Optimal Energy Management of Hybrid Power System with Two-Scale Dynamic Programming

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Abstract—Hybrid power system (HPS) is the power system consists of renewable energy sources and traditional energy sources used together to increase system efficiency and reduce operation cost. Energy management is one of the main issues in operating the HPS, which needs to be optimized with respect to the current and future change in generation, demand, and market price, particularly for HPS with strong renewable penetration. Optimal energy management strategies such as dynamic programming (DP) may become significantly suboptimal under strong uncertainty in prediction of renewable generation and utility price. In order to reduce the impact of such uncertainties, a two-scale dynamic programming scheme is proposed in this study to optimize the operational benefit based on multi-scale prediction. The proposed idea is illustrated with a simple HPS which consists of wind turbine and battery storage with grid connection. The system is expected to satisfy certain load demand while minimizing the cost via peak-load shaving. First, a macro-scale dynamic programming (MASDP) is performed for the long term period, based on long term ahead prediction of hourly electricity price and wind energy (speed). The battery state-of-charge (SOC) is thus obtained as the macro-scale reference trajectory. The micro-scale dynamic programming (MISDP) is then applied with a short term interval, based on short term-hour ahead auto-regressive moving average (ARMA) prediction of hourly electricity price and wind energy. The nodal SOC values from the MASDP result are used as the terminal condition for the MISDP. The simulation results show that the proposed method can significantly decrease the operation cost, as compared with the single scale DP method.

Index Terms: Hybrid Power Systems, Energy Management, Dynamic Programming, Wind Energy, Battery Storage

I. INTRODUCTION

Renewable energy, such wind, solar, hydro, geothermal, biomass among others, has become a critical aspect of development for power generation due to the concern for energy and environmental sustainability. The major disadvantage in renewable generation is their nature of uncertainty and intermittency. In order to meet the load demand for all time, it is common for renewable generation systems to be integrated with conventional energy sources (e.g. diesel generator) and storage (e.g. battery, ultra-capacitor, and compressed air), or the utility grid. Such

system configuration is well known as the hybrid power system (HPS). Figure 1 illustrates a simple HPS with wind generation, battery storage and grid connection.

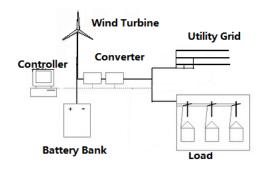


Fig. 1. Illustration of Hybrid Power System

Design and operation of the HPS has been studied from different aspects, such as optimal sizing of subsystems [1] [2], power quality control [3] [4], and energy management [5] [6]. Energy management involves optimization of the energy flow among individual components within a given period such that the operating cost can be minimized. For example, a hierarchical fuzzy based optimization method was proposed in [5] to manage energy flows for a wind-solar power system. In [6], a self-optimization method was developed with multiobjective and discrete optimization approaches for a hybrid energy storage system. In [7], battery memory effect is considered when making control strategy for hybrid power systems. In addition to the knowledge of components/ subsystems behavior (models), a successful scheme of energy management often relies on prediction of renewable generation (e.g. wind and solar), load demand, and market utility price. However, accuracy of these predictions are usually duration dependent, i.e. the longer the prediction window, the worse the prediction accuracy. From the prediction quality standpoint, only the short-term prediction should be used, however, due to the relatively slow dynamics and operating capacity of certain HPS components, e.g. storage units, short-term optimization thus obtained could be significantly suboptimal for a longer time period, e.g. diurnal.

In this paper, a two-scale dynamic optimization approach is applied to the HPS energy management problem, which aims to balance the conflicts between the long term and short term optimization for HPS, and to reduce the impact of uncertainty in the relevant predictions. The idea originated from a recent work on the trip based power management for plug-in hybrid electric vehicle conducted by our group [8]. The system is expected to satisfy a specific load demand while minimizing the cost via peak-load shaving. First, a

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macro-scale dynamic programming (MASDP) is performed for the long term period, based on long term prediction of hourly electricity price and wind energy (speed). The storage setpoints (e.g. battery state-of-charge (SOC)) is thus obtained as the macro-scale reference trajectory. The micro-scale dynamic programming (MISDP) is then applied along with the actual system operation by dividing the total operational period into a number of short intervals. The micro-scale DP problem is solved by reinforcing the terminal storage setpoint as terminal condition for the relevant optimization problem. The auto-regressive moving average (ARMA) method was adopted for the prediction of hourly electricity price and wind energy. The proposed method is evaluated on a simple HPS with wind power generation, battery storage and grid connection.

