Robust Economic Model Predictive Control with Zone Control ⋆

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Abstract: This paper presents a robust economic Model Predictive Control (EMPC) formulation for discrete-time uncertain nonlinear systems. The proposed controller not only ensures that the closed-loop system is robust to disturbances, but also ensures that the economic performance does not deteriorate in the presence of the disturbances. The key idea is to have the controller track a robust control invariant subset of the state space with specified economic properties at all times, and within the zone optimize the process economics. To this end, we introduce the notion of risk factor in the controller design and provide an algorithm to determine the economic zone to be tracked. The risk factor determines the conservativeness of the controller. Our proposed controller is computationally less demanding as it only makes use of the system model without disturbances. A nonlinear CSTR example is presented to demonstrate the performance of the proposed formulation.

Keywords: Robust economic model predictive control; Process control; Nonlinear systems; Zone control; Robust control invariance

1. INTRODUCTION

Nonlinear Model Predictive Control (MPC) with general objective known as economic Model Predictive Control (EMPC) has received significant attention in recent times. The objective function (which may not necessarily be quadratic) of EMPC generally reflects some economic performance criterion such as profit maximization or heat minimization, which is in contrast with tracking or stabilizing MPC where the objective is a positive definite quadratic function. The integration of process economics in the control layer makes EMPC of interest in many areas especially in the process industry. As such, there has been a significant number of applications of EMPC (Liu et al. (2015); Decardi-Nelson et al. (2018); Zhang et al. (2020); Griffith et al. (2017)). To address stability and computational issues, several formulations of EMPC have been proposed (Angeli et al. (2011); Liu and Liu (2016); Ellis et al. (2014)).

It is worth mentioning that majority of current results on EMPC focused on deterministic systems without considering the presence of uncertainties or disturbances. Uncertainties are unavoidable in real world applications and arise as a result of imperfect models or unmeasured disturbances. The presence of uncertainties in the control system can result in performance degradation and/or loss of feasibility which can lead to loss of stability. Moreover, it is not fully understood how the presence of disturbances or uncertainties affect EMPC. In the context of stabilizing MPC, several different concepts have been introduced to address the problems arising from the presence of uncertainties (See Mayne (2016) for a survey on robust and stochastic MPC as well as their associated challenges).

However, as pointed out in Bayer et al. (2014), simply transferring robust MPC techniques into an EMPC framework could result in poor economic performance. This is due to the fact that robust MPC techniques have been designed to reject all disturbances to achieve its desired goal which may not be the case for EMPC. This is because some disturbances can lead to better economic performance. To this end, some results on robust EMPC has been proposed. Lucia et al. (2014) presented a robust EMPC formulation based on scenario tree approach. Tube-based formulations with and without stochastic information have also been proposed in Bayer et al. (2014, 2016). The above formulations are either computationally demanding or restrictive since they inherit robust MPC techniques. Thus, simple robust EMPC formulations which does not compromise or hinder the performance of EMPC are desired.

In this work, we present a novel EMPC formulation for controlling constrained nonlinear systems subject to unmeasured but bounded disturbances. The proposed formulation incorporates economic risk in the controller design using the concept of zone control. The concept of zone control is not new. Zone MPC have been reported in several areas such as diabetes treatment (Grosman et al. (2010)), anaemia management (McAllister et al. (2018)), control of building heating systems (Privara et al. (2011)), control of irrigation systems (Mao et al. (2018)) and coal-fired boiler-turbine generating system (Zhang et al. (2020)). In the context of MPC literature, zone control is often dismissed as a trick to avoid feasibility issues and has received less attention in terms of theoretical analysis. A recent exposition on the stability analysis of MPC with

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generalized zone tracking (Liu et al. (2019)) paves the way for further applications of zone control. Our idea is to drive the states of the system to a zone which is a robust control invariant subset of the state space with specified economic properties. By creating a zone objective, the desired closed-loop economic performance in the presence of uncertainties can be specified in the controller design using a tuning parameter known as a risk factor. We demonstrate the effectiveness of the proposed controller using a nonlinear chemical process example.

