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An Adaptive Predictor Corrector Strategy for Output Feedback Control of Nonlinear Hybrid Process Systems

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Abstract— **This work presents a methodology for model-based output feedback control of uncertain nonlinear hybrid process systems using an adaptive predictor corrector strategy. A hybrid monitoring scheme is initially developed to facilitate the identification of the active mode at any given time using the measured output. A set of stabilizing output feedback controllers are then synthesized to robustly stabilize the constituent modes where appropriate state estimators are used. To stabilize each mode with minimal sensor-controller communication, a predictive model of each mode is embedded within the corresponding state feedback controller to provide an estimate of the process state which is used during periods of communication suspension. To determine when the communication must be restored, the evolution of the state estimate for the active mode is monitored and the corresponding state estimator is prompted to send its estimate to update the model state only when some update criteria are satisfied. The key idea is to use the model as a predictor and to use the Lyapunov stability constraint for each mode as a criterion for adaptively correcting the model predictions. The implementation of the proposed methodology is demonstrated using a simulated model of a chemical reactor with multiple operating modes.**

I. INTRODUCTION

The study of hybrid systems has emerged as an active research area in process control over the past two decades. Characterized by inherent and strong interactions between continuous dynamics and discrete events, hybrid systems pose several fundamental challenges, which, together with the abundance of practical applications where hybrid dynamics are dominant, have been key driving forces behind the significant and growing body of research work in this area, including works on modeling and simulation (e.g., [1]), optimization (e.g., [2]), stability analysis (e.g., [3]–[6]), monitoring (e.g., [7]–[10]) and control (e.g., [11]–[14]).

This progress notwithstanding, a careful examination of the existing methods on control of hybrid systems shows that the hybrid control problem is typically formulated and addressed within the classical feedback control paradigm where the process outputs are assumed to be transmitted directly and flawlessly to the controller where the control actions are generated and fed back to the process. In practice, this paradigm often needs to be re-examined, partly due to the increasing complexity of the process/controller interface which features additional information-processing steps and devices that should be accounted for in the control system design. For example, with the increased reliance in recent years on networked control systems, and the emergence of applications where a large number of networked sensors and actuators are deployed, issues such as resource constraints, data losses, measurement quantization, processing and communication delays, and real-time scheduling constraints are becoming commonplace, and ultimately impose constraints on the sensor-controller communication link. While these issues have been studied extensively in the context of research work on networked control systems (e.g., see [15] for a survey of some results and references in this area), the focus of existing studies has been on systems with purely continuous or purely discrete dynamics. At this stage, systematic methods for handling these issues for combined discrete-continuous processes remain lacking.

Motivated by these considerations, we present in this work a methodology for model-based output feedback control of uncertain nonlinear hybrid process systems using an adaptive predictor corrector strategy. The approach aims to robustly stabilize the hybrid process with minimal sensor-controller information transfer. Beyond reducing the susceptibility of the control system to unexpected communication disruptions, such resource-aware control approach also helps identify the fundamental limits on the stabilizability of a given process subject to limitations on measurement availability. The hybrid control structure consists of a bank of mode observers that identify the active mode at any given time, a family of robustly stabilizing Lyapunov-based output feedback controllers, where each controller consists of a model-based state feedback controller and a state estimator, and a supervisor that monitors the evolution of each Lyapunov function and switches synchronously between the different controllers and state estimators. The mode observers are co-located with the sensors and thus receive continuous measurements from the sensors. By evaluating the residuals which capture the differences between the outputs of the mode observers and the process, this monitoring scheme will locate the active mode at any given time. Once the active mode is identified, the corresponding controller and state estimator are activated by the supervisor to stabilize the process. The state estimators are embedded in the sensors and generate an estimate of the state using the measured output. To minimize the sensor-controller information transfer, a model of each mode is embedded within the corresponding state feedback controller to generate an estimate of the process state when the mode

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is active and the observer's estimate is not available from the sensors. The model state is used to generate the control action and is updated by the state estimate when the sensor-controller communication is established. The key idea of this approach is to use the model as a predictor and to use the Lyapunov stability constraint for each mode as a criterion for adaptively correcting the model predictions. This naturally leads to a state-dependent time-varying communication logic which allows the plant to respond adaptively to changes in operating conditions by increasing or decreasing the sensor-controller information transfer. In addition, this approach ultimately leads to an efficient utilization of the sensor-controller link in the sense that data are transmitted only when necessary to maintain closed-loop stability.

The rest of the paper is organized as follows. In Section II, the mathematical model for the class of systems under consideration is described. Following the design of the mode observers in Section III, the synthesis of the Lyapunov-based output feedback controllers is presented in Section IV, and the design of the adaptive predictor corrector strategy in Section V. Finally, the theoretical results are illustrated using a hybrid chemical reactor example in Section VI.

