

# Integration of Planning, Scheduling, and Control for Multi-product Chemical Systems under Preventive Maintenance

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**Abstract:** This study presents a framework to perform simultaneous planning, scheduling, and control (iPSC) under preventive maintenance (PM) for multi-product continuous manufacturing units. PM is an essential part of the chemical industry as they provide robustness to the operation and minimize potential plant shutdowns. Flexible planning periods are also investigated in this work since they can improve process economics. A case study featuring a multi-product CSTR is presented to illustrate the benefits of the proposed framework. The results show that flexible planning periods can lead to considerable economic improvement compared to fixed planning periods. Also, an appropriate PM program combined with flexible planning periods provides the opportunity to optimize the maintenance time instead of fixing those times *a priori* using process heuristics.

**Keywords:** Preventive Maintenance, Planning, Scheduling and Control, Flexible Planning Periods.

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## 1. INTRODUCTION

Enterprise-wide optimization (EWO) plays a vital role in the current competitive industry, providing a powerful tool to enable profitable and sustainable operations during workflows. Key aspects considered in EWO are planning, scheduling, and control (Grossmann, 2012). Often, problems involving planning, scheduling, and process control are considered hierarchically and solved independently. While easy to implement, hierarchical decision-making tools between these layers have significant limitations, such as process infeasibility and performance losses.

To improve the flexibility and performance of the hierarchical framework, integration strategies including integrated planning and scheduling (iPS), integrated scheduling and control (iSC), and integrated design and control (iDC) has been proposed (Rafiei and Ricardez-Sandoval, 2020). The economic benefits pointed out by these strategies motivate the development of more comprehensive decision-making strategies involving more layers in the manufacturing process. To this end, frameworks addressing the integration of planning, scheduling, and control (iPSC) have been reported. (Charitopoulos, Dua and Papageorgiou, 2017) proposed a decomposition method to solve a monolithic iPSC model. (Dias and Ierapetritou, 2020) proposed data-driven methodologies and surrogate models to solve iPSC problems. A closed-loop iPSC framework that enables online solutions under disturbances has also been reported (Charitopoulos, Papageorgiou and Dua, 2019). Those studies considered a fixed planning period in their formulations, i.e. the planning horizon is divided into equal-length planning periods. Also, the length of each planning period is a fixed parameter in their formulations, which could limit the possibility of finding more economically attractive solutions.

In a multi-layer integrated system, system reliability is critical. To keep the operation running at the target level and achieve the expected profits, maintenance is needed. Maintenance

plays an essential role to ensure product availability, reliability, and product quality. Often, maintenance in a chemical process can be divided into corrective maintenance (CM) and preventive maintenance (PM). CM happens after a failure occurs and aims to restore the system to an operational state. PM is performed before the units in the system experience a breakdown, i.e. PM aims to reduce machine degradation and the risk of failure (Shafiee and Chukova, 2013). Studies have previously considered production and maintenance schemes. The integration of PM and production scheduling for a batch unit in a single machine has been reported by (Cassady, C. R., & Kutanoglu, 2005). Recently, a study focused on a joint model involving production, quality control and PM showed that an integrated approach offers better economic performance than CM (Cheng and Li, 2020). While previous studies brought significant advances to consider the integration of production and maintenance; those works have not yet considered PM in the context of iPSC.

This study presents a novel iPSC framework that explicitly accounts for PM. The iPSC model is formulated as a mixed-integer nonlinear programming (MINLP) problem. Planning periods in this model are flexible, i.e. the planning horizon is not defined *a priori* as it is done in current iPSC formulations; thus, the solutions obtained by the present framework are expected to improve the process economics. To our knowledge, this is the first study that explores PM and flexible planning periods in a monolithic iPSC framework.

## 2. MONOLITHIC MODEL

This section presents the mathematical framework for the integration of planning, scheduling, and optimal control under PM and with flexible planning periods. (Charitopoulos, Dua and Papageorgiou, 2017) presented a monolithic model for iPSC. That model assumes that the machine continuously operates without providing a time slot to perform PM, which can reduce the risk of a machine breakdown during operation. Also, planning periods are defined as *a priori*, which may limit

the model flexibility. In the present model, PM combined with flexible planning periods obtained from optimization are considered. As shown in Fig. 1, a flexible planning period ( $s$ ) consists of a flexible manufacturing period (MP) and PM. MP consists of the start-up time, production time, transition time and shut-down time needed to provide PM to a machine. Once a PM has been completed, it represents the end of a planning period. Both MP and PM in each planning period are obtained from optimization. Since we consider the planning periods optimization variables, the present model selects the optimal time slot to perform each PM. The problem statement and assumptions considered in this model are as follows:

