

# Active Fault Diagnosis for Hybrid Systems Based on Sensitivity Analysis and EKF

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**Abstract**—An active fault diagnosis (AFD) approach for different kinds of faults is proposed. The AFD approach excites the system by injecting a so-called excitation input. The input is designed off-line based on a sensitivity analysis in order that the maximum sensitivity for each individual system parameter is obtained. Using the maximum sensitivity results in better precision in the estimation of the corresponding parameter. The fault detection and isolation is done by comparing the nominal parameters with those estimated by an extended Kalman filter. In this study, Gaussian noise is used as the input disturbance as well as the measurement noise for simulation. This method is implemented on a large scale livestock hybrid ventilation model which was obtained during previous research.

## I. INTRODUCTION

THE performance of modern control systems typically depends on a number of strongly interconnected components. Component faults may degrade the performance of the system or even result in a loss of functionality. In applications such as climate control systems for livestock buildings, this is unacceptable as it may lead to a loss of animal life. The methods for detection and isolation of component faults are either passive or active. Passive fault diagnosis (PFD), without acting upon the system decides if a fault has occurred based on observations of the system input and output. In active fault diagnosis (AFD), a diagnoser generates a so-called excitation input, which shapes the input to the system, in order to decide whether the output represents normal or faulty behaviour and if it is possible to decide which kind of fault has occurred. There are two perspectives for the benefit of AFD. The first one is to identify the faults that may be hidden due to the regulatory actions of controllers during the normal operation of the system. The second is to isolate the faults in systems with slow responses. Fault diagnosis of hybrid systems attracts the attentions of researchers because complex industrial systems involve both discrete and continuous components. Examples of AFD for linear system are in [3], [11], [12], [13] and [16]. AFD of hybrid systems has been addressed in [2], [5], [7], [6], [15] and [17]. In [17] and [7] the AFD approach is based on generating the excitation inputs online, and using model predictive control (MPC). The idea of AFD in [5] is quite different and uses selectively blocking or executing controllable events such that the fault detection is faster and more precise. In [2] the problem is addressed as a

## Nomenclature

$k, leak, c, d$	constants
$a$	opening angle of the inlets
$\Delta P_{inlet}$	the pressure difference across
	the opening area of the inlet
$i$	the zone number
$\rho$	the outside air density
$V_{ref}$	the ambient wind speed
$C_P$	the wind pressure coefficient
$H$	Height
$H_{NLP}$	NLP stands for the neutral pressure level
$P_i$	pressure inside zone $i$
$g$	gravity constant
$V_{fan}$	fan voltage of the chimney in the stable
$C_{P_{outlet}}$	the wind pressure coefficient
$T_i$ and $T_o$	temperature inside and outside the stable
$m_1, m_2$	constants,
$q_{i-1,i}^{st}, q_{i,i+1}^{st}$	stationary flows between two adjacent zones
$Q_{in,i}, Q_{out,i}$	heat transfer by mass flow
$Q_{i-1,i}$	heat exchange from zone $i-1$ to zone $i$
$Q_{i,i-1}$	heat exchange from zone $i$ to $i-1$
$Q_{conv}$	convective heat loss through the building envelope and described as
	$UA_{wall}(T_i - T_o)$
$Q_{source}$	the heat source
$\dot{m}$	mass flow rate
$c_P$	heat capacity
$F_A$	actuator faults
$N_i$	regions neighbouring region $i$
$\hat{\theta}, \theta^*, \theta_N$ and $\theta \in \mathcal{R}^l$	the estimated, true, nominal and running parameter vectors of the system
$v(k)$ and $w(k)$	disturbance and measurement noises
$y_m$ and $y$	output prediction and the measurement
$\zeta, \xi$	a white Gaussian sequence
$\sigma, \lambda$	singular and eigenvalue
$f_i$	vector fields of the state space description.
$g_i$	a known function.

