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Optimal Design of Power-Split Transmissions for Hydraulic Hybrid Passenger Vehicles

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***Abstract*— Hydraulic hybrid vehicles are inherently power dense. Power-split or hydro-mechanical transmissions (HMT) have advantages over series and parallel architectures. In this paper, an approach for optimizing the configuration and sizing of a hydraulic hybrid power-split transmission is proposed. Instead of considering each mechanical configuration consisting of combinations of gear ratios, a generalized kinematic relation is used to avoid redundant computation. This captures different architectures such as input coupled, output coupled and compound configurations. Generic kinematic relations are shown to be mechanically realizable. Modal operation of the transmission is introduced to reduce energy loss. The Lagrange multiplier method for computing the optimal energy management control is shown to be computationally efficient for use in transmission design iterations. An optimal design case study indicates improvement in fuel economy and smaller component sizes for the compound and input coupled power-split configurations.**

Keywords: Hybrid vehicles, hydraulics, power-split, engine management, optimal control, Lagrange multiplier.

I. INTRODUCTION

Hydraulic hybrid vehicles are equipped with an accumulator for energy storage. Similar to Hybrid Electric Vehicles (HEVs), they reduce fuel consumption and emission by regenerating braking power, and by allowing the engine and transmission components to operate more efficiently. The three most common categories of hydraulic hybrid vehicle architectures are the series, parallel and power-splits. Series hybrids are advantageous in decoupling the engine operation from the vehicle load and speed completely so that the engine can operate at its most efficient point whenever it is turned on. However, all the energy is transmitted through the hydraulic pump/motors which are relatively inefficient compared to mechanical gears. Hence, fuel economy is highly dependent on the efficiencies of the hydraulic components. Parallel hybrids, in contrast, are advantageous that a significant portion of the power is transferred through the highly efficient mechanical path. However, the engine's operating speed is constrained to be related to the vehicle speed so that it cannot be operated as efficiently. Power-split hybrids can decouple engine operation from vehicle load/speed, and also permits a portion of the energy to be transmitted via the mechanical path.

Non-hybridized power-split transmissions provide a continuously variable transmission (CVT) ratio between the input (engine) and output (vehicle) [2], [6], [7]. Power-split transmissions are hybridized when the hydraulic pump/motors or electric motor/generators are connected to energy storage devices (hydraulic accumulators or electric batteries) ([1], [3], [4], [5]). Hybridization allows engine power to be different from the vehicle power. This extra degree of freedom enables braking energy to be recaptured and the engine to operate at power levels that are more efficient. The engine could potentially be downsized for mean instead of peak power, further improving the fuel economy.

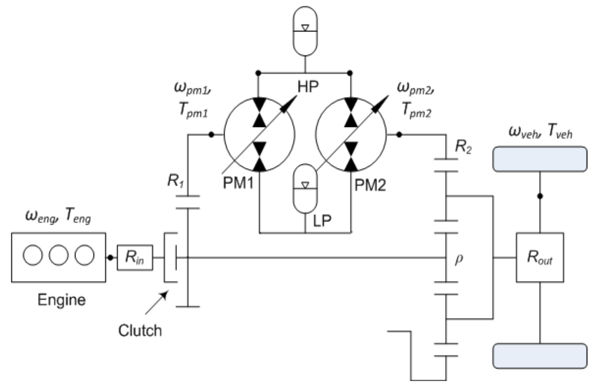

Fig. 1. Input coupled power-split configuration

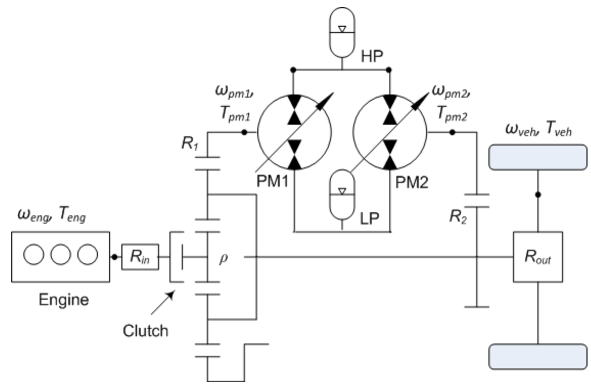

Fig. 2. Output coupled power-split configuration

There are many kinds of power-splits including [7]: input coupled power-splits (e.g. used in the hydraulic hybrid vehicle in [3]) and the output coupled power-splits (e.g. Toyota Prius) that require one planetary gear set; and compound power-splits ([1], [2], [6]) that require two planetary gear sets. In [1], [2], [6] where an extra planetary gear set is used, it is only to provide discrete gear shift. This paper considers, as in [7], a more general case where both planetary gear sets act as power combination devices.

