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Adaptive Operational Space Control of Redundant Robot Manipulators

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***Abstract*— The *operation space formulation* requires model certainty to completely decouple the null space and operational space dynamics. In this paper, we present an adaptive operational space control for redundant robots that does not require exact knowledge of the robot inertial parameters. The use of the inertia matrix as the weighting matrix in the generalized inverse of the Jacobian leads to nonlinearly parameterized task space dynamics. We show that the nonlinear parametrization can be expressed as ratios of linearly parameterized numerator and denominator terms. Based on this, we construct a control Lyapunov function to eliminate some of the denominator terms during control design, leaving behind a linearly parameterized form that can be easily compensated for. For uncertainties that cannot be transformed into linearly parameterized form, we consider them as time-varying uncertainties and dominate them based on the fact that they are bounded, without knowledge of the bounds. Asymptotic tracking performance of the end-effector is achieved. Simulation results are shown to illustrate the control performance.**

I. INTRODUCTION

Task space formulation of robot dynamics is appealing as it permits intuitive specification of desired robot behavior with respect to the external environment, and facilitates the design of feedback control to achieve the desired behavior. For non-redundant robot manipulators, the Jacobian matrix is, in general, invertible, thus allowing end-effector velocities to be mapped to joint velocities. Furthermore, the task space dynamics are linearly parameterized, facilitating the design of adaptive control in the task space via familiar techniques [1], [2], [3], [4]. However, redundant robots pose a much more difficult problem, as the Jacobian matrix is non-invertible. To tackle this problem, one of many forms of generalized inverse can be used to map end-effector velocities to joint velocities.

The *operational space control* formulation [5] uses the inertia matrix as the weighting matrix in the generalized inverse of the Jacobian. This choice of weighting matrix minimizes the instantaneous kinetic energy as well as the acceleration energy, and leads to a unique dynamically consistent inverse that decouples the operational space from the null space dynamics [5], [6]. The approach essentially overcomes the limitations of inverse kinematics based redundancy resolution, including the neglect of robot dynamics when specifying desired joint motion, and the presence of joint space drift when the end-effector is repeatedly tracing a closed path in task space [7].

However, operation space formulation requires model certainty to completely decouple null space and operational space forces. In the presence of model uncertainties, degradation of control performance has been shown and analyzed [8], [9]. To improve control performance, multi-rate operational control [9] has been proposed, in which joint command is computed using the operational space formulation in an outer loop, and the joint space robot dynamics is feedback linearized in an inner loop. Other methods of adaptive task space control for redundant robots, but which depart from the *operational space control* formulation [5], can be found in [10], [11], [12]. These methods employ some forms of pseudo-inverse for kinematic resolution of the desired joint trajectories, followed by control of robot dynamics to track the desired joint trajectories.

In this paper, we present an adaptive operational space control that does not require exact knowledge of the robot inertial parameters. The key challenge is that the task space dynamics for redundant robots induced by the inertia-weighted generalized inverse Jacobian (i.e. under the operational space control formulation) is nonlinearly parameterized. As such, standard adaptive control approaches requiring linear parametrization of robot dynamics, typically used in non-redundant robots, cannot be applied for redundant robots. By analyzing the structure of the inertia-weighted generalized inverse, we show that the nonlinear parametrization can be expressed as fractions of linearly parameterized terms. With this insight, we modify the control Lyapunov function to eliminate some of the denominator terms during Lyapunov synthesis, leaving behind linearly parameterized numerator terms that can be easily compensated for. We also employ robust domination design to dominate uncertainties that cannot be transformed into linearly parameterized form. In particular, we treat the terms as time-varying uncertainties, and dominate them based on fact that they are bounded, but without knowledge of good estimates of the bounds.

II. DYNAMICS OF REDUNDANT ROBOTS

Consider a kinematically redundant robot manipulator described by:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{1}$$

where $M(q) \in \mathbb{R}^{n\times n}$ is a symmetric positive definite matrix, $C(q,\dot{q})\dot{q} \in \mathbb{R}^n$ the Coriolis and centrifugal forces, $G(q) \in \mathbb{R}^n$ the gravitational forces, $q \in \mathbb{R}^n$ the robot joint position, and $\tau \in \mathbb{R}^n$ the joint force. The terms $M(q)$, $C(q,\dot{q})$, and $g(q)$ contain uncertain dynamic parameters.

The task is to control the end-effector position, $x(t) \in \mathbb{R}^m$, described by the forward kinematics $x = F(q)$, to track a desired trajectory $x_d(t)$ that is bounded and twice

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differentiable in time. The important kinematic relations between the task and joint space are

$$\dot{x} = J(q)\dot{q} \quad (2)$$

$$\ddot{x} = J(q)\ddot{q} + \dot{J}\dot{q} \quad (3)$$

where $J(q) = dF(q)/dq$ is the Jacobian matrix.

