

Adaptive Tube-Enhanced Multi-Stage Nonlinear Model Predictive Control^{*}

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Abstract: A robust adaptive controller for nonlinear plants with parametric uncertainties, additive disturbances, and state estimation errors based on the tube-enhanced multi-stage (TEMS) nonlinear model predictive (NMPC) framework is proposed. In TEMS NMPC, primary multi-stage NMPC is used to achieve robustness against the uncertainties which have a large effect on the evolution of the state of the plant, and ancillary multi-stage NMPC is used to track the predictions of the primary controller to counteract the effect of the small uncertainties. We propose updating, at each time step, the uncertainty set considered by the scenario trees of the primary and ancillary controllers with a tighter non-falsified uncertainty set, which results from solving a guaranteed parameter estimation (GPE) optimization problem. This produces significant performance improvements over the non-adaptive implementation as will be shown on the Williams-Otto continuous stirred tank reactor (CSTR) case study.

Keywords: Adaptive control, Nonlinear model predictive control, Robust NMPC, Multi-stage NMPC, Process control.

1. INTRODUCTION

Model predictive control (MPC) has become the most used advanced process control strategy, especially in the process control community because of its ability to handle large multi-variable systems and to respect state and input constraints in a non-conservative manner. Moreover, the process performance can be optimized directly (rather than indirectly via tracking of set points) within its framework (Engell, 2007). However, since it is a model-based strategy, the accuracy of the model is crucial for the stability and satisfactory performance of the controlled system. A plethora of research has been dedicated to the study of the nominal robustness of MPC (using nominal MPC despite the presence of uncertainties) such as in (Limon Marruedo et al., 2002; Picasso et al., 2010). However, when the uncertainties in the model are not sufficiently small, robust MPC formulations must be employed, such that robust constraint satisfaction and acceptable performance are achieved despite the uncertainties.

The first robust MPC algorithm is the open-loop min-max MPC (Campo and Morari, 1987; Zheng and Morari, 1993), where a sequence of control inputs that minimizes the worst case cost that can result from all the possible realization sequences of the uncertainty while satisfying the state and input constraints is computed. The deficiency of the open-loop min-max MPC formulation is its excessive conservatism which is a direct consequence of ignoring the feedback information in the optimization problem formulation. Motivated by the need to reduce the conservatism, closed-loop min-max MPC formulations were developed

(Lee and Yu, 1997; Scokaert and Mayne, 1998), where the feedback information is considered explicitly in the formulation of the optimization problem, and as a result, a sequence of control policies is computed rather than a sequence of control inputs, which can considerably reduce the conservatism of the approach. However, finding a solution for the closed-loop min-max NMPC optimization problem is very difficult, if at all possible.

Attempting to simplify the receding horizon nonlinear optimal control problem, tube-based NMPC algorithms were developed (Mayne et al., 2011; Yu et al., 2013; Falugi and Mayne, 2014; Villanueva et al., 2017). In (Mayne et al., 2011; Falugi and Mayne, 2014) a nominal primary NMPC with tightened state and input constraints is used to track the required target and an ancillary NMPC is used to track the predictions of the primary controller in order to provide robustness against the plant uncertainties. While this is intuitive and computationally appealing, if the uncertainties in the model have a large effect on the state evolution of the plant, the required constraint tightening for this approach can be very conservative as was shown for case studies in (Subramanian et al., 2018; Abdelsalam et al., 2020b).

Multi-stage MPC, first proposed for the linear case in (Muñoz de la Peña et al., 2005) and for the nonlinear case in (Dadhe and Engell, 2008; Lucia and Engell, 2012), is a less conservative approach to the robust MPC problem, where the model uncertainty is represented by a scenario tree of future evolution, and the availability of feedback information is considered explicitly in the formulation of the optimization problem, where also a tree of future control moves is computed. The objective function of

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the optimization problem is to minimize the weighted average of the costs of all considered scenarios. Stabilizing formulations for multi-stage NMPC were proposed in (Lucia et al., 2020; Abdelsalam et al., 2020a). However, as the number of considered uncertainties and the prediction horizon increase, the size of the optimization problem grows quickly and may become unmanageable.

To mitigate the rapid growth of the problem size with the number of considered uncertainties, tube-enhanced multi-stage (TEMS) NMPC was proposed in (Subramanian et al., 2018). In TEMS NMPC, a primary multi-stage NMPC with tightened state and input constraints is employed to handle the significant uncertainties, and an ancillary multi-stage controller tracks the predictions of the primary controller in order to robustify the scheme against the small uncertainties. To further alleviate the computational burden of the approach, simplified tube-enhanced multi-stage (STEMS) NMPC was proposed in (Abdelsalam et al., 2020b), where standard NMPC rather than multi-stage NMPC was employed as the ancillary controller which in an adaptive manner tracks one of the scenarios of the primary multi-stage NMPC. However, for both TEMS NMPC and STEMS NMPC, the measurement knowledge was not used to update the uncertainties that are considered in the multi-stage scenario tree(s).

