

# Advanced Traveler Information System with Communication Constraints

Rohit Kumar and David A. Castañón

**Abstract**—The Advanced Traveler Information Systems (ATIS), which provide real-time information about public transit to the commuters have gained tremendous popularity in last few years. The accuracy of such a system depends on the location information of the bus at the base station. Consequently bus uses a fixed transmission policy of communicating periodically to base station. This periodic communication is generally expensive, and it is in fact one of the main factors that prevents aggressive deployment of such systems. In this paper we first propose new architecture and develop new framework to study the ATIS in a communication constrained environment and then propose three different techniques for implementation of the system that tries to minimize loss in accuracy in this constrained environment. Simulation examples are provided in the end to support the proposed theory.

## I. INTRODUCTION

Advances in technologies like wireless communication, Global Positioning System (GPS), and computational power have directly impacted the transportation industry [1]. These technologies have led to the development of ATIS [2], [3], [4]. These systems provide the commuters with time of arrival and current location of the public transit, and it helps them to better plan their trip. These systems have received even more attention as governments are trying to encourage people to use public transport dependence on oil [5].

For an ATIS, each transit has a GPS receiver and a transmitter that transmits the location information of the bus to the base station [6], [7]. The base station then uses this location information and advanced algorithms to estimate the time of arrival (ETA) for rest of the stops on the itinerary [8], [9], [10], [11], [12]. A block diagram of the existing ATIS is shown in the Fig. 1(a).

In the block diagram, Predictor uses location information of the bus to predict estimated time of arrival for subsequent stops on the route. Existing methods employ protocols of transmissions that are fixed a priori, either transmitting location information either every  $k^{th}$  time instant or after every  $m$  meters [13], [14]. The problem with these protocols is that they do not account for the traffic condition and time of journey. For example, if the bus is running when there is less traffic on the road, the number of communications to the base station can be reduced. Excessive communication is one of the main hurdles for implementation of ATIS systems, especially in developing countries. A transit authority (TriMet in Portland, Oregon) that did not account for communication

cost in its planning had to withdraw its implementation due to the cost [5]. Proposals to use wire line communications have been dismissed because there is no flexibility to change routes or change bus stops, and it also causes problems in maintenance.

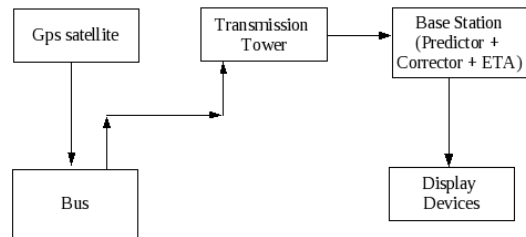


Fig. 1. Figure showing the block diagram of the ATIS system. The predictor and corrector are the predict and update steps of the conventional filter. ETA are the algorithms that are used to estimate the expected time of arrival for the future stops on the itinerary.

In our previous work [15], we developed hybrid system models and estimation algorithms for accurate tracking of the location of buses at the depot as they traveled along routes with heterogeneous traffic conditions. This required periodic communication of bus location measurements to the depot. In this paper we propose a new architecture for ATIS that allows buses to determine when transmissions are required, in order to reduce communications. We develop algorithms for controlling the occurrence of bus transmissions that try to maintain the accuracy of the bus location estimates at the base station under communication constraints, using alternative optimization approaches.

The rest of this paper is organized as follows: Section II presents background model and estimation algorithms used for tracking locations of buses. Section III describes the problem of determining when buses should communicate their local estimates to the base station. In section IV we present three alternative approaches for controlling when to transmit from buses to the base station. Section V has evaluations of the proposed algorithms on a simulated example with three traffic links. The paper ends with concluding remarks in Section VI.

## II. BACKGROUND

In this section we present the model that is used to define dynamics of the bus on the road and estimation algorithm that will be used for tracking. These models and estimation algorithms were described in [15]; we provide a brief review here.