**Given:**

- A single-stage, multi-product continuous chemical process.
- A production planning horizon ( $H$ ).
- Bounds of the MP in each planning period ( $s$ ).
- Bounds of the production time to a product for each  $s$ .
- Demands for each product at the end of the planning horizon, and their corresponding selling prices.
- Dynamic model describing the transient operation of the unit.
- Raw material, operating, backorder and inventory costs.
- The start-up and shut-down time of the unit at the beginning and the end of each planning period.

**Determine:**

- Optimal length of each planning period ( $s$ ).
- Optimal production sequences.
- Production time and cost for each product in each period ( $s$ ).
- Optimal PM time in each planning period ( $s$ ).
- Inventory and backorder level for each product at the end of the planning horizon.
- Optimal dynamic trajectories to transition between products.

The present model assumes the following: i) The filling time and shut-down time of the unit are fixed. ii) Each product can only be assigned once in a planning period. iii) Each PM assumes that the machine will return to the status “as good as new” after each PM (i.e., aging is not considered). iv) All the products lower than or equal to product demands are sold right after the planning horizon.

As shown in Fig. 1, the planning horizon is divided into  $S$  periods, which are optimization variables; hence, each planning period ( $s$ ) may be of different lengths. These planning periods are modeled as discrete-time points. The detailed schedule of  $N$  products within each period ( $s$ ) is presented as a continuous process and the dynamic trajectories from product  $i$  to product  $j$  can be found from optimization;  $x(t)$  denotes the state variables of the system. MP consists of a preset time range defined by a lower bound ( $mtlow$ ) and an upper bound ( $mtup$ ). The production sequence of each product is determined at the scheduling level. At the end of each MP, the

machine is emptied (turned off) so that PM can be performed. The length of each PM is related to the duration of the MP.

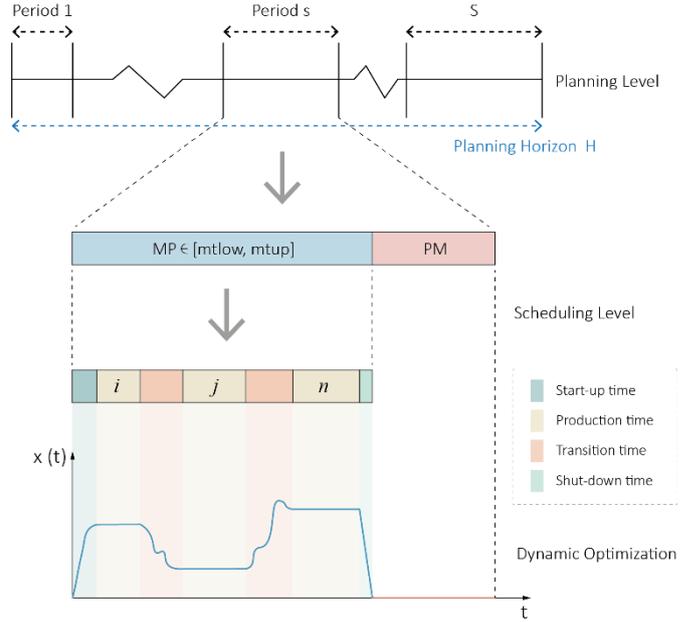


Fig. 1 Structure of the PSC model under PM constraints with flexible planning periods.

As shown in Fig. 1, PM is the last step considered in a planning period. Once PM is completed, the machine needs a reset to a specific level of operation, which is the start-up process of the next planning period. Fig. 1 also shows the sequence at the dynamic optimization level: i) start-up, ii) production time and dynamic transitions among the different products manufactured during the period, iii) shut down, and iv) PM.