Based on the idealized operational space formulation [5], the applied joint force

$$\tau_{id} = J^T(q)f + (I_n - J^T\bar{J}^T)\tau_n \quad (4)$$

where $\tau_n \in \mathbb{R}^n$ is any null space torque designed to achieve subtasks such as joint limit and singularity avoidance, I_n the $n \times n$ identity matrix, and

$$\bar{J} = M^{-1}J^T(JM^{-1}J^T)^{-1} \quad (5)$$

is the dynamically consistent generalized inverse that decouples the operational space dynamics from the null space dynamics, and minimizes the instantaneous kinetic energy as well as the acceleration energy [5], [6].

From (1), (3), and (4), the idealized operational space dynamics can be written as:

$$M_x\ddot{x} + \bar{J}^T(C_q\dot{q} + G) = f \quad (6)$$

where the coefficient matrices are defined as

$$M_x = (JM^{-1}J^T)^{-1} = \bar{J}^T M \bar{J} \quad (7)$$

$$C_q = C - M\bar{J}\dot{J} \quad (8)$$

$$f = \bar{J}^T \tau \quad (9)$$

and the following properties hold.

Property 1: The inertia matrix M_x is symmetric positive definite.

Throughout this paper, we denote $M_{x_{ij}}$ as the (i,j)th element of the matrices M_x.

In the presence of uncertainty in the inertial parameters, the generalized inverse $\bar{J}$ correspondingly becomes uncertain. An intuitive countermeasure is to replace (4) with

$$\tau = J^T(q)f + (I_n - J^T\hat{\bar{J}}^T)\tau_n \quad (10)$$

where $\hat{\bar{J}}$ is an estimate of $\bar{J}$. In this case, complete decoupling is no longer achieved, since $\bar{J}$ is dependent on the inertial parameters. As a result, the operational space dynamics (1), (3), and (10) become

$$\begin{aligned} M_x\ddot{x} + \bar{J}^T(C_q\dot{q} + G) &= f + \bar{J}^T(I_n - J^T\hat{\bar{J}}^T)\tau_n \\ &= f - \tilde{\bar{J}}^T\tau_n \quad (11) \end{aligned}$$

where $\tilde{\bar{J}} = \hat{\bar{J}} - \bar{J}$. It is apparent that the mismatch between $\hat{\bar{J}}$ and $\bar{J}$ contributes directly to coupling of null space and operational space dynamics.

It should also be noted that the left hand side of (11) is *nonlinearly parameterized* due to the presence of M^{-1} in $\bar{J}$, as seen in (5). As such, standard adaptive control approaches for robot manipulators based on linear parametrization cannot be applied to solve the adaptive operational space control problem for redundant robot manipulators.

To apply joint force τ according to (10), an estimate $\hat{\bar{J}}$ of the dynamically consistent generalized inverse $\bar{J}$, is required. However, $\bar{J}$ is also nonlinearly parameterized as follows:

$$\bar{J} = \frac{1}{d(q,\phi_d)}\bar{J}_n(q,\phi_n) \quad (12)$$

with the scalar d and each element of the matrix $\bar{J}_n$ linearly parameterized:

$$d(q,\phi_d) = \psi_d(q)\phi_d \quad (13)$$

$$\bar{J}_n(q,\phi_n) = \begin{bmatrix} \psi_{p_{11}}(q)\phi_n & \cdots & \psi_{p_{1m}}(q)\phi_n \\ \vdots & \ddots & \vdots \\ \psi_{p_{n1}}(q)\phi_n & \cdots & \psi_{p_{nm}}(q)\phi_n \end{bmatrix} \quad (14)$$

where $\psi_d(q) \in \mathbb{R}^{l_d}$, $\phi_d \in \mathbb{R}^{l_d}$, $\psi_{p_{ij}}(q) \in \mathbb{R}^{l_n}$, $\phi_n \in \mathbb{R}^{l_n}$. As such, for any $\rho \in \mathbb{R}^n$, we have

$$\bar{J}_n^T(q,\phi_n)\rho = \psi_n(q,\rho)\phi_n \quad (15)$$

where $\psi_n(q,\rho) \in R^{m\times l}$ is a regressor matrix.

Property 2: The scalar $\psi_d(q)\phi_d$ is always positive $\forall q \in \mathbb{R}^n$.

Proof: From (5), we see that $\psi_d(q)\phi_d$ is the product of the determinants of M and JMJ^T. Since M and JMJ^T are both positive definite, it follows that $\psi_d(q)\phi_d > 0$ $\forall q \in \mathbb{R}^n$. ■

Property 3: There exists a compact set $\Omega_d \in \mathbb{R}^{l_d}$ such that $\psi_d(q) \in \Omega_d$ $\forall q \in \mathbb{R}^n$.