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D. Castañón and R. Kumar are with the Department of Electrical Engineering, Boston University

We assume that bus route is known a priori, and can be divided into a sequence of traffic links  $m = 1, \dots, M$ . Each link has different traffic conditions, and is modeled with different dynamics that governs the evolution of continuous state associated with the bus location and speed. Let  $m(k)$  denote the link index that bus is in at time  $k$ . Then, the model at time  $k$  on link  $m(k)$  is described in state space form as:

$$\mathbf{x}(k+1) = \mathbf{F}_{m(k)}\mathbf{x}(k) + \mathbf{W}_{m(k)}(k) \quad (1)$$

$$\mathbf{y}(k) = \mathbf{h}_{m(k)}(\mathbf{x}(k)) + \mathbf{V}_{m(k)}(k) \quad (2)$$

$$m(k) = g(\mathbf{x}(k)) \quad (3)$$

where  $\mathbf{F}_{m(k)}$  is a matrix governing the linear state dynamics,  $\mathbf{h}_{m(k)}$  is a nonlinear measurement equation, and  $g$  is an integer valued map that identifies the current link at time  $k$ . The processes  $\mathbf{W}_{m(k)}(k)$  and  $\mathbf{V}_{m(k)}(k)$  are link dependent, modeled as mutually independent zero-mean white Gaussian noise with zero mean and variance  $\mathbf{Q}_{m(k)}(k)$  and  $\mathbf{R}_{m(k)}(k)$ , respectively.

In our model, each link has a different average velocity that governs the flow of traffic on that. To account for this, we model velocity of the vehicle by an *Ornstein-Uhlenbeck* process, which is specified in continuous time as

$$dV_t = -\lambda(V_t - V_0) + \sigma dW_t \quad (4)$$

with  $dW_t$  is the differential of Brownian motion. In (4),  $V_t$  is velocity at time  $t$ ,  $\lambda$  is rate of convergence to average speed, and  $V_0$  is average velocity of the link. Adding position state to the continuous dynamics and then discretizing it with time step  $h$  yields the following discrete time model for the link parameters:

$$\mathbf{x}(k+1) = \mathbf{F}_{m(k)}^{(h)}\mathbf{x}(k) + \mathbf{B}_{m(k)}^{(h)}u_{m(k)} + \mathbf{W}_{m(k)}^{(h)}(k) \quad (5)$$

where  $\mathbf{F}_{m(k)}^{(h)} = e^{\tilde{\mathbf{F}}_{m(k)}h}$ ,  $\mathbf{B}_{m(k)}^{(h)} = \int_0^h e^{\tilde{\mathbf{F}}_{m(k)}s}\tilde{\mathbf{B}}ds$ ,  $h$  is the sampling interval, and  $\mathbf{W}_{m(k)}^{(h)}(k)$  is white Gaussian noise with zero mean and variance  $\mathbf{Q}_{m(k)}^{(h)}$ . The expressions for  $\mathbf{F}_{m(k)}^{(h)}$ ,  $\mathbf{B}_{m(k)}^{(h)}$  and  $\mathbf{Q}_{m(k)}^{(h)}$  are given below.

$$\mathbf{F}_{m(k)}^{(h)} = e^{\tilde{\mathbf{F}}_{m(k)}h} = \mathbf{I}_{2 \times 2} + \tilde{\mathbf{F}}_{m(k)} \frac{(e^{\lambda_{m(k)}h} - 1)}{\lambda_{m(k)}}$$

$$\mathbf{B}_{m(k)}^{(h)} = \int_0^h e^{\tilde{\mathbf{F}}_{m(k)}s}\tilde{\mathbf{B}}ds = \begin{bmatrix} \frac{1}{\lambda_{m(k)}^2}(e^{\lambda_{m(k)}h} - 1) - \frac{h}{\lambda_{m(k)}} \\ \frac{1}{\lambda_{m(k)}}(e^{\lambda_{m(k)}h} - 1) \end{bmatrix}$$

$$\mathbf{Q}_{m(k)}^{(h)}(1,1) = \frac{h}{\lambda_{m(k)}^2} + \frac{1}{\lambda_{m(k)}^3} \left( \frac{e^{2\lambda_{m(k)}h} - 1}{2} - 2(e^{\lambda_{m(k)}h} - 1) \right)$$

$$\mathbf{Q}_{m(k)}^{(h)}(1,2) = \frac{1}{\lambda_{m(k)}^2} \left( \frac{e^{2\lambda_{m(k)}h} - 1}{2} - (e^{\lambda_{m(k)}h} - 1) \right)$$

$$\mathbf{Q}_{m(k)}^{(h)}(2,2) = \frac{e^{2\lambda_{m(k)}h} - 1}{2\lambda_{m(k)}}$$