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In [1], a methodology to design an optimal hydraulic hybrid power-split transmission is presented. It involves generating all permutations of design configurations, screening them through a mechanical feasibility check, and evaluating and determining the optimal parameters for the mechanically feasible ones. With two planetary gear sets and two clutches, there are 1,152 possible candidate configurations that need to be considered individually. This three-step-methodology can be quite computationally intensive.

In this paper, a computationally efficient methodology is proposed to design hydraulic hybrid power-split transmissions with optimal efficiencies. Both the kinematics of the mechanical transmission and the sizes of the hydraulic components are optimized. The methodology utilizes the insight that there are many design configurations that are mechanically distinct, but they are kinematically equivalent and thus have the same performance. By considering the kinematic relation between the various components rather than individual gear sets, evaluation of redundant configurations can be avoided. The realization into actual gear sets needs only be performed on the final design. Various vehicle operating modes are introduced for power-split architectures to reduce loss. Whereas dynamic programming is generally needed [10] to evaluate the optimal control and the fuel economy for each design, a Lagrange multiplier method where only the *net* change in the state of charge (SOC) is constrained, is used to improve computational efficiency [8], [3].

The rest of this paper is organized as follows: Section II describes the different power-split architectures. Section III explains the generalized modeling approach in this study. Section IV discusses the restricted operating modes. Section V describes the optimization process and the Lagrange multiplier method used in this design. Section VI and VII present the optimization results and discussions. Lastly, Section VIII contains concluding remarks.

II. Hydraulic Hybrid Power-split Transmissions

Hydro-mechanical transmission (HMT) uses a set of hydrostatic transmission and at least one planetary gearset to realize the power-split feature. Additional planetary gearsets and clutches can also be used to achieve discrete gear shift, similar to a conventional automatic transmission. However, for simplicity they are not considered in this paper. There are two basic power-split configurations, i.e. input coupled and output coupled transmissions. Despite the differences in architecture, a power-split transmission can be interpreted as a four-port device with power flows between the engine, wheel and the two pump/motors [4].

A. Input coupled architecture

An input coupled transmission (Fig.1) splits the power from the engine with a fixed gear into a mechanical and a hydraulic transmission path. The hydraulic path is modulated by the accumulator and the resultant power is recombined with the mechanical power via a planetary power combination device. The kinematic relationship between the speed and torque of the pump/motors ($\omega_{pm\,1/2}, T_{pm\,1/2}$) with those of the engine (ω_{eng}, T_{eng}) and of the vehicle (wheel) (ω_{veh}, T_{veh}) can be expressed as follows:

$$\begin{pmatrix}\omega_{pm1}\\ \omega_{pm2}\end{pmatrix} = \begin{pmatrix} r_{11} & 0\\ r_{21} & r_{22}\end{pmatrix}\begin{pmatrix}\omega_{eng}\\ \omega_{veh}\end{pmatrix} \tag{1a}$$

$$\begin{pmatrix}T_{pm1}\\ T_{pm2}\end{pmatrix} = \begin{pmatrix} -1/r_{11} & r_{21}/(r_{11}r_{22})\\ 0 & -1/r_{22}\end{pmatrix}\begin{pmatrix}T_{eng}\\ T_{veh}\end{pmatrix} \tag{1b}$$

where the gear ratios can be physically decomposed into $r_{11} = R_1 R_{in}$, $r_{21} = -R_2 R_{in}\rho_2$, $r_{22} = R_2 R_{out}(1+\rho_2)$ and R_1, R_2 are the fixed gear ratios on the pump/motors, R_{in} and R_{out} are the fixed gear ratios from the input (engine) and to the output shafts, ρ_2 is the radius-ratio of the sun and ring of the planetary gear. Unit 1 is the 'torquer' as it adds torque to or subtracts torque from the engine as shown in Eq.(1b), and Unit 2 is the 'speeder' as it alters the engine operating speed as shown in Eq.(1a).

