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Nonlinear Aircraft Modeling and Controller Design for Target Tracking

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Abstract—**In order to provide a novel perspective for videography of high speed sporting events, a highly capable trajectory tracking control methodology is developed for small scale Unmanned Aerial Vehicles (UAVs). The proposed controller combines dynamic inversion with linear tracking control using the internal model approach, and relies on a trajectory generating exosystem generated from available target training data. Three different aircraft models are presented each with increasing levels of complexity, in an effort to identify the simplest controller that yields acceptable performance. The dynamic inversion and linear tracking control laws are derived for each model, and simulation results are presented for tracking of elliptical and periodic trajectories.**

I. Introduction

Many sporting events exist for which a real-time third-person perspective of the action would be exhilarating for spectators to observe. Such a view, located directly behind and above the athlete, remains difficult to achieve due to the high speed of the action, the rapid changes in direction and the resulting safety concerns that arise from tracking sporting action at close range, particularly with manned aircraft. The use of small autonomous aerial vehicles may open the door to such a perspective for a wide variety of sporting events, such as downhill skiing, mountain and road biking and race car driving.

The problem of tracking an arbitrary moving ground target can be simplified in this application by assuming a known track that the target will follow. The problem can then be converted to one of nonlinear output tracking with a known exosystem, subject to disturbances and variations in time.

A sequence of three nonlinear aircraft models of increasing complexity are presented. All three models are demonstrated to be feedback linearizable, and a common linear tracking control methodology is employed on each. The first model is a simple 3 degree of freedom (DOF) model that is used to establish the possibility of using nonlinear techniques for trajectory tracking, but the model does not capture, for example, nonlinearities in the relationship between lift, drag and vehicle speed or angle of attack. The second follows Hauser and Hindman [1], and describes a coordinated flight regime where the sideslip angle is deliberately maintained at zero, restricting the possible trajectories of the aircraft, but improving on the first model in terms of fidelity. The final model is a more classical model of aircraft dynamics first presented by Lane and Stengel [2], relying on aerodynamic derivatives to define the forces and moments.

The tracking control approach used involves first performing feedback linearization and then applying linear tracking control using the internal model principle of Francis and Wonham [3]. For this, linear exosystems are defined which allow tracking of elliptical and periodic trajectories which can be fit to existing track data for third-person perspective tracking. An alternative to this approach which relaxes the need for feedback linearization would be to define nonlinear tracking controllers per Isidori and Byrnes [4], however this method requires the solution of a partial differential equation for each trajectory under consideration. Khalil's robust tracking control could also be considered [5], however it relies on high-gain assumptions to ensure tracking which is impractical to implement.

The requirement of feedback linearization has led to the selection of a particular subset of aircraft models, and has resulted in the omission of valid models [6], [7] which include coupling between forces and angles. The result of such coupling, which does indeed exist in practice, is that the system becomes non-minimum phase, although slightly so, as was demonstrated by Tomlin et al. [7], and therefore is no longer feedback linearizable. For non-minimum phase systems Devasia, Chen and Paden [8] presented a method for nonlinear tracking based on non-causal inversion. This aspect of the modeling and controller design remains an area for future work.

This paper proceeds as follows. Section (II) presents the three aircraft models in order of increasing complexity, and some representative linear target models are presented. The tracking controllers to be used with each model are defined in Section (III), and simulation results are presented in Section (IV). Finally, a brief discussion of future directions is included in the final Section.

II. Aircraft and Target Models

Four distinct coordinate frames are used to define the aircraft models in order to specify forces and moments in a straightforward manner. Vehicle motion is defined relative to an inertial frame $\mathbf{I}$. Aerodynamic forces and moments are defined in the wind frame $\mathbf{F_w}$, i.e., vehicle carried axes aligned with the direction of the oncoming freestream velocity, or in a vehicle carried vertical frame $\mathbf{F_v}$. Finally, the body frame $\mathbf{F_b}$, used solely in the third model, is needed for defining the equations of motion of the vehicle.