Property 4: Let $\det(\bullet)$ be the determinant of $(\bullet)$. Then, the denominator d in (13) is given by:

$$d = (\det(M))^{n-1}\det(JJ^T) \quad (16)$$

Proof: Denote, by $\mathrm{adj}(\bullet)$, the adjugate matrix of $(\bullet)$. From (5), we have

$$\begin{aligned} \bar{J} &= M^{-1}J^T(JM^{-1}J^T)^{-1} \\ &= \frac{\mathrm{adj}(M)J^T}{\det(M)}\left(\frac{J\ \mathrm{adj}(M)J^T}{\det(M)}\right)^{-1} \\ &= \frac{\mathrm{adj}(M)J^T\mathrm{adj}\left(J\ \mathrm{adj}(M)J^T\right)}{\det(J\ \mathrm{adj}(M)J^T)} \quad (17) \end{aligned}$$

Thus, from (12), we see that

$$\begin{aligned} d &= \det(J\ \mathrm{adj}(M)J^T) \\ &= \det(\mathrm{adj}(M))\det(JJ^T) \\ &= (\det(M))^{n-1}\det(JJ^T) \end{aligned}$$

and we conclude that Property 4 holds. ■

III. Adaptive Operational Space Control

In the previous section, two groups of nonlinearly parameterized uncertainties have been described, one appearing in the generalized inverse $\bar{J}$ in the applied torque (10), and another in the left hand side of the operational space dynamics (11). We tackle the former by using a modified quadratic control Lyapunov function to directly handle the nonlinear parametrization of $\bar{J}$. On the other hand, the latter is dealt with by first rearranging the dynamics into a linear time-varying parametrization form, followed by dominating the time-varying, but bounded, parameters via an approach similar to [13], [12].

We employ backstepping design methodology [14] to construct an adaptive operational space control for robot manipulator (1). In the first step, denote $z = x - x_d$, $\nu = \dot{x} - \alpha$, and consider the quadratic function $V_1 = (1/2)z^T z$, which has the time derivative

$$\dot{V}_1 = z^T(\nu + \alpha - \dot{x}_d) \tag{18}$$

Design the stabilizing function as

$$\alpha = \dot{x}_d - K_z z \tag{19}$$

where $K_z > 0$, and we obtain

$$\dot{V}_1 = -z^T K_z z + z^T \nu \tag{20}$$

In the second step, we consider the quadratic function

$$\begin{aligned} V_2 &= V_1 + \frac{d}{2}\nu^T M_x \nu + \frac{1}{2\gamma}\tilde{\beta}^2 + \frac{1}{2}\tilde{\phi}_d^T \Gamma_d^{-1}\tilde{\phi}_d \\ &\quad + \frac{1}{2}\tilde{\phi}_n^T \Gamma_n^{-1}\tilde{\phi}_n + \frac{1}{2}\tilde{\phi}_g^T \Gamma_g^{-1}\tilde{\phi}_g \end{aligned} \tag{21}$$

where $(\tilde{\bullet}) = (\hat{\bullet}) - (\bullet)$, ϕ_g a vector of unknown parameters of $\bar{J}_n^T G$, $\Gamma_d = \Gamma_d^T > 0$ and $\Gamma_n = \Gamma_n^T > 0$ are constant matrices, $\gamma > 0$ a constant, and $d(q, \phi_d) > 0$, defined in (13), is introduced to eliminate nonlinear parametrization, since $\bar{J}_n = d\bar{J}$ is linearly parameterized.

The time derivative of V_2 is given by

$$\begin{aligned} \dot{V}_2 &= \nu^T d[f - \tilde{\bar{J}}^T \tau_n - \bar{J}^T(C_q\dot{q} + G) - M_x\dot{\alpha}] \\ &\quad + \frac{d}{2}\nu^T \dot{M}_x \nu + \frac{\dot{d}}{2}\nu^T M_x \nu + \nu^T z + \frac{1}{\gamma}\tilde{\beta}\dot{\hat{\beta}} \\ &\quad + \tilde{\phi}_d^T \Gamma_d^{-1}\dot{\hat{\phi}}_d + \tilde{\phi}_n^T \Gamma_n^{-1}\dot{\hat{\phi}}_n + \tilde{\phi}_g^T \Gamma_g^{-1}\dot{\hat{\phi}}_g - z^T K_z z \end{aligned}$$

Substituting $\tilde{\bar{J}} = \hat{\bar{J}} - \bar{J}$ and using the identity $M_x = \bar{J}^T M \bar{J}$ leads to

$$\begin{aligned} \dot{V}_2 &= \nu^T d[f - \hat{\bar{J}}^T \tau_n + \bar{J}^T(\tau_n - C_q\dot{q} - G - M\bar{J}\dot{\alpha}) \\ &\quad + \frac{1}{2}\dot{M}_x \nu] + \frac{\dot{d}}{2}\nu^T M_x \nu + \nu^T z + \frac{1}{\gamma}\tilde{\beta}\dot{\hat{\beta}} \\ &\quad + \tilde{\phi}_d^T \Gamma_d^{-1}\dot{\hat{\phi}}_d + \tilde{\phi}_n^T \Gamma_n^{-1}\dot{\hat{\phi}}_n + \tilde{\phi}_g^T \Gamma_g^{-1}\dot{\hat{\phi}}_g - z^T K_z z \end{aligned}$$