B. Output coupled architecture

An output coupled transmission (Fig.2) is configured in a reversed arrangement to the input coupled architecture. Engine power is split with the planetary power-split device in the hydraulic and mechanical paths, and recombined with a fixed gear at the output shaft. The power in the hydraulic path is again modulated by the accumulator power. Its kinematic relationship is represented by:

$$\begin{pmatrix}\omega_{pm1}\\ \omega_{pm2}\end{pmatrix} = \begin{pmatrix} d_{11} & d_{12}\\ 0 & d_{22}\end{pmatrix}\begin{pmatrix}\omega_{eng}\\ \omega_{veh}\end{pmatrix} \tag{2a}$$

$$\begin{pmatrix}T_{pm1}\\ T_{pm2}\end{pmatrix} = \begin{pmatrix} -1/d_{11} & 0\\ d_{12}/(d_{11}d_{22}) & -1/d_{22}\end{pmatrix}\begin{pmatrix}T_{eng}\\ T_{veh}\end{pmatrix} \tag{2b}$$

where $d_{11} = -R_1 R_{in}\rho_1$, $d_{12} = R_1 R_{out}(1+\rho_1)$, $d_{22} = R_2 R_{out}$ and ρ_1 is the radius-ratio of the sun and ring of the planetary gear. In contrast with input coupled transmission, Unit 1 is the 'speeder' while Unit 2 is the 'torquer'.

III. Generalized Transmission Modeling

In a typical process for designing a power-split hybrid transmission such as in [1], [3], a specific architecture (e.g. input coupled, output coupled) or the connection between the gear sets [1], is chosen first and then the specific parameters are optimized to achieve certain performance and / or overall system efficiency. In this paper, we consider a generalized transmission's kinematic relationship:

$$\begin{pmatrix}\omega_{pm1}\\ \omega_{pm2}\end{pmatrix} = \underbrace{\begin{pmatrix} g_{11} & g_{12}\\ g_{21} & g_{22}\end{pmatrix}}_{G}\begin{pmatrix}\omega_{eng}\\ \omega_{veh}\end{pmatrix} \tag{3}$$

$$\begin{pmatrix}T_{pm1}\\ T_{pm2}\end{pmatrix} = -G^{-T}\begin{pmatrix}T_{eng}\\ T_{veh}\end{pmatrix} \tag{4}$$

where $G \in \Re^{2\times 2}$ is nonsingular and the elements of the matrix are arbitrary. The lower and upper triangular matrices in (1a) and (2a) for the input coupled and output coupled configurations can be considered as special cases.