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Throughout this section, $x \in \mathbb{R}^3$ denotes the aircraft position in an inertial frame, with $\dot{x}$ and $\ddot{x}$ the velocity and acceleration in inertial coordinates. In the sense of Euler angles, the frames $\mathbf{F_w}$ and $\mathbf{F_b}$ are related by the rotation sequence ($-\beta$, α, 0) as seen in Fig. (1). α and β denote the aerodynamically important angles, angle of attack and sideslip, respectively. The rotation from $\mathbf{F_w}$ to $\mathbf{I}$ is defined by $R \in SO(3)$ and is used in the second model. The 3-2-1 Euler angle sequence (ψ, γ, ϕ) gives the orientation of $\mathbf{F_w}$ relative to $\mathbf{F_v}$. Here, γ denotes the flight path angle of climb and ψ denotes the flight path heading. Angular velocities of the $\mathbf{F_b}$ frame are represented by (p, q, r) whereas (p_w, q_w, r_w) give the angular velocity of the $\mathbf{F_w}$ frame.

A. Simple Flight Model

The first model presented assumes that direct control over the rates of change of velocity and flight path angles of climb and heading is possible. Let $V := \|\dot{x}\|$ and treat $\dot{V}$, $\dot{\gamma}$, $\dot{\psi}$ as control inputs where γ and ψ are the two flight path angles introduced above. Then the kinematics of the simple flight model can be modeled as

$$\dot{x} = \begin{bmatrix} V\cos(\gamma)\cos(\psi) \\ V\cos(\gamma)\sin(\psi) \\ -V\sin(\gamma) \end{bmatrix} \tag{1}$$

$$\dot{V} = u_1, \ \dot{\gamma} = u_2, \ \dot{\psi} = u_3, \ y = x \tag{2}$$

where u_1, u_2 and u_3 are control inputs and y is the output. This model entirely ignores, among other things, the roll dynamics of the vehicle, as well as any coupling between the three inputs.

B. Coordinated Flight Model

The second model depends on the assumption that the aircraft is restricted to coordinated flight, which requires that β, the sideslip angle, be zero for all time. In practice, coordinated flight is possible by implementing regulation of sideslip using the aircraft rudder. Let $v^w \in \mathbb{R}^3$ and $a^w \in \mathbb{R}^3$ denote, respectively, the aircraft velocity and acceleration expressed in the wind frame $\mathbf{F_w}$. The rotation rates in wind axes are $\omega = (\omega_1^w, \omega_2^w, \omega_3^w) \in \mathbb{R}^3$ and the evolution of R in time is described by

$$\dot{R} = R\hat{\omega}, \ \hat{\omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}. \tag{3}$$

The assumption of coordinated flight can be imposed with the constraint

$$\dot{x} = VRe_1 \tag{4}$$

where $e_1 = (1, 0, 0)^\top$. The control inputs are taken as the aircraft forward acceleration a_1^v, the upward acceleration a_3^v and the roll rate ω_1. Let $g^w = R^\top g$ be the gravity vector $g = (0, 0, g_3)^\top$ rotated into $\mathbf{F}_w$. Then, differentiating Eqn. (4), we obtain

$$\ddot{x} = g + Ra^w \tag{5}$$

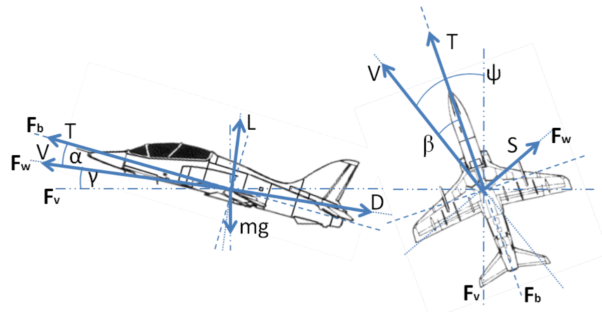


Fig. 1. Axes, flight angles, and forces.

where $a^w = (a_1^w, 0, a_3^w)$ and the control inputs are $u = (u_1, u_2, u_3) = (\omega_1^w, a_1^w, a_3^w)$. The coordinated flight constraint imposes conditions on the evolution of the rotation rates in $\mathbf{F_w}$,

$$\omega_2^w = -\frac{a_3^w + g_3^w}{V}, \ \omega_3^w = \frac{g_2^w}{V}. \tag{6}$$

In summary, the coordinated flight model is given by Eqns. (3) to (5) with states $(x_1, x_2, x_3, \dot{x}_1, \dot{x}_2, \dot{x}_3, R)$. The state space dimension is 7 instead of 9 because the coordinated flight requirement imposes additional constraints on the motion of R [1].