II. PRELIMINARIES

We consider switched uncertain nonlinear systems described by the following state-space representation:

$$\begin{aligned} &\dot{x}(t) = A_i x(t) + f_i(x(t)) + B_i u_i(t) + W_i(x(t))\theta_i(t) \\ &y(t) = Cx(t), \quad t \in [t^k_{i,\mathrm{in}}, t^k_{i,\mathrm{out}}) \\ &i(t) \in \mathcal{I} = \{1, 2, \cdots, N\}, \quad k \in \mathbb{N} \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ denotes the vector of continuous-time state variables, $u_i \in \mathbb{R}^m$ denotes the vector of manipulated inputs associated with the i-th mode, $\theta_i \in \Theta_i \subset \mathbb{R}^q$ denotes the vector of bounded uncertain variables that takes values in a nonempty compact convex subset of $\mathbb{R}^q$ and describes parametric uncertainty and/or time-varying external disturbances, $y \in \mathbb{R}^p$ denotes the vector of output variables. A_i, B_i, $W_i(\cdot)$ and C are $n\times n$, $n\times m$, $n\times q$ and $p\times n$ matrices, respectively. $f_i : \mathbb{D} \to \mathbb{R}^n$ denotes a Lipschitz map and $\mathbb{D} \subset \mathbb{R}^n$ is a domain that contains the origin $x = 0$. The switching signal $i : [0, \infty) \to \mathcal{I}$ is assumed to be a piecewise continuous (from the right) function of time, i.e., $i(t^k) = \lim_{t\to t^{k+}} i(t)$ for all k, implying that only a finite number of switches are allowed on any finite interval of time. It is assumed that all entries of the vector field $f_i(\cdot)$, and the $n \times q$ matrix $W_i(\cdot)$ are sufficiently smooth. We also assume that the state, x, does not jump when the system switches between modes, which means that x is everywhere continuous. The notations $t^k_{i,\mathrm{in}}$ and $t^k_{i,\mathrm{out}}$ are used to denote the k-th time that the i-th mode is switched in and out, respectively. Referring to the quasi-linear structure considered in (1), it should be noted that while this form is chosen here to simplify the analysis and design calculations to be presented later, this structure is quite common in many practical systems such as chemical processes where material and energy flows (which usually depend linearly on the state) are coupled with chemical reactions (whose rates typically depend nonlinearly on the state).

III. MODE IDENTIFICATION AND MODE TRANSITION DETECTION

Given that different modes within a switched system exhibit different dynamics, different controllers will be needed to stabilize the different modes. The ability to activate the correct controller at the right time requires the development of a mechanism for identifying the active operating mode and detecting possible transitions from one mode to another. Failure to perform synchronous process/controller transitions could lead to performance degradation or even instability. To address this problem, we construct the following set of mode observers (inspired in part by the ideas in [16], [17]) to replicate the dynamics of each mode:

$$\begin{aligned} &\dot{z}_i = L_i z_i + K_i y + (M_i - H_i)Cf_i(\zeta_i) \\ &\zeta_i = z_i + H_i y, \ r_i = M_i y - \zeta_i, \ i \in \mathcal{I} \end{aligned} \tag{2}$$

where $z_i \in \mathbb{R}^n$ is the state of the i-th mode observer, $\zeta_i \in \mathbb{R}^n$ is the observer output, $r_i \in \mathbb{R}^n$ is the residual, and L_i, K_i, M_i and H_i are design matrices. The following proposition summarizes the conditions that the mode observers of (2) must satisfy in order to allow the identification of the active mode and the detection of mode transitions in the presence of uncertainties.

Proposition 1. *Consider the switched uncertain nonlinear system of* (1) *and the bank of mode observers in* (2) *where L_i, K_i, M_i and H_i are chosen such that for all $i \in \mathcal{I}$:*

$$(M_i - H_i)CA_i - L_i(M_i - H_i)C - K_i C = 0 \tag{3a}$$

$$(M_i - H_i)CB_i = (M_i - H_i)CW_i(x) = 0 \tag{3b}$$

$$(M_i - H_i)C \neq 0 \tag{3c}$$

Without loss of generality, let mode ε, for some $\varepsilon \in \mathcal{I}$, be the active one over any time interval $[t^k_{\varepsilon,\mathrm{in}}, t^k_{\varepsilon,\mathrm{out}})$, $k \in \mathbb{N}$. Then, the evolution of the corresponding residual, $r_\varepsilon(t)$, for $t \in [t^k_{\varepsilon,\mathrm{in}}, t^k_{\varepsilon,\mathrm{out}})$, is governed by:

$$\dot{r}_\varepsilon = L_\varepsilon r_\varepsilon + (M_\varepsilon - H_\varepsilon)C[f_\varepsilon(x) - f_\varepsilon(\zeta_\varepsilon)] \tag{4}$$

while the residual for any inactive mode j, where $j \in \mathcal{I} - \{\varepsilon\}$, satisfies the following evolution equation:

$$\begin{aligned} \dot{r}_j = {} & L_j r_j + (M_j - H_j)C[(A_\varepsilon - A_j)x + f_\varepsilon(x) - f_j(\zeta_j)] \\ & + (M_j - H_j)C[B_\varepsilon u_\varepsilon + W_\varepsilon(x)\theta_\varepsilon] \end{aligned} \tag{5}$$

Sketch of proof. The derivative of the i-th residual can be computed from $\dot{r}_i = M_i C\dot{x} - \dot{\zeta}_i$, where $\dot{x}$ and $\dot{\zeta}_i$ are given by (1) and (2), respectively. Then, after applying the conditions (3a)-(3c), (4) and (5) can be obtained. □

The following theorem characterizes the expected behavior of the mode observer residuals corresponding to the active and inactive modes in terms of time-varying bounds that will serve as the basis for identifying the active mode.