discrete event system and a finite state machine is used to guide the identification. In this paper, as in [13] and [14], we design the excitation inputs for AFD in an off-line mode. A benefit of off-line input design is that the online computational efforts of the fault diagnoser can focus only on the detection/isolation problem. This benefit is considerable when the system comprises a large number of inputs. Our approach embarks from a sensitivity analysis in order to generate the inputs. Here, the amplitude and frequency of the inputs are defined such that the maximum sensitivity value for each parameter of the system is obtained. Note that it is also possible to limit the value of the input signal by defining a boundary on the signal in the sensitivity analysis problem. Shaping the input according to the sensitivity analysis allows faults in the parameters to be easily identified. Finding the highest sensitivity for each parameter is a non-convex optimization problem. In order to solve a non-convex

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optimization problem with classical approaches, it must be reformulated as a convex problem. This reformulation is possible as long as some necessary conditions are satisfied, which is not always feasible. Hence we have used a genetic algorithm (GA) to solve the problem. The excitation inputs are applied in open loop and the required parameters of the system are estimated by the extended Kalman filter (EKF). By comparing the normal with the estimated parameters of the system, different incipient and severe faults can be identified. Note that it is not desirable to disturb a system continuously, therefore at first, the abnormal behaviour of the system is observed by a common PFD method, and then the AFD approach is applied over a shorter period. The climate control problem, used in the current research, is stable in open loop mode and application of the AFD over a short period does not destabilize it. However, for systems which are unstable in open loop, stabilization guarantees should be considered in the AFD algorithm. These guarantees are provided by the satisfaction of stability constraints.

This paper is organized as follows. Section II presents the preliminaries and problem formulation. The design of the input using sensitivity analysis is discussed in Section III. Section IV is dedicated to the EKF setup. An example is presented in Section V, and the experimental setup is discussed in VI. The results are given in Section VII, while the conclusion is presented in the last section.

## II. PRELIMINARIES AND PROBLEM FORMULATION

### A. State-Input Dependent Nonlinear Switching Systems

The class of systems considered here are hybrid nonlinear systems with uncontrollable state-input dependent switching:

$$x(k+1) = f_i(x(k), u(k), k, \theta, F_A, v(k)), \text{ for } \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \in \mathcal{X}_i \quad (1)$$

$$y_m(k) = Cx(k) + F_s + w(k), \quad (2)$$

where  $F_A$  and  $F_s$  are the actuator and sensor faults,  $u(k) \in \mathbb{R}^m$  is the control input and  $x(k) \in \mathbb{R}^n$  is the state, and  $y_m(k) \in \mathbb{R}^p$  is the output. All variables are at time  $k$ , the sets

$$\mathcal{X}_i \triangleq \{ [x(k)^T u(k)^T]^T \mid \mathbf{g}_i(x, u) \leq K_i, i = 1, \dots, s \} \quad (3)$$

are manifolds (possibly un-bounded) in the state-input space,  $\theta \in \mathcal{R}^l$  is the parameter vector,  $v(k)$  and  $w(k)$  are the disturbance and measurement noise respectively,  $f_i$  are vector fields of the state space description, and  $\mathbf{g}_i$  is a known function. Here, it is assumed that the hybrid system is continuous:

$$f_i(x_*(k), u_*(k)) = f_j(x_*(k), u_*(k)) \quad j \in N_i \quad (4)$$

where  $(x_*(k), u_*(k))$  are the sampling points corresponding to the boundary between two neighbouring regions and  $N_i$  is the region neighbouring region  $i$ . Here, only the actuator fault is considered; however, we believe that it is also possible to

detect the sensor fault by this parameter estimation technique and this will be considered in future work.

### B. General Problem of AFD and Main Work

In the current research, the system parameters are related to the actuators. Assume that a faulty actuator is used rarely during the normal operation of the system, and hence has little effect on the system response. Consequently, its parameter is not identified correctly and a fault is not detected. In order to detect correctly the faulty behaviour of the system, a sequential input signal over a finite time interval is applied to the system. At the end of the interval, a fault isolation algorithm is executed to isolate the fault. Excitation of the system by the input leads the actuator to affect the system response; therefore the parameter may be estimated more precisely and the fault becomes observable. The main work is separated into two parts:

- 1) Design of the excitation input, off-line and relying on so-called sensitivity analysis in order that the maximum sensitivity for each individual system parameter is obtained.
- 2) Deriving the fault isolation algorithm, based on estimation of the system parameters with EKF and comparing those parameters with the normal values. The values are considered as a prior knowledge of the system.