C. Full Aerodynamic Model

In the final model, aerodynamic coefficients are included that account for airflow and control surface deflections.

1) Aerodynamic Moments: The moments can be expressed as,

$$\mathcal{L} = C_l \frac{1}{2}\rho V^2 sb, \ \mathcal{M} = C_m \frac{1}{2}\rho V^2 sc, \ \mathcal{N} = C_n \frac{1}{2}\rho V^2 sb \tag{7}$$

where $\mathcal{L}, \mathcal{M}, \mathcal{N}$ represent roll, pitch and yaw moments respectively and C_l, C_m and C_n represent the nondimensional coefficients which can be represented as,

$$\begin{aligned} C_l &= f_{C_l}(\alpha, \beta, \delta R, \delta A) \\ C_m &= f_{C_m}(\alpha, \delta E) \\ C_n &= f_{C_n}(\alpha, \beta, \delta A, \delta R) \end{aligned} \tag{8}$$

where δE, δA and δR are the aircraft elevator, aileron and rudder deflections respectively.

2) Aerodynamic Forces: Forces in the $\mathbf{F_w}$ frame are

$$\begin{aligned} L &= C_L \frac{1}{2}\rho V^2 s + T\sin\alpha \\ D &= C_D \frac{1}{2}\rho V^2 s - T\cos\alpha\cos\beta \\ S &= -C_S \frac{1}{2}\rho V^2 s + T\cos\alpha\sin\beta. \end{aligned} \tag{9}$$

where T is the thrust, L is the lift force, D is the drag force and S is the side force and C_L, C_D and C_S are the corresponding nondimensional aerodynamic coefficients. These depend on α and β as shown below,

$$C_L = f_{C_L}(\alpha), \ C_D = f_{C_D}(\alpha), \ C_S = f_{C_S}(\alpha, \beta) \tag{10}$$

Note, the direct effects of control surface deflection on the forces are neglected. It can be shown that if the effects of control surface deflection are directly included in the equations for forces, the inverse dynamics will request unnecessarily large amounts of control inputs that could lead to an unstable system [2].

The thrust, T, can be approximated by $T = T_{max}\delta T$, where δT is the throttle setting and T_{max} is the maximum engine thrust. A more complicated engine model can be used here without loss of generality (see, for example, Ducard et al. [9]).

3) Equations of Motion: Ignoring the effects of Earth's curvature (flat-earth approximation [6]) and using combined wind and body frames for forces, angles and angular rates [2], the equations of motion for an aircraft can be defined as follows.

Velocity, V, flight path climb angle, γ, and flight path heading, ψ, from section (II-A) can now be further defined in terms of forces and angular velocities

$$\begin{aligned} \dot{V} &= -\frac{D}{m} - g\sin\gamma \\ \dot{\gamma} &= q_w\cos\phi - r_w\sin\phi \\ \dot{\psi} &= (q_w\sin\phi + r_w\cos\phi)\sec\gamma \end{aligned} \tag{11}$$

where the angles and angular velocities are in the $\mathbf{F_w}$ frame and $V = \|\dot{x}\|$ as defined earlier.

The angle of attack, α, the side-slip angle, β and wind-axes roll, ϕ are functions of angular velocities (in both $\mathbf{F_b}$ and $\mathbf{F_w}$ frames) and take the form

$$\begin{aligned} \dot{\alpha} &= q - q_w\sec\beta - p\cos\alpha\tan\beta - r\sin\alpha\tan\beta \\ \dot{\beta} &= r_w + p\sin\alpha - r\cos\alpha \\ \dot{\phi} &= p_w + q_w\sin\phi\tan\gamma + r_w\cos\phi\tan\gamma. \end{aligned} \tag{12}$$

The moment equations are best described in the $\mathbf{F_b}$ frame to simplify the relationship to control inputs

$$\begin{aligned} \dot{p} &= \frac{1}{I_x}(\mathcal{L} + I_{zx}(\dot{r} + pq) + (I_y - I_z)qr) \\ \dot{q} &= \frac{1}{I_y}(\mathcal{M} + I_{zx}(r^2 - p^2) + (I_z - I_x)rp) \\ \dot{r} &= \frac{1}{I_z}(\mathcal{N} + I_{zx}(\dot{p} - qr) + (I_x - I_y)pq) \end{aligned} \tag{13}$$

where inertia, I, is given by

$$I = \begin{bmatrix} I_{xx} & 0 & -I_{zx} \\ 0 & I_{yy} & 0 \\ -I_{zx} & 0 & I_{zz} \end{bmatrix} \tag{14}$$

