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A Model Predictive Controller of Plastic Sheet Temperature for a Thermoforming Process

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***Abstract*— This paper presents a method to control the surface temperature of a plastic sheet using model predictive control (MPC). Although control techniques have been developed for the heating phase of the thermoforming process, oven heater temperatures in the thermoforming industry are still largely adjusted by trial and error based on the experience of the operator. MPC is one of the advanced methods for process control that has been used in different plants since the 1980s. Even though the MPC controller can handle a multivariable process, the large number of computations makes it difficult to apply to large systems such as multi-zone temperature control in a thermoforming machine. In this paper, the design of a model predictive controller is reported and implemented on a complex thermoforming oven with a large number of inputs and outputs for precise control of sheet temperatures under hard constraints on heater temperature and their rates.**

I. INTRODUCTION

PLASTIC products are increasingly supplanting products made of conventional and expensive materials such as aluminum, glass, wood and paper because of their numerous advantages. This encourages researchers to develop cost-effective and accurate controllers for polymer forming processes [1-6]. The thermoforming process consists of three phases, namely heating, forming, and solidification. The first and most important part in thermoforming is heating the sheet to the softening temperature, which is the heating phase. As heating is the first phase of the process, the remaining phases depend on the outcome of the heating phase.

At present, in some cases standard proportional-integral-derivative (PID) controllers are used. But this type of controller cannot handle process constraints such as the maximum and minimum heater temperatures, as well as limited heating and cooling rates of the heaters. Because of the limited heating and cooling rate, it is typically observed that the heaters fail to track the control inputs from the controller. Moreover, PID controllers do not take into consideration the model information to calculate the optimal control input. PID controllers also have a serious drawback in controlling an MIMO system as the inverse heating problem (IHP) has to be solved in real time to decouple the system. Even though PID controllers usually work well in near steady-state conditions, the plastic sheet will melt before it reaches steady state, forcing the controller to operate in transient condition within the cycle time [8]. Some researchers have developed cycle-to-cycle control techniques to control the thermoforming process [6,9].

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The idea is to use information from past cycles to help the closed-loop system better track the desired trajectory across cycles. Control during sheet reheat is also complicated by the fact that there is a high level of uncertainty surrounding the process, particularly with the material properties. The fact that this is a multi-input, multi-output (MIMO) problem with a high degree of coupling between inputs and outputs also introduces additional complexity. Moreover, environmental conditions may change between cycles because of the nonlinearity and time dependency of the system that make the controller delay in achieving the correct input signal. Sometimes a cycle-to-cycle controller converges very slowly resulting in lots of discarded parts.

MPC has some nice features. For example, it can automatically compensate for process interaction and measure the disturbances as well as handle difficult process dynamics, e.g., dead-time dominant. Another important advantage of this type of control is its ability to cope with hard constraints on controls and states. So MPC can optimize the performance by allowing for operation close to the system constraints.

An MPC predicts future control inputs solving the optimization problem over an output horizon. This involves the minimization of a cost function using the model of the system at each sampling instant. The computation of the optimization problem at every sampling instant may require complex calculations demanding a very fast processor [10]. Due to the online optimization at every sampling instant, MPC has not been an effective technique to deal with large multivariable constrained systems that increase computational complexity in solving the optimization problem. Moreover, although several issues like stability, feasibly and performance of linear MPC control are well developed and understood [11, 12], much work needs to be done in the field of nonlinear MPC to make it popular in industry. Thus, the large system size and the presence of nonlinearities in the heating phase of thermoforming seem to have discouraged control engineers from using optimal control techniques for this kind of system. Recently, some works [13-16] have developed explicit solutions of the optimization problem and proposed a new framework to deal with a nonlinear system as a combination of piecewise affine hybrid systems. These results extend the applicability of the MPC controller to low-cost, slow processors and improve software adaptability and ratability in real-time implementation. Multi-parametric quadratic programming helps in solving the model predictive optimization problem offline which reduces the real-time computational burden of the controller. Thus, in this paper, we explore MPC using the explicit solution of the optimization problem for temperature control of a thermoforming machine. In section II, we introduce MPC for controlling the process whereas in section III and IV, we discuss the multi-parametric quadratic programming used to solve the online optimization with an offline strategy in the development of MPC for the heating phase of thermoforming process. Section V reviews the model of the heating phase whereas in section VI, a multi-parametric MPC is developed for the heating phase of the thermoforming process. The performance of the controller is investigated thereafter.

