j,k}^i$ is the disturbance estimate; and $\mathbf{u}_{min}, \mathbf{u}_{max}$ are the lower and upper input bounds, respectively.

The embedded MPC subproblems assume a standard QDMC algorithm (Garcia and Morshedi, 1986). The objective function, (9), is a target tracking objective with an input move penalty. The process model uses a linear state-space model in (10)-(12) with control horizon m and prediction horizon p . The inputs are constrained in (14) and the input move change defined in (13).

2.4 Initialization, Update, and Feedback

The primary optimization problem and the embedded MPC subproblems interact by the primary problem providing initial conditions and a disturbance estimate to the MPC, and the MPC providing input moves to the primary problem. These interactions occur for every scenario $i = 1, \dots, S$ and at every time step j along the DRTO prediction horizon.

Equation (6) shows that the set-points used at the MPC level are determined from the reference trajectories calculated in the DRTO problem. Specifically, the set-points are a subset of the reference trajectories, as defined in (15) and (16).

$$\tilde{\mathbf{y}}_{j,k}^{SP} = \mathbf{y}_{j+k}^{Ref} - \mathbf{y}_{ss} \quad (15)$$

$$\tilde{\mathbf{u}}_{j,k}^{SP} = \mathbf{u}_{j+k}^{Ref} - \mathbf{u}_{ss} \quad (16)$$

Where $\mathbf{y}_{ss}, \mathbf{u}_{ss}$ are the steady state values of the outputs and inputs, respectively. The steady state values are subtracted because the MPC variables are in deviation form, as the MPC uses a linear state-space model, while the DRTO variables are not.

The disturbance estimate for QDMC is normally computed as the difference between the measured and predicted outputs. Here, the MPC subproblems instead use the DRTO primary problem output prediction as a proxy for a measured output, shown in (17). The disturbance estimate is assumed constant for the length of each MPC subproblem in (18).

$$\tilde{\mathbf{d}}_{j,0}^i = (\mathbf{y}_j^i - \mathbf{y}_{ss}) - C \tilde{\mathbf{x}}_{j,0}^i \quad j = 0, \dots, N - 1 \quad (17)$$

$$\tilde{\mathbf{d}}_{j,k}^i = \tilde{\mathbf{d}}_{j,0}^i \quad k = 1, \dots, p \quad j = 0, \dots, N - 1 \quad (18)$$

The initial states for the first MPC subproblem are the MPC predicted states at the first time step in the previous DRTO execution ($\tilde{\mathbf{x}}_{1,0,prev}^i$), shown in (19), and for each subsequent MPC subproblem the initial states are the predicted states in the last MPC execution, (20). Similarly, the previous MPC input (required for the change in input computation) for the first MPC is the last implemented input move for the actual plant, (21), and is the last MPC computed optimal input move for all subsequent MPC subproblems (22).

$$\tilde{\mathbf{x}}_{0,0}^i = \tilde{\mathbf{x}}_{1,0,prev}^i \quad (19)$$

$$\tilde{\mathbf{x}}_{j,0}^i = \tilde{\mathbf{x}}_{j-1,1}^i \quad j = 1, \dots, N - 1 \quad (20)$$

$$\tilde{\mathbf{u}}_{0,-1}^i = \mathbf{u}_{-1} - \mathbf{u}_{ss} \quad (21)$$

$$\tilde{\mathbf{u}}_{j,-1}^i = \tilde{\mathbf{u}}_{j-1,0}^i \quad j = 1, \dots, N - 1 \quad (22)$$

In (5), the DRTO inputs are determined by the MPC subproblems. The DRTO uses the first computed MPC input move of each MPC subproblem as the input value for that scenario and DRTO time step, as shown in (23).

$$\mathbf{u}_j^i = \tilde{\mathbf{u}}_{j,0}^i + \mathbf{u}_{ss} \quad j = 0, \dots, N - 1 \quad (23)$$

2.5 Solution Strategy

The primary DRTO optimization problem and the MPC subproblems make up a single multilevel optimization problem, where the objective function is (1) and (2)-(8) are the DRTO constraints. Equations (9)-(14) make up the inner MPC subproblems and are included as constraints in the primary DRTO problem. The two optimization problems are related by further constraints in (15)-(23), such that the solution of the primary problem depends on the solutions of the MPC subproblems.