**2.1 Flexible planning period and Preventive Maintenance**

As mentioned above, the length of a planning period  $s$  ( $PP_s$ ) consists of a flexible length of MP ( $mpt_s$ ) and PM ( $mt_s$ ).

$$PP_s = mpt_s + mt_s \quad (1)$$

where  $mpt_s$  is defined as follows:

$$mpt_s = Ms_s + \sum_i T_{is} + \sum_i \sum_j T_{ijs}^{Tr} + Me_s \quad (2)$$

where  $T_{ijs}^{Tr}$  represents the transition time from product  $i$  to product  $j$  during planning period  $s$  (see section 2.3);  $T_{is}$  is the production time of product  $i$  in period  $s$ ;  $Me_s$  and  $Ms_s$  are the shut-down and start-up times during period  $s$ , respectively.  $Me_s$ ,  $Ms_s$  and  $mt_s$  are defined in the next section. For each product  $i$  manufactured during period  $s$ , there is a minimum and a maximum allowed production time, i.e.

$$\theta_s^{lo} E_{is} \leq T_{is} \leq mtup E_{is} \quad \forall i, s \quad (3)$$

where  $\theta_s^{lo}$  represents a user-defined minimum production time allowed for product  $i$  on period  $s$ .  $E_{is}$  is a binary variable that defines if product  $i$  is assigned for production in period  $s$  (further details are shown in the next section). Constraint (3) is enforced to avoid the need to transition over multiple products for short production times. Note that the upper bound for each product  $i$  in period  $s$  must be lower than or equal to the upper bound of each manufacturing time,  $mtup$ . The

following constraint enforces that each MP is performed within a specific time range to minimize the occurrence of any potential machine breakdowns, i.e.

$$mtlow \leq mpt_s \leq mtup \quad (4)$$

Both  $mtlow$  and  $mtup$  are specified *a priori* and can be obtained from historical process data or process heuristics, e.g. by inspecting the shortest and longest time at which breakdowns have occurred for similar units in the past.

As shown in (5), each PM ( $mt_s$ ) is a function of the MP ( $mpt_s$ ). To simplify the analysis,  $mt_s$  is assumed to be linearly proportional to  $mpt_s$ ;  $mt_0$  is a user-defined parameter that corresponds to the maximum duration of a PM, i.e., when the operation of the unit has reached the maximum available manufacturing time ( $mtup$ ).

$$mt_s = mt_0 \frac{mpt_s}{mtup} \quad (5)$$

Further, all planning periods must take place within the  $H$ , i.e.

$$\sum_s PP_s \leq H \quad (6)$$

The present model assumes that the machine must be shut down and start-up before and after a PM. The machine start-up time is determined as follows:

$$Ms_s = ft + T_{0is}^{Tr} \quad (7)$$

$$T_{0is}^{Tr} = f(x_0, x(t), u_0, u(t)) \quad (8)$$

$Ms_s$  represents the start-up time for planning period  $s$ .  $ft$  is the time needed to load the unit to a certain level that is suitable for the operation, e.g. in the case of a CSTR,  $ft$  represents the time needed to fill-in the unit to a certain liquid level and can be calculated as a function of the reaction volume and the inlet flowrate.  $T_{0is}^{Tr}$  is the time needed to complete the transition from an initial state  $x_0$  and initial input  $u_0$  to a steady-state operating condition, which corresponds to the state of the first product that needs to be produced in that period.  $T_{0is}^{Tr}$  depends on the system's states  $x(t)$  and inputs  $u(t)$ , e.g. in the case of CSTR,  $T_{0is}^{Tr}$  is the time needed to transition from an initial concentration in the reactant species to the steady-state concentration in those same species that correspond to the first product produced in that period. More details about  $T_{0is}^{Tr}$  are given in section 2.3. In the present work, the shut-down time ( $Me_s$ ) is assumed equal to  $ft$  since it is assumed that the time needed to empty the unit is the same required to load the unit to an operational level, e.g. in the case of a CSTR, the unit can be emptied at the same rate as filled in the start-up process if the liquid level remains constant throughout the production of the multiple products.

When PM is executed, the unit is idling; thus, there will be a loss of an opportunity to produce products. We consider this as a loss of production cost LPC (\$/h), i.e.

$$LPC = \frac{\sum_i ((C_{s_i} - C_{pr_i}) r_i)}{\sum_i r_i} \quad (9)$$

where  $C_{s_i} - C_{pr_i}$  is the unit profit (\$/mol), which represents the unit selling price minus the unit production costs of product  $i$ .  $r_i$  is the production rate of product  $i$ . Equation (10) is the maintenance cost  $MC$  (\$). Note that  $\sum_s mt_s$  represents the total PM time (h).  $LC$  (\$/h) is the labor cost and  $FC_{PM}$  (\$) is a fixed cost for PM; both terms are user-defined.