II. Multi-parametric quadratic MPC for heating phase of thermoforming

MPC is a good choice for controlling the heating phase. Although the heating phase of thermoforming machines is a slow process, the number of system equations is high when multiple sensors are used. Thus, much computation and expensive hardware is required to implement MPC for the system due to the online minimization of the cost function. Even though an advanced nonlinear programming algorithm for optimization may be used, the speed and accuracy of the solution is not guaranteed. Multi-parametric quadratic programming was introduced to compute the online optimization offline to express the solution as a combination of affine functions of the state and input to overcome the implementation problem of the conventional MPC controller [13]. The solution is computed offline and the controller obtains its control move based on the value of the state using some affine function, which eases the computational burden of online optimization. In this paper, multi-parametric quadratic MPC for heating phase of the thermoforming machine is proposed. The model developed for the heating phase of the thermoforming machine is nonlinear [8]. So the model equations are linearized around the operating point (x^*, u^*) to control the system using multi-parametric quadratic MPC.

$$A = \left.\frac{\partial f(x,u)}{\partial x}\right|_{\substack{x=x^* \\ u=u^*}} \qquad B = \left.\frac{\partial f(x,u)}{\partial u}\right|_{\substack{x=x^* \\ u=u^*}} \qquad C = \left.\frac{\partial h(x)}{\partial x}\right|_{x=x^*}$$

(1)

In the rest of the paper, we use t as the present value and $t+k$ for k-th future value predicted at time t. The linear state-space equations for the system are,

$$x(t+1) = Ax(t) + Bu(t), \qquad y(t) = Cx(t) \tag{2}$$

where $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}$. With the linearized system equation, by substituting $x(t+k)|_t = A^k x(t) + \sum_{j=0}^{k-1} A^j B u_{t+k-1-j}$, the optimization problem of MPC can be reformulated following some algebraic manipulation as [13]:

$$V(x(t)) = \frac{1}{2}x^T(t)Yx(t) + \min_U \left\{\frac{1}{2}U^T HU + x^T(t)FU\right\}$$

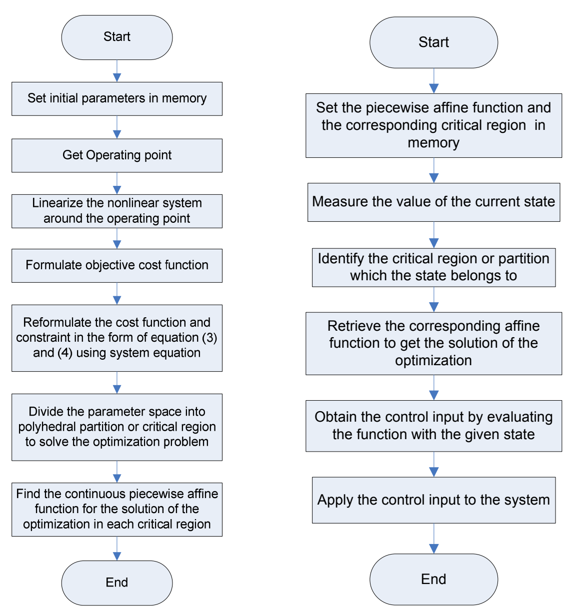
such that $GU \le W + Ex(t)$,

where, $U \triangleq [u_t, ...u_{t+N_U-1}]^T$ is the optimization vector and H, F, Y, G, W, E are obtained from the weighting matrices Q, R. Defining $z \triangleq U + H^{-1}F^T x(t)$, the problem can be transformed into