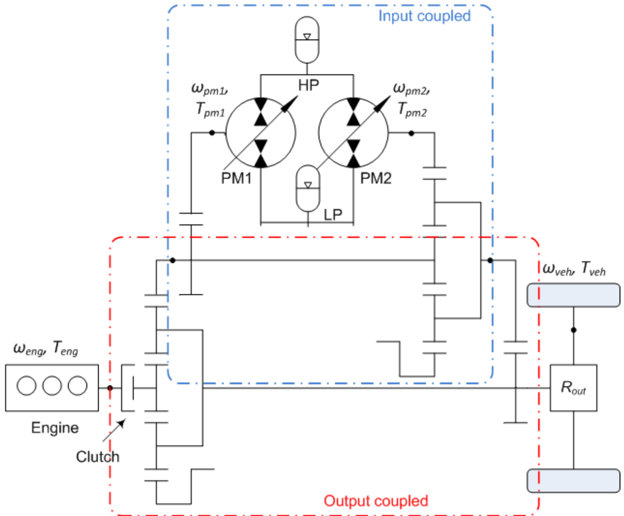

Fig. 3. Combined input-output power-split configuration

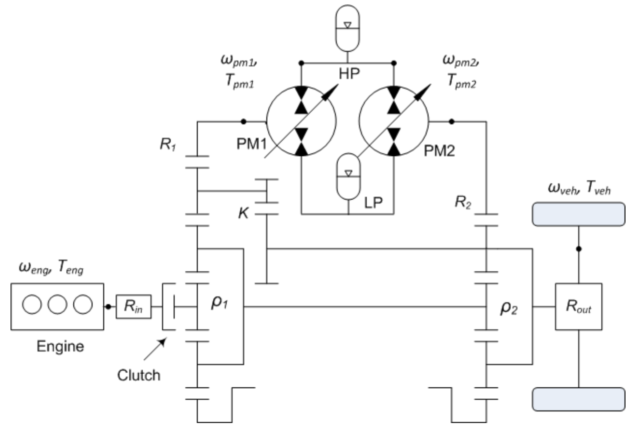

Fig. 4. Compound power-split configuration

An important question to ask is whether an arbitrary kinematic relationship in Eq.(3) can indeed be realized mechanically. The following proposition provides one realization.

Proposition 1 *An arbitrary nonsingular kinematic relation G in Eq.*(3) *can be realized by a cascade connection of an input coupled and an output coupled transmission.*

Proof: This result can be shown by LU factorizing G as a product of an upper and a lower triangular matrix:

$$\begin{pmatrix}\omega_{pm1}\\ \omega_{pm2}\end{pmatrix} = \underbrace{\begin{pmatrix} r_{11} & 0 \\ r_{21} & r_{22}\end{pmatrix}}_{\text{Input coupled}} \underbrace{\begin{pmatrix} d_{11} & d_{12} \\ 0 & d_{22}\end{pmatrix}}_{\text{Output coupled}} \begin{pmatrix}\omega_{eng}\\ \omega_{veh}\end{pmatrix} \tag{5}$$

Since the lower and upper diagonal matrices correspond to an input coupled and an output coupled configuration as shown in Eqs.(1a) and (2a), Eq.(5) can be realized by connecting the pump/motor ports of the output coupled transmission to the input/output (i.e. engine and vehicle) ports of an input coupled transmission, and the pump/motors to transmission's pump/motor ports. This is illustrated in Fig.3. ◇

LU factorization is not unique if specific values are not imposed on the diagonal elements of the triangular matrix. This non-uniqueness preserves some extra degrees of freedom in realizing the G matrix in order to satisfy other design constraints.

The mechanical realization in Fig.3 can be further simplified to a compound planetary transmission. Fig.4 shows one of the possibilities. Here, the matrix G can be physically realized as

$$G = \begin{pmatrix} R_1 & 0 \\ 0 & R_2 \end{pmatrix} \begin{pmatrix} -\rho_1 & K(1+\rho_1) \\ (1+\rho_2) & -\rho_2 \end{pmatrix} \begin{pmatrix} R_{in} & 0 \\ 0 & R_{out} \end{pmatrix} \tag{6}$$

where the middle matrix represents the ratios (ρ_1, ρ_2) of and the connection (K) between the 2 planetary gear sets, and the first and last matrices are the fixed gear ratios on the pump/motors (R_1, R_2), and the input and output (R_{in}, R_{out}). R_{out} is equivalent to the final drive ratio. Notice that an arbitrary G can be realized with some choices of the parameters. Some redundancy is preserved to allow ρ_1 and ρ_2 to be positive and to satisfy other geometric constraints. In this realization, both planetary gear sets perform power combination/split functions instead of one of them being used for discrete gear shifts only.

This generalized power-split model Eq.(6) reduces to an input coupled or an output coupled architecture by setting $\rho_1 = -1$ and $\rho_2 = -1$ respectively. Setting either $\rho = -1$ reduces the planetary gear to a fixed gear ratio and can be used for discrete gear shifts with the addition of a clutch.

Although a kinematic relation G in (3) can be realized in many ways, they affect the operation of the pump/motors, engine and the vehicle in the same way if the mechanical gears are ideal. In reality, since the losses in the hydraulic components and the engine dominate, this is a reasonable assumption. Because of this, using G as a design parameter to be optimized avoids many redundant computations.