This multilevel programming problem could be solved via several methods. In this paper, the simultaneous approach presented in Jamaludin and Swartz (2015) is used, where the two problems are solved simultaneously. This is accomplished by reformulating the embedded MPC subproblems as their first order Karush-Kuhn-Tucker (KKT) conditions, as shown in Baker and Swartz (2008). These conditions are expressed as algebraic equations and

are then included in the primary DRTO problem as constraints. Since the MPC subproblems are each a convex QP, the first order KKT conditions are necessary and sufficient for optimality.

The reformulation produces a mathematical program with complementarity constraints (MPCC). The complementarity constraints are handled using an exact penalty approach, as proposed in Ralph and Wright (2004). The product of the primal and dual KKT variables are summed and multiplied by an appropriate weighting factor and then included in the objective function. With a sufficiently large weight, the solution to the original MPCC is obtained. The resulting problem may then be solved by a standard NLP solver.

3. CASE STUDY

3.1 Single Reaction CSTR

The case study investigated is a nonlinear CSTR with one inlet stream, one outlet stream, and one reaction occurring with one reactant and one product. The reaction kinetics are described by a Michaelis-Menten equation and the differential equations for the concentration of reactant and product are shown in (24), (25) (Gao, 2012),

$$\frac{dC}{dt} = D * (C_{in} - C) - \frac{V_m * C}{K_s + C} \quad (24)$$

$$\frac{dP}{dt} = \frac{V_m * C}{K_s + C} - D * C \quad (25)$$

where C, P are the concentrations of reactant and product, respectively, C_{in} is the inlet concentration of reactant, D is the ratio of flow rate to reactor volume, V_m is the maximum reaction rate, K_s is the reaction constant, and t is time.

3.2 Translation to Optimization Formulation

For application to the DRTO formulation, the inlet concentration of reactant is used as the input ($u = C_{in}$), the concentration of reactant and product are the states ($x_1 = C$, $x_2 = P$), and the concentration of product the output ($y = P$). The differential equations are solved numerically in the DRTO using the backward Euler method resulting in the following dynamic and algebraic equations.

$$\begin{bmatrix} x_{1,j+1}^i \\ x_{2,j+1}^i \end{bmatrix} = \begin{bmatrix} x_{1,j}^i + \left(D * (u_j^i - x_{1,j+1}^i) - \frac{V_m * x_{1,j+1}^i}{K_s + x_{1,j+1}^i} \right) * \Delta t \\ x_{2,j}^i + \left(\frac{V_m * x_{1,j+1}^i}{K_s + x_{1,j+1}^i} - D * x_{2,j+1}^i \right) * \Delta t \end{bmatrix} \quad (26)$$

$$y_j^i = x_{2,j}^i + d_j^i \quad (27)$$

The algebraic equation, (27), uses the second state as the output and includes a disturbance estimate. This disturbance estimate is the scenario prediction (using the backward Euler equations) minus the last measured plant output value and is assumed constant for the duration of the optimization horizon.

The backward Euler equations above make up the nonlinear discrete time model used in the primary DRTO problem.

For the MPC subproblems, the linearized model of the same process found in Gao (2012) is discretized using a zero-order hold and then converted to state-space form.

The value of the maximum reaction rate, V_m , is treated as the uncertain parameter with a deviation of $\pm 20\%$. The DRTO uses three scenarios, with the minimum, nominal, and maximum values of V_m , respectively. The scenario weights are determined to reflect a normal distribution with minimum and maximum parameter values assumed to be two standard deviations from the mean.