$$V_z(x(t)) = \min_z \frac{1}{2} z^T Hz \tag{3}$$

$$\text{such that } Gz \le W + Sx(t), \tag{4}$$

where $S \triangleq E + GH^{-1}F^T$ and $V_z(x(t)) = V(x(t)) - \frac{1}{2}x(t)^T \left(Y - FH^{-1}F^T\right)x(t)$, and the current state $x(t) = x_o$ can be taken as a vector of parameters. If there are q inequalities in (4), then $z \in \mathbb{R}^{m.N}$, $H \in \mathbb{R}^{m.N\times m.N}$, $G \in \mathbb{R}^{q\times m.N}$, $W \in \mathbb{R}^{q\times 1}$, $S \in \mathbb{R}^{q\times n}$ and $F \in \mathbb{R}^{n\times q}$. In [14], it is shown that the explicit solution of the optimization is a continuous piecewise affine function defined over the partition of the parameter space. Following [14], we propose an algorithm for the offline computation of the optimization problem for the heating phase of the thermoforming process and hence implement it in MPC control of the process. The whole algorithm is described by the flow chart in Fig.1.



(a) (b)

Fig.1: (a) Algorithm for offline optimization of the objective function for MPC (b) Algorithm for incorporating the solution of offline optimization into the controller

III. Modeling of Sheet Reheat Phase in Thermoforming

As the MPC uses the process model to predict the output over the output horizon for an input vector and optimize the performance objective function, it is important to discuss the model used in designing the MPC. The model used in this section is primarily developed in [8]. The interested reader can get details of the model therein, but it is briefly discussed here to illustrate the proposed multi-parametric quadratic MPC. Each IR temperature sensor points at an area on the plastic sheet to perform the temperature measurement. Each such area is designated as a "zone". To facilitate modeling, we assume that there are two IR sensors for each zone of the plastic sheet, one looking at the sheet from above and the other from below (Fig.2).

To analyze the propagation of the heat inside the plastic sheet, heat transfer equations must be defined for some points inside the sheet. To do so, each zone is divided into layers throughout the thickness of the sheet (Fig.3). For each node, a differential equation describes the heat exchange of the corresponding layer. Since the surface of the plastic sheet is an important boundary of energy exchange, a node is located directly at the surface, see Fig.3. For each node, a differential equation describes the heat exchange of the corresponding layer. There are three ways (conduction, convection and radiation) to exchange energy between heaters, ambient air and nodes. Combining all three forms of heat transfer into the equation for 2M heaters, Z zones and 2 nodes for each zone, and taking the transmissivity into account in the energy transfer from the radiant heaters to the plastic sheet, the model for the k-th zone in the heating phase becomes,

$$\frac{dT_{k,top}}{dt}=\frac{2}{\rho VC_p}\left\{\begin{array}{l}\left(\frac{kA}{\Delta z}(T_{k2}-T_{k,top})\right)+h_{st}\left(T_{\infty_{top}}-T_{k,top}\right)\\+\beta_1 Q_{RT_k}+\beta_1(1-\beta_1)Q_{RB_k}\end{array}\right\}$$

$$\frac{dT_{k,bottom}}{dt}=\frac{2}{\rho VC_p}\left\{\begin{array}{l}\left(\frac{kA}{\Delta z}(T_{k,N-1}-T_{k,bottom})\right)\\+h_{sb}\left(T_{\infty bottom}-T_{k,bottom}\right)\\+\beta_1(1-\beta_1)Q_{RT_k}+\beta_1 Q_{RB_k}\end{array}\right\}$$

$$\frac{dT_{\infty,top}}{dt}=\frac{1}{\rho_{air}V_{air}C_{p_{air}}}\left\{\begin{array}{l}\sum_{j=1}^{M}A_h h_{ht}\left(\theta_j-T_{\infty,top}\right)\\+2A_{air}h_a\left(T_{air}-T_{\infty,top}\right)\\+\sum_{i=1}^{Z}A_{zone}h_{st}\left(T_{i,top}-T_{\infty,top}\right)\end{array}\right\}$$

$$\frac{dT_{\infty,bottom}}{dt}=\frac{1}{\rho_{air}V_{air}C_{p_{air}}}\left\{\begin{array}{l}\sum_{j=M+1}^{2M}A_h h_{hb}\left(\theta_j-T_{\infty,bottom}\right)\\+2A_{air}h_a\left(T_{air}-T_{\infty,bottom}\right)\\+\sum_{i=1}^{Z}A_{zone}h_{sb}\left(T_{i,bottom}-T_{\infty,bottom}\right)\end{array}\right\} \quad (4)$$