In general, the conservatism of robust MPC algorithms stems from the ignorance of the controller about the true system description. Hence, a natural extension is to refine the description of the model uncertainty by exploiting the measurement information and the available model structure, which leads to robust adaptive MPC. In (Adetola et al., 2009; Canale et al., 2013; Gonçalves and Guay, 2016) adaptive NMPC algorithms based on the min-max NMPC framework for which finding a solution is generally prohibitive, or adaptive algorithms that use the Lipschitz bounds NMPC approach (Limon et al., 2005) which can be very conservative during the uncertainty set update phase were proposed.

In this paper, we propose a new robust adaptive NMPC scheme for plants with parametric uncertainties, i.e. constant but unknown model parameters, additive disturbances and state estimation errors based on the TEMS NMPC framework. The idea is to augment the TEMS NMPC scheme with a set membership estimation approach to shrink the uncertainty set that is considered by the multi-stage primary and ancillary controllers. We adopt a guaranteed parameter estimation (GPE) approach similar to the one in (Gottu Mukkula and Paulen, 2016) which results in a hyperrectangle over-approximation of the set of uncertain parameters. The scenario trees of the primary and ancillary controllers are updated at each time step according to the uncertainty set obtained from solving the GPE optimization. We demonstrate the significant performance benefits that can be achieved by the proposed adaptive TEMS scheme on the Williams-Otto continuous stirred tank reactor (CSTR) example.

2. SYSTEM DESCRIPTION

We consider a discrete time system described by

$$x_{t+1} = f(x_t, u_t, d_t) + w_t, \quad (1)$$

$$y_t = h(x_t) + \delta_t, \quad (2)$$

where $x_t \in \mathbb{R}^{n_x}$ is the plant state, $u_t \in \mathbb{R}^{n_u}$ is the control input, $d_t \in \mathbb{D} \subset \mathbb{R}^{n_d}$ is the vector of parametric uncertainties and $w_t \in \mathbb{W} \subset \mathbb{R}^{n_x}$ is a vector of additive disturbances at time step t . The successor plant state is denoted by x_{t+1} and the nonlinear plant dynamics is described by $f : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_d} \rightarrow \mathbb{R}^{n_x}$. The plant output is denoted by $y_t \in \mathbb{R}^{n_y}$ which is determined by (2), where $h : \mathbb{R}^{n_x} \rightarrow \mathbb{R}^{n_y}$ is a (possibly) nonlinear mapping and $\delta_t \in \mathbb{\Delta} \subset \mathbb{R}^{n_y}$ is the measurement noise vector. The plant state and input are constrained to lie in the compact sets \mathbb{X} and \mathbb{U} respectively. The sets \mathbb{W} and $\mathbb{\Delta}$ are assumed to be compact and to contain the origin. The set \mathbb{D} is assumed to be a compact hyperrectangle.

Due to the unavailability of the full state information, a state estimator will be employed to reconstruct the plant state from the plant model and the output information. We denote the state estimation error by $e_t = x_t - \hat{x}_t \in \mathbb{E}_t$, where the sets \mathbb{E}_t are assumed to be compact sets that contain the origin $\forall t \geq 0$. We assume the knowledge of the initial state estimation error bound \mathbb{E}_0 .

3. THE PROPOSED ADAPTIVE TEMS NMPC

The philosophy of TEMS NMPC (Subramanian et al., 2018) is to classify the uncertainties into large and small uncertainties. By large uncertainties, we mean the uncertainties which can result in significant changes in the evolution of the plant state from the nominal evolution, and by small uncertainties, we mean the uncertainties which have a small effect on the evolution of the plant state. It will be assumed hereafter that the parametric uncertainties d_t are the large uncertainties, while the additive disturbances w_t and the state estimation errors e_t are the small uncertainties.

We assume that the parameter vector d_t is unknown but constant over time:

$$d_t = d = [\tilde{d}^1, \tilde{d}^2, \dots, \tilde{d}^{n_d}]^T. \quad (3)$$

The proposed adaptation strategy will involve only the large uncertainties d_t . We assume that initially (at $t = 0$), the unknown parameters lie in the compact hyperrectangular set \mathbb{D} . It is desired to utilize the available measurements and the available model to find at each time step a smaller compact hyperrectangle \mathbb{D}_t which is guaranteed to contain the true value of the vector of parametric uncertainties d_t , and to update the uncertainty sets that are considered by the primary and ancillary controllers accordingly.

The proposed adaptive TEMS NMPC consists of three main elements, which are the primary controller, the ancillary controller and the adaptation strategy, which are explained in the following three subsections.