Theorem 1. *Consider the systems of* (4) *and* (5) *where L_ε and L_j are chosen to be Hurwitz and $f_\varepsilon(\cdot)$ and $f_j(\cdot)$ are Lipschitz on some domain $\mathbb{D} \subset \mathbb{R}^n$ with Lipschitz constants ϑ_ε and ϑ_j, respectively. Then the systems of* (4) *and* (5) *are input-to-state stable.*

Proof. Consider the system of (4) first. Since L_ε is Hurwitz, then for any positive definite matrix S_ε, there exists a positive definite matrix P_ε that satisfies the Lyapunov equation $L'_\varepsilon P_\varepsilon + P_\varepsilon L_\varepsilon = -S_\varepsilon$. Take $V_\varepsilon(r_\varepsilon) = r'_\varepsilon P_\varepsilon r_\varepsilon$ as a Lyapunov function candidate which satisfies $\lambda_m(P_\varepsilon)\|r_\varepsilon\|^2 \leq V_\varepsilon(r_\varepsilon) \leq \lambda_M(P_\varepsilon)\|r_\varepsilon\|^2$, where $\lambda_m(\cdot)$ and $\lambda_M(\cdot)$ denote the minimum and maximum eigenvalues of a matrix, and $\|\cdot\|$ denotes the Euclidean norm of a vector or matrix. Evaluating $\dot{V}_\varepsilon$ along the trajectories of the system (4) yields:

$$\begin{aligned}\dot{V}_\varepsilon &= -r_\varepsilon' S_\varepsilon r_\varepsilon + 2r_\varepsilon' P_\varepsilon (M_\varepsilon - H_\varepsilon) C[f_\varepsilon(x) - f_\varepsilon(M_\varepsilon y - r_\varepsilon)] \\ &\le -\lambda_m(S_\varepsilon)\|r_\varepsilon\|^2 + \vartheta_\varepsilon \sigma_{\varepsilon 1}\|r_\varepsilon\|\|(I - M_\varepsilon C)x + r_\varepsilon\| \\ &\le -\lambda_m(S_\varepsilon)\|r_\varepsilon\|^2 + \vartheta_\varepsilon \sigma_{\varepsilon 1}\|r_\varepsilon\|^2 + \vartheta_\varepsilon \sigma_{\varepsilon 1}\sigma_{\varepsilon 2}\|x\|\|r_\varepsilon\|\end{aligned}$$

where $\sigma_{\varepsilon 1} \triangleq 2\|P_\varepsilon(M_\varepsilon - H_\varepsilon)C\|$ and $\sigma_{\varepsilon 2} \triangleq \|(I - M_\varepsilon C)\|$, which implies that:

$$\dot{V}_\varepsilon \le -\lambda_m(S_\varepsilon)(1 - \pi_\varepsilon)\|r_\varepsilon\|^2, \quad \forall \ \|r_\varepsilon\| \ge G_\varepsilon \|x\| \tag{6}$$

where $\pi_\varepsilon \in (0, 1)$, $G_\varepsilon \triangleq \varrho_\varepsilon \vartheta_\varepsilon \sigma_{\varepsilon 1}\sigma_{\varepsilon 2}$ and $\varrho_\varepsilon \triangleq (\lambda_m(S_\varepsilon)\pi_\varepsilon - \vartheta_\varepsilon \sigma_{\varepsilon 1})^{-1}$. By the same token, we consider $V_j(r_j) = r_j' P_j r_j$ as a Lyapunov function candidate for the system of (5), and it can be shown that $\dot{V}_j(r_j)$ satisfies the following inequality after introducing a constant $\pi_j \in (0, 1)$:

$$\dot{V}_j \le -\lambda_m(S_j)(1 - \pi_j)\|r_j\|^2, \ \forall \ \|r_j\| \ge G_j\|x\| + D \tag{7}$$

where $G_j \triangleq \varrho_j \vartheta_j \sigma_{j1}\sigma_{j2}$, $\varrho_j \triangleq (\lambda_m(S_j)\pi_j - \vartheta_j \sigma_{j1})^{-1}$, $\sigma_{j1} \triangleq 2\|P_j(M_j - H_j)C\|$, $\sigma_{j2} \triangleq \|(I - M_j C)\|$, $D \triangleq \varrho_j \sigma_{j1}\|[f_\varepsilon(x) - f_j(x) + (A_\varepsilon - A_j)x + B_\varepsilon u_\varepsilon + W_\varepsilon(x)\theta_\varepsilon]\|$. From (6) and (7), we conclude that the systems of (4) and (5), with x as input to each, are input-to-state stable. □

Remark 1. (6) and (7) imply that the residual for the active mode satisfies a time-varying bound of the following form:

$$\|r_\varepsilon(t)\| \le \beta_\varepsilon(\|r_\varepsilon(0)\|, t) + \gamma_\varepsilon G_\varepsilon \|x(t)\| \tag{8}$$

for some class $\mathcal{KL}$ function $\beta_\varepsilon(\cdot, \cdot)$, and $\gamma_\varepsilon = \sqrt{\frac{\lambda_M(P_\varepsilon)}{\lambda_m(P_\varepsilon)}}$, while the residuals for the inactive modes satisfy:

$$\|r_j(t)\| \le \beta_j(\|r_j(0)\|, t) + \gamma_j(G_j\|x(t)\| + D) \tag{9}$$

for some class $\mathcal{KL}$ function $\beta_j(\cdot, \cdot)$, and $\gamma_j = \sqrt{\frac{\lambda_M(P_j)}{\lambda_m(P_j)}}$. These bounds imply that both r_ε and r_j are ultimately bounded, but have different ultimate bounds. Therefore, if we choose $\gamma_\varepsilon G_\varepsilon = \gamma_j G_j$ (by proper selection of the design matrices), it can be seen that the ultimate bound for the residual for the active mode will always be closer to zero than those for the inactive modes. This difference is due to the term D which arises due to the differences between the dynamics of the active and inactive modes as well as the sensitivity of the residuals of the inactive modes to the uncertainties within the active mode (see Proposition 1). Based on the expected differences between the bounds in (8) and (9), one can attempt to identify the active mode and detect mode transitions. The idea would be to monitor the residuals simultaneously to determine the one that attains the smallest (steady-state) offset and declare this as the active mode. When the given pattern of residuals changes and another residual begins to exhibit minimal offset, a mode transition is declared and a new active mode is identified. Note, however, that caution must be exercised when applying these criteria since the analysis is based essentially on the ultimate bounds on the residuals (and not the residuals' actual values). Note also that until the transient terms, $\beta_\varepsilon(\cdot, \cdot)$ and $\beta_j(\cdot, \cdot)$, decay to sufficiently small values, they may still have significant contributions to the values of the residuals and thus could cause delays in identifying the active mode. Such delays can be minimized by proper choices of L_ε and L_j to ensure that the transient terms approach zero sufficiently fast (relative to the dwell time for the active mode).

Remark 2. Note that unlike the case under full-state feedback in which the residuals are evaluated using the full-state (e.g., see [10]), the residuals in (2) are evaluated using the measured output only. As can be seen from the analysis leading to (6), an important consequence of the lack of access to the full-state is that the term $[f_\varepsilon(x) - f_\varepsilon(\zeta_\varepsilon)]$ cannot simply be bounded by $\|r_\varepsilon\|$ (which is sufficient to prove convergence under state feedback); instead, this term depends also on x. Therefore, the residual for the active mode no longer converges to zero but approaches a terminal neighborhood of the origin that depends on the size of the state as well as the observer's design parameters, and this leads at most to ultimate boundedness (provided that $\|x\|$ is bounded). This introduces some ambiguity in the analysis of the residuals' behavior as it forces the designer to rely on comparing the different offsets attained by the different residuals shortly after mode switching to identify the active mode. Note, however, that from the input-to-state stability property established in Theorem 1 and the estimate in (8), if x converges to zero then so will r_ε. In this case, the active mode can be identified with certainty as the one with the residual that approaches zero.

IV. Robust Stabilization of Switched Systems Using Output Feedback Control

Once a mode transition is detected and the active mode is identified, the supervisor needs to activate the corresponding output feedback controller to robustly stabilize the hybrid system and achieve an arbitrary degree of uncertainty attenuation within the active subsystem. In this section, we describe the controller design procedure and provide an explicit characterization of the closed-loop stability properties.

A. Robust state feedback controller synthesis

Consider the switched nonlinear system of (1) and, without loss of generality, assume that the uncertainties are non-vanishing, i.e., $W_i(0) \neq 0$. We further assume the uncertainty is bounded by $\|\theta_i\| \le \theta_{b,i}$. Introducing a robust control Lyapunov function [18] for each mode $i \in \mathcal{I}$, the following robust nonlinear state feedback controller can be designed using the results in [14] (see also [19], [20]):

$$u_i = k_i(x, \theta_{b,i}, \rho_i, \chi_i, \phi_i) = \mathfrak{K}_i(x) \tag{10}$$

where $\mathfrak{K}_i(x) = -\frac{L^*_{\tilde{f}_i}V_i + \left((L^{**}_{\tilde{f}_i}V_i)^2 + \|(L_{B_i}V_i)'\|^4\right)^{1/2}}{\|(L_{B_i}V_i)'\|^2}(L_{B_i}V_i)'$ if $\|(L_{B_i}V_i)'\| \neq 0$ and $\mathfrak{K}_i(x) = 0$ if $\|(L_{B_i}V_i)'\| = 0$, $L^*_{\tilde{f}_i}V_i = L_{\tilde{f}_i}V_i + \frac{(\rho_i\|x\| + \chi_i\theta_{b,i}\|L_{W_i}V_i\|)\|x\|}{\|x\| + \phi_i}$, $L^{**}_{\tilde{f}_i}V_i = L_{\tilde{f}_i}V_i + \rho_i\|x\| + \chi_i\theta_{b,i}\|L_{W_i}V_i\|$, $L_{\tilde{f}_i}V_i = (\partial V_i/\partial x)\tilde{f}_i(x)$, $L_{B_i}V_i = (\partial V_i/\partial x)B_i$, $L_{W_i}V_i = (\partial V_i/\partial x)W_i(x)$, and the constants $\rho_i > 0$, $\chi_i > 1$ and $\phi_i > 0$ are controller tuning parameters.

Using a standard Lyapunov argument, it can be shown (see [21] for a similar proof) that there exist class $\mathcal{K}$ functions, $\alpha_i(\cdot)$ and $\psi_i(\cdot)$, such that $\dot{V}_i$ satisfies:

$$\dot{V}_i = L_{\tilde{f}_i}V_i + L_{B_i}V_i\mathfrak{K}_i(x) + L_{W_i}V_i\theta_i \le -\alpha_i(\|x\|) + \psi_i(\bar{\phi}_i) \tag{11}$$

where $\bar{\phi}_i = \frac{\phi_i}{\chi_i - 1}$. Furthermore, given any real number $\delta_i > 0$, there exists $\tilde{\phi}_i$ such that if $\bar{\phi}_i \le \tilde{\phi}_i$, $\limsup_{t\to\infty}\|x\| \le \delta_i$ and the nominal equilibrium point of the i-th mode is practically stable. This implies that the controller enforces robust closed-loop stability for each mode with an arbitrary degree of attenuation of the effects of uncertainty, and guarantees convergence to an arbitrarily small neighborhood of the nominal equilibrium point in finite time.