$$\mathbf{Q}_{m(k)}^{(h)} = \begin{bmatrix} \mathbf{Q}_{m(k)}^{(h)}(1,1) & \mathbf{Q}_{m(k)}^{(h)}(1,2) \\ \mathbf{Q}_{m(k)}^{(h)}(2,1) & \mathbf{Q}_{m(k)}^{(h)}(2,2) \end{bmatrix} \times \sigma^2$$

where  $\tilde{\mathbf{F}}_{m(t)} = \begin{bmatrix} 0 & 1 \\ 0 & \lambda_{m(t)} \end{bmatrix}$ ,  $\tilde{\mathbf{B}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ ,  $m(k)$  indicates the link number ( $m(k) \in \{1, \dots, M\}$ ),  $u_{m(k)} = -\lambda_{m(k)}V_{0m(k)}$  ( $V_{0m(k)}$  is average velocity of the  $m(k)$ <sup>th</sup> link) and  $\lambda_{m(k)}$  is the rate constant for link  $m(k)$ . The details are given model can be found in [15].

Based on this hybrid model, the results in [15] propose different nonlinear estimation algorithms for tracking the location of buses. In this paper, we will use a modification of Extended Kalman filter (EKF) for the system given in (1 - 3) proposed in [15]. The main extension is to incorporate the unknown switching time between links due to bus position uncertainty, which requires a two-step predictor that exploits the property that the proposed process model is linear. The two-step prediction algorithm is described below.

Assume that at step  $k-1$ , EKF updated estimate is  $\mathbf{x}(k-1|k-1)$  and its updated covariance  $\mathbf{P}(k-1|k-1)$ . The algorithm is implemented as follows: first step is to compute expected time remaining to switch links,  $\tau_s$ , by predicting the updated estimate using the process model. This is straightforward is given as follows:

$$\tau_s = \frac{C - x_1(k-1|k-1)}{x_2(k-1|k-1)}$$

where  $C$  is the location of the switching corner,  $\mathbf{x}(k-1|k-1) = [x_1(k-1|k-1) \ x_2(k-1|k-1)]^T$ . Then, if  $\tau_s \geq h$ , the prediction algorithm uses the current link model and generates a standard EKF prediction. In case where  $\tau_s < h$ , prediction uses a two-step process, where prediction from  $t = kh$  to  $kh + \tau_s$  uses model from the first link, and prediction from  $kh + \tau_s$  to  $(k+1)h$  uses the model from subsequent link, where discrete model matrices are adjusted appropriately to the size of the prediction intervals. The update of EKF remains the same. Relevant equations are summarized below.

$$[\mathbf{x}(k|k-1), \mathbf{P}(k|k-1)] = \text{Predict}\{\mathbf{x}(k-1|k-1), \mathbf{P}(k-1|k-1)\}$$

One step prediction equations (Away from corner):

$$\mathbf{x}(k|k-1) = \mathbf{F}_{m(k)}^{(h)}\mathbf{x}(k-1|k-1)$$

$$\mathbf{P}(k|k-1) = \mathbf{F}_{m(k)}^{(h)}\mathbf{P}(k-1|k-1)\mathbf{F}_{m(k)}^{(h)T} + \mathbf{Q}_{m(k)}^{(h)}$$

Two step prediction equations (Close to the corner)

$$\mathbf{x}(\tau_s|k-1) = \mathbf{F}_{m(k)}^{(\tau_s)}\mathbf{x}(k-1|k-1)$$

$$\mathbf{P}(\tau_s|k-1) = \mathbf{F}_{m(k)}^{(\tau_s)}\mathbf{P}(k-1|k-1)\mathbf{F}_{m(k)}^{(\tau_s)T} + \mathbf{Q}_{m(k)}^{\tau_s}$$

$$\mathbf{x}(k|k-1) = \mathbf{F}_{m(k)+1}^{(h-\tau_s)}\mathbf{x}(\tau_s|k-1)$$

$$\mathbf{P}(k|k-1) = \mathbf{F}_{m(k)+1}^{(h-\tau_s)}\mathbf{P}(\tau_s|k-1)\mathbf{F}_{m(k)+1}^{(h-\tau_s)T} + \mathbf{Q}_{m(k)+1}^{(h-\tau_s)}$$