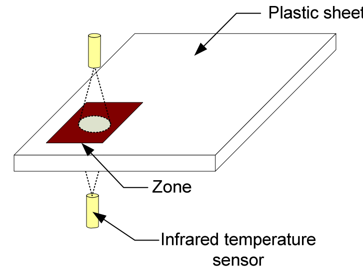


Fig.2: Zone and IR temperature sensors

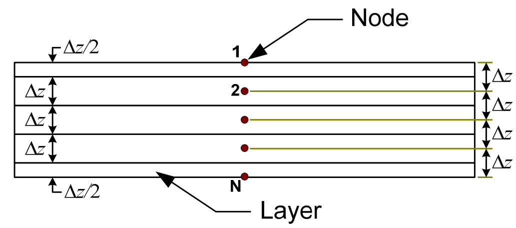


Fig. 3: Layers and nodes

where,

$$Q_{RT_k}=\sigma\varepsilon_{eff}A_h\left[\sum_{j=1}^{M}\left(\theta_j^4-T_{k,top}^4\right)F_{kj}\right]$$

$$Q_{RB_k}=\sigma\varepsilon_{eff}A_h\left[\sum_{j=M+1}^{2M}\left(\theta_j^4-T_{k,bottom}^4\right)F_{kj}\right]$$

$$\beta_1:=1-e^{-A\Delta z/2}$$

The meaning of the symbols used in the model equation can be found in reference [8]. Details of the method for calculating effective emissivity and view factors can be found in [9].

IV. Design of Multi-Parametric Quadratic MPC for Heating Phase of Thermoforming Machine

The design of the MPC controller is discussed in this section. At first, the system equation is linearized to obtain a linear system equation of the system so that multi-parametric MPC can be developed using the system equation. The next step is to incorporate the constraints into the controller. The oven heaters have maximum and minimum temperature constraints. The heating and cooling rate of the heaters also have some limitations. The conventional MPC requires an online solution of the optimization problem within a sampling period. As the size of the model of heating phase of the thermoforming process is large, it is difficult to use online optimization to implement MPC. So, the next step is to compute the optimization offline using multi-parametric programming. This recently developed technique allows solving an optimization problem offline for a constrained system within a certain range of the parameters. The solution of the optimization problem will be provided by a piecewise affine function by analyzing several properties of the geometry of the polyhedral partition and its relation to the combination of the active constraints for different polyhedral region. Then, the MPC controller based on the model is tuned in such a way that the desired performance is achieved.

(1) Linearization of the system:

The nonlinear system equations of the thermoforming process need to be linearized at an operating point of the system to incorporate the model in the design of the MPC controller. On the other hand, because of the nonlinear property of the system, the equation obtained by linearizing the system at a particular operating point may not properly sustain the properties of the actual system at another operating point far from the linearization point. Thus, the system is linearized at different operating point and different controllers are developed for each linear system. Based one the operating point, the control input will switch among different controllers. In this paper, different operating points are selected: $[T_k = 50^\circ C, \theta_j = 200^\circ C]$,

$[T_k = 100^\circ C, \theta_j = 250^\circ C]$, and

$[T_k = 150^\circ C, \theta_j = 300^\circ C]$

(2) Incorporating Constraints:

There are some input-constraints in sheet heating of the thermoforming process such as the maximum and minimum heater temperatures as well as maximum heating and cooling rates because of the limited input to the heater. The heaters are usually made of ceramic that could be damaged if heated more than 500°C, which results in a constraint in the input heater temperature. On the other hand, the heater cannot be cooled less than the environment temperature. Ajersch performed some experiments to determine the maximum rate of heating and cooling to develop a model of the heater bank [8]. Although it can give some primary idea about the maximum heating and cooling rate, these rates depend on the operating condition of the system like the input power, heat consumed by the sheet, heat consumed by oven air and oven wall (that largely depend on sheet, oven air and oven wall temperature). As the maximum electrical power input to the heater is bounded, it is quite understandable that the maximum heating rate is bounded too. The bound on the maximum heating and cooling rates of the heater depend on the amount of heat transfer to plastic sheet, oven wall by radiation and to oven air by convection. With the increase of the heater temperature, the maximum cooling rate increases as the heater can lose heat faster to the sheet, oven wall and to the environment. As the heater loss increases at higher temperature, the maximum heating rate will be reduced. On the contrary, at lower temperature of the heater the maximum heating rate increases and maximum cooling rate decreases. So the input constraints about heating and cooling rate are function of the current heater temperature. Unfortunately, MPC cannot handle this kind of input-constraints that depend on present value of the input heater temperature. But the whole operating range of the heater can be divided into different sub-range and different maximum heating and cooling rate constrains could be incorporated in the design of the controller.

(3) Reduction of the number of partitions in offline solution of multi-parametric quadratic MPC:

The number of polyhedral regions depends on a number of parameters that include system state, previous control input, reference output, measured disturbance and prediction horizon as well as the number of free input, constraint. The number of polyhedral regions also depends on the range of the parameters in the multi-parametric quadratic programming used to solve the optimization problem. Because of the large number of inputs, state variables and constraints in thermoforming, multi-parametric quadratic programming (mp-QP) results in a large number of polyhedral regions, with a piecewise affine function for each region, which is practically not possible to implement. So the next challenge in implementing MPC for this process is to reduce the number of region in the offline solution. One of the possible ways to reduce the number of regions is to reduce the size of the system input and hence reduce the number of parameters as well as number of the input constraints. The model of the heating phase of the thermoforming process has all of the constraints in its inputs. It can be proven that as the rank of the S matrix in the constraint of equation (4) is less than or equal to the number of constraints, the number of regions for piecewise affine solutions will remain the same for any number of parameters that is higher than the number of the constraints. In the case of the heating phase of this process, the number of parameters is much higher than the number of constraints. Therefore, the number of partitions or region of the parameter space defining the optimal controller is insensitive to the dimension of the parameter vector or the number of parameters involved in the mp-QP. Thus, if we can reduce the number of input constraints, the number of partitions will be reduced. In the case of the MPC design for the thermoforming machine, only two of the heaters (top heater and its opposite bottom heater) are used at a time to control the temperature of the sheet and all other heater temperature are considered to stay constant. As the temperature of a heater can change at most 1°K per second (where the actual heater temperature is within the range 350°K~700°K), it is reasonable to consider the heater temperature to be constant within a sampling period (which is 1 second). For each pair of heaters, a different MPC will be designed whereas other heater temperatures will be considered constant at the starting temperature of the sample. If there are 2M heaters in the thermoforming oven, then there will be M controllers that have just 2 inputs with the constraint applicable for those inputs. Thus, it is observed that the number of partitions or regions is significantly reduced for every MPC controller.

(4) Choosing the weight matrix of the controller

There are M controllers for the process and each controller computes reference heater temperatures for a pair of heaters. If the same weights are given for output tracking at every point's temperature of the sheet, then the controller will try to force the heater temperature in such a way that it tries to heat every point of the sheet to achieve the desired temperature. But some zones on the sheet are so far from the heater that the heater has very little influence on them, so the sensitivity of those parts of the sheet is very low with the change of heater temperature. This will force the heater to attain a very high temperature, even at the cost of a higher temperature at the nearest zone on the sheet from the heater. This could even burn some parts of the sheet. As the heaters are distributed all over the oven, every heater can be used more to heat those zones of the sheet that are closer to them. This could be attained by using an appropriate weight matrix for reference outputs. The elements of the weight matrix are chosen in such a way that the weight matrix entries for a sheet zone temperature will be inversely proportional to the distance between the sheet point and the heater.