## III. DESIGN OF EXCITATION INPUT USING GA AND SENSITIVITY ANALYSIS

The goal is to design the excitation input using sensitivity analysis for more precise parameter estimation and consequently better fault isolation. To achieve this goal, first we analyse a parameter estimation algorithm based on a least mean square (LMS) method where the measurement signal includes noise, and a criterion for better estimation by the LMS algorithm in the presence of noisy signal is shown. Then the correspondence between the parameter estimation LMS algorithm and sensitivity analysis is described. Finally the excitation input signal is designed using GA and sensitivity analysis.

Let us assume that the problem is to estimate the system parameters through the following LMS approach.

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}} P(u, y, \theta, \xi) \quad (5)$$

where the performance function  $P$  is given by

$$P(u, y, \theta, \xi) = \frac{1}{2N} \sum_{k=1}^N \epsilon^2(k, u, y, \theta, \xi) \quad (6)$$

$$\epsilon(k, \theta, \xi) = y_m(k, \theta) - y(k, \xi), \quad (7)$$

where  $\xi$  is the noise signal,  $y(k, \xi)$  is the measurement signal approximated as  $y(k, \xi) = y_m(k, \theta^*, \xi)$ ,  $y_m(k, \theta^*, \xi)$  is the output of the model when it depends on the noise signal  $\xi$ , and  $y_m(k, \theta)$  is the output of the model when it does not depend on the noise signal  $\xi$ , we assume  $\xi$  is zero. Estimated, running and true parameter vectors are

represented by  $\hat{\theta}$ ,  $\theta$ ,  $\theta^*$ . In the following we omit  $u$  and  $y$  from the notation. Consider the following definitions:

$$\theta^* = \underset{\theta}{\operatorname{argmin}} P(\theta^*, 0) \Rightarrow [D_{\theta}P](\theta^*, 0) = 0 \quad (8)$$

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}} P(\hat{\theta}, \xi) \Rightarrow [D_{\theta}P](\hat{\theta}, \xi) = 0. \quad (9)$$

Let the performance function be approximated using the first and second order terms of a Taylor series expansion with respect to  $\theta$  and  $\xi$  at  $\theta^*$  and 0:

$$P(\theta, \xi) \approx P(\theta^*, 0) + [D_{\theta}P](\theta^*, 0)(\theta - \theta^*) + [D_{\xi}P](\theta^*, 0)\xi \quad (10)$$

$$+ (\theta - \theta^*)^T [D_{\theta, \theta}P](\theta^*, 0)(\theta - \theta^*) + \xi^T [D_{\xi, \xi}P](\theta^*, 0)\xi \\ + \xi^T [D_{\theta, \xi}P](\theta^*, 0)(\theta - \theta^*) + (\theta - \theta^*) [D_{\xi, \theta}P](\theta^*, 0)\xi$$

where  $D_{\theta}P = \frac{\partial P}{\partial \theta}$  and  $D_{\theta, \xi}P = \frac{\partial^2 P}{\partial \theta \partial \xi}$ . In order to derive the parameter  $\theta$  from the smooth performance function  $P(\theta, \xi)$ , we apply the partial derivative of (9) on the performance function, the result is:

$$2[D_{\theta, \theta}P](\theta^*, 0)(\hat{\theta} - \theta^*) + \xi^T [D_{\theta, \xi}P](\theta^*, 0) \\ + [D_{\xi, \theta}P](\theta^*, 0)\xi = 0 \Rightarrow \\ H(\hat{\theta} - \theta^*) = \zeta, \quad (11)$$