Update Equations:

$$[\mathbf{x}(k|k), \mathbf{P}(k|k)] = \text{Update}\{\mathbf{x}(k|k-1), \mathbf{P}(k|k-1), \mathbf{y}(k)\}$$

$$\hat{\mathbf{m}}(k) = g(\mathbf{x}(k|k-1))$$

$$\mathbf{H} = \frac{\partial}{\partial \mathbf{x}} h_{\hat{\mathbf{m}}(k)}(\mathbf{x}(k|k-1))$$

$$\mathbf{innv}(k) = \mathbf{y}(k) - \mathbf{h}_{\hat{\mathbf{m}}(k)}(\mathbf{x}(k|k-1))$$

$$\mathbf{S}(k) = \mathbf{H}\mathbf{P}(k|k-1)\mathbf{H}^T + \mathbf{R}(k)$$

$$\mathbf{K}(k) = \mathbf{P}(k|k-1)\mathbf{H}^T\mathbf{S}^{-1}(k)$$

$$\mathbf{x}(k|k) = \mathbf{x}(k|k-1) + \mathbf{K}(k)\mathbf{innv}(k)$$

$$\mathbf{P}(k|k) = (\mathbf{I} - \mathbf{K}(k)\mathbf{H})\mathbf{P}(k|k-1)$$

### III. BUS COMMUNICATION PROBLEM

As shown in Fig.1, a bus collects GPS information and transmits it to the base station. The base station uses the received observation with an estimation algorithm such as the above EKF to estimate the current state of the bus, which is used by ETA algorithms to predict times of arrival for the rest of the stops on the itinerary.

We assume that the bus has on-board computation capability to perform its own estimation algorithm, and to determine whether its information is worth communicating to the base station. In this architecture, the base station still performs prediction between communications from the bus. However, when new communications arrive, they contain the most recent state estimate of the bus, and thus replace the bus state estimate at the base station.

Let  $P_G(k)$  be the conditional distribution of the bus state available at the base station at time  $k$ , and  $P_L(k)$  be the conditional distribution available at the bus at time  $k$ . We assume that  $P_G(k)$  is known both to the bus and the base station, because the bus can compute the same predictions as the base station. Let  $j \leq k$  denote the time of last communication from the bus to the base station. Then, in our proposed architecture,  $P_G(k) = p(\mathbf{x}(k)|\mathbf{Y}^{[0,j]})$ , and  $P_L(k) = p(\mathbf{x}(k)|\mathbf{Y}^{[0,k]})$ , where  $\mathbf{Y}^{[0,k]} = \{\mathbf{y}(1), \dots, \mathbf{y}(k)\}$ .

Let  $u(k)$  denote the decision at time  $k$  made by the bus as to whether to send its information to the base station:

$$u(k) = \begin{cases} 0 & \text{No Transmission;} \\ 1 & \text{Transmission;} \end{cases}$$

For  $u(k) = 1$ , the information at base station  $P_G(k)$  is updated with  $P_L(k)$ , i.e.,  $P_G(k) = P_L(k)$ .

The problem we would like to solve is to minimize the difference between  $P_G(k)$  and  $P_L(k)$  over time, while keeping the number of communications to a minimum. To measure distance between the conditional densities at the bus and the base station, we use the Kullback-Leibler (KL) distance, denoted by  $D_{KL}(P_G(k)||P_L(k))$ . Conceptually, we can formulate the problem of selecting the communication times as a stochastic control problem, as follows:

$$\begin{aligned} \text{Objective} \quad & \text{minimize}_u \quad E_y \left\{ \sum_{k=1}^T D_{KL}(P_G(k)||P_L(k)) \right\} \\ \text{subject to:} \quad & E_y \left\{ \sum_{k=1}^T c_1 u(k) \right\} \leq M_r; \end{aligned}$$

such that

$$P_L(k) = \text{Update}\{\text{Predict}\{P_L(k-1)\}, \mathbf{y}(k)\}$$

and

$$\begin{aligned} P_G(k) &= \text{Predict}\{P_G(k-1)\} \quad \text{if } u(k) = 0; \\ &= P_L(k) \quad \text{if } u(k) = 1. \end{aligned}$$

where  $u(k)$  is adapted to the information in  $\mathbf{Y}^{[0,k]}$ . The expectation is over the observations  $\mathbf{y}$ ,  $M_r$  is the total number of resources available and  $c_1$  is the cost of each communication. However, this problem is a stochastic control problem with combinatorial decision space, nonlinear objective and continuous measurements, which becomes intractable for most nontrivial instances. In the next section, we describe approximate solutions which are suitable for computation by the bus.