Remark 3. For switched systems with only a finite number of mode transitions, robust stabilization of the continuous modes is sufficient to guarantee stability of the overall system. However, when considering an infinite number of mode transitions over the infinite time interval, additional restrictions on the growth of each Lyapunov function for the time periods during which the corresponding mode is inactive are needed. This is typically expressed in the form of a multiple Lyapunov function stability constraint [3].

B. Observer-based output feedback control

The direct implementation of the robust feedback controller of (10) requires the availability of full-state measurements, which are seldom available in practice. To compensate for the lack of full-state measurements, an appropriate state estimator must be designed for each mode $i \in \mathcal{I}$ to provide an estimate of the state from the measured output which can then be used by the state feedback control law to compute the control action. The combination of the state feedback controller with the state estimator yields an output feedback controller of the following general form:

$$\dot{\omega}_i = \Gamma_i(\omega_i, y, \mu_i),\ u_i = \mathfrak{K}_i(\omega_i) \tag{12}$$

where $\omega_i \in \mathbb{R}^n$ is the state estimate, μ_i is an observer design parameter, and $\Gamma_i(\cdot,\cdot,\cdot)$ is a vector function. In the interest of generality, we will not limit the discussion to any particular state estimator design method; instead, we will consider any type of state estimator that satisfies the requirements set forth in the following assumption.

Assumption 1. *Referring to the i-th closed-loop subsystem of* (1) *and* (12)*, given any set of positive real numbers $\{\delta_{b,i}, \theta_{b,i}, \delta_{d,i}\}$, there exists $\phi_i^* > 0$, and for each $\phi_i \in (0, \phi_i^*]$, there exists $\mu_i^* > 0$, such that if $\bar{\phi}_i \le \phi_i^*$, $\mu_i \le \mu_i^*$, $\|x(0)\| \le \delta_{b,i}$, $\|\omega_i(0)\| \le \delta_{b,i}$ and $\|\theta_i\| \le \theta_{b,i}$, the trajectories of the closed-loop system are bounded and satisfy $\limsup_{t\to\infty}\|x(t)\| \le \delta_{d,i}$. Furthermore, given any positive real number T_i^b, there exists $\tilde{\mu}_i \le \mu_i^*$ such that if $\mu_i \in (0, \tilde{\mu}_i]$, $\|x(t) - \omega_i(t)\| \le \Upsilon_i \mu_i$ with some $\Upsilon_i > 0$ for all $t \ge T_i^b$.*

Remark 4. Assumption 1 requires that the observer designed for each mode be able to (a) ensure that the closed-loop system under the output feedback controller of (12) is stable with an ultimate bound on the closed-loop state that can be tuned via proper selection of the observer design parameter, and (b) enforce an arbitrarily fast convergence of the observer-generated estimate to the actual state by proper selection of the observer design parameter. These requirements will facilitate the design and implementation of the adaptive predictor-corrector strategy described in the next section. Typical examples of state estimators satisfying Assumption 1 include high-gain observers (e.g., see [14], [22]) where μ_i scales inversely with the observer gain. Note that convergence of the estimation error below some desired level is ensured only after a short period of time T_i^b which can be made arbitrarily small by proper selection of μ_i.

The following proposition provides an explicit characterization of the stability properties of each mode under the corresponding output feedback controller.

Proposition 2. *Consider the i-th closed-loop subsystem of* (1) *and* (12) *where Assumption 1 holds with $\bar{\phi}_i \le \phi_i^*$, $\mu_i \le \tilde{\mu}_i$, $\|x(0)\| \le \delta_{b,i}$, $\|\omega_i(0)\| \le \delta_{b,i}$ and $\|\theta_i\| \le \theta_{b,i}$. Then there exists a class $\mathcal{K}$ function $\varsigma_i(\cdot)$ such that for all $t \ge T_i^b$:*

$$\dot{V}_i(x(t)) \le -\alpha_i(\|x(t)\|) + \psi_i(\bar{\phi}_i) + \varsigma_i(\mu_i) \tag{13}$$

Proof. Evaluating $\dot{V}_i(x)$ along the trajectories of (1) and (12) and then applying (11) yields:

$$\dot{V}_i(x) \le -\alpha_i(\|x\|) + \psi_i(\bar{\phi}_i) + \|L_{B_i}V_i(x)\|\|\mathfrak{K}_i(\omega_i) - \mathfrak{K}_i(x)\|$$