The remainder of the paper is organized as follows. In Section II, the system configuration and component models are presented. The prediction of electricity price and wind power generation is discussed in Section III, Section IV presents the detailed procedure of the two-scale dynamic programming for the energy management, with the simulation result given in Section V. The paper closes in section VI with a conclusion for the whole paper.

II. SYSTEM CONFIGURATION AND MODELS

A. System Configuration and Components

A hybrid energy system usually combines the renewable (such as wind and solar) and more reliable conventional power generation sources (such as grid supported by fossil fuel power plant or deisel generator) together to enhance system reliability, power quality and operational efficiency, balancing the energy supply and minimizing the energy cost on supplying system load. Storage units are used to reinforce power availability, power quality and to reduce the net operational cost via peak load shaving.

In this study, the HPS used as an illustrative example consists of a 10 kW wind turbine generator, a 200 Ah lithium-ion battery pack with connection to electricity grid. The HPS is used to satisfy the power need for a residential building.

The wind power generation is calculated as:

$$\overline{P_{w}} = \rho \left[\frac{2}{3} D \right]^{2} \overline{U}^{3} \tag{1}$$

where $\overline{P_w}$ is the average wind power, ρ is the air density, D is the rotor diameter, and \overline{U} is the average wind speed in a time period. The rotor diameter is 8m and the efficiency of the wind turbine is assumed to be 80%.

Lithium-ion (Li-ion) has been considered as a good choice for energy storage, since it has high energy density, high specific energy and no memory effect. Rechargeable Li-ion battery packs have been widely used for portable applications, hybrid vehicles, and more recently well attended for energy storage of renewable generation. The simplified Li-ion battery bank model can be described as a nonlinear circuit. Regardless temperature, the state of charge (SOC) of the

battery can be calculated by

$$SOC(t) = \frac{1}{C} \int_{0}^{t} i(\tau)d\tau + SOC_{ini}$$
 (2)

where C is the battery pack capacity. SOC_{ini} is the initial SOC, and i(t) is the current of the battery, positive for charge and negative for discharge

The HPS has grid connection which implicitly relies on fossil fuel power plants. The electricity price fluctuates in a given time interval, e.g. an hour, according to the total supply and consumption of the grid. In this study, is obtained from the published data at the Midwest Independent Transmission System Operator (MidwestISO) website [9]. The HPS can trade electricity with the grid at any time; it may buy or sell electricity from or to the grid.

B. System Load

Most HPS are used to supply loads. Some standalone HPSs supplies local residential buildings, others supplies utilities, such as cell phone station. In our study, the system load is used to supply residential buildings.

C. System Controller

The main task for the HPS controller is to perform the appropriate actions to manage the energy flow with the objective of minimizing the operating cost. The HPS controller includes the following functionalilities:

- Store the generated wind energy to the battery;
- Supply the generated wind energy to the system load;
- Sell the generated wind energy to the grid:
- Buy the energy from the grid and store to the battery;
- Buy the energy from the grid and use it to supply the system load;
- Sell the energy from the battery to the grid;
- Fetch the energy from the battery and use it to supply the system load;

III. PREDICTION OF ELECTRICITY PRICE AND WIND POWER GENERATION

The dynamic optimization for HPS energy management requires the prediction of the hourly electricity price and wind power with certain period, e.g. 24 hour or 2 two 3 hours. Both electricity price and wind speed are obtained as time sequence. There have many prediction schemes developed for the prediction of the electricity price and wind speed [10][11].