### IV. PROPOSED ALGORITHMS

As an initial step in developing approximate algorithms, we will assume that conditional densities  $P_G$  and  $P_L$  are well-approximated by Gaussian densities, and thus can be represented by their respective conditional means and covariances. The KL distance for two  $N$ -dimensional Gaussian distributions  $\mathcal{N}(\mu_0, \Sigma_0)$ , and  $\mathcal{N}(\mu_1, \Sigma_1)$  is given as [16]:

$$\begin{aligned} D_{KL}(\mathcal{N}_0||\mathcal{N}_1) &= \frac{1}{2} \left( \log_e \frac{|\Sigma_1|}{|\Sigma_0|} + \text{tr}(\Sigma_1^{-1}\Sigma_0) + \right. \\ &\quad \left. (\mu_1 - \mu_0)^T \Sigma_1^{-1} (\mu_1 - \mu_0) - N \right) \end{aligned}$$

This distance is readily computed in terms of the statistics generated by EKF algorithm used in tracking and predicting the current bus state.

#### A. Parametric Policy

The first policy we propose is a parametric policy for communication, based on the idea that bus should communicate when the distance between its conditional state distribution and the distribution at the base station exceeds a threshold  $\eta$ . Thus, as long as the bus is following the expected schedule, there is little need for additional transmissions. The resulting optimization is

$$\begin{aligned} \text{minimize}_\eta \quad & E_y \left\{ \sum_{k=1}^T D_{KL}(P_G(k)||P_L(k)) \right\} \\ \text{such that} \quad & E_y \left\{ \sum_{k=1}^T c_1 u(k) \right\} \leq M_r, \end{aligned}$$

where

$$u(k) = \begin{cases} 0 & \text{if } D_{KL}(P_G(k)||P_L(k)) \leq \eta; \\ 1 & \text{otherwise.} \end{cases}$$

where  $c_1$  is the cost of communication. Due to the monotonicity of the cost with respect to communications, the optimal threshold is one which uses the maximum communication while meeting the constraint with equality. Optimizing for the threshold  $\eta$  can be done off-line using Monte-Carlo simulations and learning, with a combination of stochastic approximation techniques. The threshold  $\eta$  selected off-line is fixed *a priori* and remains fixed for the remaining schedule.

#### B. Fixed Horizon Optimization

In this approach, we try to solve of problem to determine the time of communication using an on-line optimization framework to minimize the difference between global ( $P_G$ ) and local information ( $P_L$ ) subject to communication constraint. We require the selection of one communication time every  $p$  time steps, and we solve the problem of which time step is the best one to communicate. We divide the total travel time into windows of size  $p$  each such that only one transmission is permitted per window, i.e.,  $T = M_r \times p$ . Since  $p \ll T$ , the resulting optimization problem is significantly reduced and can be handled on-line. This approach is similar to the current practice of periodic communications, but gives the bus some flexibility in choosing the time of communication within a window based on its information. Note that the search space for each window is simply of size  $p$ .

The optimization problem for each window is given by

$$\text{minimize}_{\gamma \in \Gamma} E_y \left\{ \sum_{k=1}^p D_{KL}(P_G(k) || P_L(k)) \right\} \quad (6)$$

where each window has different initial conditions. The problem in (6) still requires expectation over future measurements obtained by the bus, making it unsuitable for real-time computation. Instead, we propose an open-loop model where the conditional means of the Gaussian conditional distributions  $P_G(k)$  and  $P_L(k)$  evolve according to the unobserved dynamics (i.e. with no measurement updates) while the conditional covariance of  $P_L(k)$  evolves as the Riccati equation of the EKF. This certainty equivalence approximation allows computation of the expectation in (6) in terms of known means and covariances that depend only on the time of communications. The resulting deterministic optimization problem becomes

$$\text{minimize}_{\gamma \in \Gamma} \sum_{k=1}^p D_{KL}(P_G(k) || P_L(k)) \quad (7)$$

Let  $\gamma_{opt}$  be the solution to (6). The communication schedule is formulated at the beginning of each window as shown in Fig. 2(a). Consequently even though the base station receives new information from the bus, it cannot use the new information immediately for re-planning. The next communication schedule is only developed at the beginning of the next window using the currently available information at that instant (which includes the new information due to transmission in the last window). Note that  $P_G$  and  $P_L$  at the end of planning phase and at the end of implementation phase are different distributions. This is because  $P_L$  has been updated with new observation and  $P_G$  has been updated with  $P_L$  at the time of scheduled transmission during the fixed window. Hence, a new plan is developed in light of the new available information. Note that the communication constraint is explicitly satisfied in this case as the total available resources are divided equally among each window.