Assumption 1 and the Lipschitz continuity of $\mathfrak{K}_i(\cdot)$ allow us to conclude that there exist two positive real constants Π_i and Ξ_i such that $\|L_{B_i}V_i(x)\| \le \Pi_i$ (since x is bounded) and $\|\mathfrak{K}_i(\omega_i) - \mathfrak{K}_i(x)\| \le \Xi_i\|\omega_i - x\|$. Therefore,

$$\dot{V}_i(x) \le -\alpha_i(\|x\|) + \psi_i(\bar{\phi}_i) + \varsigma_i(\mu_i) \tag{14}$$

where $\varsigma_i(\mu_i) \triangleq \Pi_i\Xi_i\Upsilon_i\mu_i$. □

V. Adaptive predictor corrector strategy

The implementation of the control law in (12) requires the continuous availability of the state estimate from the state estimators embedded within the sensors. To reduce sensor-controller information transfer as much as possible without sacrificing stability, a model of each mode is embedded within the corresponding controller to provide it with an estimate of the state when the state estimate is not transmitted. Feedback from the sensors to the controller is performed by updating the model state using the observer's estimate whenever communication is restored. Under this architecture, the control law for each mode is implemented as follows:

$$\begin{aligned} &u_i(t) = \mathfrak{K}_i(\widehat{x}(t)),\ i \in \mathcal{I} \\ &\dot{\widehat{x}}(t) = \mathfrak{A}_i x(t) + \mathfrak{F}_i(\widehat{x}(t)) + \mathfrak{B}_i u_i(t),\ t \in [t_i^j, t_i^{j+1}) \\ &\widehat{x}(t_i^j) = \omega_i(t_i^j),\ j \in \mathbb{N} \end{aligned} \tag{15}$$

where $\widehat{x}$ is the state of the model for the i-th mode, $\mathfrak{A}_i$, $\mathfrak{F}_i(\cdot)$ and $\mathfrak{B}_i$ model A_i, $f_i(\cdot)$ and B_i in (1), respectively, t_i^j is used to denote the j-th time instant that the model state embedded in i-th controller is updated using the state estimate transmitted by the i-th state estimator. When the sensor-controller communication is suspended, the control action is evaluated based on $\widehat{x}$. Computing $\dot{V}_i(x)$ along the trajectories of the system of (1) subject to (15) yields:

$$\begin{aligned} \dot{V}_i(x(t)) \le &-\alpha_i(\|x(t)\|) + \psi_i(\bar{\phi}_i) + \varsigma_i(\mu_i) \\ &+ L_{B_i}V_i(x)[\mathfrak{K}_i(\widehat{x}_i(t)) - \mathfrak{K}_i(\omega_i(t))] \end{aligned} \tag{16}$$

for all $t \ge T_i^b$. As can be seen from (16), an additional term appears on the right-hand side when the communication is suspended. This term, which arises due to the discrepancy between $\widehat{x}$ and ω_i, could change the expected decay rate of V_i or even result in the growth of V_i, hence instability of the system. When this difference is large enough to dominate the negative term of the right-hand side, communication must be restored so that $\widehat{x}$ can be updated using ω_i and the undesirable behavior of the system due to the plant-model mismatch can be corrected to regain stability. This suggests that the behavior of $V_i(x)$ can be used as a threshold for suspending and terminating communication. However, owing to the unavailability of full-state measurements, we need to examine $V_i(\omega_i)$ and its derivative instead. Specifically, from

Assumption 1 and the continuity of $\alpha_i(\cdot)$ and $\dot{V}_i(\cdot)$, we can conclude that given $\mu_i > 0$, there exist class $\mathcal{K}$ functions $\gamma_i(\cdot)$ and $\kappa_i(\cdot)$ such that, for all $t \geq T_i^b$:

$$\|x-\omega_i\| \leq \Upsilon_i\mu_i \Rightarrow \begin{cases} \left|\alpha_i(\|x\|) - \alpha_i(\|\omega_i\|)\right| \leq \gamma_i(\mu_i) \\ \left|\dot{V}_i(x) - \dot{V}_i(\omega_i)\right| \leq \kappa_i(\mu_i) \end{cases} \quad (17)$$

Substituting the above estimates into (16), we obtain:

$$\begin{aligned} \dot{V}_i(\omega_i(t)) \leq & -\alpha_i(\|\omega_i(t)\|) + \psi_i(\bar{\phi}_i) + \varpi_i(\mu_i) \\ & + L_{B_i}V_i(x)[\mathfrak{K}_i(\widehat{x}(t)) - \mathfrak{K}_i(\omega_i(t))] \end{aligned} \quad (18)$$

for all $t \geq T_i^b$, where $\varpi_i(\cdot) \triangleq \varsigma_i(\cdot) + \gamma_i(\cdot) + \kappa_i(\cdot)$ is a class $\mathcal{K}$ function. The following theorem describes how the above bound on $\dot{V}_i(\omega_i)$ can be employed for suspending and restoring the communication.

Theorem 2. *Consider the system of* (1)*, for which* $V_i(x)$*,* $i \in \mathcal{I}$*, satisfy* (13) *when the state estimate generated by the state estimator for the active mode is transmitted continuously to the corresponding model embedded in the corresponding controller. Let* $\widehat{x}(t) = \omega_i(t)\ \forall\, t \in [0, T_i^b]$*, and let* $t_i^{j-} > T^b$ *be the* j*-th time that* $\dot{V}_i(\omega_i)$ *satisfies the following bound:*

$$\dot{V}_i(\omega_i(t_i^{j-})) > -\alpha_i(\|\omega_i(t_i^{j-})\|) + \psi_i(\bar{\phi}_i) + \varpi_i(\mu_i) \quad (19)$$

where $\omega_i(t_i^{j-}) = \lim_{t \to t_i^{j-}} \omega_i(t)$*, then the update law given by* $\widehat{x}(t_i^j) = \omega_i(t_i^j)$ *ensures that* $\dot{V}_i(x_i(t_i^j)) \leq -\alpha_i(\|x(t_i^j)\|) + \psi_i(\bar{\phi}_i) + \upsilon_i(\mu_i)$*, for some class* $\mathcal{K}$ *function* $\upsilon_i(\cdot) > \varpi_i(\cdot)$*.*