In this study, the ARMA is adopted. The ARMA model has two components: autoregressive and moving average, which can be characterized by

$$X_{t} = \sum_{j=1}^{p} a_{j} X_{t-j} + \sum_{k=0}^{q} b_{k} e_{t-k}$$
 (3)

where X is the time-series data sequence, and e_t is a white noise process characterized by zero mean and variance σ . The equation states that a realization of the time-series X(t) depends on a linear combination of the past observations plus

a moving average of e_i . The model in Eq. (3) is known as an ARMA(p,q) process, where p is the order of the autoregressive process of X on itself, and q is the order of the moving-average error term.

In order to find the appropriate model order (p, q), we go through an order identification process. We try different of groups values of the order of the ARMA model, p and q, and fit the ARMA model one by one with the hourly data of the electricity price in June 2009, and then select the order p, q for which ARMA model has the best Akaikes Information Criterion (AIC) value [12]. The prediction results are shown as Fig 2 and 3.

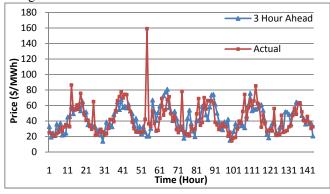


Fig.2. 3-Hour Ahead Electricity Price Prediction

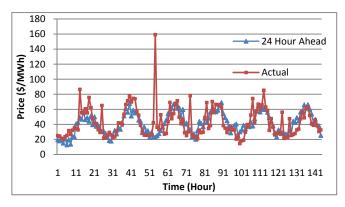


Fig. 3. 24-Hour Ahead Electricity Price Prediction

The longer the prediction window is, the larger prediction error it has. From Fig. 4 and 5, the results clearly show that shorter term prediction (3-hour period in this study) is better than long term prediction (24-hour period in this study).

Similar procedure is applied to select the wind speed forecast model with the hourly wind speed training data from June 1st through June 25th 2009 in Milwaukee, Wisconsin from Wheather Underground [13]. The prediction result of one day is shown in Fig. 4.

Because the unpredictable nature of wind, the data-driven based forecast model can have relatively good prediction for 3-hour prediction. The error gets larger for larger prediction window. For one-day ahead wind prediction, data driven methods (such as ARMA) would not work. In comparison, meteorological model based approach could give relatively more reasonable prediction [11]. Implementation of such prediction model is under way. As a

simplified treatment, we adopt the one-day ahead wind power prediction from MidwestISO for the macro-scale prediction used in the optimization process to be described in next section.

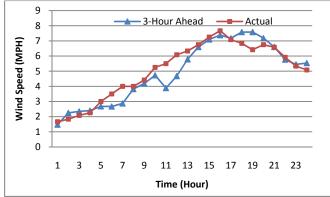


Fig.4. 3-Hour Ahead Wind Prediction of One Day in June 2010

IV. TWO-SCALE DYNAMIC PROGRAMMING BASED HPS ENERGY MANAGEMENT

A. Dynamic Programming Based HPS Energy Management

Dynamic programming is a global optimization approach for nonlinear dynamic systems. In HPS energy management, the optimal control strategy is to minimize the system cost by managing the energy flows among generation resources, storage units, loads and the grid.

In the discrete-time format, the hybrid power system model can be expressed as:

$$x(k+1) = f[x(k), u(k)] \tag{4}$$

where x(k) is the state vector of the system, such as load, incoming renewable energy, electricity, and battery SOC. Vector u(k) represent the control variables of energy flows among different components of HPS. The optimization problem relevant to the energy management of HPS is to find the control input u(k) so as to minimize the following cost function:

$$J = \sum_{k=0}^{N-1} L[x(k), u(k)]$$

$$= \sum_{k=0}^{N-1} [p(k)][u_{W2G}(k) + u_{B2G}(k) + u_{G2L}(k)]$$
 (5)

where:

- p(k) is the electricity price at time interval k.
- u_{W2G}(k) is the amount of wind power sold to grid at time interval k, which is positive because selling energy to grid will make profit..
- u_{B2G}(k) is the amount of battery power traded with grid at time interval k, which can be positive or negative. The value is positive if energy flows from battery to grid; it would be negative if energy flows from grid to battery.
- $u_{G2L}(k)$ is the amount of grid power purchased by load at time interval k, which is positive because buying

energy from grid will increase cost..