### C. Receding Horizon Optimization

The fixed horizon policy reduces the computational complexity by dividing the optimization problem into individual windows. However, this policy is unable to adapt to unexpected errors during the planned period that may require more frequent communications, as one has to wait until the end of the current window before additional communications can be scheduled.

To overcome this drawback we propose a receding horizon approach that uses current information to plan communications for next  $p$  time steps. It then implements the recommended action for the next time step, updates the local and global probability density estimates  $P_G(k), P_L(k)$ , and then plans for the next  $p$  steps (see Fig. 2(b)). In this case we limit the plans to select only one communication per window. Therefore the planning problem for a window is given as:

$$\text{minimize}_{\gamma \in \Gamma} E_y \left\{ \sum_{k=1}^p D_{KL}(P_G(k) || P_L(k)) \right\} \quad (8)$$

As in fixed horizon optimization, the bus does not have access to future observation values at the planning time. In order to avoid excessive computations, we again use a

certainty equivalence approach and assume that the mean of the conditional distributions evolves open-loop using the predictor EKF equations. Since observation is the only random quantity in the problem formulation, this transforms a stochastic optimization problem to a deterministic one.

In order to account for limited communications, we introduce a time-varying penalty term on the objective function as

$$\text{minimize}_u \sum_{k=1}^p D_{KL}(P_G(k) || P_L(k)) + \sum_{k=1}^p \check{\lambda}(k) c_1 u(k) \quad (9)$$

where penalty  $\check{\lambda}(k)$  is time varying, and chosen to encourage satisfaction of the cost constraints in (6).

For our receding horizon algorithm, we choose  $\check{\lambda}(k) = \frac{L}{\delta}$  where  $L$  is a tuning parameter chosen via Monte-Carlo simulations and  $\delta$  is the time difference between current time and the time that last transmission took place. Thus, the cost of communications  $\check{\lambda}$  decreases inversely proportional to the time between successive transmissions. This discourages use of new transmissions close to previous transmissions unless new information has created a large distance between the conditional densities  $P_G(k), P_L(k)$ . However, as the time from the previous transmissions increases, it is less costly to use communications.

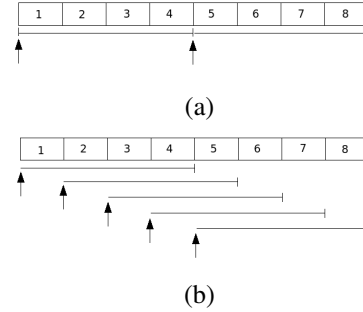


Fig. 2. Optimization set for a window length of 4. The arrow indicates the point when re-planning takes place. (a) Fixed Horizon (b) Receding Horizon

## V. SIMULATION EXAMPLE

In this section we illustrate the performance of the above algorithms as compared with alternative approaches using simulated examples. In this example we take a linear system of three links as shown in Fig. 3. The dynamics of each link is modeled with process model from section II, with following parameters: L1:  $V_0 = 1500, \lambda = -1, \sigma^2 = 300$ ; L2:  $V_0 = 2000, \lambda = -1, \sigma^2 = 450$ ; L3:  $V_0 = 1750, \lambda = -1.5, \sigma^2 = 350$ . The distances are measured in feet and time in minutes. We use parameters that are closer to those expected in urban traffic. In Fig. 3, each street has a different elevation ( $[\theta_{L_1}, \theta_{L_2}, \theta_{L_3}] = [\frac{\pi}{4}, 0, \frac{\pi}{3}]$  rads) with respect to the reference coordinate system and the measurements are made via a GPS receiver. The GPS is assumed to have variance of  $500^2$  along  $x$  and  $y$  directions. In addition, we assume that the average velocity in each link is unknown, and must be estimated as part of the state variable  $\mathbf{x}(k)$  based on the observations