3.1 Primary controller

In TEMS NMPC, a primary multi-stage NMPC is used to handle the parametric uncertainties. A scenario tree is generated using a discrete uncertainty set \mathbb{D}^d sampled from the set \mathbb{D} . The set $\mathbb{D}^d := \{d^1, d^2, \dots, d^s\}$ contains a finite number s of vectors. Although for nonlinear systems, the parameters that give the worst case scenarios can be found anywhere in the set \mathbb{D} , however, as noted in (Srinivasan et al., 2003), it is likely that they lie on the boundaries

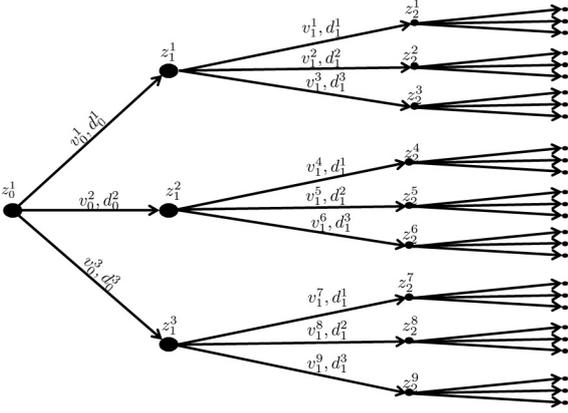


Fig. 1. Scenario tree representation for multi-stage NMPC.

of the parameter set. Therefore, the set \mathbb{D}^d should at least contain the extreme realizations of the uncertainty, i.e. the vertices of the set \mathbb{D} . The evolution of the state of the primary system is described by

$$z_{t+1} = f(z_t, v_t, \bar{d}_t), \quad (4)$$

where $z_t \in \mathbb{Z} \subseteq \mathbb{X}$ is the primary system state, $v_t \in \mathbb{V} \subseteq \mathbb{U}$ is the primary system input, $\bar{d}_t \in \mathbb{D}^d$ is the uncertainty conceived by the primary system. The sets \mathbb{Z} and \mathbb{V} are the tightened state and input constraints. As shown in Figure 1, the branches of the tree model the different realizations of the uncertainty, which together with the scenario-dependent inputs of the primary system, give rise to different evolution of the primary system state for each scenario. The reaction to the feedback information is modeled explicitly, because at each stage (prediction step) in the scenario tree, different inputs are computed for the different predicted states (tree nodes). However, as the same information is used, the inputs generated at each tree node must be the same, which is called the non-anticipativity constraint. To reduce the rapid growth of the problem size with the prediction horizon, the branching in the scenario tree can be limited to a certain prediction step called the robust horizon N_R (Lucia and Engell, 2012), and the uncertainty is assumed to be constant afterwards. The optimization problem of the primary controller is formulated as follows:

$$\min_{v_k^j \forall (j,k) \in I_{N-1}} V_N^p(z_t) \quad (5a)$$

subject to

$$z_{k+1}^j = f(z_k^{p(j)}, v_k^j, d_k^{r(j)}), \quad \forall (j, k+1) \in I, \quad (5b)$$

$$z_{k+1}^j \in \mathbb{Z}, \quad \forall (j, k+1) \in I, \quad (5c)$$

$$v_k^j \in \mathbb{V}, \quad \forall (j, k) \in I_{N-1}, \quad (5d)$$

$$v_k^j = v_k^l, \text{ if } z_k^{p(j)} = z_k^{p(l)}, \quad \forall (j, k), (l, k) \in I_{N-1}, \quad (5e)$$

$$V_N^p(z_t) = \sum_{i=1}^{N_s} \omega_i^p \tilde{V}_i^p, \quad (6)$$

where with some abuse of notation, z_t denotes the current primary system state (at time step t) and z_k^j denotes a predicted primary system state at the future time step $t+k$. The set I contains all the indices of the scenario-tree, N is the prediction horizon, N_s denotes the number of scenarios (which depends on the considered discrete realizations of the uncertainties and the robust horizon), the set I_{N-1} denotes the set of occurring indices until

prediction step $N-1$, ω_i^p is the respective scenario weight for the primary controller, z_{k+1}^j is the predicted primary system state which is determined by (5b) and depends on its parent state $z_k^{p(j)}$, the primary system input v_k^j and the considered realization of the uncertainty $d_k^{r(j)} \in \mathbb{D}^d$. The primary system states and inputs are constrained according to (5c) and (5d). The non-anticipativity constraints are enforced by (5e). The scenario cost is given by

$$\tilde{V}_i^p = \sum_{k=0}^{N-1} \ell^p(z_{k+1}^j, v_k^j), \quad (7)$$

$\forall z_{k+1}^j, v_k^j$ in scenario i , where ℓ^p is the stage cost of the primary controller. The optimal predicted inputs and states are denoted by $v_k^{*j}, \forall (j, k) \in I_{N-1}$ and $z_k^{*j}, \forall (j, k) \in I$. At time step t , let the set $Z_r(t) := \{z_1^{*1}, z_1^{*2}, \dots, z_1^{*s}\}$ denote the optimal states predicted at the first stage by the scenario tree. The scenario tree of the primary controller is initialized at $t=0$ with the initial state estimate \hat{x}_0 , and is initialized $\forall t \geq 1$ with the state in $Z_r(t-1)$ which has the minimum Euclidean distance to \hat{x}_t . As a result, $\bar{d}_t \in \mathbb{D}^d$ in (4), $\forall t \geq 0$. Note that the actual vector of parameters $d_t \in \mathbb{D}$ might be different from $\bar{d}_t \in \mathbb{D}^d$.