Finally, the kinematic model in Eqn. (1) can be used to generate the aircraft trajectory. In Eqns. (11) to (13), angular velocities in the $\mathbf{F_w}$ frame (p_w, q_w, r_w) can be expressed as functions of the system states $(V, \gamma, \psi, \alpha, \beta, \phi, p, q, r)$ as

$$\begin{aligned} p_w &= p\cos\alpha\cos\beta + (q - \dot{\alpha})\sin\beta + r\sin\alpha\cos\beta \\ q_w &= \frac{1}{mV}(L - mg\cos\gamma\cos\phi) \\ r_w &= -\frac{1}{mV}(S - mg\cos\gamma\sin\phi) \end{aligned} \tag{15}$$

In summary, Eqns. (11) to (13) define the 9 states of the aircraft, driven by the control inputs, δT, δE, δA and δR through the moment and force definitions of Eqns. (7) to (10).

D. Target Models

The internal model approach to tracking requires a linear time-invariant (LTI) exosystem for generating trajectories. This exosystem can be used to approximate target trajectories using trial run data before the actual sporting event. During real-time tracking, any deviations by the target from these reference trajectories should be rejected by the controller as potentially small disturbances.

Two such exosystems, as used in simulations for this work, are defined below. The first one generates a fixed altitude elliptical trajectory which can find many uses in aerial reconnaissance or auto racing applications. The second, a periodic trajectory generator, can be used to chase targets up or downhill along a periodic path.

1) Elliptical Trajectory: The desired elliptical trajectory is given by

$$x_{ref} = \begin{bmatrix} 100\cos(t) \\ 50\sin(t) \\ 5 \end{bmatrix} \tag{16}$$

where $x_{ref} = (x_{1ref}, x_{2ref}, x_{3ref}) \in \mathbb{R}^3$ represents the desired aircraft position in inertial coordinates. This trajectory can be generated by the following system

$$\dot{w}_1 = w_2,\ \dot{w}_2 = -w_1,\ \dot{w}_3 = w_4,\ \dot{w}_4 = -w_3,\ \dot{w}_5 = 0 \tag{17}$$

with initial conditions $w_1(0) = 100, w_2(0) = 0, w_3(0) = 0, w_4(0) = 50$ and $w_5(0) = 5$ and with $w_1 = x_{1_{ref}}$, $w_3 = x_{2_{ref}}$, and $w_5 = x_{3_{ref}}$.

2) Basic Ski Slope: A simple ski slope trajectory can be generated using a combination of periodic functions for V_{ref}, γ_{ref}, ψ_{ref} and β_{ref}. Setting V_{ref}, γ_{ref} as constants and β_{ref} to zero gives a trajectory with a constant climb angle that maintains zero sideslip through aggressive turns. These turns can be formulated using any dynamically feasible time varying functions for ψ_{ref} as follows

$$V_{ref} = 150,\ \gamma_{ref} = 5,\ \psi_{ref} = 5\cos(\frac{t}{10}),\ \beta_{ref} = 0 \tag{18}$$

The reference trajectories from Eqn. (18) can be generated by the exosystem

$$\dot{w}_1 = \dot{w}_2 = \dot{w}_5 = 0,\ \dot{w}_3 \quad = w_4,\ \dot{w}_4 = -\frac{1}{100}w_3 \tag{19}$$

where $w_1(0) = 150, w_2(0) = 5, w_3(0) = 5, w_4(0) = 0$, $w_5(0) = 0$ and $w_1 = V_{ref}$, $w_2 = \gamma_{ref}$, $w_3 = \psi_{ref}$, and $w_5 = \beta_{ref}$. This exosystem provides a simple example of how downhill ski slope trajectories can be represented by a linear exosystem, which can be extended to more complex reference trajectories as needed.