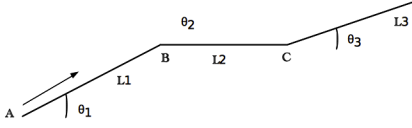


Fig. 3. Examples Set up. L1,L2 and L3 are the links. A, B and C are the corners with coordinates  $(x_1^c, y_1^c)$ ,  $(x_2^c, y_2^c)$ , and  $(x_3^c, y_3^c)$ , respectively. The arrow indicates the direction of travel.

of actual position based on progress against traffic. The observation equation is given as:

$$\mathbf{y}(k) = \mathbf{H}(k)\mathbf{x}(k) + \mathbf{M}(k) + \mathbf{v}(k)$$

where  $\mathbf{H}(k) = \begin{bmatrix} \cos\theta_{m(k)} & 0 & 0 \\ \sin\theta_{m(k)} & 0 & 0 \end{bmatrix}$ , and  $\mathbf{M}(k) =$

$\begin{bmatrix} x_{m(k)}^c - d_{m(k)}\cos\theta_{m(k)} \\ y_{m(k)}^c - d_{m(k)}\sin\theta_{m(k)} \end{bmatrix}$ ,  $d_{m(k)}$  is the sum of link lengths prior to the current link. These equations can be easily derived using geometry. For our problem ( $T = 60, M_r = 8, p = 8, c_1 = 1$ ). For the parametric policy, we ran 100 simulations for each  $\eta$  ( $\eta$  is increased in steps of 0.02). We find that  $\eta_o = 0.65$  satisfies the average communication constraints and yields the best average KL distance. Hence  $\eta_o$  is used as the threshold for real-time examples.

We will compare our results against periodic policy where base station is updated with conditional distribution of the bus every  $s^{th}$  time instant, i.e,

$$P_G(k) = \begin{cases} P_L(k) & \text{if } \frac{k}{s} \text{ is an integer;} \\ = \text{Predict}\{P_G(k-1)\} & \text{otherwise.} \end{cases}$$

For the fixed horizon problem, we have 7 windows. So only one transmission is allowed in each window and one transmission is used for hand-shake. For the receding horizon implementation, we perform Monte-carlo simulations off line to determine the value of  $L$  such that communication constraints are satisfied. We find that  $L = 10$  works well for our problem.

We once again like to explicitly highlight some features of our system. This is a push protocol system where the bus does the planning and decides the time of communication. Only bus has access to the true observation  $y$ . The base station is updated with latest estimation of the filter at the bus whenever a transmission takes place. The bus has the access to system equations of the base station. Since the filter is implemented with an EKF, the conditional distributions are means and covariances of the Gaussian distributions.

For the parametric policy, once the threshold is decided the implementation is relatively simple. The bus has its own current information and information available at the base station. Whenever the difference is greater than the precomputed threshold, a transmission takes place and the base station is updated. For fixed horizon and receding horizon optimization, the means of the  $P_G$  and  $P_L$  are predicted forward to determine the link change (change in the dynamical system) during the planning and implementation phase and then distribution is propagated accordingly.

The communication policy for a random simulation is shown in Fig. 4 and tracking accuracy are given in Fig. 5, 6, 7, 8 . We present average KL distance between the bus and base station for the trip. We believe KL distance is better statistic to measure the loss in accuracy due to communication constrained environment. Lower the average KL distance implies that base station has better understanding of bus. The estimated time of arrival for rest of the stops on the itinerary will be directly proportional to the accuracy of current information available at the base station. However we have listed Root Mean Square (RMS) and sum of the trace of the variances in Table I for readers interested in these statistics. Table I lists results of 200 Monte-Carlo simulations.

Type	Mean x	mean v	tr(Variance)	Avg. KL Dist.
KF	200	21.79	$0.6998 \times 10^5$	-
Periodic	226	19.57	$1.8654 \times 10^5$	1.1432
Parametric	223	20.80	$1.5592 \times 10^5$	0.25359
Fix. Horizon	254	20.13	$1.4378 \times 10^5$	1.1735
Rec. Horizon	222	20.41	$1.7581 \times 10^5$	0.30957

TABLE I  
RMS ERROR FOR DIFFERENT ALGORITHMS.  $x$  IS MEASURED IN FEET  
AND  $v$  IS MEASURED IN FEET/MINS.