- $u_{\rm B2G}(k)$ can be both positive and negative.

Minimization of the operational cost needs to satisfy the following inequality constraints:

$$u_{W2B}(k) - u_{B2G}(k) = P_{charge}(k)$$
 (6.1)

$$u_{B2L}(k) + u_{B2G}(k) = P_{discharge}(k)$$
 (6.2)

$$u_{G2L}(k) + u_{W2L}(k) + u_{B2L}(k) = P_L(k)$$
 (6.3)

$$u_{W2G}(k) + u_{W2L}(k) + u_{W2R}(k) = P_W(k)$$
 (6.4)

$$u_{W2B}(k), u_{W2G}(k), u_{W2L}(k), u_{B2L}(k), u_{G2L}(k) \ge 0$$
 (6.5)

where

- $u_{\text{W2B}}(k)$ is the amount of wind power store to the battery at time interval k, which is positive.
- $u_{W2L}(k)$ is the amount of wind power supply to the load at time interval k, which is positive.
- $-u_{\rm B2L}(k)$ is the amount of battery power supply to the load at time interval k, which is positive
- $P_L(k)$ is the system load at time interval k.

 $P_W(k)$ is the amount of incoming wind power at time interval k, which should be used for supplying the load, storing in the battery and selling to the grid.

 $P_{charge}(k)$ is the total charging amount to the battery at time interval k., which subjects to the battery model.

 $P_{discharge}(k)$ is the total discharging amount from the battery at time interval k., which subjects to the battery model.

Equation (6.3) implies that at any time interval, the system load must be meet with the total amount of u_{G2L} , u_{W2L} and u_{B2L} . The complete battery model can be described as:

$$P_{charge}(k) = \int_{0}^{T} i(t)U_{(soc)}dt$$
 (7.1)

$$P_{discharge}(k) = \int_{0}^{T} i(t)U_{(soc)}dt$$
 (7.2)

$$P_{discharge}(k) = \int_{0}^{T} i(t)U_{(soc)}dt$$
 (7.3)

$$I_{Disch \arg eMax} \le i(t) \le I_{Ch \arg eMax}$$
 (7.4)

$$SOC_{\min} \le SOC(k) \le SOC_{\max}$$
 (7.5)

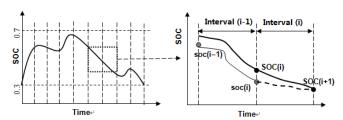
where i(t) is the battery current. It will be positive when the battery is being charged, and negative when the battery is being uncharged. Due to the bus limitation, the charging and discharging current have their maximum limitation. In our system, both the maximum charging and discharging current are 100 A. SOC(k) is the battery state of charge at time k, and subscripts min and max refer to the minimum and maximum value of the relevant variables, respectively. In our study, the SOC_{min} is 0.3 and SOC_{max} is 0.7.

An effective way to solve the above cost function numerically is to do the quantization and interpolation. For continuous state space and control space, the state and control values are first discredited into finite grids. At each step of the optimization search, the cost function $J_k[x(k)]$ is evaluated only at the grid points of the state variables. If the next state x(k+1) does not fall exactly on a quantized value, then the value of $J_k[x(k+1)]$. At each step, the backward DP with interpolation method was used.

B. Two-Scale DP for HPS Energy Management

Both of electricity price and incoming wind power for each future time interval must be predicted in order to perform optimization by dynamic programming for the future operation. As discussed in section III, the long term ahead prediction of both electricity price and incoming wind power is not enough reliable because of the large prediction window. Optimization based on long term prediction always produce sub-optimal results. On the other hand, although the short term prediction is more reliable, however, due to the relatively slow dynamics and operating capacity of certain HPS components, e.g. storage units, short-term optimization thus obtained could be significantly suboptimal for a longer time period, e.g. diurnal.