where  $H = [D_{\theta, \theta}P](\theta^*, 0)$ , and  $\zeta = \xi^T [D_{\theta, \xi}P](\theta^*, 0) + [D_{\xi, \theta}P](\theta^*, 0)\xi$ .  $\zeta$  is the noisy signal, thus its large error should cause a small error in  $\hat{\theta} - \theta^*$ . This means that the condition number of matrix  $H$  should be small [14]. The condition number of the matrix  $H$  is:

$$\kappa(H) = \frac{\sigma_{max}(H)}{\sigma_{min}(H)}, \quad (12)$$

where  $\sigma_{max}$  and  $\sigma_{min}$  are the maximum and minimum of the singular values of the Hessian matrix  $H$ . In fact, assuming a small value of the condition number  $\kappa(H)$ , the LMS algorithm is able to estimate the parameter of the system more precisely in the presence of the noise.

Here, we specify the importance of (12) from the sensitivity analysis point of view. According to [9], a larger value of sensitivity for parameter  $\theta$  leads to a smaller deviation of  $\theta$  from the true value  $\theta^*$  generates significant deviation in the value of  $\epsilon$ . This fact results in more precise parameters estimation, as it is obvious from (5) to (7), and as discussed in detail in [9]. For obtaining high sensitivity for the entire system parameters, the ratio of maximum to minimum sensitivity should be small, i.e.,

$$R = \frac{S_{max}}{S_{min}} = \frac{\sqrt{\lambda_{max}}}{\sqrt{\lambda_{min}}} = \frac{\sigma_{max}(H)}{\sigma_{min}(H)} \quad (13)$$

where the sensitivity is  $S = \frac{\partial \epsilon}{\partial \theta}$  and  $\lambda$  is the eigenvalue of the Hessian matrix of  $H$ . As is obvious, the ratio in (13) is equal to the condition number (12), which shows the correspondence between the sensitivity analysis and parameter estimation algorithm based on the LMS approach.

In the following, we assume the input is a sinusoidal signal and its amplitude  $\alpha$  and frequency  $f$  are designed so that the

minimum  $R$  is obtained:

$$U = \alpha \sin(2\pi ft) \quad (14)$$

$$(\alpha, f) = \underset{\alpha, f}{\operatorname{argmin}} R \quad (15)$$

$$s.t. \begin{cases} (1) \\ \alpha_{min} \leq \alpha \leq \alpha_{max} \\ f_{min} \leq f \leq f_{max} \end{cases}$$

where  $\alpha_{min}$  and  $\alpha_{max}$  are minimum and maximum values of  $\alpha$ , and  $f_{min}$  and  $f_{max}$  are the minimum and maximum values of  $f$ . In some cases, it may be necessary to consider more than one periodic signal in  $U$  for estimation of different parameters.

Equation (15) is non-convex and non-differentiable. To solve the problem with classical approaches, the problem must be changed to a convex problem by defining some constraints. Obtaining these constraints is not always feasible and is considered an open issue in the literature: see [10]. Using evolutionary search algorithms such as GA avoids having to change the problem to a convex one. As the optimization problem is calculated off-line, the computational effort is not important. The reader is referred to [4] for more details of the GA.

#### IV. THE EKF SETUP

The aim of using the EKF is to estimate the parameters after exciting the system by the designed inputs. The abnormal behaviour of the system is detected from the estimated parameters.

According to a current literature survey about Kalman filtering (KF), [1] and [18], the EKF is similar to the parameter estimation procedure using the LMS approach as in (5). Hence, the result of the sensitivity analysis for parameter estimation problems based on the LMS approach is also relevant for the EKF.

The performance of the EKF depends on the matrix  $P$ . This matrix is independent of the system inputs, when the system operating point is constant, as in the stationary Kalman filter. The EKF algorithm approximates the nonlinear system by a Taylor series expansion around an operating point for every sample instant. If the operating point changes in each sample due to the input, the covariance matrix will depend on the input. The excitation input changes the operating point such that the covariance matrix rapidly decreases to zero. However, large variations are in most cases not desirable over long periods. Hence, at first an abnormal behaviour of the system is observed by a common PFD method. Then the AFD algorithm is applied for a short interval to identify different faults and those hidden during normal operation of the system.