$$MC = \sum_s mt_s (LPC + LC) + FC_{PM} \quad (10)$$

## 2.2 Scheduling problem

In each planning period, the first and the last products produced during MP are determined using (11) and (12). The binary variables  $F_{is}$  and  $L_{is}$  indicate the first and last product to be produced, respectively. Setting  $F_{is}$  ( $L_{is}$ ) to 1 indicates that product  $i$  is the first (last) product in the period  $s$ , and zero otherwise.

$$\sum_i F_{is} = 1 \quad \forall s \quad (11)$$

$$\sum_i L_{is} = 1 \quad \forall s \quad (12)$$

Equation (13) ensures that the first product can be considered only if it is assigned in period ( $s$ ).

$$F_{is} \leq E_{is} \quad \forall i, s \quad (13)$$

A similar equation is used to present that the last product  $i$  can be considered only if it is assigned in planning period  $s$ , i.e.

$$L_{is} \leq E_{is} \quad \forall i, s \quad (14)$$

Multiple products may be produced within each planning period, thus requiring the need to transition from product  $i$  to product  $j$ . To model the transition process between two products within the same planning period, the binary variable  $Z_{ijs}$  is introduced to indicate if product  $j$  is produced right after product  $i$  in period  $s$ . Equation (15) denotes that a product  $j$  needs to experience a transition from another assigned product  $i$ , unless it is the first product to be processed in period  $s$ .

$$\sum_{i \neq j} Z_{ijs} = E_{js} - F_{js} \quad \forall j, s \quad (15)$$

Equation (16) ensures that once a product  $i$  is assigned to the planning period  $s$ , there is a transition from product  $i$  to another assigned product  $j$ , unless product  $i$  is the last product.

$$\sum_{i \neq j} Z_{ijs} = E_{is} - L_{is} \quad \forall i, s \quad (16)$$

Equation (17)-(19) enforces the production sequence. the integer variable  $O_{is}$  is an integer variable that denotes the production sequence during the planning period  $s$ .  $O_{is}$  is also used to enforce that each product can only be produced once at most in a planning period.  $M$  is a sufficiently large (big-M) number chosen from preliminary trials.

$$O_{js} - (O_{is} + 1) \geq -M(1 - Z_{ijs}) \quad \forall i, j, s, i \neq j \quad (17)$$

$$O_{is} \leq ME_{is} \quad \forall i, s \quad (18)$$

$$F_{is} \leq O_{is} \leq \sum_i E_{is} \quad \forall i, s \quad (19)$$

## 2.3 Dynamic optimization and linking constraints

The generic continuous dynamic model is as follows:

$$\frac{dx}{dt} = f(x(t), u(t)) \quad (20)$$

$$x(t)|_{t=0} = x_0 \quad (21)$$

$$x^L \leq x(t) \leq x^U, u^L \leq u(t) \leq u^U \quad (22)$$

where  $x(t) \in R^{n_x}$  denotes the state variables;  $u(t) \in R^{n_u}$  is the manipulated variables;  $x^L, u^L$  and  $x^U, u^U$  are the corresponding bounds on each of these variables.  $x_0$  is the initial value of the state variables. Numerical discretization techniques are applied to solve (20). In this work, Orthogonal Collocation on Finite Elements (OCFE) is used to discretize the differential equation (20); thus, the dynamic optimization problem is transformed into a nonlinear programming (NLP) problem by approximating the control and state profiles, across the finite elements, with a family of orthogonal polynomials such as Lagrange or Legendre polynomials. The time domain is discretized into finite elements( $f$ ) with collocation points( $c$ ) considered within each finite element.

Based on the above descriptions, the dynamic optimization of variables  $T_{0is}^{Tr}$  and  $T_{ijs}^{Tr}$  are defined. The formulations of these two variables are similar; for brevity, we only present the formulation of  $T_{ijs}^{Tr}$ . The corresponding discretization of this variable in the time domain is as follows:

$$t_{ijsfc}^{ocfe} = ((f - 1) + Rts_c)T_{ijs}^{Tr}/N_f \quad (23)$$

where  $Rts_c$  is the roots of the orthogonal polynomial used in the OCFE, and  $N_f$  is the cardinality of the set of finite elements. Continuity of the state variables across adjacent finite elements is enforced as per (24).  $f$  and  $c$  are the indexes of finite elements and collocation points, respectively;  $m$  and  $n$  are the indexes of state variables and control inputs, respectively.