In order to balance the conflicts between the long term and short term optimization, a two-scale dynamic optimization approach is applied to the HPS energy management problem. This two-scale dynamic programming includes two dynamic programming: long term ahead macro-scale dynamic programming (MASDP) and short term ahead micro-scale dynamic programming (MISDP).



(a) Macro-scale Optimal SOC Profile (b)Zoomed View of Two Time Intervals

Fig. 5. Illustration of MASDP and MISDP

The macro-scale dynamic programming (MASDP) firstly does long term optimization based on long term prediction of incoming wind power and the hourly electricity price. After that, the long term operation of the energy flows management is obtained. The battery SOC reference trajectory of the macro-scale optimization is shown in Fig. 5(a).

Then, the long term is divided into a number of equal-distance time intervals. When the time get close to a time interval, short term ahead electricity price and incoming wind power is predicted. Then, the micro-scale dynamic programming (MISDP) is applied based on the short term prediction results. As shown in Fig. 5(b), assume that the system is about to complete the operation in the $(i-1)^{-th}$ time interval, and the operation for the i^{-th} time interval is needed. The bold solid line indicates the long term optimal SOC profile obtained by MASDP, denoted as upper case notation "SOC(i)", while the actual SOC history in the $(i-1)^{-th}$ time interval is shown as the thin solid line, denoted as the lower case notation "Soc(i)." For the DP based energy management

algorithm for the i^{th} interval, the actual value of battery state-of-charge soc(i) will be used as the initial value, and the SOC(i+1), the battery state-of-charge value at the end of the time interval in the macro-scale optimal profile, will be used as the terminal value. The dynamic programming algorithm is then applied based on the short-term operation for the time interval i. The optimal energy management strategy will result in the predicted SOC profile as the bold dashed line in Fig. 5(b). As the system runs, such a process will be repeated for each time interval in sequence. Thus, with the relative reliable short term prediction results, the optimization of MISDP will result a better operation strategy reinforcing the long term operation based on MASDP.

C. Long-Term Ahead Macro-Scale Dynamic Programming

The long-term macro-scale dynamic programming does optimization based on long-term hourly prediction of incoming wind power and the hourly electricity price within the long term range. For the MASDP, the cost function (5) has to be rewritten to:

$$J = \sum_{k=0}^{N-1} p_{macro}(k) [u_{W2G}(k) + u_{B2G}(k) + u_{G2L}(k)]$$
 (8)

where $\stackrel{\wedge}{p}_{macro}(k)$ is the long term ahead predicted electricity price at time interval k.

Besides, one constraint (6.4) should be rewritten to:

$$u_{W2G}(k) + u_{W2L}(k) + u_{W2R}(k) = \hat{P}_{W-macro}(k)$$
 (9)

where $\hat{P}_{W-macro}(k)$ is the long term ahead predicted incoming wind power at time interval k.

In this study, the macro time range is 24 hours. The prediction model has been introduced in section II, where the ARMA model is used to predict 24-hour ahead electricity price. The 24-hour ahead wind power prediction is referred from MidwestISO.

D. Short Term Ahead Micro-Scale Dynamic Programming

This short term ahead micro-scale dynamic programming is based on the short time ahead predictions of electricity price and wind speed, to do optimization on each time interval. Within each time interval, the cost function which we want to minimize should be rewritten as:

$$J_{Micro} = \sum_{k=0}^{T-1} [p_{micro}(k)][u_{W2G}(k) + u_{B2G}(k) + u_{G2L}(k)]$$
 (10)

where T is the time duration within one time interval, $p_{micro}(k)$ is the short term ahead predicted electricity price at time interval k. Besides, the constraint (6.4) is rewritten as

$$u_{W2G}(k) + u_{W2I}(k) + u_{W2R}(k) = \stackrel{\wedge}{P}_{W-micro}(k)$$
 (11)

where $\hat{P}_{W-micro}(k)$ is the short term ahead predicted incoming wind power at time interval k. In our study, the time period for the micro-scale DP is set to be 3 hours.

V. SIMULATION RESULT

The proposed idea is evaluated with the wind and electricity price data obtained on June 26th 2010. The hourly end-user electricity usage in California, Fig 6, is used as example for illustrative purpose [14].