(5)Tuning parameters of the controller:

The parameters of the controller, such as output prediction horizon, control horizon and constraint horizon length are tuned in this step such that the controller provides its desired performance. With the increase of the horizon length, the performance improves at the cost of an increase in the number of constraints that will increase the number of polyhedral regions. So the complexity of the final piecewise affine functions for the MPC controller increases dramatically, characterizing a tradeoff between performance and computational complexity. The length of the output prediction horizon, control horizon and constraint horizon length are chosen such that it is the smallest number giving a convenient number of polyhedral regions as well as providing the desired performance. After computing the controller using multi-parametric programming, the optimum control input command heater temperature to the system will be obtained as an affine function of system state, previous control input, reference output and measured disturbance.

V. SIMULATION RESULTS

The effectiveness of the proposed MPC controller for the thermoforming heating process is investigated extensively in simulation. First, a simulation model is developed using Matlab/Simulink. Then, the performance levels of the proposed and conventional methods are compared using the developed model. The oven consists of top and bottom heater. Each heater consists of 6 (3x2) heater banks. There are 9 equidistant sensors (3x3) considered on each side of the sheet. The conventional PI controller and cycle-to-cycle iterative learning controller (ILC) are used to compare the performance with the proposed MPC controller. After the design and development of the MPC controller, each pair of control inputs will be formulated in an explicit expression of 30 system state variables (18 outputs, 2 air temperatures and 10 other inputs), 2 previous control inputs and 18 reference outputs. The output sheet temperature at the sensor points for the first 10 cycles with a cycle duration of 700s are shown in fig.4 for PI, ILC and MPC controllers, respectively. A ramp with maximum amplitude 130°C is used as command sheet temperature for PI and MPC controller whereas a constant 130°C is used for ILC to heat the sheet to a uniform sheet temperature at 130°C at the end of the cycle. At the beginning of each cycle, the sheet is entered into the oven. So the temperature of the sheet is same as the environment temperature of the industry at the start of the cycle and the temperature of the sheet increases over the cycle. The controllers control the heater temperature of the oven to have a uniform 130°C temperature over the whole sheet. It is observed that the PI controller gives the largest deviation of 10°C.