3.2 Ancillary Controller

The task of the ancillary controller is to counteract the effect of the small uncertainties which are assumed to be the additive disturbances and the state estimation errors. This is accomplished by employing multi-stage NMPC with the same tree structure as for the primary controller whose objective is to track the optimal state and input predictions of the primary controller for all the considered scenarios. The ancillary controller is initialized at each time step with the state estimate \hat{x}_t . The optimization problem of the ancillary controller is formulated as follows:

$$\min_{u_k^j \forall (j,k) \in I_{N-1}} V_N^a(\hat{x}_t) \quad (8a)$$

subject to

$$\hat{x}_{k+1}^j = f(\hat{x}_k^{p(j)}, u_k^j, d_k^{r(j)}), \quad \forall (j, k+1) \in I, \quad (8b)$$

$$u_k^j \in \mathbb{U}, \quad \forall (j, k) \in I_{N-1}, \quad (8c)$$

$$u_k^j = u_k^l, \text{ if } \hat{x}_k^{p(j)} = \hat{x}_k^{p(l)}, \quad \forall (j, k), (l, k) \in I_{N-1}, \quad (8d)$$

$$V_N^a(\hat{x}_t) = \sum_{i=1}^{N_s} \omega_i^a \tilde{V}_i^a, \quad (9)$$

where ω_i^a is the respective scenario weight for the ancillary controller, \hat{x}_{k+1}^j is the predicted plant state which is determined by (8b) and depends on its parent state $\hat{x}_k^{p(j)}$, the input u_k^j and the realization of the uncertainty $d_k^{r(j)}$. The predicted inputs are constrained by (8c), and (8d) is the non-anticipativity constraints for the ancillary controller. The scenario cost for each of the scenarios of the ancillary controller is

$$\tilde{V}_i^a = \sum_{k=0}^{N-1} \ell^a(\hat{x}_{k+1}^j - z_{k+1}^{*j}, u_k^j - v_k^{*j}), \quad (10)$$

$\forall \hat{x}_{k+1}^j, u_k^j$ in scenario i , where ℓ^a is the stage cost of the ancillary controller. The optimal predicted inputs and states are denoted by $u_k^{*j}, \forall (j, k) \in I_{N-1}$ and $\hat{x}_k^{*j}, \forall (j, k) \in I$, and u_0^{*1} is applied to the plant.

As can be seen in (8), the predicted states are unconstrained in the ancillary controller optimization problem. This is because, by the presented formulation, the plant state trajectory will lie in a tube around the primary system state trajectory, and hence by the adequate choice of the constraint tightening (the sets \mathbb{Z} and \mathbb{V}) for the primary controller, the plant state x_t will satisfy the original state constraints \mathbb{X} of the plant.

3.3 Adaptation of the scenario trees

In this subsection, we will detail how the available measurements will be used in conjunction with the plant model to update the sampled uncertainties used in the scenario trees of the primary and ancillary controllers, i.e. the scenario trees of the primary and ancillary controllers at each time step will be generated using a new discrete finite set \mathbb{D}_t^d . As mentioned earlier, we assume the knowledge of the initial state estimation error bound \mathbb{E}_0 , and hence $x_0 \in \hat{x}_0 \oplus \mathbb{E}_0$. At time step t , a sequence of $t + 1$ measurement vectors $\{y_0, y_1, \dots, y_t\}$ and a sequence of t applied inputs $\{u_0, u_1, \dots, u_{t-1}\}$ are available. At each time step, $\forall t \geq 1$, lower and upper bounds on each of the uncertain parameters \tilde{d}^i ($i \in \{1, 2, \dots, n_d\}$) in the vector of uncertain parameters (3), will be determined by solving two guaranteed parameter estimation (GPE) optimization problems similar to (Gottu Mukkula and Paulen, 2016). The constraints for both the lower bound and upper bound GPE problems are:

$$x_{k+1} = f(x_k, u_k, d) + w_k, \quad \forall k \in \{0, 1, \dots, t-1\}, \quad (11a)$$

$$y_k = h(x_k) + \delta_k, \quad \forall k \in \{0, 1, \dots, t\}, \quad (11b)$$

$$\delta_k \in \Delta, \quad \forall k \in \{0, 1, \dots, t\}, \quad (11c)$$

$$w_k \in \mathbb{W}, \quad \forall k \in \{0, 1, \dots, t\}, \quad (11d)$$

$$x_0 \in \hat{x}_0 \oplus \mathbb{E}_0, \quad (11e)$$

$$d \in \mathbb{D}_{t-1}, \quad (11f)$$

The optimization problem for determining the lower bound on the parameter \tilde{d}^i at time step t is defined as follows:

$$\min_{x_0, \delta_k, w_k, d} \tilde{d}^i, \quad \text{subject to (11)} \quad (12)$$