III. Tracking Controllers

A. Inverse Dynamics and Output Tracking

Consider a system of the form

$$\begin{aligned}\dot{x} &= f(x) + \sum_{i=1}^{m} g_i(x) u_i \\ y_1 &= h_1(x), \ldots, \; y_m = h_m(x)\end{aligned} \tag{20}$$

where $f(x), g_1(x), \ldots, g_m(x)$ are smooth vector fields and $h_1(x), \ldots, h_m(x)$ are smooth real-valued functions defined in a domain $D \subset \mathbb{R}^n$. By definition, the system in Eqn. (20) with output $y = (y_1, \ldots, y_m)$ has a vector relative degree of $\{r_1, \ldots, r_m\}$ at $x_0 \in D$ if

$$\begin{aligned} L_{g_j} L_f^k h_i(x) = 0 \\ \forall j \in \{1, \ldots, m\}, \quad \forall k < r_i - 1, \quad \forall i \in \{1, \ldots, m\}\end{aligned} \tag{21}$$

for all x in a neighborhood of x_0 and the matrix

$$B^*(x) = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1(x) & \ldots & L_{g_m} L_f^{r_1-1} h_1(x) \\ L_{g_1} L_f^{r_2-1} h_2(x) & \ldots & L_{g_m} L_f^{r_2-1} h_2(x) \\ \vdots & \vdots & \vdots \\ L_{g_1} L_f^{r_m-1} h_m(x) & \ldots & L_{g_m} L_f^{r_m-1} h_m(x) \end{bmatrix} \tag{22}$$

is nonsingular at $x = x_0$ [10], [11].

Differentiating each component y_i of the output r_i times and using the output and its derivatives to partially define a coordinate transformation defined in a neighborhood of x_0 yields a system of the form

$$\begin{aligned} \dot{\xi}_1^i = \xi_2^i, \; \ldots, \; \dot{\xi}_{r_i-1}^i = \xi_{r_i}^i \\ \dot{\xi}_{r_i}^i = L_f^{r_i} h_i(x) + \sum_{j=1}^{m} L_{g_j} L_f^{r_i-1} h_i(x) u_j \end{aligned} \tag{23}$$

for all $i \in \{1, \ldots, m\}$. If $\sum_{i=1}^m r_i = n$, then the output y and its derivatives completely specify a coordinate transformation $\xi = T(x)$ where, in transformed coordinates, the system has the form as in Eqn. (23). However, in the case where $\sum_i r_i < n$, the coordinate transformation $T(x)$ is only partially defined by y and its derivatives. In order to complete the coordinate transformation we must find $n - \sum_i r_i$ additional functions and augment them to the output and its derivatives. In doing so we can obtain a valid coordinate transformation $(\eta, \xi) = T(x)$ defined in a neighborhood of x_0. In general one cannot impose any particular structure to the additional states η and their evolution will be described by a general equation of the form

$$\dot{\eta} = q(\xi, \eta) + p(\xi, \eta) u. \tag{24}$$

Setting $\eta = 0$ and $u = 0$ yields $\dot{\eta} = q(\xi, 0)$ which are the zero dynamics of the system in Eqn. (20). The system in Eqn. (20) is minimum phase if these zero dynamics have an asymptotically stable equilibrium point in the domain of interest.

When the output y yields a vector relative degree $\{r_1, \ldots, r_m\}$ with $\sum_i r_i < n$, then one can appeal to differential flatness or dynamic feedback linearization to obtain a system that is linear in transformed coordinates via the use of dynamic feedback. Specifically, in dynamic feedback linearization, sometimes called dynamic extension, state variables are introduced in the controller that correspond to a chain of integrators in order to obtain a relative degree

$$\sum_{i=1}^{m} \tilde{r}_i = n + v \tag{25}$$

where $\tilde{r}_i$, $i \in \{1, \ldots, m\}$ is the number of derivatives of y_i that must be taken before a control input appears and v is the number of additional states introduced in by the controller. In other words, by adding integrators in the controller, the appearance of the control input in the derivatives of y is delayed until the desired relative degree is obtained.

In either the static case when $\sum_i r_i = n$, or the dynamic case when $\sum_i \tilde{r}_i = n + v$, it is possible to define a regular feedback transformation that when applied to the system in Eqn. (23) yields a controllable linear system with m chains of integrators $\xi_1, \ldots, \xi_n$. The highest order derivatives from the system can be grouped together to obtain

$$\begin{bmatrix} \dot{\xi}_{r_1}^1 & \dot{\xi}_{r_2}^2 & \ldots & \dot{\xi}_{r_m}^m \end{bmatrix}^\top = A^*(x) + B^*(x) u \tag{26}$$

where $B^*(x)$ (also known as the decoupling matrix) is as defined in Eqn. (22) and

$$A^*(x) = \begin{bmatrix} L_f^{r_1} h_1(x) & \ldots & L_f^{r_m} h_m(x) \end{bmatrix}^\top. \tag{27}$$