Proof. From (16), it can be seen that if (19) is satisfied, then $L_{B_i}V_i(x)[\mathfrak{K}_i(\widehat{x}) - \mathfrak{K}_i(\omega_i)] > 0$. If the state of the model is updated such that $\widehat{x}(t_i^j) = \omega_i(t_i^j)$, then we obtain $\dot{V}_i(\omega_i(t_i^j)) \leq -\alpha_i(\|\omega_i(t_i^j)\|) + \psi_i(\bar{\phi}_i) + \varpi_i(\mu_i)$. And thus, by applying (17), we conclude that there exists a class $\mathcal{K}$ function $\upsilon_i(\cdot) \triangleq \varpi_i(\cdot) + \gamma_i(\cdot) + \kappa_i(\cdot) > \varpi_i(\cdot)$ such that $\dot{V}_i(x(t_i^j)) \leq -\alpha_i(\|x(t_i^j)\|) + \psi_i(\bar{\phi}_i) + \upsilon_i(\mu_i)$. □

Remark 5. When using the update law specified in Theorem 2, communication is re-established when (19) is satisfied, and therefore the update times are state-dependent. Immediately after the update, the model estimation error is reset to zero and the system behavior is corrected. Compared with a static communication policy (with a fixed update period), the proposed adaptive predictor corrector strategy is able, on the one hand, to respond promptly whenever the plant is influenced by unexpected external disturbances, and, one the other hand, to suspend the communication for as long as the prescribed threshold on $\dot{V}_i(\omega_i)$ is not breached so that the utilization of the sensor-controller link can be minimized. Note that continuous communication is initially needed in order to allow the observer estimation error to become sufficiently small so that x can be reliably inferred from ω_i.

VI. Simulation study: application to a chemical reactor with multiple operating modes

To illustrate the implementation of the proposed methodology, we consider a well-mixed continuous stirred tank reactor (CSTR) where three irreversible elementary exothermic reactions of the forms $A \xrightarrow{k_{10}} D$, $A \xrightarrow{k_{20}} U$ and $A \xrightarrow{k_{30}} R$ take place in parallel, where A is the reactant, D is the desired product, and U, R are undesired byproducts. The reactor has three operating modes: in mode 1, pure A is provided at flow rate F_1, molar concentration C_{A1} and temperature T_{A1}; in mode 2, species A enters the reactor at flow rate F_2, molar concentration C_{A2} and temperature T_{A2}; in mode 3, one more stream with A at flow rate F_3, molar concentration C_{A3} and temperature T_{A3} is introduced. Transitions between different operating modes are dictated by the change in the operating requirements. A jacket is used to remove or provide heat to the reactor. Under standard modeling assumptions, a hybrid model of the process can be derived from material and energy balances, and takes the following form:

$$\begin{aligned} \dot{C}_A &= \sum_{\tau=1}^{3} \sigma_\tau(t)\frac{F_\tau}{V}(C_{A\tau} - C_A) - \sum_{l=1}^{3} r_l(C_A, T) \\ \dot{C}_D &= -\sum_{\tau=1}^{3} \sigma_\tau(t)\frac{F_\tau}{V}C_D + k_{10}\exp(-\frac{E_1}{RT})C_A \\ \dot{T} &= \sum_{\tau=1}^{3} \sigma_\tau(t)\frac{F_\tau}{V}(T_{A\tau} - T) - \sum_{l=1}^{3} \frac{\Delta H_l}{\rho c_p} r_l(C_A, T) + \frac{Q}{\rho c_p V} \end{aligned}$$

where $r_l(C_A, T) = k_{l0}\exp\left(-\frac{E_l}{RT}\right)C_A$, C_A denotes the concentration of A, T denotes the reactor temperature, $\tau \in \{1, 2, 3\}$ is the feed stream index, $\sigma_\tau(t)$ can either be 0 or 1, representing removal or addition of a new stream, V is the reactor volume, k_{l0}, E_l, ΔH_l, where $l \in \{1, 2, 3\}$, denote the pre-exponential constants, the activation energies, and the enthalpies of the three reactions, ρ and c_p are the density and heat capacity of the fluid in the reactor, R is the gas constant, Q is the rate of heat transfer to the reactor. Using typical values for the process parameters, the reactor with $Q = Q_i^{\mathrm{nom}}$ (Q_i^{nom} is the nominal value of the rate of heat transfer for the i-th mode) usually has three equilibrium points for the i-th mode: two locally asymptotically stable and one unstable. The control objective here is to stabilize each mode at its unstable equilibrium point $(C_A^{1s}, C_D^{1s}, T^s) = (3.59\,\mathrm{mol/L}, 0.41\,\mathrm{mol/L}, 388.57\,\mathrm{K})$, $(C_A^{2s}, C_D^{2s}, T^s) = (3.91\,\mathrm{mol/L}, 0.20\,\mathrm{mol/L}, 388.57\,\mathrm{K})$, and $(C_A^{3s}, C_D^{3s}, T^s) = (4.11\,\mathrm{mol/L}, 0.14\,\mathrm{mol/L}, 388.57\,\mathrm{K})$ in the presence of parametric uncertainties in the enthalpies of the reaction (which is simulated by $0.001\Delta H_l^{\mathrm{nom}}\sin(t)$) and in the inlet temperature of the first stream T_{A1} (which is 5 K lower than the value used in the controller synthesis). The manipulated input used is $u_i = Q - Q_i^{\mathrm{nom}}$ and measurements of C_D are assumed to be available. To facilitate the design process, we define the displacement variables $x = [x_1\, x_2\, x_3]' = [C_A - C_A^{1s}\ C_D - C_D^{1s}\ T - T^s]'$ so that the steady state of the first mode x_1^s is placed at the origin.