where $\mathbb{D}_0 = \mathbb{D}$ and \tilde{d}^i is the i th element in the vector d given in (3). Accordingly, the optimizer determines an initial plant state x_0 (which has to satisfy (11e)), a disturbance sequence $\{w_k\}$ (which has to satisfy (11d)), a noise sequence $\{\delta_k\}$ (which has to satisfy (11c)) and a parameter vector d (which has to satisfy (11f)) that results in the minimum possible value of the unknown parameter \tilde{d}^i given the known applied input sequence $\{u_k\}$ and the known measurement sequence $\{y_k\}$ (which have to satisfy (11a) and (11b)). As a result, the lower bound is the optimal value of \tilde{d}^i which results from solving (12). In a similar fashion, the optimization problem for determining the upper bound on the parameter \tilde{d}^i is defined as follows:

$$\min_{x_0, \delta_k, w_k, d} -\tilde{d}^i, \quad \text{subject to (11)} \quad (13)$$

and the upper bound is the optimal value of \tilde{d}^i which results from solving (13). The lower and upper bounds will be denoted by $\tilde{d}_l^i(t)$ and $\tilde{d}_u^i(t)$ respectively. Hence, at each time step t , $\tilde{d}^i \in [\tilde{d}_l^i(t), \tilde{d}_u^i(t)]$. A new hyperrectangle \mathbb{D}_t is determined using the closed intervals $[\tilde{d}_l^i(t), \tilde{d}_u^i(t)]$, $\forall i \in \{1, 2, \dots, n_d\}$, and by construction and due to using (11f) in the constraints of both (12) and (13), $\mathbb{D}_t \subseteq \mathbb{D}_{t-1} \subseteq \mathbb{D}_0 = \mathbb{D}$. The discrete finite set \mathbb{D}_t^d can then be obtained by

including the vertices of \mathbb{D}_t , and for the remaining vectors in the set \mathbb{D}_t^d , any sampling criterion can be used. For example, if at $t = 0$, the set $\mathbb{D}_0^d = \mathbb{D}^d$ consists of the vertices of the set $\mathbb{D}_0 = \mathbb{D}$ and a nominal parameter vector, which is the average of the vertices of the set \mathbb{D}_0 , then the nominal parameter vector will be the average of the vertices of the set \mathbb{D}_t , for each $t \geq 1$

The size of the optimization problems (12) and (13) increases at each time step because we are using only the set \mathbb{E}_0 . If we assume the knowledge of the sets \mathbb{E}_t , $\forall t \geq 1$, then (12) and (13) can be solved in a receding horizon fashion. Let N_A denote the horizon length, then at each time step, \mathbb{E}_t will be used in (12) and (13) instead of \mathbb{E}_0 , and only the last $N_A + 1$ measurement vectors will be used instead of all the past $t + 1$ measurement vectors $\forall t \geq N_A$. However, we will continue by assuming only the knowledge of \mathbb{E}_0 , and (12) and (13) will be solved only for $t \in \{1, 2, \dots, T_{max}\}$, for a predefined value of T_{max} , and the sets \mathbb{D}_t and \mathbb{D}_t^d will be fixed from T_{max} on.

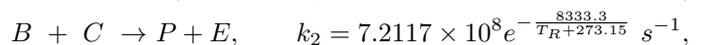
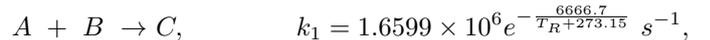
According to the above, for each uncertain parameter \tilde{d}^i , two optimization problems are solved at each time step to determine the upper and lower bounds on this parameter. Therefore, $2n_d$ optimization problems must be solved in total. The $2n_d$ optimization problems are independent, and hence can be solved in parallel.

4. CONTROLLER IMPLEMENTATION

The controller can be implemented as per Algorithm 1. Steps A1-4 are carried out only at $t = 0$. This is because at $t = 0$, $z_0 = \hat{x}_0$ and therefore the ancillary controller optimization problem (if solved) will produce the same solution (optimal state and input trajectories) as the primary controller. Therefore, only the primary controller optimization problem is solved at $t = 0$. Steps B1-8 execute the adaptation of the scenario trees of the primary and the ancillary controllers, and are performed only when $1 \leq t \leq T_{max}$. Steps C1-7 effectuate the primary and ancillary controllers, where the input applied to the plant is generated by the ancillary controller in step C4. Note that steps B1-5 can be solved in parallel depending on the available computational resources.

5. CASE STUDY: WILLIAMS-OTTO CSTR

We compare the performance of the proposed adaptive TEMS NMPC with the non-adaptive version of the TEMS NMPC for the Williams-Otto CSTR benchmark problem (Williams and Otto, 1960), where the following reactions occur:



The reaction rates are of the form

$$k_i = a_i \times e^{-\frac{b_i}{T_R + 273.15}}.$$

We consider the same CSTR model used in (Abdelsalam et al., 2020b). The differential equations of the model are omitted here for brevity. The mass fractions of the six components are denoted by X_A, X_B, X_C, X_E, X_G and X_P , and the reactor and jacket temperatures are denoted

by T_R and T_J . The control inputs are the inlet flow rates F_A and F_B , and the jacket cooling water inlet temperature T_{Jin} . The values of the model parameters can be found in (Williams and Otto, 1960).