2. PROBLEM SETUP

In this work, we consider finite dimensional discrete-time nonlinear process systems of the form

$$x(t+1) = \hat{f}(x(t), u(t), w(t)) \tag{1}$$

which can be further decomposed into the form

$$x(t+1) = f(x(t), u(t)) + g(x(t), w(t)) \tag{2}$$

In Equations 1 and 2, $x \in R^n$ is the system state, $u \in R^m$ is the control input and $w(t) \in R^p$ denotes the disturbance affecting the system at the current time $t \in I_{\geq 0}$. Also, $\hat{f}$ is the nonlinear system with disturbances, f is the process system without disturbances and g is the mismatch caused by the presence of disturbances. The system state and input vectors are restricted to be in the coupled non-empty compact convex sets of the form

$$(x(t), u(t)) \in Z \subseteq X \times U \tag{3}$$

and the disturbance vector is bounded, i.e. $w(t) \in W$ for all $t \in I_{\geq 0}$. The sets U and W are required to contain the origin in their interior. We assume that the functions $f : R^n \times R^m \to R^n$ and $g : R^n \times R^p \to R^n$ are continuous.

The primary control objective is to find a stabilizing feedback control law $u(t) = \mu(x(t))$ that renders the closed loop system (2) feasible and minimizes the average economic cost over the infinite horizon T

$$\limsup_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \ell_e(x(t), u(t)) \tag{4}$$

where $l_e : R^n \times R^m \to R$ is a general economic cost function which may not necessarily be quadratic and positive definite with respect to an equilibrium point.

As a result of the presence of the disturbances, it is in general difficult to determine the control inputs. One conceptual approach to determine the optimal feedback control law is to solve a min-max optimization problem (Mayne (2016)). While this approach is conceptually appealing, it suffers from high computational demand which makes it impractical to implement. In this paper, we propose an EMPC scheme based on zone tracking and the nominal model. The zone to be tracked can be considered as an economic trust region where the controller ensures that the system states stay inside all the times. This makes our proposed approach similar to other trust-region based approaches such as the Lyapunov-based EMPC techniques (Heidarinejad et al. (2012)). However, our formulation introduces economic risk factor in the controller design thus implicitly considers an upper bound on the asymptotic average performance of the closed-loop system.

3. ROBUST EMPC FRAMEWORK

In this section, we present the proposed EMPC algorithm that tracks an economic zone to ensure that the dynamics of (2) is restricted to an economic zone whenever possible and within the zone optimize the process economics. Thus, we transform the EMPC problem to tracking an economically viable zone (irrespective of the disturbances).

Let us first assume that such an economic zone has been created and is denoted as X_e. The procedure to create such an economic zone will be discussed in Section 4. We first define (x_s, u_s) as the economically optimal steady state in the target economic zone X_e. That is

$$(x_s, u_s) = \arg\min \ \ell_e(x, u) \tag{5a}$$
$$s.t. \quad x = f(x, u) \tag{5b}$$
$$(x, u) \in X_e \times U \tag{5c}$$

Without loss of generality, we assume that (x_s, u_s) uniquely solves the steady state optimization problem.

3.1 Robust Economic MPC formulation

With information about the current state $x(t)$, our proposed controller uses the nominal model

$$z(k+1) = f(z(k), v(k)), \quad z(0) = x(t), \quad k = 0, \ldots, N-1 \tag{6}$$

to find a control sequence $\mathbf{v} = \{v(0), \ldots, v(N-1)\}$ and associated state sequence $\mathbf{z} = \{z(0), \ldots, z(N)\}$ that minimizes the cost function

$$V_N(x(t), \mathbf{v}) = \sum_{k=0}^{N-1} \ell_e(z(k), v(k)) + \ell_z(z(k)) \tag{7}$$

over the prediction horizon of N time steps. Here, $z(t) \in X \subseteq R^n$ and $v(t) \in U \subseteq R^m$ are the nominal state vector and computed control input vector respectively. Also, ℓ_z is a zone tracking penalty term which is defined as:

$$\ell_z(z) = \min_{z^z} \quad c_1(\|z - z^z\|_1) + c_2(\|z - z^z\|_2^2) \tag{8a}$$
$$s.t. \quad z^z \in X_e \tag{8b}$$