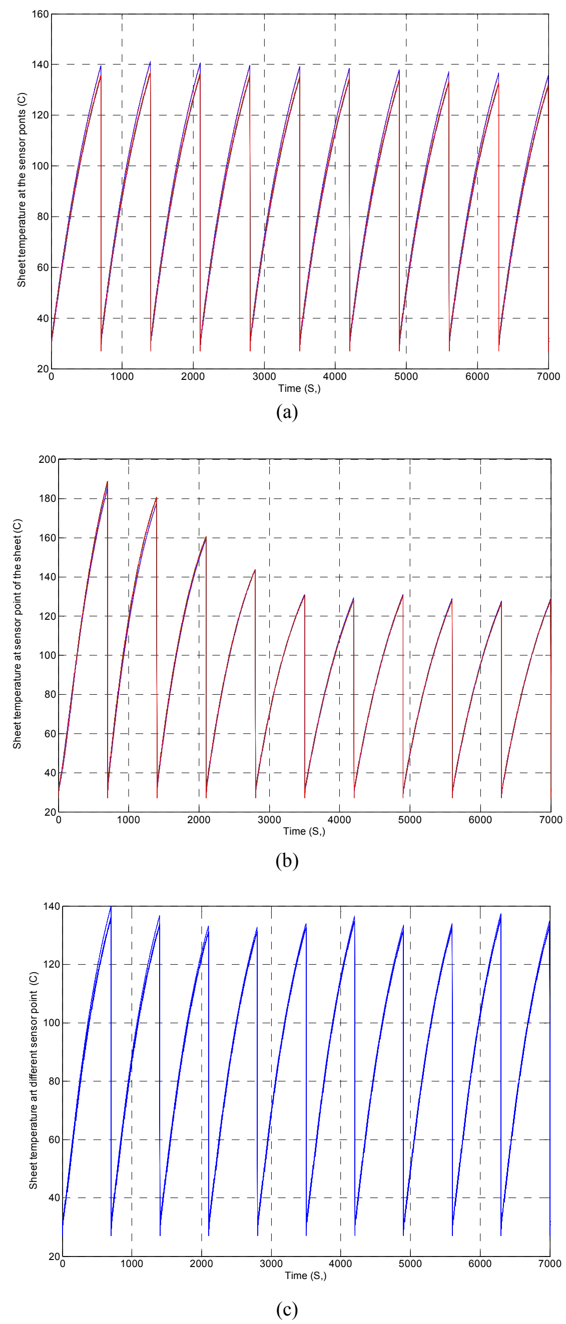


Fig.4: The sheet temperature at sensor location for (a) PI (b) ILC (c) MPC

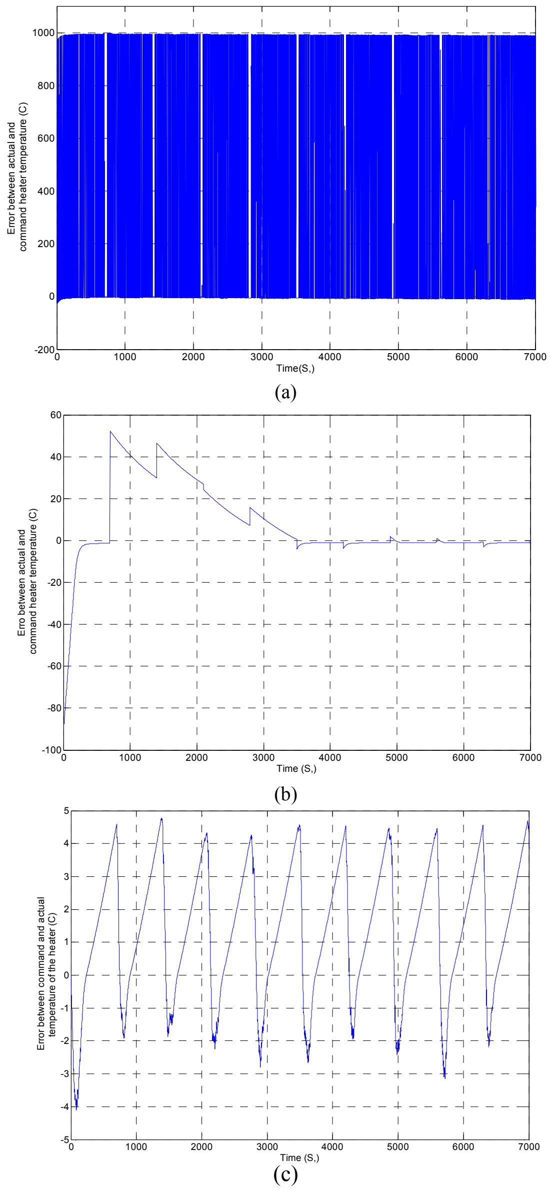


Fig.5: Error between control input and the actual temperature of a heater for (a) PI (b) ILC (c) MPC

The ILC controller gives very bad performance in the first few cycles, but it is getting better along the following cycles. In contrast, the proposed MPC controller gives a better performance as the sheet temperature obtained at the end of the cycle remains pretty close to the desired temperature. In Fig. 5, it is observed that the heater temperature of the PI controller cannot follow the command heater temperature from the controller output as the controller did not consider the constraint of the heater in calculating the control input. So the error between the command heater temperature and actual heater temperature is as high as 1000°C. In case of ILC, the error is large at the beginning but it gets better with time, even though the controller did not consider the constraint of the heater in calculating the control input. But it takes five cycles to attain the command heater temperature. In the case of MPC, the heater can follow the command heater temperature from the controller and the error is as low as 5°C.

VI. Conclusion

In this paper, a step-by-step approach is proposed for the development of a multi-parametric MPC for the thermoforming process. The main challenges in the deployment of the MPC controller for the process are discussed. The explicit implementation of the MPC controller, in the form of a piecewise affine control law computed offline, obviates the need for online optimization.

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