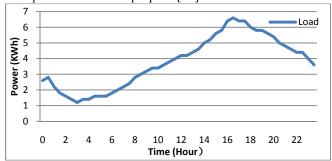


Fig.6. Hourly Electricity Load for Residential Building

The electricity price prediction of 3-hour ahead, 24-hour ahead and actual electricity price is shown in Fig. 7. The predicted wind energy, which is calculated from predicted wind speed by Eq. (1), is shown in Fig. 8.

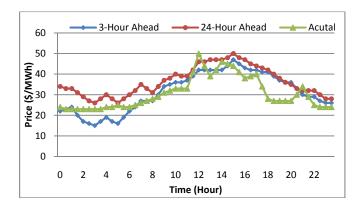


Fig.7. Hourly Electricity Price Predictions of 3-Hour Ahead, 24-Hour Ahead and Actual Electricity Price

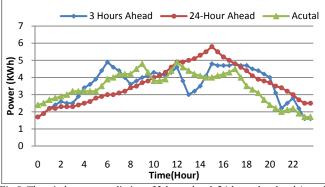


Fig. 8. The wind energy prediction of 3-hour ahead, 24-hour ahead and Actual incoming wind energy

The optimal system operation is calculated from section IV. The battery SOC of MASDP and MCSDP is shown in Fig. 9

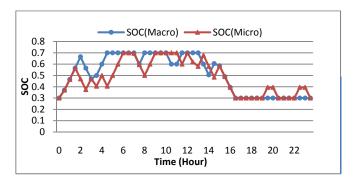


Fig.9. The Battery State of Charge of Macro SOC and Micro SOC

Table I shows the finally total daily cost using different methods for the above simulation case, along with four other cases. Notice that the "Non-optimal" column corresponds to a simply method that the HPS simply satisfies the load demand at each time instant, with no optimization with regard to past and/or future system behavior. It will buy energy from grid if wind energy does not meet the load, and it will sell extra wind energy to the grid after feed the load. The "True Optimum" case is the result based on application of dynamic programming to the wind/utility profile, as if such information is perfectly available to the system. The "24-hour ahead Macro DP" indicates the result of applying the control policy derived via DP based on 24-hour ahead prediction to the actual wind/utility profiles. The difference between this column and that of the "True Optimum" is clearly shown, which is due to the inaccuracy of prediction.

The "Two-scale DP" results show improvement in cost reduction, with the last column showing the relative saving in percentage. The improvement is clearly shown, with the average of 13% and standard deviation of 8.37%. The variation is due to the varying discrepancy between macro-scale (24-hour ahead) and micro-scale (3-hour ahead) prediction. Although better assessment would be possible with longer term evaluation, the effectiveness of the proposed scheme is clearly demonstrated.

Table I
The total daily cost of different methods

	Non-	"True"	24-hour	Two	Relative
Case	Optimal	Optimum	ahead	scale	Saving
			Macro DP	DP	(%)
1	\$0.622	\$0.398	\$0.475	\$0.450	5.56
2	\$0.658	\$0.340	\$0.595	\$0.569	4.57
3	\$0.367	\$0.046	\$0.372	\$0.332	12.05
4	\$0.387	\$0.032	\$0.380	\$0.307	23.78
5	\$1.080	\$0.220	\$0.925	\$0.777	19.05

VI. CONCLUSION

This paper proposes a two-scale dynamic programming method for the energy management of the hybrid energy system. First, a macro-scale dynamic programming is performed for the long term period, based on long term ahead prediction of hourly electricity price and wind energy. The battery state-of-charge is thus obtained as the macro-scale reference trajectory. The micro-scale dynamic programming is then applied within a short term interval, based on short term-hour ahead ARMA prediction of hourly electricity price and wind energy. The nodal SOC values from the macro planning result are used as the terminal condition for the micro planning. The proposed method is tested on the designed HPS. The simulation results show that the proposed method can significantly decrease the operation cost, compared with the 24-hour ahead Macro DP method.

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