IV. Multi-mode operations of the hybrid HMT

In normal power-split operation, the engine and both pump/motors are all operating. To reduce engine idling losses, as well as volumetric and friction losses in pump/motors when they are not heavily utilized, we allow the drive-train options to declutch the engine or to disengage individual pump/motors with valves. We refer to these combinations as modes.

These modes can be divided mainly into 2 categories: "engine-on" and "hydraulic-only". For the input coupled or output coupled cases, there are 6 modes. With "engine-on", `HMT` mode refers to the case when both pump/motors cooperate to achieve power-split operation. `Parallel` modes refer to the cases when either one of the two hydraulic units is either shut-off or free-spinning. With "hydraulic-only", one of the hydraulic units can be shut-off or free-spin (`P/M-1 or 2 only`) or both pump/motors cooperate (`P/M 1&2`). In the compound power-split case, there are 11 combinations since each pump/motor can be locked up or free-spin. Table I summarizes all possible modes available for the 3 power-split architectures.

TABLE I

MODE FOR DIFFERENT ARCHITECTURES (* USED IN THIS STUDY)

Input coupled	Modes	Comments
mode 1*	HMT	Power-split
mode 2*	Parallel-1	Lock-up P/M-2
mode 3	Parallel-2	Freespin P/M-1
mode 4*	P/M-1 only	Lock-up P/M-2
mode 5*	P/M-2 only	Lock-up P/M-1
mode 6	P/M-1&2 only	Generally not used
Output coupled	**Modes**	**Comments**
mode 1*	HMT	Power-split
mode 2	Parallel-1	Freespin P/M-2
mode 3*	Parallel-2	Lock-up P/M-1
mode 4*	P/M-1 only	Charging only
mode 5*	P/M-2 only	-
mode 6	P/M-1&2 only	Generally not used
Compound	**Modes**	**Comments**
mode 1*	HMT	Power-split
mode 2*	Parallel-1a	Lock-up P/M-2
mode 3	Parallel-1b	Freespin P/M-2
mode 4	Parallel-2a	Lock-up P/M-1
mode 5	Parallel-2b	Freespin P/M-1
mode 6*	P/M-1a only	Lock-up P/M-2
mode 7	P/M-1b only	Freespin P/M-2
mode 8*	P/M-2a only	Lock-up P/M-1
mode 9	P/M-2b only	Freespin P/M-1
mode 10	P/M-1&2a only	Generally not used
mode 11	P/M-1&2b only	Generally not used

For simplicity, only four modes, `HMT`, `P/M-1 only`, `P/M-2 only` and the `parallel` using the "torquer" pump/motor, are considered. Other modes are neglected because they will likely not be efficient. For example, the `parallel` mode using the "speeder" pump/motor will not allow the engine to operate as efficiently; and the `P/M-1&2 only` mode will incur inefficiencies due to power recirculation or low pump/motor displacements.

V. HYBRID POWER SPLIT TRANSMISSION OPTIMIZATION

A. Transmission Parameterization

In this paper, we determine the hydraulic hybrid power-split transmission design that maximizes fuel economy. The vehicle weight, engine (size and efficiency map), and drive-cycles are assumed to be given. The hydraulic hybrid power-split transmission will be parameterized by $(G, D_{max,1}, D_{max,2})$ which are the kinematic relation $G \in \Re^{2\times 2}$ in Eq.(3) and the maximum displacements of the two pump/motors.

In order to compare different power-split architectures, three separate cases are considered: input coupled (G is lower triangular), output coupled (G is upper triangular), and compound (G is a full matrix) architectures. Once the optimal G is determined, it can be realized mechanically such as in Sections II and III.

To parameterize pump/motors of different sizes, we use a typical 28cc variable displacement bent-axis pump/motor as a reference and assume that the torque and flow scale linearly with the maximum displacement D_{max} at a given speed (ω), pressure (P) and displacement ratio $d \in [-1, 1]$ (such that $d \cdot D_{max}$ is the commanded pump displacement). In other words, the mechanical and volumetric efficiency maps with respect to (ω, P, d) are those of the reference 28cc pump/motor.