As can be seen from values of the parameters b_1 , b_2 and b_3 in the reaction rates, the parameter b_1 has the largest effect, and for that reason we consider inexact knowledge of the parameter b_1 which is considered to be uncertain by $\pm 6\%$ from its nominal value which is 6666.7. We consider this as the parametric uncertainty which is considered in the scenario tree of the primary and ancillary controllers. Hence, $d_t \in [0.94, 1.06]$. We assume random but bounded additive disturbances on each of the eight plant states. The bounds of the additive disturbances are $\pm 5 \times 10^{-4}$ for the concentrations, ± 0.2 for T_R and ± 0.01 for T_J . These random additive disturbances are added to the solution of the differential equations at each time step. The three mass fractions X_E , X_G and X_P are measured with measurement errors of ± 0.05 . Furthermore, T_R and T_J are measured with measurement errors of $\pm 0.3^\circ\text{C}$. The state estimator used is the extended Kalman filter (EKF). The assumed initial state estimation errors are $e_0 \in [-0.05, 0.05]$ for X_A and X_B , $e_0 \in [-0.03, 0.03]$ for X_C and X_E , $e_0 \in [-0.01, 0.01]$ for X_G and X_P and $e_0 \in [-2.5, 2.5]^\circ\text{C}$ for T_R and T_J . The sampling time for the controllers and for the EKF is $T_s = 30$ seconds. The prediction horizon for the controllers is $N = 20$, and the multi-stage controllers have a robust horizon $N_R = 1$. The constraints on the states and inputs are, $60^\circ\text{C} \leq T_R \leq 90$, $0.5 \text{ kg s}^{-1} \leq F_A \leq 10 \text{ kg s}^{-1}$, $0.5 \text{ kg s}^{-1} \leq F_B \leq 10 \text{ kg s}^{-1}$, $15^\circ\text{C} \leq T_{Jin} \leq 100^\circ\text{C}$. The primary and ancillary multi-stage controllers for TEMS NMPC and adaptive TEMS NMPC consider the minimum, nominal and maximum values of d , which are $\{0.94, 1.0, 1.06\}$ at $t = 0$. The scenario trees of the adaptive TEMS are adapted as was explained earlier, where the adaptation occurs only for the first five time steps, i.e. $T_{max} = 5$. The objective function of the primary controllers is an economic objective which is the maximization of the instantaneous profit given by:

$$8.7 \times 3600 ((0.66X_P + 0.015X_E)(F_A + F_B) - 0.044F_A - 0.066F_B)$$

in $\$ \text{ hr}^{-1}$, where the 8.7 is a correction factor for the purchasing power of the USD from the year 1960 (when the original paper was published (Williams and Otto, 1960)) to the year 2020, 0.66 and 0.015 are the sale prices in $\$ \text{ kg}^{-1}$ for products P and E in the year 1960, 0.044 and 0.066 are the costs in $\$ \text{ kg}^{-1}$ for reactants A and B in the year 1960, and the 3600 is for the conversion from $\$ \text{ s}^{-1}$ to $\$ \text{ hr}^{-1}$. The ancillary controllers are tuned to track the primary system states and inputs with the stage cost:

$$10\Delta X_P^2 + 2\Delta X_E^2 + 5\Delta F_A^2 + 10\Delta T_R^2 + \Delta F_B^2 + 10^{-3}\Delta T_{Jin}^2,$$

where Δx is the deviation from the primary system state or input. All the scenarios are assumed to be equally weighted in both the primary and ancillary controllers. The constraints for the primary and ancillary controllers are shown in Table 1. Table 2 shows the steady state profits that are achieved by applying TEMS NMPC and adaptive TEMS NMPC for different values of the uncertain parameter, which shows a significant profit increase that can be achieved by adaptive TEMS NMPC. This is a result of the better knowledge gained by the adaptive TEMS NMPC scheme about the uncertain parameter. Figure 2 shows the evolution during the first five time steps, of the minimum,