with $c_1 \in R_{\geq 0}$, $c_2 \in R_{\geq 0}$ being weights on the l_1 norm and the squared l_2 norm respectively, and z^z is a slack variable. The zone tracking cost characterizes the deviation of the system states from the zone. Thus, our EMPC optimization problem is a multi-objective optimization problem which seeks to first minimize the deviation of the system's states from X_e as well as minimize the economic objective. Once the system states enter the economic zone, the zone tracking penalty vanishes and the economic cost is optimized. To ensure that the zone is given a higher priority, large weights on the zone penalty are used. The presence of the l_1 norm ensures that the economic zone is exactly tracked.

At each sampling time, the following dynamic optimization problem $\mathcal{P}_N(x(t))$ is solved:

$$\min_{\mathbf{v}} \quad V_N(x(t), \mathbf{v}) \tag{9a}$$
$$s.t. \quad z(k+1) = f(z(k), v(k)), \quad k = 0, \ldots, N-1 \tag{9b}$$
$$z(0) = x(t) \tag{9c}$$
$$z(k) \in X, \quad k = 0, \ldots, N-1 \tag{9d}$$
$$v(k) \in U, \quad k = 0, \ldots, N-1 \tag{9e}$$
$$z(N) = x_s \tag{9f}$$

In the optimization problem above, Equation 9c is the initial state constraint, Equation 9f is a terminal equality constraint and Equations 9d and 9e are the constraints on the state and inputs respectively.

Remark 1. Our proposed robust EMPC formulation is not restricted to only pointwise terminal constraints. The terminal equality constraint in (9) can be relaxed in the current formulation to a much larger terminal set which is robust control invariant. However, if steady state operation is the best operating strategy for the process, then appropriate terminal cost and terminal set that ensures closed-loop stability needs to be obtained. For ease of exposition, we opted for pointwise terminal constraint in our formulation.

The solution of $\mathcal{P}_N(x(t))$ denoted $\mathbf{v}^*$ gives an optimal value of the cost $V_N^0(x(k))$ and at the same time $u(t) = v^*(0)$ which is injected into (2). The prediction horizon is shifted forward by one sampling time once information about $x(t+1)$ is known and the optimization problem $\mathcal{P}_N(x(t+1))$ is solved to find $u(t+1)$.

An input sequence $\mathbf{v}$ is termed feasible for initial state $x(t)$ if the corresponding state sequence $\mathbf{z}$ generated by the nominal system $z(k+1) = f(z(k), v(k))$ with initial condition $z(0) = x(t)$ together satisfy the constraints of the optimal control problem. We denote the feasibility region of (9) by Z_N i.e.

$$Z_N = \{(z(0), \mathbf{v}) | \exists z(1), \ldots, z(N) : z(k+1) = f(z(k), v(k)) \in Z, \forall k \in I_0^{N-1}, z(0) = x(t), z(N) = x_s\}$$

The projection of Z_N onto R^n is defined as the set of admissible states X_N i.e.

$$X_N = \{z | \exists u \in U : (z, u) \in Z_N\}$$

3.2 Recursive Feasibilty

While the use of only the nominal model makes our proposed robust EMPC formulation computationally attractive, the presence of disturbances may compromise the feasibility of the optimization problem of (9). This must therefore be addressed. It is easy to show that the optimization problem (9) is recursively feasible for the nominal system (see Liu and Liu (2018)). However, since the disturbances are not considered in the optimization problem, constraint satisfication of the initial state is not guaranteed even if the initial optimization problem is feasible. Hence the problem may become infeasible at some point in time. Since infeasibility is caused by the presence of hard state constraints, one approach is to relax them to soft constraints as done in Yang et al. (2015). This is the approach we use in this work to ensure feasibility of the optimization problem at all times $t \in I_{\geq 0}$.

4. ECONOMIC ZONE

In the previous section, we have presented a robust EMPC formulation with a ficticious zone objective. Thus, the ability of our proposed controller to perform as expected hinges on effective determination of the economic zone X_e to be tracked. The question on how to approapriately select the economic zone still remains. In this section, we introduce the concept of risk factor in the controller design and present an algorithm for determining the economic zone X_e for our proposed controller.