B. Synthesis of the optimal control

In order to evaluate the fuel economy of each transmission design, it is necessary to develop the control that optimizes its performance. In [3], a 3-level control architecture is proposed for hydraulic hybrid power-split vehicles where the drive-cycle dependent high-level controls the accumulator power, the transmission dependent mid-level translates high-level command into optimal desired engine and pump/motor operating points that satisfy the commanded vehicle torque, and the low-level achieves these operating conditions.

Because an optimal control is needed for each design, a computationally efficient approach is needed. To this end, the engine operating points for each operating mode in Section IV are restricted a priori. In `HMT` mode, the engine operates only at the most efficient 'sweet spot', with excess or deficit power absorbed or compensated by the accumulator. In `Parallel` mode, the engine will operate at the most efficient torque for the vehicle speed. These are chosen to maximize engine efficiency and are found to be valid from preliminary studies. The mid-level control in [3] that would require a costly static optimization becomes a known map, and the high level decision variables reduce from the continuous set of accumulator power into a finite set of operating modes.

Further simplifying assumptions are made on the accumulator to improve computation efficiency. It is assumed that: 1) the system operating pressure is constant; 2) the accumulator capacity is not constrained. To ensure that the engine supplies all the energy, the net change in accumulator energy is constrained to be zero. Thus, the high-level (energy management) control is formulated as:

$$\min_{\mathtt{mode}(\cdot)} \int_{t_0}^{t_f} Loss(t, \mathtt{mode}(t)) \cdot dt$$
$$\text{subject to} \int_{t_0}^{t_f} Pow_{acc}(t, \mathtt{mode}(t))\, dt = 0 \tag{7}$$

where $Loss(t, \mathtt{mode})$ is the total losses in the engine and the hydraulic components, and $Pow_{acc}(t, \mathtt{mode})$ is the accumulator power, if an operating `mode` is applied to satisfy the drive-cycle speed and torque at time t. The constrained optimization problem in (7) can be solved by use of the scalar Lagrange multiplier λ as [3]:

$$\max_{\lambda} \min_{\mathtt{mode}(\cdot)} \int_{t_0}^{t_f} Loss(t, \mathtt{mode}(t)) + \lambda \cdot Pow_{acc}(t, \mathtt{mode}(t)) dt \tag{8}$$

This algorithm is referred to as the Lagrange multiplier method. It is computationally efficient because the inner minimization can be done inside the integral and the outer maximization is one-dimensional. The optimized λ can be related to fuel equivalence in the ECMS approach in [9]. If accumulator capacity constraints or dynamics are included, the solution (e.g. with Dynamic Programming) would be significantly more time consuming.

TABLE II

OPTIMAL DESIGNS FOR THE 3 POWER-SPLIT ARCHITECTURES

Architecture	Input coupled
Matrix G	$\begin{pmatrix} 1.0175 & 0 \\ 2.0660 & -8.3570 \end{pmatrix}$
P/Ms' size	P/M-T=27.7cc P/M-S=28.8cc
City/Highway/Combined	78.6/56.1/64.2 [mpg]
Architecture	Output coupled
Matrix G	$\begin{pmatrix} 1.2768 & -4.0424 \\ 0 & 4.7239 \end{pmatrix}$
P/Ms' size	P/M-S=23.9cc P/M-T=39.1cc
City/Highway/Combined	72.7/54.9/61.2 [mpg]
Architecture	Compound
Matrix G	$\begin{pmatrix} 0.9810 & 0.6400 \\ 2.0573 & -8.3764 \end{pmatrix}$
P/Ms' size	P/M-1=24.5cc P/M-2=24.7cc
City/Highway/Combined	79.5/56.1/64.5 [mpg]

Finally, the achieved fuel economy can be computed from the minimized loss and the drive cycle energy requirements. However, to ensure that the design is drivable regardless of the accumulator charge, we check if it is drivable in the `HMT` mode whenever a positive torque is required. Infinite loss or 0 mpg fuel economy is assigned to the transmission if the drivability test fails.

C. Transmission optimization procedure

The optimization process is summarized below:

1) Initialize transmission kinematics and pump/motor sizes (G, D_{max1}, D_{max2}).
2) Check `HMT` mode drivability requirements. Goto Step 5 if fails.
3) Solve optimal control problem in (8)
4) Evaluate the achieved fuel economy
5) Generate new (G, D_{max1}, D_{max2}) using standard optimization algorithm (Matlab's `fminsearch`). Repeat Steps 2, 3, 4 until convergence.