Since $d\bar{J}^T = \bar{J}_n^T$, we have

$$\begin{aligned} \dot{V}_2 &= \nu^T [d(f - \hat{\bar{J}}^T \tau_n + \frac{1}{2}\dot{M}_x \nu) + \bar{J}_n^T(\tau_n - C_q\dot{q} \\ &\quad - G - M\bar{J}\dot{\alpha}) + \frac{\dot{d}}{2}M_x \nu + z] + \frac{1}{\gamma}\tilde{\beta}\dot{\hat{\beta}} \\ &\quad + \tilde{\phi}_d^T \Gamma_d^{-1}\dot{\hat{\phi}}_d + \tilde{\phi}_n^T \Gamma_n^{-1}\dot{\hat{\phi}}_n + \tilde{\phi}_g^T \Gamma_g^{-1}\dot{\hat{\phi}}_g - z^T K_z z \end{aligned}$$

Now, we design the end-effector force as

$$f = \hat{\bar{J}}^T \tau_n + f_u \tag{22}$$

where $\hat{\bar{J}}^T = (\psi_n \hat{\phi}_n)/(\psi_d \hat{\phi}_d)$ and f_u is to be designed later. This yields

$$\begin{aligned} \dot{V}_2 &= \nu^T[df_u + \bar{J}_n^T(\tau_n - G) - Y(q, \dot{q}, \dot{\alpha}, \nu, \phi) + z] \\ &\quad + \frac{1}{\gamma}\tilde{\beta}\dot{\hat{\beta}} + \tilde{\phi}_d^T \Gamma_d^{-1}\dot{\hat{\phi}}_d + \tilde{\phi}_n^T \Gamma_n^{-1}\dot{\hat{\phi}}_n + \tilde{\phi}_g^T \Gamma_g^{-1}\dot{\hat{\phi}}_g \\ &\quad - z^T K_z z \end{aligned} \tag{23}$$

where

$$\begin{aligned} Y(q, \dot{q}, \dot{\alpha}, \nu, \phi) &:= \bar{J}_n^T(C_q\dot{q} + M\bar{J}\dot{\alpha}) \\ &\quad - \frac{1}{2}(d\dot{M}_x + \dot{d}M_x)\nu \end{aligned} \tag{24}$$

and ϕ is a vector of uncertain parameters. From (23), the term $\bar{J}_n^T(\tau_n - G)$ is linearly parameterized in ϕ_n, according to (14) and the fact that G is also linear-in-the-parameters:

$$\bar{J}_n^T(\tau_n - G) = \psi_n(q, \tau_n)\phi_n + \psi_g(q)\phi_g \tag{25}$$

and thus, the parameters ϕ_n and ϕ_g can be adaptively estimated. However, the term Y is still nonlinearly parameterized with respect to ϕ.

To circumvent this difficulty, we treat the matrices $\bar{J}_n^T M \bar{J}$, $\dot{d}M_x$, $d\dot{M}_x$, and $\bar{J}_n^T C_q$ as time-varying uncertainties, and dominating them based on the bounds of the time-varying parameters. While $\bar{J}_n^T M \bar{J}$ is always bounded for all $q \in \mathbb{R}^n$, $\dot{d}M_x$, $d\dot{M}_x$ and $\bar{J}_n^T C_q$ depend linearly on $\dot{q}$. The latter does not pose a problem as $\dot{q}$ can be factored into the regressor.

To this end, define

$$\begin{aligned} A &:= -d\dot{M}_x - \dot{d}M_x \\ B &:= \bar{J}_n^T C_q \end{aligned} \tag{26}$$

and let $a_{ij} \in \mathbb{R}^n$ and $b_{ij} \in \mathbb{R}^n$, for $i, j = 1, ..., m$, be defined such that

$$\begin{aligned} \dot{q}^T a_{ij} &= A_{ij} \\ \dot{q}^T b_{ij} &= B_{ij} \end{aligned} \tag{27}$$

Thus, we write

$$\begin{aligned} -(\dot{d}M_x + d\dot{M}_x)\nu/2 &= \psi_A(\nu, \dot{q})\theta_A(t) \\ \bar{J}_n^T C_q \dot{q} &= \psi_B(\dot{q})\theta_B(t) \\ \bar{J}_n^T M \bar{J}\dot{\alpha} &= \psi_C(\dot{\alpha})\theta_C(t) \end{aligned} \tag{28}$$

where the regressors are defined in (29)-(30) with $\otimes$ denoting the Kronecker product. The time-varying parameters are given by

$$\begin{aligned} \theta_A(t) &= [\theta_{A_1}^T,\ ...,\ \theta_{A_m}^T]^T \\ \theta_{A_i}(t) &= [a_{i1}^T,\ ...,\ a_{im}^T]^T, \quad i = 1, ..., m \\ \theta_B(t) &= [\theta_{B_1}^T,\ ...,\ \theta_{B_m}^T]^T \\ \theta_{B_i}(t) &= [b_{i1}^T,\ ...,\ b_{in}^T]^T, \quad i = 1, ..., m \\ \theta_C(t) &= [\theta_{C_1}^T,\ ...,\ \theta_{C_m}^T]^T \\ \theta_{C_i}(t) &= d[M_{x_{ii}},\ ...,\ M_{x_{im}}]^T, \quad i = 1, ..., m \end{aligned} \tag{31}$$