Since B^* is nonsingular near x_0, the feedback law

$$u = B^*(x)^{-1}(v - A^*(x)), \tag{28}$$

where $v \in \mathbb{R}^m$ is an auxiliary control input, is well-defined in a neighborhood of x_0. When the controller in Eqn. (28) is applied to Eqn. (20) a controllable linear system is obtained

$$\begin{aligned} \dot{\xi} &= \begin{bmatrix} \dot{\xi}_1^1 & \ldots & \dot{\xi}_{r_1}^1 & \ldots & \dot{\xi}_1^m & \ldots & \dot{\xi}_{r_m}^m \end{bmatrix}^\top \\ &= A_1 \xi + B_1 v. \end{aligned} \tag{29}$$

Using the internal model approach by Wonham and Francis [3], controllers can now be designed that will track trajectories generated by LTI exosystems like those presented in Eqns. (17) and (19). To this end, this system is augmented by

$$\dot{w} = A_2 w, \; e = D_1 \xi + D_2 w \tag{30}$$

where ξ is the state of the system in Eqn. (29), w is the state of the exosystem and e is the tracking error. The problem is solvable if and only if A_2 has only unstable eigenvalues, (A_1, B_1) is stabilizable and there exist matrices X and U such that

$$D_1 X + D_2 = 0, \; A_1 X - X A_2 + B_1 U = 0 \tag{31}$$

Once X and U are found, the resulting controller takes the form

$$v = F_1 q + F_2 w \tag{32}$$

where F_1 is chosen such that $A_1 + B_1 F_1$ is Hurwitz and $F_2 = U - F_1 X$.

B. Simple Flight Model

The kinematic aircraft model from Section (II-A) has a well defined vector relative degree of $\{2,2,2\}$ at each point in the regions of interest for our application. To see this, note that the decoupling matrix B^* is given by

$$B^* = \begin{bmatrix} \cos\gamma\cos\psi & -V\cos\psi\sin\gamma & -V\cos\gamma\sin\psi \\ \cos\gamma\sin\psi & -V\sin\gamma\sin\psi & V\cos\gamma\cos\psi \\ -\sin\gamma & -V\cos\gamma & 0 \end{bmatrix} \quad (33)$$

This matrix is singular if and only if $V^2\cos\gamma = 0$, which corresponds to either a zero velocity or a climb angle of $\frac{\pi}{2}$. The decoupling matrix is therefore invertible during normal flight conditions. We conclude that the output $y = x$ yields a well-defined relative degree almost everywhere and $\sum_{i=1}^{3} r_i = 6$ which implies that the system is feedback linearizable via a static controller. The corresponding coordinate transformation, as discussed in section (III-A), is given by

$$\begin{gathered} \left[\ \xi_1^1 \quad \xi_2^1 \quad \xi_1^2 \quad \xi_2^2 \quad \xi_1^3 \quad \xi_2^3\ \right]^\top = \\ \left[\ x_1 \quad \dot{x_1} \quad x_2 \quad \dot{x_2} \quad x_3 \quad \dot{x_3}\ \right]^\top \end{gathered} \quad (34)$$

and is valid in a neighborhood of any point at which $V \neq 0$ and $\gamma \neq \frac{\pi}{2}$. The static feedback linearizing control law is given by

$$u = {B^*}^{-1} v \quad (35)$$

because $A^* = 0$ in this case. The auxiliary control input v can be designed for tracking an elliptical trajectory. The exosystem in Eqn. (17) results in a tracking error

$$e = \left[\ w_1 - \xi_1^1 \quad w_3 - \xi_1^2 \quad w_5 - \xi_1^3\ \right]^\top. \quad (36)$$

By appropriately defining D_1 and D_2 based on Eqn. (36), the matrices X and U can be found by solving Eqn. (31). Furthermore, F_1, can be suitably picked depending on the magnitude of gains required for minimizing e and v can then be computed using Eqn. (32).