VI. RESULTS

The proposed design approach is applied to a 1000kg compact vehicle (including 300kg for the hybrid transmission) similar to the one presented in [3], with a 21kW diesel engine with peak efficiency of 29%. The combined EPA urban and highway cycle is used to evaluate the fuel economy. A constant system pressure of 13.8MPa (2000psi) is assumed.

Table II shows the optimal input coupled, output coupled and compound power-splits designs. As expected, the compound architecture achieves the highest fuel economy, the input coupled design achieves $\sim 0.5\%$ less, and the output coupled achieves $\sim 5\%$ less. The kinematics matrix G of the compound design is very close to that of the optimal input-coupled design. The combined pump/motor sizes of the compound design is smaller - 13% and 21% less than the input coupled and output coupled designs. Table III shows two possible realizations of the optimized G matrix for the compound power-split design according to Fig. 4.

Fig.5 shows the optimal distribution, throughout the drive cycle, of the operating modes for compound power-split

TABLE III

TWO REALIZATIONS OF THE OPTIMAL COMPOUND DESIGN ACCORDING TO FIG. 4. NEGATIVE RATIOS IMPLY INTERNAL GEARS.

Ratios	ρ_1	ρ_2	R_1	R_2	R_{in}	R_{out}	K
Design 1	0.50	0.75	-1.96	1.18	1.00	9.50	-0.023
Design 2	0.75	0.75	-2.62	2.35	0.50	4.75	-0.029

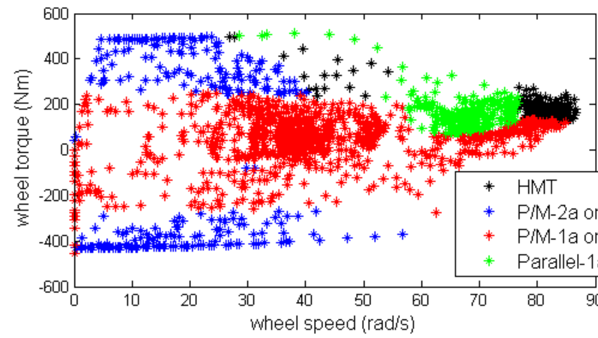

Fig. 5. Compound power-split modes distribution

design in Table II. The "engine-on" modes (i.e. `HMT` and `parallel`) occur mainly at high vehicle speeds. This accounts for ∼33% of the cycle time, consistent with the engine power at peak efficiency and the mean power requirement. At lower vehicle speeds and during braking, "hydraulic-only" modes are preferred. "(S)peeder" pump/motor (P/M-2) only is preferred at high torques whereas "(T)orquer" pump/motor (P/M-1) only mode is preferred at lower torques. Similar distributions can be generated for the input coupled and output coupled designs.

VII. DISCUSSIONS

The optimal design procedure utilizes several simplifying assumptions to increase computational efficiency. We examine the impact of some of these assumptions here.

Effect of accumulator pressure: The optimal designs in Table II were obtained by assuming that the system pressure is at 13.8MPa (2000 psi). This is the low pressure limit and is used to ensure that the pump/motors are sized conservatively. The fuel economy of these designs at other system pressures are evaluated and shown in Fig. 6. For all 3 architectures, fuel economies decrease by the similar amount as system pressure increases. This is because the pump/motors tend to operate at the less efficient lower displacements and increased leakage at higher pressures. In actual driving, pressure varies between highest and lowest limits. It is expected that the actual fuel economy will fall between the estimates at the highest and lowest pressures.