which evolve in compact sets Ω_{θ_A}, Ω_{θ_B}, and Ω_{θ_C} $\forall t \geq 0$

$$\begin{aligned} \theta_A(t) &\in \Omega_{\theta_A} \subset \mathbb{R}^{nm^2} \\ \theta_B(t) &\in \Omega_{\theta_B} \subset \mathbb{R}^{mn^2} \\ \theta_C(t) &\in \Omega_{\theta_C} \subset \mathbb{R}^{l_M} \end{aligned} \tag{32}$$

$$\psi_A(\nu,\dot q)=\begin{bmatrix}(\nu\otimes\dot q)^T & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & (\nu\otimes\dot q)^T\end{bmatrix},\qquad \psi_B(\dot q)=\begin{bmatrix}(\ [\dot q^2],\ [\dot q\dot q]\ ) & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & (\ [\dot q^2],\ [\dot q\dot q]\ )\end{bmatrix} \tag{29}$$

$$\psi_C(\dot\alpha)=\begin{bmatrix}\dot\alpha^T & 0 & \cdots & \cdots & 0\\ \dot\alpha_1({}^m e_2) & \dot\alpha_{2:m}^T & \cdots & \cdots & 0\\ \vdots & \vdots & \ddots & & \vdots\\ \dot\alpha_1({}^m e_{m-1}) & \dot\alpha_2({}^{m-1}e_{m-2}) & \cdots & \dot\alpha_{m-1:m}^T & 0\\ \dot\alpha_1({}^m e_m) & \dot\alpha_2({}^{m-1}e_{m-1}) & \cdots & \dot\alpha_{m-1}({}^2 e_2) & \dot\alpha_m\end{bmatrix},\quad \dot\alpha=(\dot\alpha_1,...,\dot\alpha_m)^T \tag{30}$$

$$[\dot q^2]=(\dot q_1^2,\ \dot q_2^2,\ \cdots,\ \dot q_n^2),\qquad [\dot q\dot q]=(\dot q_1\dot q_2,\ \dot q_1\dot q_3,\ \cdots,\ \dot q_{n-1}\dot q_n)$$

$${}^k e_i=(p_1,\cdots,p_j,\cdots,p_k),\qquad p_j=\begin{cases}1 & \text{if } j=i\\ 0 & \text{otherwise}\end{cases},\qquad j=1,2,...,k,$$

where $l_M=m(m+1)/2$ by virtue of the symmetry of M_x. As a result, we can show that Y in (24) can be linearly parameterized in terms of bounded time-varying parameters, as follows:

$$Y=\psi(\dot q,\nu,\dot\alpha)\theta(t) \tag{33}$$

where

$$\psi=(\psi_A,\psi_B,\psi_C)\,,\quad \theta=\left(\theta_A^T,\theta_B^T,\theta_C^T\right)^T \tag{34}$$

Since $\theta_i(t)\in[\underline\theta_i,\overline\theta_i]\ \forall t\geq 0,\ i=A,B,C$, we obtain the bound $\|\theta(t)\|\leq\beta$, where

$$\beta\ :=\ \left(\sum_{i=1}^{L}\max\left(\underline\theta_i^2,\overline\theta_i^2\right)\right)^{1/2} \tag{35}$$

and $L=2l_M+mn(m+n)$. Note that β does not need to be known, but it is adaptively estimated in the controller.

The parameterizations in (13), (25), (28) and (33) lead to:

$$\begin{aligned}\dot V_2 &= \nu^T[-f_u\psi_d\tilde\phi_d+f_u\psi_d\hat\phi_d+\psi_n\phi_n+\psi_g\phi_g-\psi\theta(t)\\ &\quad +z]+\frac{1}{\gamma}\tilde\beta\dot{\hat\beta}+\tilde\phi_d^T\Gamma_d^{-1}\dot{\hat\phi}_d+\tilde\phi_n^T\Gamma_n^{-1}\dot{\hat\phi}_n\\ &\quad +\tilde\phi_g^T\Gamma_g^{-1}\dot{\hat\phi}_g-z^TK_zz\\ &= -z^TK_zz\nu^T[f_u\psi_d\hat\phi_d+\psi_n\phi_n+\psi_g\phi_g-\psi\theta(t)\\ &\quad +z]+\frac{1}{\gamma}\tilde\beta\dot{\hat\beta}+\tilde\phi_d^T(\Gamma_d^{-1}\dot{\hat\phi}_d-\psi_d^T\nu^Tf_u)\\ &\quad +\tilde\phi_n^T\Gamma_n^{-1}\dot{\hat\phi}_n+\tilde\phi_g^T\Gamma_g^{-1}\dot{\hat\phi}_g-z^TK_zz\end{aligned}$$