C. Coordinated Flight Model

The coordinated flight model with 7 states $(x_1, x_2, x_3, \dot{x}_1, \dot{x}_2, \dot{x}_3, R)$ and output $y = (x_1, x_2, x_3)$ as described in Section (II) has a well defined vector relative degree of $\{2,2,2\}$ but $\sum_{i=1}^{3} r_i = 6 < 7$ and therefore static feedback cannot be used to feedback linearize this system. In order to obtain a fully linear system in transformed coordinates, two new states need to be added to the system by adding integrators on two of the inputs. The resulting system will have 9 states $(x, \dot{x}, R, a_1^w, a_3^w)$ with a well defined vector relative degree of $\{\tilde{r}_1, \tilde{r}_2, \tilde{r}_3\} = \{3,3,3\}$. The system can equivalently be expressed in coordinates $(x_1, \dot{x}_1, \ddot{x}_1, x_2, \dot{x}_2, \ddot{x}_2, x_3, \dot{x}_3, \ddot{x}_3)$ as shown in [1]. The new control inputs are given by

$$u = R \left[\ \dot{a}_1^v \quad w_1 \quad \dot{a}_3^v\ \right]^\top. \quad (37)$$

The A^* and B^* matrices can be formed using Eqns. (27) and (22) respectively which results in

$$A^* = R \begin{bmatrix} w_2 a_3^v \\ w_3 a_1^v \\ -w_2 a_1^v \end{bmatrix},\ B^* = R \begin{bmatrix} 1 & 0 & 0 \\ 0 & -a_3^v & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (38)$$

Finally, a feedback linearizing control law is generated using Eqn. (28) and (37). A linear tracking controller can now be designed for v as before. The tracking error, e, for the elliptical exosystem is also given by Eqn. (36).

D. Full Aerodynamic Model

Taking $y = (y_1, \ldots, y_4) = (V, \gamma, \psi, \beta)$ as the output we get $r_1 = r_2 = r_3 = r_4 = 1$. The sum of these relative degrees is less than the number of states (9) in the system which is a direct result of the throttle input (δT) appearing in the force terms, L, D and S. Using the previous discussion on zero dynamics, this implies 5 uncontrollable and potentially unobservable internal states. Extending the system with two additional states and replacing δT with $\ddot{\delta T}$ as the new control input ensures differential flatness by delaying the appearance of the control inputs in the output derivatives. The new control input is

$$u = \left[\ \ddot{\delta T} \quad \delta E \quad \delta A \quad \delta R\ \right]^\top. \quad (39)$$

With these re-defined control inputs, the decoupling matrix is given by

$$B^* = \begin{bmatrix} \dddot{V}_{\delta T} & \dddot{V}_{\delta E} & \dddot{V}_{\delta A} & \dddot{V}_{\delta R} \\ \dddot{\gamma}_{\delta T} & \dddot{\gamma}_{\delta E} & \dddot{\gamma}_{\delta A} & \dddot{\gamma}_{\delta R} \\ \dddot{\psi}_{\delta T} & \dddot{\psi}_{\delta E} & \dddot{\psi}_{\delta A} & \dddot{\psi}_{\delta R} \\ 0 & 0 & \ddot{\beta}_{\delta A} & \ddot{\beta}_{\delta R} \end{bmatrix} \quad (40)$$

where each entry is expressed using partial derivatives of $\dddot{V}, \dddot{\gamma}, \dddot{\psi}$ and $\ddot{\beta}$ with respect to each control input.

The matrix in Eqn. (40) was checked, numerically, to be nonsingular in the region of interest when tracking the basic ski slope reference trajectories from Section (II-D2). This implies that the system yields a well defined vector relative degree of $\{\tilde{r}_1, \tilde{r}_2, \tilde{r}_3, \tilde{r}_4\} = \{3,3,3,2\}$. Using Eqn. (28), and an appropriate A^* term (not shown here for brevity), a feedback linearizing controller is formed. For tracking control, the error can be defined as

$$e = \left[\ w_1 - \xi_1^1 \quad w_2 - \xi_1^2 \quad w_3 - \xi_1^3 \quad w_5 - \xi_1^4\ \right]^\top \quad (41)$$

where ξ_1^1, ξ_1^2, ξ_1^3 and ξ_1^4 refer to V, γ, ψ and β respectively. The controller can then be completed, with results for a BAC-221 high performance aircraft shown in the next section.

IV. Simulation Results

Simulation results for trajectory tracking control of the simple flight model are shown in Fig. (2). The aircraft position achieves perfect tracking of the elliptical exosystem described in section (II-D), and also rejects errors in initial conditions smoothly. To implement this controller, the aircraft is assumed to have a low level autopilot which would provide the required V, γ and ψ to track the trajectory. The scale of the reference ellipse and the time to traverse it can be adjusted based on the performance limitations of the aircraft.