Fault isolation relies on a simple algorithm. The algorithm isolates the fault  $F_i$  according to the residual generator  $r_i = \hat{\theta}_i - \theta_{Ni}$ , where  $\theta_{Ni}$  is the nominal value of the  $i$ th parameter of the system, which is assumed as prior knowledge of the system, and  $\hat{\theta}_i$  is the parameter estimated by the EKF. The fault isolation algorithm is given in Table I. If  $r_i$  is greater

TABLE I  
FAULT ISOLATION ALGORITHM

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**Algorithm 1**  
For  $i = 0$  to  $l$   
  IF  $r_i = \left| \hat{\theta}_i - \theta_{Ni} \right| > \delta$   
     $F = F_i$   
  End IF  
End For

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than a predefined threshold  $\delta$ , the system is subject to the fault  $F_i$ .

## V. EXAMPLE

The AFD algorithm is applied to the climate control system of a live-stock building, which was obtained during previous research, [8]. The general schematic of the large scale live-stock building equipped with its climate control system is illustrated in Fig. 1. In a large stable, the indoor airspace is incompletely mixed; therefore it is divided into conceptually homogeneous parts called zones. Due to the indoor and outdoor conditions, the airflow direction varies between adjacent zones. Therefore, the system behaviour is represented by a finite number of different dynamic equations. The model is intended to be a realistic representation of internal temperatures for all multi-zone types of livestock buildings. The model is divided into subsystems as follows:

### A. Inlet Model

An inlet is built into an opening in the wall. The following approximated model for airflow,  $q_i^{in}$  into the zone  $i$  is used.

$$q_i^{in} = k_i(a_i + leak)\Delta P_{inlet}^i \quad (16)$$

$$\Delta P_{inlet}^i = 0.5C_P V_{ref}^2 - P_i + \rho g \left(1 - \frac{T_o}{T_i}\right)(H_{NLP} - H_{inlet}) \quad (17)$$

where  $P_i$  is the pressure inside zone  $i$ ,  $k_i$  and  $leak$  are constants,  $a_i$  is the opening angle of the inlets,  $\Delta P_{inlet}^i$  is the pressure difference across the opening area and wind effect,  $\rho$  is the outside air density,  $V_{ref}$  is the wind speed,  $C_p$  stands for the wind pressure coefficient.  $H$  stands for the height and  $H_{NLP}$  is the neutral pressure level which is calculated from

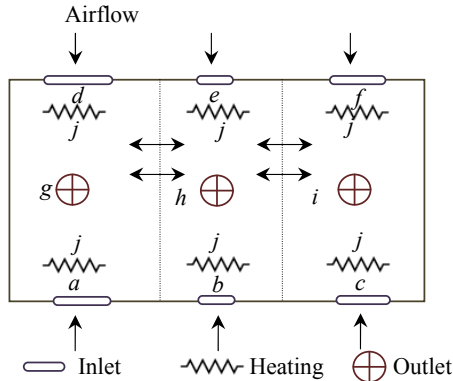


Fig. 1. The top view of the test stable

the mass balance equation.  $T_i$  and  $T_o$  are the temperature inside and outside the stable and  $g$  is the gravity constant.

### B. Outlet Model

The outlet is a chimney with an electrically controlled fan and plate inside. A simple linear model for the airflow out of zone  $i$  is given by:

$$q_i^{out} = V_{fan}^i c_i - d_i \Delta P_{outlet}^i \quad (18)$$

$$\Delta P_{outlet}^i = 0.5C_P V_{ref}^2 - P_i + \rho g \left(1 - \frac{T_i}{T_o}\right)(H_{NLP} - H_{outlet}) \quad (19)$$