Following the methodology presented in Sections III and IV, three mode observers and Lyapunov-based output feedback controllers are designed where the Lyapunov function for each mode is chosen to be: $V_i(\mathcal{X}) = \|\mathcal{X} - x_i^s\|^2, i \in \{1, 2, 3\}$ where $\mathcal{X}$ can represent the process state x, the model state $\widehat{x}$ or the state estimate ω_i. Since we consider only a finite number of mode transitions in this example, stability of each continuous closed-loop subsystem is sufficient to ensure stability of the overall switched system. Due to space limitations, the synthesis details for mode observers, controllers, state estimators and dynamic models are omitted.

To demonstrate the implementation of the proposed strategy, the reactor is initialized in mode 1 and the corresponding controller is activated to robustly stabilize the system. Fig. 1 shows that all the states are able to approach the desired

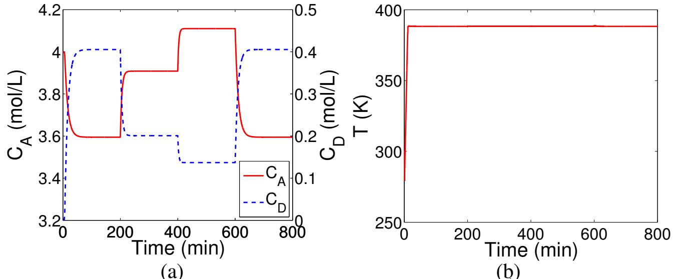

Fig. 1. Evolution of the concentration of the reactant A and the desired product D (1(a)), and the reactor temperature (1(b)).

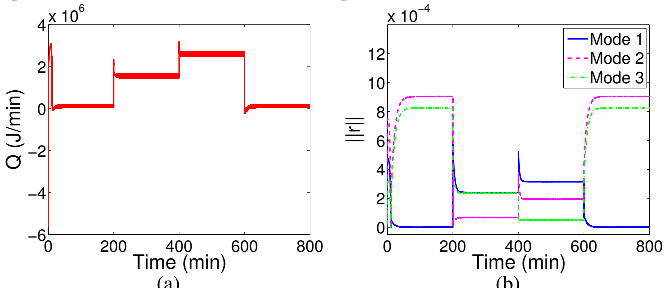

Fig. 2. Evolution of the rate of heat transfer (2(a)) and the residuals of the mode observers (2(b)) as the reactor switches from mode 1 to mode 2 to mode 3 and back to mode 1 at 200, 400, and 600 min, respectively.

steady state values very quickly and stay within a small neighborhood of the desired equilibrium point. Fig. 2(b) shows the residuals for the three mode observers. For $0 \leq t < 200\,\mathrm{min}$, the residual for mode observer 1 converges quickly and stays closest to the origin after a short period of time, which indicates that mode 1 is active during this time interval. At $t = 200.01\,\mathrm{min}$, the residual for mode observer 2 starts to have the minimal value, thus indicating that operation has been switched to mode 2 (the actual mode switching occurs at $t = 200\,\mathrm{min}$, and the time required for convergence is very short since the eigenvalues of the matrices L_i are quite large in magnitude). After the mode transition is detected, the output feedback controller for mode 2 is activated to stabilize the process. Subsequent transitions to mode 3 and mode 1 follow a similar pattern. Note that within each mode, the controller relies on the model predictions until the state estimate satisfies (19) at which time the model state is updated using the state estimate. As a measure of the extent of sensor-controller communication, we define an "average update rate" for each mode as the ratio between the time used for updates and the entire amount of time during a certain interval. Based on this definition, and after 800 minutes of operation, the average update rates for the four periods are 6.16%, 6.77%, 7.00% and 6.05%. Note that even when the process state converges to the terminal set, updates are still necessary from time to time because there is no feedback link between the plant and the controller when the sensor-controller communication is suspended and thus the process state is not guaranteed to remain within the required neighborhood of the unstable equilibrium point. The effects of greater plant-model mismatches and unexpected disturbances on the control system performance are also investigated by setting the value of T_{A1} used in the dynamic models to be $310\,\mathrm{K}$ and introducing disturbances that are simulated by $40\,\mathrm{K}$ step increases in the inlet temperature for all feed streams for $t \in [500\,\mathrm{min}, 503\,\mathrm{min})$. The controller is still able to keep the state within a slightly larger neighborhood of the nominal equilibrium point (the figures are omitted due to space limitations) but the average update rates for the four periods now are 13.50%, 13.76%, 14.56% and 13.76%. This shows that, the effects of the greater plant-model mismatches and unexpected disturbances are compensated by more frequent communication because the model prediction errors tend to grow faster.

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