$$x_{mijfs}^{ocfe\,init} = x_{mijf-1}^{ocfe\,init} + T_{ijs}^{Tr} h^{ocfe} \sum_{c=1}^c \Omega_{cN_c} \dot{x}_{mijf-1,c} \quad (24)$$

where  $h^{ocfe}$  is the step of each finite element, i.e.

$$h^{ocfe} = \frac{1}{N_f} \quad (25)$$

Moreover,  $\Omega_{cN_c}$  is calculated following the Radau IV quadrature and  $\dot{x}_{mijfs}$  is the computation of the numerical value of the derivative of the  $m^{\text{th}}$  state, i.e.

$$\dot{x}_{mijfs} = f^m(x_{mijfs}^{ocfe}, u_{mijfs}^{ocfe}, \sigma) \quad \forall n, m, i, j, s, f, c \quad (26)$$

Equation (27) determines the state variables across the discretized time domain, where  $cp$  is the index of collocation points to show the different points with index  $c$ :

$$x_{mijsc}^{ocfe} = x_{mijf}^{ocfe\,init} + T_{ijs}^{Tr} h^{ocfe} \sum_{cp=1}^c \Omega_{cp} \dot{x}_{mijscp} \quad (27)$$

To connect the dynamic optimization model to the planning and scheduling model, linking variables are required:  $x_{mijis}^{\text{in}}, u_{nijis}^{\text{in}}, x_{mijis}^{\text{fin}}, u_{nijis}^{\text{fin}}$ . The first two variables are the state and manipulated variables at the beginning of the transition whereas the last two are similarly for the end of the transition in period  $s$ . Eq. (28)-(29) show the discretized state variable ( $x_{mijis}^{\text{ocfe}\,init}$ ) and manipulated variable ( $u_{nijis}^{\text{ocfe}}$ ) of the first finite element and first collocation point is equal to the initial value

at the beginning of the transition. At the end of the transition, the state of the system ( $x_{mijis}^{\text{fin}}$ ) is equal to the discretized state variable  $x_{mijisN_fN_c}^{\text{ocfe}}$  at the last collocation point ( $N_c$ ) of the last finite element ( $N_f$ ), as shown in (30). Equation (31) represents the same condition for the discretized manipulated variable.

$$x_{mijis}^{\text{in}} = x_{mijisf}^{\text{ocfe}\,init} \quad \forall m, i, j, s, i \neq j, f = 1 \quad (28)$$

$$u_{nijis}^{\text{in}} = u_{nijisfc}^{\text{ocfe}} \quad \forall n, i, j, s, i \neq j, f = 1, c = 1 \quad (29)$$

$$x_{mijis}^{\text{fin}} = x_{mijisN_fN_c}^{\text{ocfe}} \quad \forall m, i, j, s, i \neq j \quad (30)$$

$$u_{nijis}^{\text{fin}} = u_{nijisN_fN_c}^{\text{ocfe}} \quad \forall n, i, j, s, i \neq j \quad (31)$$

Since  $x_{mijis}^{\text{in}}$  and  $x_{mijis}^{\text{fin}}$  correspond to the initial and final values of the transitions between different products, they are also related to the steady-state value when the production occurs. As shown in (32), the initial value of the  $m^{\text{th}}$  state variable at the beginning of a transition is equal to the steady-state of the product  $i$  ( $x_{mi}^{\text{ss}}$ ) that was processed before the transition. The same condition is stated for the manipulated variable in (33). Equation (34) indicates that the transition between two products is terminated once the system has reached the next steady-state, i.e. the final value of the state variable is equal to the steady-state of product  $j$  ( $x_{mj}^{\text{ss}}$ ), which is the next product in the production sequence. Equation (35) states the same condition for the manipulated variable.

$$x_{mijis}^{\text{in}} = x_{mi}^{\text{ss}} Z_{ijs} \quad \forall m, i, j, s, i \neq j \quad (32)$$

$$u_{nijis}^{\text{in}} = u_{ni}^{\text{ss}} Z_{ijs} \quad \forall n, i, j, s, i \neq j \quad (33)$$

$$x_{mijis}^{\text{fin}} = x_{mj}^{\text{ss}} Z_{ijs} \quad \forall m, i, j, s, i \neq j \quad (34)$$

$$u_{nijis}^{\text{fin}} = u_{nj}^{\text{ss}} Z_{ijs} \quad \forall n, i, j, s, i \neq j \quad (35)$$

#### 2.4 Objective function

The objective function of this iPSC model aims to maximize profits. Alternative expressions, e.g. minimization of costs, can also be considered. The profits are calculated as the difference between revenue ( $Rev$ ) and total costs ( $Toc$ ), i.e.