The objective for this case study is profit maximization, where the reactor accumulates revenue only when the product concentration is within a specified quality band, and an input cost is subtracted. The discrete nature of the quality band is approximated with a product of hyperbolic tangent functions.

$$\phi = \sum_{i=1}^M w(i) * \sum_{j=0}^N 10 * R_{1,j}^i * R_{2,j}^i - u_j^i \quad (28)$$

$$R_{1,j}^i = \frac{1}{2} \tanh \left(\gamma (y_j^i - (1 - \delta) y_j^{tar}) \right) + \frac{1}{2} \quad (29)$$

$$R_{2,j}^i = \frac{1}{2} \tanh \left(\gamma \left((1 + \delta) y_j^{tar} - y_j^i \right) \right) + \frac{1}{2} \quad (30)$$

This product above approximates $R_1 * R_2 \approx 1$ if $(1 - \delta) y_j^{tar} \leq y \leq (1 + \delta) y_j^{tar}$.

Additionally, the set-point trajectories are constrained such that the set-point can only change value every four time steps, rather than every time step. This serves to smooth the transition and slightly decreases the size of the optimization problem.

The DRTO and MPC parameters are listed in Table 1.

Table 1. Relevant optimization, MPC, and case study parameter values

Parameter	Description	Value	Units
N	Optimization horizon	40	-
Δt_{DRTO}	DRTO time step	2	h
p	MPC prediction horizon	10	-
m	MPC control horizon	2	-
Δt_{MPC}	MPC time step	1	h
Q	Output deviation weight	10	-
R	Input move penalty weight	1	-
S	Input deviation weight	0	-
y_{min}^{SP}	Output set-point lower bound	0	g/L
y_{max}^{SP}	Output set-point upper bound	5.724	g/L
u_{min}	Input lower bound	0	g/L

u_{max}	Input upper bound	10	g/L
γ	Hyp tan weight	100	-
D	Ratio of flow rate to volume	0.4	h^{-1}
V_m	Maximum reaction rate	0.5	g/L·h
K_s	Reaction constant	0.2	g/L

3.3 Subcase 1: Small Transition with Constrained Output

The first subcase investigated involves a relatively small transition of 0.1 g/L for the product concentration, with a tight quality band. The initial conditions for the problem are $u_0 = 1$ g/L, $x_{1,0} = 0.276$ g/L, and $x_{2,0} = 0.724$ g/L. The value of y^{tar} is 0.826 g/L, with $\delta = 0.01$.

For this subcase, an additional one-sided linear economic penalty is added to the objective function for exceeding a product concentration of 0.84 g/L. This is included to represent an additional cost in the process required to correct a product concentration substantially higher than intended. The weight on this penalty term is -100 .

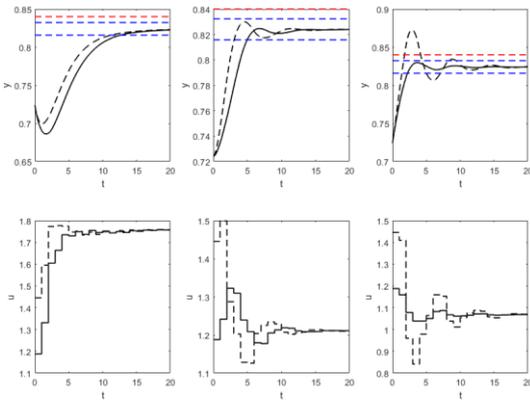


Fig. 2. Input and output trajectories for subcase 1.

In Figure 2 (and Figure 3 below), the minimum, $V_m = 0.4$ (left), nominal, $V_m = 0.5$ (centre), and maximum, $V_m = 0.6$ (right) uncertain parameter value scenarios are shown. The solid line is the robust DRTO trajectory, the black dashed line is the nominal CL DRTO trajectory, the blue dashed lines are the bounds of the profit band, and the red dashed line is the upper bound that triggers the additional economic penalty.