Considering the bound on $\theta(t)$ according to (35), we have

$$\begin{aligned}\dot V_2 &\leq \nu^T(f_u\psi_d\hat\phi_d+\psi_n\phi_n+\psi_g\phi_g+z)+\|\nu^T\psi\|\beta\\ &\quad +\frac{1}{\gamma}\tilde\beta\dot{\hat\beta}+\tilde\phi_d^T(\Gamma_d^{-1}\dot{\hat\phi}_d-\psi_d^T\nu^Tf_u)+\tilde\phi_n^T\Gamma_n^{-1}\dot{\hat\phi}_n\\ &\quad +\tilde\phi_g^T\Gamma_g^{-1}\dot{\hat\phi}_g-z^TK_zz\end{aligned} \tag{36}$$

We design f_u as follows:

$$f_u=\frac{1}{\psi_d\hat\phi_d}\left(-K_\nu\nu-z-\psi_{ng}\hat\phi_{ng}-\frac{\psi\psi^T\nu\hat\beta^2}{\|\nu^T\psi\|\hat\beta+\varepsilon\|\nu\|^2}\right) \tag{37}$$

where $\psi_{ng}=[\psi_n,\psi_g]$, $\hat\phi_{ng}=[\hat\phi_n^T,\hat\phi_g^T]^T$, $K_\nu>0$, and ε is a positive constant satisfying

$$\varepsilon<\lambda_{\min}(K_\nu) \tag{38}$$

Note that the third term on the right hand side adaptively compensates for $\psi_n\phi$, and the fourth term dominates $\|\nu^T\psi\|\beta$ in(36). The joint torque applied to the motors is given by (10). Substituting (37) into (36) yields

$$\begin{aligned}\dot V_2 &\leq \|\nu^T\psi\|\beta-\frac{\|\nu^T\psi\|^2\hat\beta^2}{\|\nu^T\psi\|\hat\beta+\varepsilon\|\nu\|^2}+\frac{1}{\gamma}\tilde\beta\dot{\hat\beta}\\ &\quad +\tilde\phi_d^T(\Gamma_d^{-1}\dot{\hat\phi}_d-\psi_d^T\nu^Tf_u)+\tilde\phi_n^T(\Gamma_n^{-1}\dot{\hat\phi}_n-\psi_n^T\nu)\\ &\quad +\tilde\phi_g^T(\Gamma_g^{-1}\dot{\hat\phi}_g-\psi_g^T\nu)-z^TK_zz-\nu^TK_\nu\nu\\ &\leq \frac{\|\nu^T\psi\|\hat\beta\varepsilon\|\nu\|^2}{\|\nu^T\psi\|\hat\beta+\varepsilon\|\nu\|^2}+\tilde\beta\left(\frac{1}{\gamma}\dot{\hat\beta}-\|\nu^T\psi\|\right)\\ &\quad +\tilde\phi_d^T(\Gamma_d^{-1}\dot{\hat\phi}_d-\psi_d^T\nu^Tf_u)+\tilde\phi_n^T(\Gamma_n^{-1}\dot{\hat\phi}_n-\psi_n^T\nu)\\ &\quad +\tilde\phi_g^T(\Gamma_g^{-1}\dot{\hat\phi}_g-\psi_g^T\nu)-z^TK_zz-\nu^TK_\nu\nu\end{aligned} \tag{39}$$

Since $ab/(a+b)\leq a$ for any $a,b>0$, it can be shown that

$$\begin{aligned}\dot V_2 &\leq \tilde\beta\left(\frac{1}{\gamma}\dot{\hat\beta}-\|\nu^T\psi\|\right)+\tilde\phi_d^T(\Gamma_d^{-1}\dot{\hat\phi}_d-\psi_d^T\nu^Tf_u)\\ &\quad +\tilde\phi_n^T(\Gamma_n^{-1}\dot{\hat\phi}_n-\psi_n^T\nu)+\tilde\phi_g^T(\Gamma_g^{-1}\dot{\hat\phi}_g-\psi_g^T\nu)\\ &\quad -z^TK_zz-\nu^T(K_\nu-\varepsilon I_m)\nu\end{aligned} \tag{40}$$

where I_m is the $m\times m$ identity matrix.

Finally, the adaptation laws are designed as

$$\dot{\hat\beta}=\gamma\|\nu^T\psi\| \tag{41}$$

$$\dot{\hat\phi}_d=\begin{cases}\Gamma_d\psi_d^T\nu^Tf_u, & \text{if } \hat\phi_d\in\mathrm{int}(\Phi)\\ & \text{or if } \hat\phi_d\in\partial\Phi\\ & \quad\text{and } \psi_d\Gamma_1\psi_d^T\nu^Tf_u\geq 0\\ 0, & \text{otherwise}\end{cases} \tag{42}$$

$$\dot{\hat\phi}_g=\Gamma_g\psi_g^T\nu \tag{43}$$

$$\dot{\hat\phi}_n=\Gamma_n\psi_n^T\nu \tag{44}$$

where $\text{int}(\Phi)$ and $\partial\Phi$ are the interior and boundary of the set Φ, defined by

$$\Phi := \bigcap_{\psi_d \in \Omega_d} \{\hat{\phi}_d \in \mathbb{R}^l : \psi_d \hat{\phi}_d > 0\} \quad (45)$$

and Ω_d a compact set from Property 3. The projection algorithm (42) guarantees that the following conditions are satisfied:

$$\begin{aligned} 0 &\leq \psi_d \hat{\phi}_d \\ 0 &\geq \tilde{\phi}_d^T (\Gamma_d^{-1} \dot{\hat{\phi}}_d - \psi_d^T \nu^T f_u) \end{aligned} \quad (46)$$

Furthermore, by choosing $\hat{\beta}(0) > 0$, (41) ensures that $\hat{\beta}(t) > 0 \; \forall t \geq 0$.