$$Profit = Rev - Toc \quad (36)$$

The total cost represents the addition of the production cost ( $Pr$ ), inventory cost ( $Inv$ ), backorder cost ( $Bac$ ), raw material consumption cost ( $Rc$ ), transition cost ( $Trc$ ), and maintenance cost ( $MC$ ), i.e.

$$Toc = Pr + Inv + Bac + Rc + Trc + MC \quad (37)$$

Formulations for  $Rev$ ,  $Pr$  and  $Rc$  were adopted from a previous study (Charitopoulos, Dua and Papageorgiou, 2017).

Eq. (38)-(40) describe the transition cost  $Trc$ , which consists of the transition cost at the start-up stage ( $Tr_0$ ), and the transition cost from product  $i$  to  $j$  ( $Tr_1$ ).

$$Trc = Tr_0 + Tr_1 \quad (38)$$

$$Tr_0 = \sum_n \sum_i \sum_s \sum_f \sum_c u_{n0isfc}^{\text{ocfe}} T_{0is}^{Tr} h^{ocfe} \Omega_{cN_c} \quad (39)$$

$$Tr_1 = \sum_n \sum_i \sum_j \sum_s \sum_f \sum_c u_{n i j s f c}^{ocfe} T_{i j s}^{Tr} h^{ocfe} \Omega_{c N_c} \quad (40)$$

Equation (41) shows the inventory cost, which is calculated as the summation of the inventory level ( $IV_i$ ) of every product  $i$ , multiplied by the unit production price  $C_{iv}$  (\$/mol).

$$Inv = \sum_i C_{iv} IV_i \quad (41)$$

When the demands for every product ( $D_i$ ) cannot be met, a backorder penalty cost is allowed, i.e.

$$Bac = \sum_i C_{bo} BO_i \quad (42)$$

where  $C_{bo}$  (\$/mol) represents the unitary backorder cost and  $BO_i$  presents the backordered product  $i$  at the end of the planning horizon.

The amount (mol) of product  $i$  produced within each planning period  $s$  ( $Pr_{is}$ ), and the products in the entire planning horizon ( $Pr_i$ ), can be calculated as follows,

$$Pr_{is} = r_i T_{is} \quad \forall i, s \quad (43)$$

$$Pr_i = \sum_s Pr_{is} \quad (44)$$

where  $r_i$  is the parameter to represent the production rate of product  $i$ . Both the backorders ( $BO_i$ ) and inventory level ( $IV_i$ ) are determined at the end of the planning horizon as follows:

$$BO_i = (D_i - Pr_i) Y_{t_i} \quad \forall i \quad (45)$$

$$IV_i = (D_i - Pr_i) * (Y_{t_i} - 1) \quad \forall i \quad (46)$$

where the binary variable  $Y_{t_i}$  is defined as follows:

$$Y_{t_i} = \begin{cases} 1, & \text{there is a backorder of product } i \\ 0, & \text{there is an inventory of product } i \end{cases} \quad \forall i \quad (47)$$

We employ the iPSC model considering PM and the flexible planning period to solve simultaneous planning, scheduling and control problem. Then OCFE is used for the discretization of the dynamics of the system for optimal control. Linking constraints are applied to connect the planning and scheduling model to the dynamic optimization model. Based on the above descriptions, the monolithic iPSC MINLP model considered in this work is as follows:

$$\max. \text{Profit} = Rev - \text{Toc}$$

s.t. eq. (1)-(19) Planning and scheduling

eq. (20)-(27) Dynamic transition

eq. (28)-(35) Linking constraints

### 3. CASE STUDY

The performance of the proposed iPSC model with PM and flexible planning period presented in the previous section was tested using a case study featuring a single multiple-input and multiple-output (MIMO) non-isothermal multi-product CSTR unit adopted from the literature (Charitopoulos, Dua and Papageorgiou, 2017). The reactions are exothermic; thus, a cooling jacket is used to maintain the unit's temperature within limits. The dynamic model is shown in (48)–(49). In this work, the planning period  $H$  is set to 2 weeks. The inlet concentration of the reactant ( $C_{ao}$ ) is assumed to be constant.