Table 2. Actual profit values for the nominal and robust DRTO for the first subcase

Scenario	Nominal	Robust	Percent Improvement
Minimum	51.48	38.70	-24.8
Nominal	143.6	124.7	-13.2
Maximum	98.08	155.4	58.4
Average	97.72	106.3	8.78

Table 2, above, shows that, in the first subcase, the robust DRTO displays improved performance in terms of economics on average over the nominal DRTO. Additionally, the nominal DRTO exceeds the economic penalty upper bound trigger in the maximum scenario while the robust DRTO does not. This is accomplished by exhibiting less aggressive performance in all three scenarios, as can be seen in Figure 2, resulting in reduced economic performance in the minimum and nominal scenarios because the output reaches the profit band later than in the nominal DRTO case. However, this less aggressive performance reduces overshoot in the maximum scenario, improving economic performance largely by avoiding exceeding the upper bound trigger of the economic penalty. For this subcase, the average solution time for the robust DRTO was 4.3 seconds, with a maximum time of 15.7s, compared to the average nominal DRTO time of 0.4s, with a maximum of 1.3s. These solutions were obtained using a 3.2GHz INTEL CORE i7-8700 processor with 8GB RAM running Windows 10.

3.4 Subcase 2: Large Transition with Unconstrained Output

The second subcase investigated involves a larger transition of 0.276 g/L for the product concentration, with a looser profit band and no upper bound economic penalty term. The simulation time is also increased to 40 h to accommodate the larger transition magnitude. The initial conditions for the problem are the same as for subcase 1. The value of y^{tar} is 1 g/L, with $\delta = 0.05$.

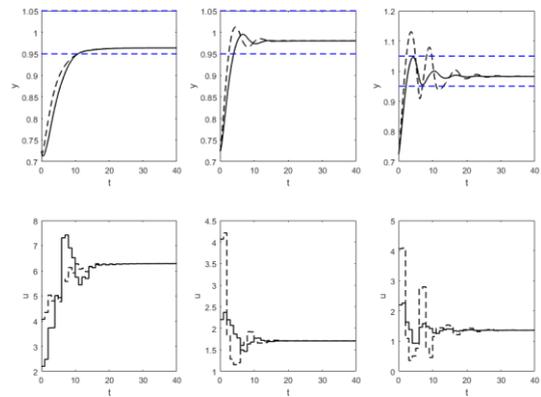


Fig. 3. Input and output trajectories for subcase 2.

Table 3. Actual profit values for the nominal and robust DRTO for the second subcase

Scenario	Nominal	Robust	Percent Improvement
Minimum	48.03	50.45	5.04
Nominal	301.1	288.0	-4.35
Maximum	251.0	321.0	27.9
Average	200.0	219.8	9.90

The second subcase shows similar performance of the robust DRTO as the first subcase, as seen in the economic data in Table 3. The robust DRTO is less aggressive in all three scenarios, seen in Figure 3, leading to only slight improvement in performance in the minimum scenario and a reduction in performance in the nominal scenario. In the maximum scenario, the robust DRTO is able to maintain the product concentration in the profit band for more time than the nominal DRTO because the nominal DRTO is overshooting the band, thus improving on the economic performance. Additionally, the robust DRTO decreases the settling time required by the nominal DRTO in the maximum scenario. On average, the robust DRTO again improves on the single scenario DRTO. For this subcase, the average solution time for the robust DRTO was 23.9 seconds, with a maximum of 68.9s, compared to the average nominal DRTO time of 2.6s, with a maximum of 8.5s, using the same system as before.

4. CONCLUSIONS

In this work, a robust CL DRTO algorithm is presented and tested. The formulation models the dynamics of the system in real-time while predicting the future MPC action in response to set-point trajectories provided by the DRTO. It does this for multiple uncertain plant models to predict a range of possible plant behavior. Within each of these scenarios, it performs the CL MPC prediction and is thus able to determine the optimal set-point trajectory for the MPC-plant system across multiple scenarios.

The robust CL DRTO formulation is tested against a similar algorithm which also exhibits CL prediction of future MPC responses but does not consider multiple uncertain plant scenarios. In a case study involving a nonlinear CSTR, the robust CL DRTO is shown to outperform the nominal CL DRTO in terms of expected profit generation across three scenarios of possible plant behaviour. It accomplishes this by generally sacrificing performance in one or two of the three scenarios which then allows for improved performance in the third scenario by a larger margin, thus improving performance overall.