Simulation result for the coordinated flight model with the elliptical exosystem is shown in Fig. (3). The aircraft rapidly reduces the initial errors and achieves perfect tracking. As with the simple flight model, this controller can be used where an autopilot is already present that can command the

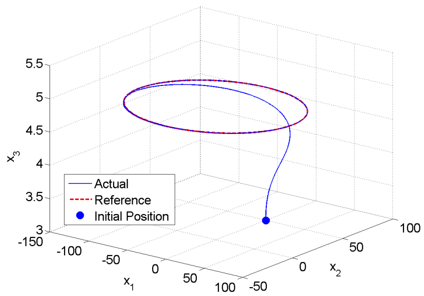


Fig. 2. Simulation results for the simple flight model.

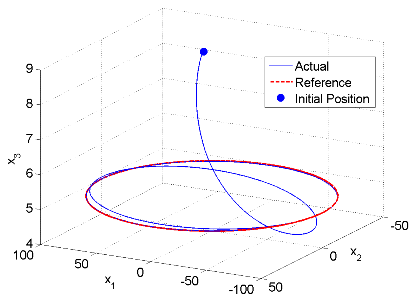


Fig. 3. Simulation results for the coordinated flight model.

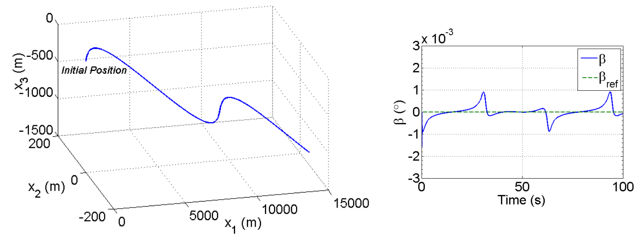
required forward and vertical acceleration and the required roll rate a_1^v, a_3^v, w_1, respectively. Since the model doesn't account for aircraft limitations, these control outputs can be adjusted by altering the flight path as required. As an example, an autopilot could be developed that would regulate the sideslip to zero at all times through rudder deflection, δR. Similarly, a_1^v and a_3^v can be generated through throttle control, δT, and elevator deflection, δE, respectively. w_1 can be produced by actuating the ailerons, δA.

A realistic BAC-221 aerodynamic model built using data from the Royal Aircraft Establishment [12] can now be tested with the proposed control strategy and the basic ski model exosystem. The final trajectory result is shown in Fig. (4(a)), which very precisely tracks the reference trajectory. Note, the scale on the graph is much larger than a ski hill to accommodate the limitations on a full size BAC-221. The sideslip angle can be seen in Fig. (4(b)). Here the controller maintains coordinated flight by actively regulating the angle to zero through a combined effort of δR and δA. Note that during the sharp turns, peaks can be seen which implies that some sideslip ($< 0.001°$) does inevitably occur.

One drawback in the proposed controller, when applied to the full aerodynamic model, is that it is not robust to small deviations from the reference. This implies a small domain of attraction around the trajectory. Hence, this approach maybe difficult to implement in practice, but could be addressed by adding a robust control component to the linear controller.

V. Conclusions

In this work, a nonlinear controller is designed which achieves trajectory tracking for three progressively complex



(a) Aggressive trajectory. (b) β regulation.

Fig. 4. Simulation results for the BAC-221.

models of an aircraft. Each of these models can be used to design autopilots provided that an underlying low level controller is already present that ensures that the input requirements are met. The controllers for the basic and coordinated flight models can be scaled up or down based on the type of aircraft being used. For a small scale UAV, where the thrust to weight ratio is usually more than one, fairly aggressive trajectories should be within reach. The controller for the full aerodynamic model needs to be built ground up for the specific aircraft after a thorough system identification. It may also require the inclusion of a robust control component to achieve tracking in non-ideal conditions. However, this controller would only require a simple underlying controller to meet control surface deflections as opposed to the other two models which require controllers for flight path angles and accelerations.

Future work on this project involves flight tests using all three aircraft models implemented on a Kadet Senior small scale UAV. The aircraft will include an onboard real-time system with sensors and state estimators together with a ground control station to perform fully controlled aggressive trajectory tracking.

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