4.1 Risk Factor

While the proposed controller is general, one has to determine an economic zone which ensures stability without compromising on the process economics. To proceed with the discussion, let us recall the following definitions in set invariance theory (Blanchini (1999)):

Definition 2. (Positively invariant set). A set $\Omega \subset X$ is said to be a forward or positively invariant set of the system $x^+ = f(x)$ if for every $x \in \Omega$, $f(x) \in \Omega$.

Definition 3. (Robust control invariant set). A set $\Omega \subset X$ is said to be a robust control invariant set (RCIS) for system (1) and constraint set (3) if for every $x \in \Omega$, there exist a feedback control law $u = \mu(x) \in U$ such that Ω is forward invariant for the closed-loop system $\hat{f}(x, u, w)$ for all $w \in W$.

The idea is to determine the economic zone as a subset of the robust control invariant subset of the state space X with desired economic features. This way, not only will tracking the economic zone ensure the process operates with desired economic performance, but it will ensure stability of the process system. To this end, we introduce the concept of risk factor $\delta \in R$ in the determination of the economic zone. The risk factor is a scalar tuning parameter which determines the size of the economic zone X_e. This intend determines the conservativeness of the controller. The larger the value, the larger the size of X_e and the less conservative the controller and vice versa. This implies that the higher the value, the more risky decisions we allow the controller to make.

4.2 Economic Zone Determination

The algorithm for determining the economic zone builds on the graph-based robust control invariant set algorithm developed by Decardi-Nelson and Liu (2021). In the algorithm, the state space is quantized with the help of finite covering, $\mathcal{C} = \{B_1, \ldots, B_l\}$, of the state space X. The finite covering $\mathcal{C}$ is a collection of closed sets known as cells or boxes $B_i, i = 1, \ldots l$, such that

$$X \subseteq \cup_{B_i \in \mathcal{C}} B_i \tag{10a}$$

$$B_i \cap B_j = \emptyset, \ \forall B_i, B_j \in \mathcal{C} \text{ with } i \neq j \tag{10b}$$

Following the quantization, the system dynamics is approximated using a directed graph. Graph investigations are then carried out on the directed graph to determine the cells that make up the largest robust control invariant set. We denote by $\mathcal{C}_r$ the cells that approximate the largest RCIS. The algorithm for determining the economic zone X_e is summarized in Algorithm 1.

Algorithm 1. Determination of economic zone

1: **Input:** g, U, W, ℓ_e, $\mathcal{C}_r$, δ
2: **Output:** X_e
3: Initialize $\mathcal{C}_e$ as empty array
4: **for** B in $\mathcal{C}_r$ **do**
5:     **if** $\forall x \in B \exists u \in U : \ell_e(x + g(x, w), u) \leq \delta \forall w \in W$ **then** Add B to $\mathcal{C}_e$
6:     **end if**
7: **end for**

8: $X_e \leftarrow \cup_{B \in \mathcal{C}_e} B$
9: **return** X_e

The choice of the selection criterion in Algorithm 1 stems from the fact that every state within the robust control invariant set is a potential initial state as well as a potential end state after one time-step. We focus on the latter since our proposed controller does not consider the disturbances. Hence, for any potential end state given by the nominal system, we know that the disturbance will be applied in the real system. By considering the effect of the disturbance on the states within the economic zone, we want to guarantee that the economic performance of the closed-loop system is bounded above by the risk factor δ. It is easy to see that as the risk factor increases, the economic zone will essentially converge to the largest robust control invariant set of (1) contained in X.

5. ILLUSTRATIVE EXAMPLE

In this section, we demonstrate the efficacy of our proposed controlled using a chemical process. We describe the chemical process example used in our analysis. Subsequently, we consider the impact of the risk factor on the asymptoptic economic performance of our proposed controlled and then finally compare the performance of our proposed controller to that of tracking MPC and traditional economic MPC.