Substituting (41)-(44) into (40), we obtain

$$\dot{V}_2 \leq -z^T K_z z - \nu^T (K_\nu - \varepsilon I_m)\nu \quad (47)$$

which is negative semidefinite in light of (38). We are now ready to summarize the main results.

Theorem 1: Consider a redundant robot manipulator (1) subjected to the adaptive control law (10), (22), (37) and (41)-(44). For $\tau_n \in \mathcal{L}_\infty$, the origin of the end-effector error system, $(z, \nu) = 0$, is uniformly asymptotically stable.

Proof: From the negative semidefiniteness of $\dot{V}_2$ in (47), we conclude, by the LaSalle-Yoshizawa Theorem, that the origin $(z, \nu) = 0$ is uniformly stable, and that $\lim_{t\to\infty}(-z^T K_z z - \nu^T (K_\nu - \varepsilon I_m)\nu) = 0$, which implies that $\lim_{t\to\infty} z(t) = \lim_{t\to\infty} \nu(t) = 0$. ■

IV. SIMULATION

Consider a model of the dynamics of a 3-link robot with 3 actuated revolute joints [15] moving on a horizontal plane. This configuration allows a single degree of redundancy for position tracking tasks involving the end-effector. All numerical values in this section are in S.I. units.

Specifically, we are concerned with tracking a desired trajectory in operational space, described by:

$$\begin{aligned} x_{d_1}(t) &= 0.1\cos(t/4) \\ x_{d_2}(t) &= 2 + 0.1\sin(t/4) \end{aligned}$$

For ease of illustration, we consider that the center of mass of each link coincides with its corresponding joint, and that the lengths of the links are all equal, specifically $l_i = 1.0$, $i = 1, 2, 3$. From Property 4, we obtain

$$\psi_d = [d_J(q)\cos^4(q_2), d_J(q)\cos^2(q_2), d_J(q)] \quad (48)$$

where $d_J(q) := \det(J(q)J^T(q))$. For masses $m_1 = m_2 = m_3 = 0.5$, and moments of inertia $I_{zz_1} = 1.5$, $I_{zz_2} = 1.0$, and $I_{zz_3} = 0.5$, we estimate the bounds for d_J as $0 \leq d_J(q) \leq 8$. Then, we have, from (48), the bounds for ψ_d as $0 \leq \psi_{d_i} \leq 8$, $i = 1, 2, 3$. As such, the projection algorithm (42) is implemented with

$$\begin{aligned} \Phi = \{\hat{\theta} \in \mathbb{R}^3 : \hat{\theta}_1 + \hat{\theta}_2 \geq 0,\ \hat{\theta}_1 + \hat{\theta}_3 \geq 0,\ \hat{\theta}_2 + \hat{\theta}_3 \geq 0, \\ \hat{\theta}_1 + \hat{\theta}_2 + \hat{\theta}_3 \geq 0,\ \hat{\theta}_i \geq 0,\ i = 1, 2, 3\} \end{aligned} \quad (49)$$

For simplicity, the null torque is specified as $\tau_{n_1} = 0.1\sin t$, $\tau_{n_2} = 0.1\cos(t)$, $\tau_{n_3} = 0.02\cos t$. The design parameters are selected as $K_z = 2I$, $K_\nu = 4I$, $\gamma = 0.5$, $\Gamma_d = 0.1I$, $\Gamma_n = 0.8I$, $\varepsilon = 1.6$. The initial conditions for the simulation are $q(0) = (\pi/4, \pi/4, \pi/4)^T$, $\dot{q}(0) = 0$, $\hat{\beta}(0) = 0.01$, $\hat{\phi}_d(0) = (0.5, 0.5, 0.5)^T$, and $\hat{\phi}_n(0) = 0$. Under these design and initial conditions, the control law (10), (22), (37), and the adaptation laws (41)-(44) are implemented.

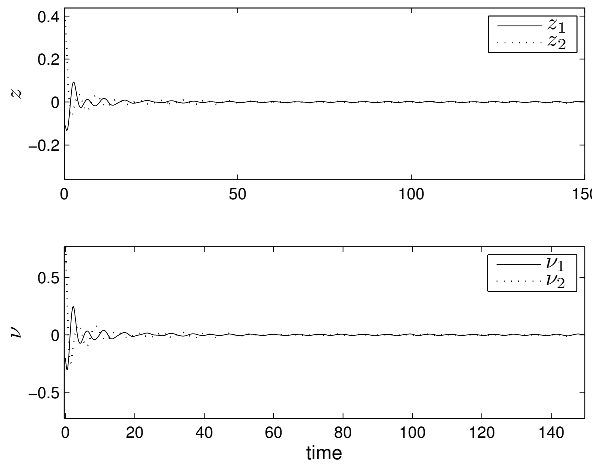

Fig. 1. The tracking errors z and ν converge to a neighborhood of 0.