$$\sum_{i=1}^3 q_i^{in} \rho \frac{\Delta P_{inlet}^i}{|\Delta P_{inlet}^i|} + \sum_{i=1}^3 q_i^{out} \rho = 0 \quad (20)$$

where  $c_i$  and  $d_i$  are constants,  $V_{fan}^i$  is the fan voltage, and the number of zones is three. The stationary flows,  $q_{i-1,i}^{st}$  and  $q_{i,i+1}^{st}$ , through the zonal border of two adjacent zones is given by:

$$q_{i-1,i}^{st} = m_1(P_{i-1} - P_i) \quad (21)$$

$$q_{i,i+1}^{st} = m_2(P_i - P_{i+1}) \quad (22)$$

$$q_{i-1,i}^{st} = \{q_{i-1,i}^{st}\}^+ - \{q_{i-1,i}^{st}\}^- \quad (23)$$

where  $m_1$  and  $m_2$  are constant coefficients. The use of curly brackets is defined by:

$$\{q_{i-1,i}^{st}\}^+ = \max(0, q_{i-1,i}^{st}), \quad \{q_{i-1,i}^{st}\}^- = \min(0, q_{i-1,i}^{st}) \quad (24)$$

### C. Modeling Climate Dynamics

The following formulation for the dynamical model of the temperature for each zone inside the stable is driven by thermodynamic laws. The dynamical model includes four piecewise nonlinear models which describe the heat exchange between adjacent zones:

$$M_i c_i \frac{\partial T_i}{\partial t} = Q_{i-1,i} + Q_{i,i-1} + Q_{i,i+1} + Q_{i+1,i} + Q_{in,i} + Q_{out,i} + Q_{conv,i} + Q_{source,i} \quad (25)$$

$$Q = \dot{m} c_p T_i, \quad Q_{i-1,i} = \{q_{i-1,i}^{st}\}^+ \rho c_p T_{i-1}, \quad (26)$$

$$Q_{i,i-1} = \{q_{i-1,i}^{st}\}^- \rho c_p T_i$$

where  $Q_{in,i}$  and  $Q_{out,i}$  represent the heat transfer by mass flow through the inlet and outlet, and  $Q_{i-1,i}$  denotes the heat exchange from zone  $i-1$  to zone  $i$  caused by stationary flow between zones.  $Q_{conv}$  is the convective heat loss through the building envelope,  $Q_{source,i}$  is the heat source,  $\dot{m}$  is the mass flow rate,  $c_i$  is the heat capacity, and  $M$  is the mass of the air inside zone  $i$ .

For the EKF, the state space model must be augmented by

the parameter dynamics, i.e.:

$$\dot{X} = \begin{bmatrix} \dot{T} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} f_j(T, U, q, \theta) + v \\ 0_{l \times 1} \end{bmatrix} \quad \text{for } \begin{bmatrix} T \\ U \end{bmatrix} \in \mathcal{X}_j \quad (27)$$

$$q = h_3(X, P, U, \theta) = [q_i^{in}, q_{1,2}^{st}, q_{2,3}^{st}, q_i^{out}]^T, \quad i = 1, \dots, 3 \quad (28)$$

$$h_2(P, T, U, \theta) = 0, \quad \theta = [c_1, c_2, c_3]^T, \quad (29)$$

$$U = [a_i, V_{fan}^i, Q_{heater}]^T \\ y = CT + w \quad j = 1, \dots, 4 \quad (30)$$

where  $f_j$  is dedicated to each piecewise state space model,  $h_2$  denotes the mass balance equation (20) for obtaining the indoor pressure in each zone, and  $U$  is the system input.

## VI. SIMULATION SETUP

Here, only the temperature is measured. The initial conditions are taken as follows:  $T_1 = T_2 = T_3 = 17.5$ ,  $T_o = 2$  °C,  $V_{ref} = 14$ ,  $P_1 = 5.6$ ,  $P_2 = 6$  and  $P_3 = 7$ .