5.1 Process Description and Simulation Settings

Consider a well-mixed continuously stirred tank reactor (CSTR) where a first-order irreversible reaction of the form $A \rightarrow B$ takes place. Since the reaction is endothermic, thermal energy is supplied to the reactor through a heating jacket. Assuming constant volume reaction mixture, the following nonlinear differential equations are obtained based on energy balance and component balance for reactant A:

$$\frac{dC_A}{dt} = \frac{q}{V}(C_{Af} - C_A) - k_0 \exp(-\frac{E}{RT})C_A \tag{11a}$$

$$\frac{dT}{dt} = \frac{q}{V}(T_f - T) + \frac{-\Delta H}{\rho C_p} k_0 \exp(-\frac{E}{RT})C_A + \frac{UA}{V\rho C_p}(T_c - T) \tag{11b}$$

where C_A and T denote the reactant concentration and temperature of the reaction mixture in mol/L and K respectively, T_c denotes the temperature of the coolant stream in K, q denotes the volumetric flow rate of the inlet and outlet streams of the reactor in L/min, C_{Af} denotes the concentration of reactant A in the feed stream, V denotes the volume of the reactor, k_0 denotes the reaction rate pre-exponential factor, E denotes the activation energy, R is the universal gas constant, ρ is the density of the reaction mixture, T_f is the temperature of the feed stream, C_p is the specific heat capacity of the reaction mixture, ΔH is the heat of reaction and UA is the heat transfer coefficient between the cooling jacket and the reactor. The values of the parameters used in the simulations are listed in Table 1.

Table 1. Table of parameter values

Parameter	Unit	Value
q	L/min	100.0
V	L	100.0
c_{Af}	mol/L	1.0
T_f	K	350.0
E/R	K	8750.0
k_0	min^{-1}	7.2×10^{10}
$-\Delta H$	J/mol	5.0×10^4
UA	$J/min \cdot K$	5.0×10^4
c_p	$J/g \cdot K$	0.239
ρ	g/L	1000.0

The control objective is to minimize the concentration of reactant A (i.e. maximize the concentration of reactant B) in the reactor while keeping the temperature of the reactor within $348K$ and $352K$. Thus, the cost function is multi-objective and is given by

$$\ell_e(x, u) = C_A + \begin{cases} 10 \times (348.0 - T)^2 & \text{if } T < 348.0 \\ 0 & \text{if } 348.0 \leq T \leq 352.0 \\ 10 \times (352.0 - T)^2 & \text{if } T > 352.0 \end{cases} \tag{12}$$

This can be achieved by manipulating the temperature of the coolant T_c.

Remark 4. Our proposed robust EMPC formulation is not restricted to only process control problems with zone objectives. The proposed controller is applicable to process control problems without a zone objective as well. It is only a coincidence that our example already has a zone objective.

The nonlinear model of (11) is discretized using a step-size $h = 0.1$ min to obtain a discrete-time nonlinear state space model of the form

$$x(t+1) = f(x(t)) + g(x(t))u(t) + h(x(t))w(t) \tag{13}$$

where $x = [C_A, T]^T$ is the state vector, $u = T_c$ is the input and $w = [C_{Af}, T_f]^T$ is the disturbance vector. The input and disturbances are subject to the following constraints: $285.0 \leq u \leq 315.0$, $0.9 \leq w_1 \leq 1.1$ and $348.0 \leq w_2 \leq 352.0$. The disturbances are assumed to be uniformly distributed in the constraints.

In the simulations, unless otherwise stated, the control horizons and sampling times of all controllers are $T = 20$ and $h = 0.1$ respectively. The values of c_1 and c_2 were both set at 1000.0. We assume that all the system states are measureable and that the optimal input can be computed in one sampling time. To ensure that the comparison is fair, the simulation is run for 10000 time steps. The initial state for the simulations is fixed at $[0.5, 350.0]^T$. The models used in both the plant and the controller are the discretized versions of (11).

5.2 Effects of Risk Factor

We first investigate the effects of the risk factor on the closed-loop system. This was conducted by determining the economic zones for different risk factors. This is shown in Figure 1. As can be seen, the size of the economic zone increases as the risk factor increases. This is due to the fact that by increasing the risk factor, more states are allowed to form part of the economic zone which agrees well with the cell selection criterion in Algorithm 1.