Figure 1 shows that, under the proposed adaptive operational space control, the end-effector position tracking error $z = x - x_d$, and the velocity tracking error $\nu = \dot{x} - \alpha$, both converge asymptotically to a small neighborhood of the origin, despite the presence of uncertainty in the nonlinearly parameterized model of redundant robot dynamics. The controller has also successfully compensated for the perturbing effects due to imperfect decoupling (11) of the operational and null space dynamics arising from an imperfect model of the dynamics.

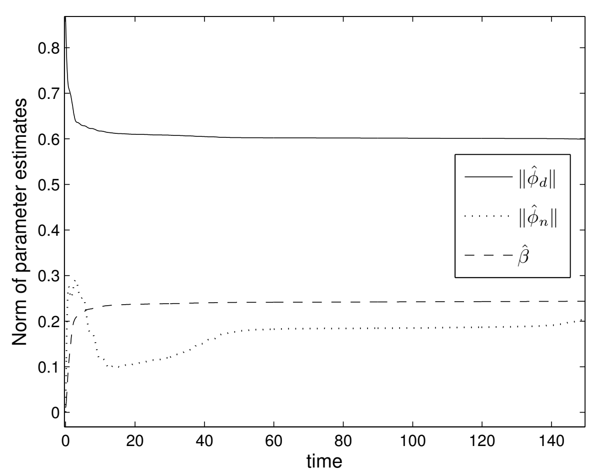

Fig. 2. The norms of the parameter estimates are convergent.

The adaptation laws result in bounded parameter estimates, as shown by the boundedness of their norms in Figure 2. Starting from some initial values, the estimates vary in response to transient error signals and eventually converge to steady state values. Under the projection algorithm (42), $\psi_d \hat{\phi}_d > 0$ is always ensured.

Apart from end-effector motion, self-motion of the redundant robot is also stable, as shown by the boundedness of the joint velocities $\dot{q}$ in Figure 3. Finally, the joint torques τ are bounded and smooth, as seen from Figure 4. The oscillations in both $\dot{q}$ and τ are induced by the sinusoidal null space torque τ_n.

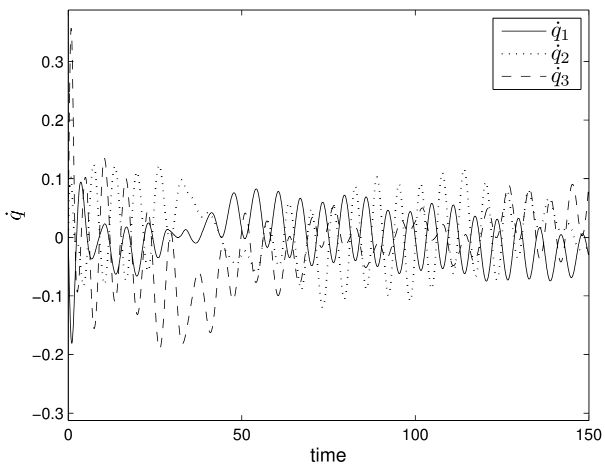

Fig. 3. The joint velocities remain bounded.

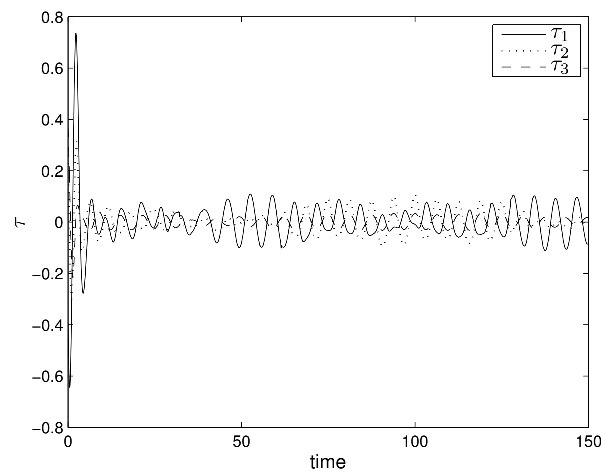

Fig. 4. The joint torques remain bounded.

V. CONCLUSIONS

This paper has presented an adaptive operational space control for redundant robots. Based on the insight that the nonlinear parametrization can be expressed as fractions of linearly parameterized terms, we have modified the control Lyapunov function to eliminate some of the denominator terms during Lyapunov synthesis, leaving behind linearly parameterized numerator terms that have been handled by adaptive control. For the terms that cannot be transformed into linearly parameterized form, we have treated them as time-varying parameters, and employed robust domination design to dominate them based on the bounds of the time-varying parameters. We have shown, via theory and simulations, that asymptotic tracking performance of the end-effector is achieved, and that all closed loop signals are bounded. Future work will involve experimental verification of the controller and investigation on the handling of multiple sub-task criteria.

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