$$\frac{d(V_1 C_b)}{dt} = V_1 k_r (C_{ao} - C_b) + F_i C_b \quad (48)$$

$$\frac{d(V_1 \rho_1 C_{p1} T_1)}{dt} = F_1 \rho_1 C_{p1} T_{10} -$$

$$F_1 \rho_1 C_{p1} T_1 + F_c \rho_c C_{pc} (T_{c0} - T_c) + V_1 k_r (C_{ao} - C_b) H \quad (49)$$

The control variables for the system are the liquid flow rate ( $F_l$ ) and the coolant flowrate ( $F_c$ ). The product concentration ( $C_b$ ) and the liquid temperature ( $T_1$ ) are state variables. Demands of each product are listed in Table 1. Note that each product corresponds to a particular concentration in  $C_b$  (not shown for brevity). The steady-state conditions (e.g. concentrations) and other parameters used in this MIMO case study are provided in (Camacho & Alba, 2013) and (Zhuge and Ierapetritou, 2015).

**Table 1. Demands of products**

Products	A	B	C	D
Demands (mol)	171000	85000	6400	35000
Products	E	F	G	H
Demands(mol)	40000	45000	22500	600

For the present study,  $mt_0$  is 5h,  $mtlow$  and  $mtup$  are 35 and 60h, respectively;  $\theta_s^{lo}$  is set to 0.5h;  $FC_{PM}$  is set to 800(\$) and the  $LC$  is 100(\$/h).

The resulting iPSC problem is formulated as an MINLP problem and was solved using the DICOPT solver in GAMS. CONOPT and CPLEX were used to solve the NLP and mixed integer programming (MIP) problems, respectively. The dynamic models were discretized using the OCFE method with 20 finite elements and each finite element includes 3 collocation points. In this case study, three test scenarios were considered to assess the performance of the proposed iPSC model: A) No PM with fixed planning periods, i.e. for  $S=6$ , the length of each planning period is 56h. B) PM with flexible planning periods. C) PM with fixed planning periods (similar to those used in Scenario A). For these scenarios, we considered three planning periods: 6, 7, and 8.

**Table 2. Results for each scenario (Unit:  $10^6$ \$)**

Scenario	S=6		S=7		S=8	
	Rev	Profit	Rev	Profit	Rev	Profit
A	11.349	8.835	11.349	8.835	11.349	8.835
B	11.138	8.603	11.136	8.601	11.132	8.597
C	11.134	8.600	11.132	8.597	11.128	8.594

As shown in Table 2, the revenue and profit for Scenario A are the same since we do not consider specific demands for each period. The profit for Scenario A is higher than Scenarios B and C, i.e., the profit for A is 2.7% and 2.73% higher than B and C when  $S=6$ . When  $S=7$  (8), the profit for A is 2.72% (2.77%) and 2.77% (2.8%) higher than that obtained for B and C, respectively. These are expected since Scenario A has more time available to manufacture products, resulting in higher revenue. However, no PM is considered; thus, it is limited since machine breakdown or aging are factors that can impact operation. The economic performance between Scenarios B and C are also compared as a function of planning periods. The profits using a flexible planning period are higher than that using fixed periods. As shown in Table 2, the profits tend to decrease as the number of planning periods increases, which is

also an indication that the selection of planning periods is not trivial and can significantly improve profits if the operation is performed using optimization.

Table 3 summarizes the results for Scenarios B and C ( $S=6$ ). Since we considered the length of MP and PM as decision variables in Scenario B, and kept them as constant parameters in Scenario C, the number of constraints and continuous variables for each scenario are slightly different. The CPU time of Scenario B is 4% lower than that obtained from Scenario C, which suggests that the complexity of both models is somewhat similar. The calculations were performed using a computer of 1.80 GHz/8GB RAM. We selected the second and third planning periods as the examples to analyse the differences in performance between Scenarios B and C. As shown in Table 3, Scenario B only produces one product in the second planning period (P2) whereas Scenario C produces three products. The lengths in PM and MP are 25.41% and 30.08% shorter in Scenario B than in Scenario C. Moreover, both scenarios produce three different products during the third planning period (P3). The lengths in PM and MP are 16.06% and 16.07% longer in Scenario B than in Scenario C. Results from these scenarios show that selecting the planning periods' length is not trivial and lead to profitable planning, scheduling, and control strategies, as shown in Table 2.