Following this we compare the closed-loop performance of the system with different economic zones or risk factors and their corresponding average economic performance

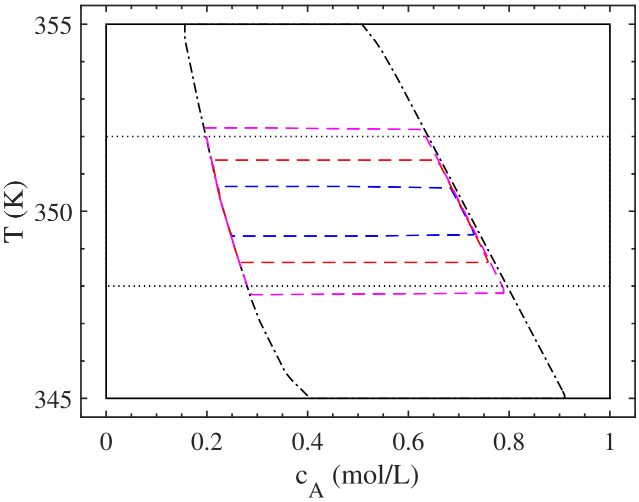

Fig. 1. Effect of risk factor on the economic zone. Hard constraints (black solid), largest robust controlled invariant set (black dash-dot), economic zones (magenta: $\delta = 50$, red: $\delta = 20$, blue: $\delta = 10$), original zone (black dots)

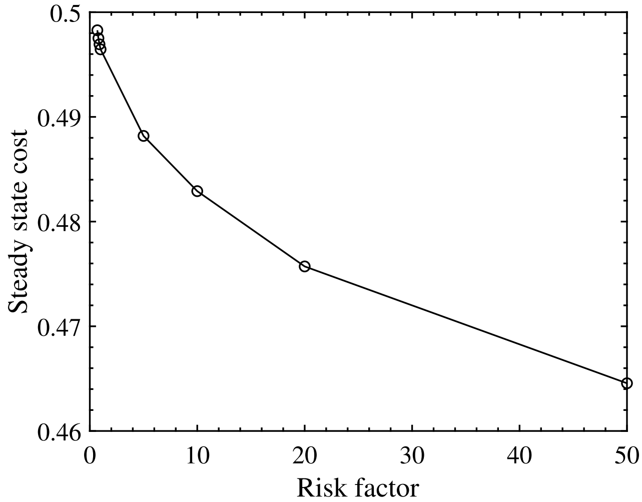

Fig. 2. Effect of risk factor on the best steady state cost in the economic zone

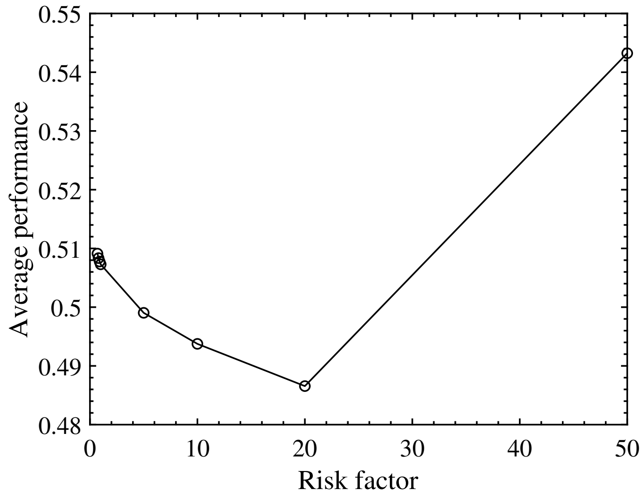

Fig. 3. Effect of risk factor on the asymptotic average performance of our proposed controller

values. This is shown in Figures 2 and 3. It can be seen in Figure 2 that the best steady state cost in the economic zone decreases as the risk factor increases. This because while the effect of uncertainties is implicitly considered in the economic zone, its effects is not considered in the steady state optimization. Hence, as expected in economic terms, the higher the risk taken, the better the potential yield. Therefore, only the best potential benefit is obtained.