Algorithm 1 Adaptive TEMS Implementation

- Require:** $\mathbb{X}, \mathbb{U}, \mathbb{D}, \mathbb{W}, \Delta, \mathbb{E}_0$.
Offline: Choose \mathbb{D}^d by including the vertices of the hyperrectangle \mathbb{D} , and for the remaining elements of \mathbb{D}^d use a pre-selected sampling criteria. Determine the sets \mathbb{Z} and \mathbb{V} by extensive simulations. Choose T_{max} . Set $\mathbb{D}_0 = \mathbb{D}$ and $\mathbb{D}_0^d = \mathbb{D}^d$.
- Online:**
- Step A1 Initialize the primary controller with \hat{x}_0 .
Step A2 Solve (5) and apply v_0^{*1} to the plant.
Step A3 Store the elements of $Z_r(0)$ and set $t = 1$.
Step A4 At the next sampling instant obtain the measurement vector y_1 .
- Step B1 Set $i = 1$.
Step B2 Solve (12) and store the lower bound $\tilde{d}_i^l(t)$.
Step B3 Solve (13) and store the upper bound $\tilde{d}_i^u(t)$.
Step B4 Set $i = i + 1$.
Step B5 **If** $i \leq n_d$: Go to step B2.
Step B6 Determine the hyperrectangle \mathbb{D}_t .
Step B7 Determine the discrete finite set \mathbb{D}_t^d from \mathbb{D}_t and the pre-selected sampling criteria.
Step B8 Set $\mathbb{D}^d = \mathbb{D}_t^d$ for the use in the primary and ancillary controllers.
- Step C1 Estimate the current plant state \hat{x}_t .
Step C2 Determine $z_1^{*j} \in Z_r(t-1)$ which has the minimum distance to \hat{x}_t and use it as the root node for the primary controller (z_1^1).
Step C3 Solve (5) and store the optimal solution sequences $\{z_k^{*j}\}$, $\{v_k^{*j}\}$ and the elements of $Z_r(t)$.
Step C4 Solve (8), apply u_0^{*1} to the plant and set $t = t + 1$.
Step C5 At the next sampling instant obtain the new measurement vector y_t .
Step C6 **If** $t \leq T_{max}$: Store u_0^{*1} as u_{t-1} , store y_t and go to step B1.
Step C7 Go to step C1.
-

Table 1. Constraints for the primary and ancillary controllers of TEMS and adaptive TEMS.

	Primary Controller	Ancillary Controller	Units
F_A	(0.6 – 9.7)	(0.5 – 10)	kg s^{-1}
F_B	(0.6 – 9.7)	(0.5 – 10)	kg s^{-1}
T_{Jin}	(19 – 97)	(15 – 100)	$^\circ\text{C}$
T_R	(60.5 – 88.5)	None	$^\circ\text{C}$

nominal and maximum values of the uncertain parameter (the set \mathbb{D}_t^d) considered by the scenario trees of the primary and ancillary controllers of the adaptive TEMS NMPC when the actual value of the uncertain parameter is $d = 0.94$. The set \mathbb{D}_t^d evolved from $\{0.94, 1.0, 1.06\}$ to $\{0.94, 0.943, 0.946\}$. Note that since the actual value of the parameter is 0.94, the lower bound in the set \mathbb{D}_t^d never changed. For the case when the actual uncertain parameter value was $d = 1.0$, the set \mathbb{D}_t^d evolved from $\{0.94, 1.0, 1.06\}$ to $\{0.975, 0.991, 1.007\}$, and for the case when the actual uncertain parameter value was $d = 1.06$, the set \mathbb{D}_t^d evolved from $\{0.94, 1.0, 1.06\}$ to $\{1.025, 1.042, 1.06\}$ (the plots of the evolution of the minimum, nominal and maximum parameter values for the cases when the actual parameter value was $d = 1.0$ and $d = 1.06$ are omitted here for lack

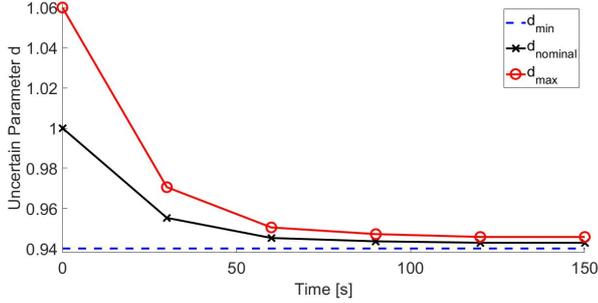


Fig. 2. The evolution of the values of the uncertain parameter considered by the scenario trees of the adaptive TEMS NMPC, when the actual value of the uncertain parameter is 0.94.

Table 2. Steady state profits for different values of the uncertain parameter.

d	TEMS	Adaptive TEMS	Units
0.94	7285	7483	$\$ \text{ hr}^{-1}$
1.0	3255	3617	$\$ \text{ hr}^{-1}$
1.06	932	1405	$\$ \text{ hr}^{-1}$

of space). It should be noted that the final uncertainty set $\mathbb{D}_{T_{max}}^d$ might not be the same for all simulation runs for the same actual value of the parameter because it depends on the actual and random additive disturbances and measurement errors. 500 simulations were performed, scanning the parameter uncertainty range with a step size of 0.005, and random but bounded generation of the additive disturbances, measurement errors and initial state estimation errors as explained earlier in this section. All the simulations were implemented using CasADi (Andersson et al., 2019) for automatic differentiation, and IPOPT (Wächter and Biegler, 2006) for solving the resulting non-linear optimization problems. Figure 3 shows the trajectories of the mass fractions of the profitable components (X_E and X_P) and the profit dynamics for the systems controlled by the TEMS and the adaptive TEMS NMPC when the actual value of the uncertain parameter is 1.06, while Figure 4 shows the corresponding trajectories of the control inputs. As can be seen, the adaptive TEMS NMPC managed to increase the production of the profitable components which resulted in a larger steady state profit when compared with the TEMS NMPC. Note also, that system controlled by the adaptive TEMS NMPC started making profits earlier than the system controlled by the TEMS NMPC, as can be seen from the profit plot in Figure 3 (the profits started becoming positive earlier). Due to better knowledge of the adaptive TEMS NMPC about the the uncertain parameter (as shown in Figure 2), the adaptive TEMS NMPC acted less cautiously than the TEMS NMPC by using higher inlet cooling water temperature T_{Jin} as shown in Figure 4. Figure 5 shows the trajectories of the reactor temperature T_R for the system controlled by the TEMS NMPC (upper figure) and the system controlled by adaptive TEMS NMPC (lower figure). As can be seen, both controllers result in the constraint satisfaction for all the simulation runs. Moreover, it can be seen that the spread of the trajectories of the reactor temperature for the system controlled by the adaptive TEMS NMPC is smaller, because the adaptive TEMS NMPC has a better knowledge about the actual uncertain parameter and hence it was acting less cautiously by heating the reactor