Two kinds of inputs are implemented in the simulation, one designed based on sensitivity analysis and one chosen arbitrarily. Their amplitude  $\alpha$  and frequency  $f$  are given in Table II. As is seen from the table, there are ten inputs in the system. Inputs 1 to 6 belong to the angle of the inlets. The value of 0 represents a closed inlet and 1 represents a fully open inlet. Inputs 7 to 9 belong to the voltage of the fans and they change from 0 to 7. The last input belongs to the temperature of the heating system and it changes from 0 to 40. The proposed AFD approach is implemented on a simulated full scale live-stock building with a slow dynamic behaviour and a sample time of 5 minutes. In such systems, the fault is sometimes hidden during normal operation of the system due to the control actions, or the fault may influence the response of the system only very slowly. Here, the AFD approach is used for a sanity check of the actuators, such as the inlets, fans, and heating system. In the winter due to the cold weather there is no need for full time ventilation mechanism, therefore the controller closes the inlets and turns off the fans or excites them very slowly, and without AFD, it may take a long time to detect the abnormal behaviour of the actuators. In the following, the algorithm is applied to detection/isolation of fault in the fans. The procedure consists of two parts. First, the input designed off-line using sensitivity analysis is applied to the system over a time horizon  $h$  as;  $U = \{U(0), \dots, U(h)\}$ , and the parameters of the system are estimated by the EKF. Then, the residual which is the discrepancy between the normal and estimated parameters is examined at the end of the time horizon  $h$ . In order to simulate realistic conditions, two Gaussian noises with standard deviation 0.5 and 0.4 are considered as an input disturbance and measurement noise

## VII. RESULTS

The results of the AFD algorithm are illustrated in Figs. 2 and 3. In Fig. 2, the temperature of each zone and the real and estimated parameters of the fans are shown. As can be seen, the EKF tracks the fan parameters correctly before

TABLE II  
AMPLITUDE AND FREQUENCY OF THE INPUT SIGNALS

inputs	$\alpha$ with sensitivity	$f$ with sensitivity	$\alpha$ without sensitivity	$f$ without sensitivity
1,3 4,6	0.7	$10^{-3}$	0.7	$10^{-7}$
2,5 7,9	0.7 7	$2 \times 10^{-3}$ $2 \times 10^{-3}$	0.7 2	$2 \times 10^{-7}$ $0.2 \times 10^{-7}$
8 10	7 20	$0.08 \times 10^{-3}$ $0.01 \times 10^{-3}$	2 20	$0.08 \times 10^{-7}$ $0.2 \times 10^{-7}$

the occurrence of any fault. After 3.5 hours, it is assumed that fan 1 and fan 3 are stuck, and they are turned off. At first there is a considerable discrepancy between the estimate and the real values, then this discrepancy decreases quickly, indicating that the algorithm is able to detect that fan 2 is in a healthy condition and the other fans are faulty. It is seen in Fig. 2 that there is a small discrepancy between the estimated and real values, which can be considered as an admissible boundary, where it is possible to distinguish between a faulty and a healthy condition. One of the necessary conditions for stabilizing the EKF is that the extended system must be uniformly completely observable [19] which is tested by looking at the observability matrix. The EKF algorithm approximates the nonlinear model by a first order Taylor series expansion at every sample instant. Therefore, the observability matrix for the linear model is calculated in each sample. The observations confirm that the matrix is always full rank.

Next, the simulation is executed with different inputs without applying the sensitivity analysis. Fig. 3 shows that there is a large discrepancy between the estimated and real parameters, in which it is not possible to infer whether a fan is in a faulty or healthy condition. Here, the condition number of the observability matrix according to (12) is calculated, which has the value of  $3.0269 \times 10^6$  for the input from the sensitivity analysis and the value of  $7.3806 \times 10^6$  without sensitivity. It is obvious that the condition number obtained by the input from the sensitivity analysis has a smaller value, which shows that the input defined by the sensitivity analysis leads to better estimation of the parameters.