However, the closed-loop performance is very different from the best steady state results. It can be seen in Figure 3 that as the risk factor increases, the average closed-loop performance decreases up to a certain risk factor where it increases sharply. This is due to the fact that the effect of the disturbances are being considered in the closed-loop system. This conforms well in economic terms since the higher the risk taken the greater the potential for economic gain or loss. Thus, for different set of disturbance sequence, it is possible to obtain a very high gain. A lower risk value therefore determines how conservative a controller should be. This shows that our proposed controller should be carefully tuned to ensure that the risk is not too large to result in poor average economic performance. One way to make better decision about the risk is to incorporate the probability distribution of the disturbances into the selection of the risk factor.

5.3 Comparison with Tracking MPC and EMPC

Following the analysis of the effects of the risk factor on the controller performance, we compare our proposed controller with risk factor of 20.0 to that of conventional EMPC and tracking MPC. The tracking MPC is made to track the best steady state point with respect to the economic objective without the zone tracking terms. The steady state values with and without the economic zone are $(x_s, u_s) = ([0.476\ 351.367]^T, 299.592)$ and $(x_s, u_s) = ([0.465\ 352.000]^T, 299.412)$ respectively. The tracking MPC quadratic cost is given as follows

$$\ell(x,u) = |x - x_s|_Q + |u - u_s|_R \qquad (14)$$

where $Q = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$ is the weight on the states and $R = 0.0$ is the weight on the input. The selected tuning parameters

The results of the comparison is shown in Table 2. As can be seen, our proposed controller gave a lower average economic cost compared to the tracking and the conventional EMPC. To understand why this is so, Figure 4 has been provided. Figure 4 shows the state, input and economic trajectories of the closed-loop system under the three controllers. It can be observed that our proposed EMPC forces the system to operate at a temperature below the $352.0K$ thus allowing room for the disturbances to occur without any significant effects on the economics. This results in a fairly stable process economics. The two other controllers on the other hand, does not consider the disturbances and therefore operate close to $352.0K$. Thus, the effects of the disturbances causes the system to operate in an expensive zone which results in a much higher cost.

Table 2. Average economic cost for the different controllers

Controller	Average cost
Tracking MPC	0.53149
Conventional EMPC	0.53153
Proposed EMPC	0.47143

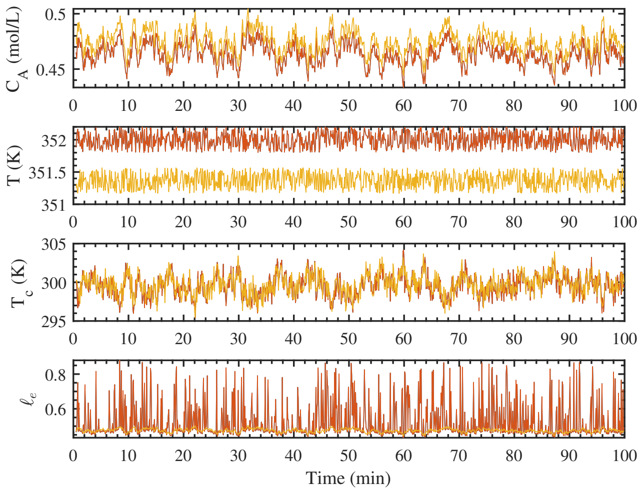

Fig. 4. State, input and economic cost profiles of the CSTR process under tracking MPC (blue), conventional EMPC (red) and our proposed EMPC (yellow)

6. CONCLUDING REMARKS

In this work, we have proposed a robust economic model predictive control framework for general nonlinear systems which essentially ensures that the asymptotic average performance of the closed-loop system does not deteriorate in the presence of disturbances. They key idea is to bind the operation of the system under the proposed controller in a subset of the state space which is not only robust control invariant but also has desired economic properties. To achieve this, a ficticious zone tracking term is added to the economic objective resulting in a multi-objective optimization problem. With a large penalty on the zone, the controller seeks to keep the states of the system in the economic zone whenever possible and within the zone, optimize the process economics. The concept of risk factor is introduced in the controller design and algorithm for determining the economic zone is also presented. The simulation results demonstrate the efficacy of our propopsed approach.

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