As expected, we note that receding horizon which accounts for the new information at each step outperforms fixed horizon and is comparable to parametric approach. However one would have expected for fixed horizon to perform better than the parametric policy but that was not the case in majority of the experiments which basically shows that one can achieve good performance in constrained environment by choosing the transmission threshold appropriately. The threshold was well trained to the simulations and hence it is can perform better than receding horizon as well. However we do not expect parametric approach to perform better than receding horizon in general as the later adapts to the real-time information while for parametric, threshold is chosen a priori. If one takes a closer look at the fixed horizon optimization schedule it almost looks like a periodic policy if it was not for the change in dynamical system on different links or for some unlikely observations.

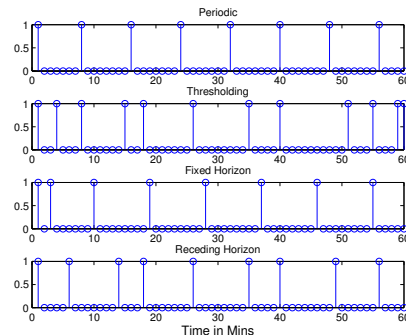


Fig. 4. Transmission Schedule for different protocols. An impulse function at any time indicates that base station is updated by the bus.

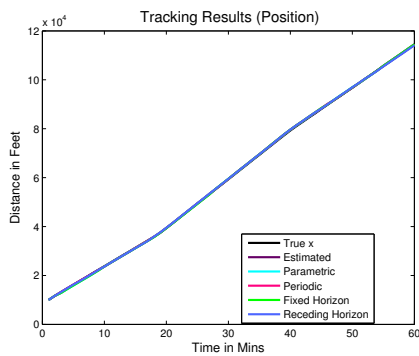


Fig. 5. Tracking Results for position

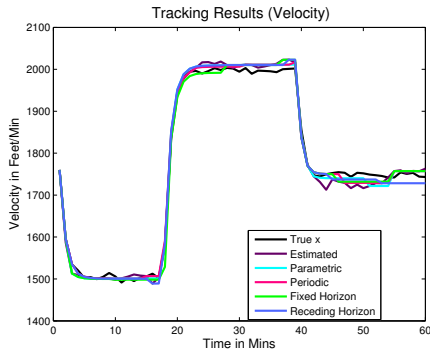


Fig. 6. Tracking Results velocity

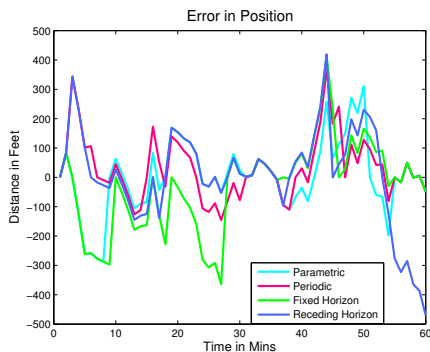


Fig. 7. Error in Position

## VI. CONCLUSION

In this paper we present a new push architecture for Advanced Traveler Information Systems based on buses determining when to communicate their position information to the base station. We also developed new algorithms that enable buses to determine when to communicate with the base station, and evaluated them using multi-link simulations. Our experiments show that the proposed algorithms reduce the number of transmission by more than 75% without much loss in estimation accuracy at the base station.

While our current work is based on simulations, we are pursuing approaches to test these algorithms using experimental data collected from real systems. Other directions for future research include enhancing the bus motion model to incorporate planned stops. The current motion models are

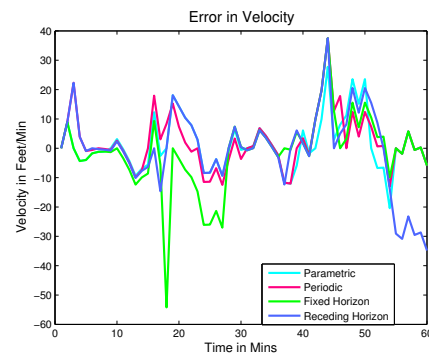


Fig. 8. Error in Velocity

accurate for transit between stops, but do not model the approach, stay and departure stages of bus stops. In addition, one can envision alternative architectures where the base station can request information from the bus based on its knowledge provided by other buses.

Our results provide evidence that ATIS can perform in a communication constrained environment without significant loss in accuracy, which will enable fielding of lower cost ATIS systems.

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