## VIII. CONCLUSIONS AND FUTURE WORK

This paper proposed a method for active fault detection and isolation in hybrid systems, which is based on off-line design of the excitation signal using sensitivity analysis. Deriving the signals off-line reduces the computational burden on the AFD algorithm. The problem of designing the inputs is formulated as a non-convex optimization problem for obtaining the maximum sensitivity for each individual system parameter and it was solved by a genetic algorithm (GA). The simulation results illustrate that the EKF converges quickly to the real parameters with the input from the sensitivity analysis; while it is unable to converge correctly to the parameters when the inputs are not provided by the sensitivity analysis.

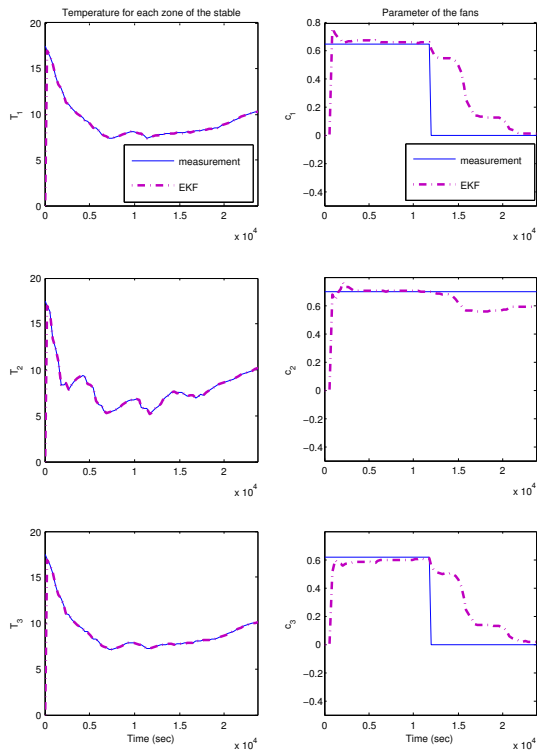


Fig. 2. The real and estimated values by EKF for indoor temperature and parameter of the fan for each zone of the stable. The excitation input is designed by sensitivity analysis.

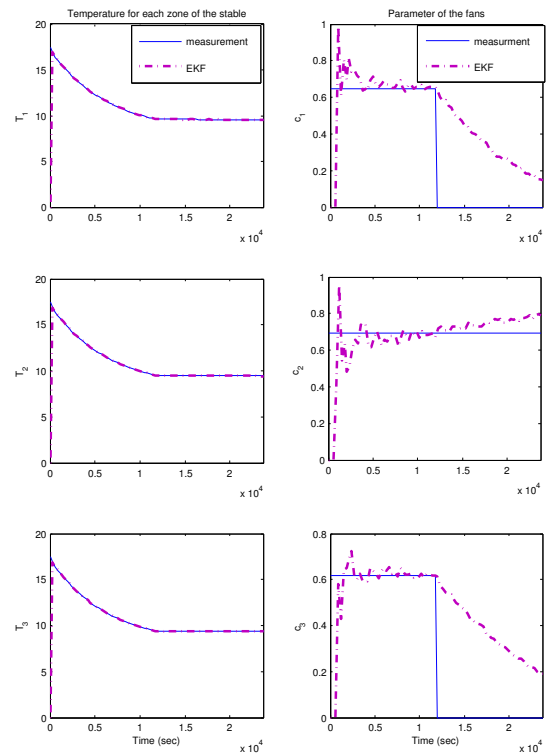


Fig. 3. The real and estimated values by EKF for indoor temperature and parameter of the fan for each zone of the stable. The excitation input is chosen arbitrary without sensitivity analysis.

The required assumption for the AFD method is that the value of the system parameter is known and the system is only subject to actuator fault. This method is more beneficial in comparison with a bank of EKF where prior knowledge about the system faults and a model for each individual fault are required. Dedicating a model for each fault is computationally expensive for a system with a large number of sensors and actuators which can also be subject to different kinds of faults. In the future, the AFD approach will be applied to closed loop systems, where the faulty model is assumed as a stochastic process and a necessary and sufficient condition for exponential stability of the system is derived.

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