**Table 3. Results: Scenarios B and C ( $S=6$ )**

	B	C
Constraints	126,694	126,681
Continues variables	161,997	161,991
Discrete variables	542	542
CPU time (s)	276.95	289.03
Optimal schedule	P2: F P3: C → A → D	P2: E → A → B P3: G → H → F
Transition time(s)	P2: 0 P3: 0.024, 0.051	P2: 0.032, 0.013 P3: 0.077, 0.031
Length of PM(h)	3.012, 5	4.038, 4.038
Length of planning period(h)	39.157, 65	56, 56

Fig. 2 shows the dynamic transition processes from products C to A during the third planning period in scenario B. As shown in this figure, a smooth transition from a high  $C_b$  (product C) to a low  $C_b$  (product A) is performed by simultaneously adjusting the outlet flowrates  $F1$  and  $Fc$ .

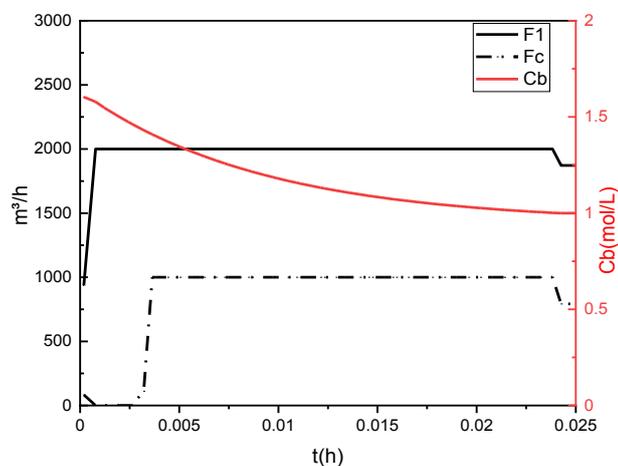


Fig. 2. Dynamic transition from product C to product A

## 4. CONCLUSIONS

An iPSC model that considers flexible PM and MP was presented in this study. The proposed formulation takes the form of an MINLP problem. This novel iPSC model was formulated by considering the length of MP and PM in each planning period as decision variables, which may result in higher profits and attractive manufacturing strategies. Results indicate that the flexible planning period with time-based PM has clear benefits to improve profits compared to the fixed planning period with PM. The results also show that the number of planning periods impact the profits (for a fixed planning horizon). Future work will focus on extending the model from single unit to multiple units and explore uncertainties on different layers of the process. In this work, we used a simplified time-based PM; a condition-based predictive maintenance will be explored in the future. Also, alternative methods that take into account plant aging will be considered as part of the future work.

## REFERENCES

- Camacho, E. and Bordons Alba, C., (2013) *Model Predictive Control*. London: Springer London, Limited.
- Cassady, C. R., & Kutanoglu, E. (2005) 'Integrating preventive maintenance planning and production scheduling for a single machine', in *IEEE Transactions on Reliability*, pp. 304–309.
- Charitopoulos, V. M., Dua, V. and Papageorgiou, L. G. (2017) 'Traveling salesman problem-based integration of planning, scheduling, and optimal control for continuous processes', *Industrial and Engineering Chemistry Research*, 56(39), pp. 11186–11205.
- Charitopoulos, V. M., Papageorgiou, L. G. and Dua, V. (2019) 'Closed-loop integration of planning, scheduling and multi-parametric nonlinear control', *Computers and Chemical Engineering*, 122, pp. 172–192.
- Cheng, G. and Li, L. (2020) 'Joint optimization of production, quality control and maintenance for serial-parallel multistage production systems', *Reliability Engineering and System Safety*. Elsevier Ltd, 204(January), p. 107146.
- Dias, L. S. and Ierapetritou, M. G. (2020) 'Integration of planning, scheduling and control problems using data-driven feasibility analysis and surrogate models', *Computers and Chemical Engineering*. Elsevier Ltd, 134.
- Grossmann, I. E. (2012) 'Advances in mathematical programming models for enterprise-wide optimization', *Computers and Chemical Engineering*. Elsevier Ltd, 47, pp. 2–18.
- Rafiei, M. and Ricardez-Sandoval, L. A. (2020) 'New frontiers, challenges, and opportunities in integration of design and control for enterprise-wide sustainability', *Computers and Chemical Engineering*. Elsevier Ltd, 132.
- Shafiee, M. and Chukova, S. (2013) 'Maintenance models in warranty: A literature review', *European Journal of Operational Research*. Elsevier B.V., 229(3), pp. 561–572.
- Zhuge, J. and Ierapetritou, M. G. (2015) 'An Integrated Framework for Scheduling and Control Using Fast Model Predictive Control', *AIChE Journal*, 59(4), pp. 215–228.