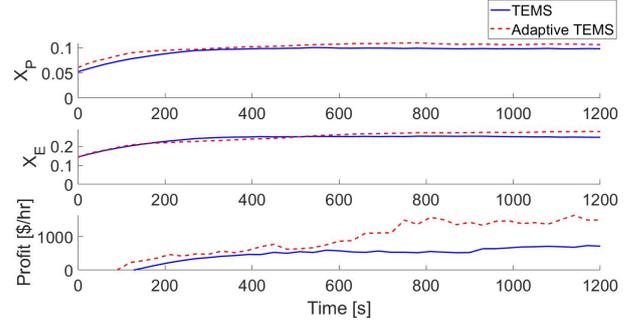


Fig. 3. Trajectories of the mass fractions X_E , X_P , and the profit dynamics for one simulation run when the actual uncertain parameter value is 1.06.

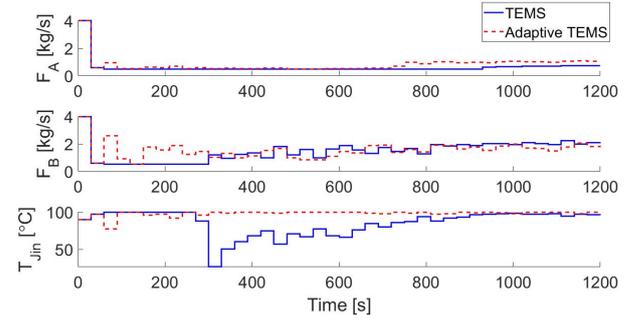


Fig. 4. Trajectories of the inlet flow rates F_A , F_B , and the inlet cooling water temperature T_{Jin} for one simulation run when the actual uncertain parameter value is 1.06.

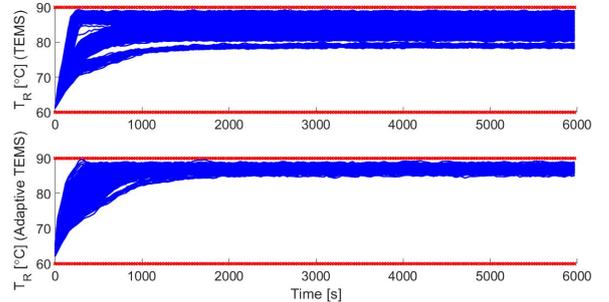


Fig. 5. Trajectories of reactor temperature T_R for the 500 simulations for the TEMS (upper figure) and the adaptive TEMS (lower figure).

temperature more than the TEMS NMPC. The average steady state profit over the 500 simulations, achieved by adaptive TEMS NMPC is 3982 $\$ \text{ hr}^{-1}$, while for TEMS NMPC, the average steady state profit is 3561 $\$ \text{ hr}^{-1}$, which means that the adaptive TEMS NMPC provides 12% increase of profit over the TEMS NMPC.

The significant benefits achieved by the proposed adaptive TEMS NMPC comes at the expense of an increased computational demand only in the first T_{max} time steps (five time steps in this case study), due to the GPE optimization problems. However, it is possible to solve the GPE optimization problems in between the the sampling times and update the set \mathbb{D}_t^d at the following time step $t + 1$ instead of at time step t , i.e. updating the set \mathbb{D}_t^d will be lagging by one sampling interval from what we proposed and implemented. By that, the online computational demand of the adaptive TEMS NMPC will be the same as that of

the non-adaptive TEMS NMPC, at the expense of delaying the performance gains by one sampling interval.

6. CONCLUSION

We proposed a robust adaptive NMPC scheme for processes with parametric uncertainties, additive disturbances and state estimation errors. The proposed adaptive controller uses TEMS NMPC to handle the different types of the uncertainties, while the uncertainty set considered by the scenario trees of the TEMS NMPC is updated based on the available measurements and the plant model to improve the knowledge of the controller about the significant uncertainties in the plant and hence further alleviate the conservatism which inevitably results from robust control under uncertainty. It was shown for the case study presented that adaptive TEMS NMPC results in considerable performance gains over the non-adaptive version.

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