Constraints on accumulator size: The Lagrange multiplier method to compute the optimal control efficiently only ensures zero net use of accumulator power but does not constrain the instantaneous state-of-charge (SOC). As shown in Fig. 7(a), the resulting control uses an impractical accumulator capacity of $2.5MJ$. This large accumulator discharges in the urban portion of the drive-cycle and

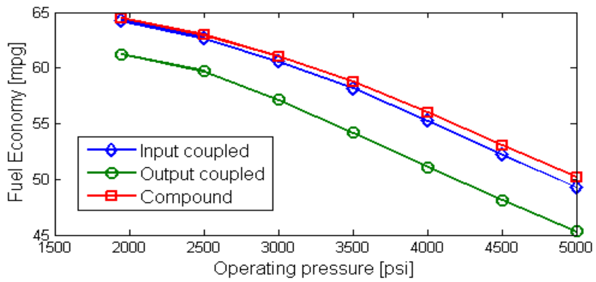

Fig. 6. Fuel economy of different architectures at various system pressures.

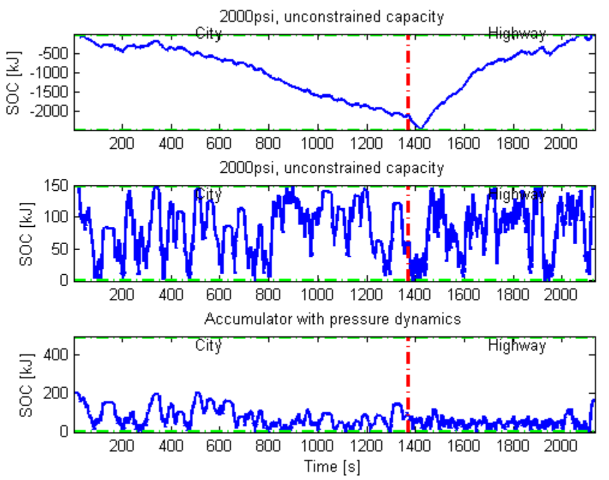

Fig. 7. Accumulator SOC over the combined drive cycle for the compound design in Table II: a) at constant pressure with unconstrained accumulator capacity; b) at constant pressure with 150kJ accumulator capacity constraint; c) with an isothermal accumulator

recharges during the highway portion. To check the effect on fuel economy if a practical constraint on the SOC is imposed, dynamic programming (DP) is performed for the optimal compound design in Table II with a reasonable 150kJ SOC (approximately 20 liters volume) constraints. Fig. 7(b) shows that with the constraint, the accumulator is discharged and recharged repeatedly throughout the drive-cycle to stay within the limits. Fuel economy decreases only from 64.5mpg to 63.1mpg.

Effect of accumulator pressure dynamics The constant pressure assumption used in the Lagrange multiplier method has neglected the actual accumulator dynamics that pressure decreases as energy depletes. To evaluate the effect of accumulator pressure dynamics, DP is applied to the optimal compound design in Table II coupled with an isothermal accumulator. The SOC over the drive-cycle is shown in Fig.7(c). The fuel economy is decreased from 64.5mpg, estimated for a constant low system pressure, unconstrained capacity case, to 63.0 mpg. Although the accumulator pressure is allowed to reach 35MPa, the DP results tend to keep the accumulator pressure low so that the fuel economy is closer to the low pressure estimate rather than the high pressure one in Fig. 6.

Computation times: The simplifying assumptions made to compute the optimal control increase computational efficiency significantly. On a basic PC, it takes only $\sim$2 sec. to evaluate a design and $\sim$15 minutes to completely optimize an architecture ($\sim$450 design iterations). In contrast, dynamic programming would take $\sim$15 mins. to evaluate only one design. The analysis above suggests that the simplifying assumptions have relatively minor impact on the estimate of fuel economy so that they can be used in rapid iterative design.

VIII. Conclusions

This paper has presented an efficient approach for optimizing the configuration and pump/motor sizes of a hydraulic hybrid power-split transmission. It utilizes a generalized kinematic relationship of the transmission to avoid redundant computation of mechanically different but kinematically equivalent configurations. A full kinematic matrix is shown to be realizable by a compound configuration. Modal vehicle operations are proposed to reduce loss. By neglecting the pressure dynamics and accumulator size constraints, the Lagrange multiplier method can be used to solve the optimal control problem necessary to evaluate each design. Simulations show that these simplifications have minor impacts on the estimated fuel economies of the optimized designs.

A case study on a compact sized vehicle indicates that the optimized compound power-split and input coupled power-split have better fuel economy and require smaller pump/motors than an optimized output coupled power-split.

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