In future work, the robust CL DRTO formulation will be tested on more complex case studies involving multiple inputs and outputs. The effect of number of modelled scenarios will also be investigated, as well as the performance of the formulation in the event of multiple uncertain parameters. The formulation will also be adjusted to accommodate approximation methods which will seek to reduce computation time with minimal loss of performance.

REFERENCES

Baker, R. and Swartz, C.L.E. (2008). Interior point solution of multilevel quadratic programming problems in constrained model predictive control applications. *Ind. Eng. Chem. Res.*, 47(1), 81-91.

Bemporad, A. and Morari, M. (1999). Robust model predictive control: A survey. In *Robustness in Identification and Control* (207-226). Springer, London.

Darby, M.L., Nikolaou, M., Jones, J. and Nicholson, D. (2011). RTO: An overview and assessment of current practice. *J. Process Control*, 21(6), 874-884.

Ellis, M. and Christofides, P.D. (2014). Integrating dynamic economic optimization and model predictive control for optimal operation of nonlinear process systems. *Control Engineering Practice*, 22, 242-251.

Garcia, C.E., Morshedi, A.M. (1986). Quadratic programming solution of dynamic matrix control (QDMC). *Chem. Eng. Commun.*, 46, 73-87.

Gao, L. (2012, August). Modeling and dynamics analyses of immobilized CSTR bioreactor using transfer function model. In *2012 International Symposium on Information Technologies in Medicine and Education* (Vol. 2, 692-695). IEEE.

Jamaludin, M.Z. and Swartz, C.L.E. (2015). A bilevel programming formulation for dynamic real-time optimization. *IFAC-PapersOnLine*, 48(8), 906-911.

Jamaludin, M.Z. and Swartz, C.L.E. (2017a). Approximation of closed-loop prediction for dynamic real-time optimization calculations. *Comp. Chem. Eng.*, 103, 23-38.

Jamaludin, M.Z. and Swartz, C.L.E. (2017b). Coordination of Distributed MPC Systems via Dynamic Real-time Optimization. *IFAC-PapersOnLine*, 50(1), 6184-6189.

Kadam, J.V., Schlegel, M., Marquardt, W., Tousain, R.L., Van Hessem, D.H., van den Berg, J. and Bosgra, O.H. (2002). A two-level strategy of integrated dynamic optimization and control of industrial processes—a case study. In *Computer Aided Chemical Engineering* (Vol. 10, 511-516). Elsevier.

Li, H. and Swartz, C.L.E. (2019). Dynamic real-time optimization of distributed MPC systems using rigorous closed-loop prediction. *Comp. Chem. Eng.*, 122, 356-371.

Lucia, S., Finkler, T., Basak, D. and Engell, S. (2012). A new robust NMPC scheme and its application to a semi-batch reactor example. *IFAC Proceedings Volumes*, 45(15), 69-74.

Mastragostino, R., Patel, S. and Swartz, C.L.E. (2014). Robust decision making for hybrid process supply chain systems via model predictive control. *Comp. Chem. Eng.*, 62, 37-55.

Qin, S.J. and Badgwell, T.A. (2003). A survey of industrial model predictive control technology. *Control Engineering Practice*, 11(7), 733-764.

Ralph, D., Wright, S.J. (2004). Some properties of regularization and penalization schemes for MPECs. *Optimization Methods and Software*, 19, 527-556.

Swartz, C.L.E. and Kawajiri, Y. (2019). Design for dynamic operation-A review and new perspectives for an increasingly dynamic plant operating environment. *Comp. Chem. Eng.*, 128, 329-339.

Tosukhowong, T., Lee, J.M., Lee, J.H. and Lu, J. (2004). An introduction to a dynamic plant-wide optimization strategy for an integrated plant. *Comp. Chem. Eng.*, 29(1), 199-208.