

Rotational Motion Control Design for Cart-Pendulum System with Lebesgue Sampling

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Abstract—This study addresses a discretization method with Lebesgue sampling for a type of nonlinear system, and proposes a control method based on the discrete system model. A cart-pendulum system is used as this example. Applying this control method to some real system, how to implement the controller is a crucial problem. To overcome the problem, an impulsive Luenberger observer is introduced with a numerical forward mapping from the current system state to the one-step ahead state by well-known Runge-Kutta method. As the result, a cart-pendulum system with a quantizer, whose quantization interval is relatively large, can be controlled effectively. Numerical simulations are performed to verify the effectiveness of the proposed method.

I. INTRODUCTION

Discrete-time control has nice properties and is natural for real systems with respect to the implementation viewpoint. This assertion is based on difficulty to realize an arbitrarily short sampling interval. That means a control law described in continuous-time systems basically cannot be implemented to real systems as it is, with the exception of analog devices use. Hence, digital control systems with a time-invariant and constant sampling interval are usually utilized to implement a desired control law by some digital devices including computers, DSPs and FPGAs [1].

Recently some interesting extensions on the digital control are discussed. Lebesgue sampling is one of such topics. In usual digital control, the update of the control input is performed every sampling interval, and the sampling interval is given by chopping the time axis at some regular intervals. On the other hand, in the Lebesgue sampling case, the update of the control input is performed whenever the output of the system exceeds the given levels which are decided by chopping the output range like chopping the time axis in the usual digital control case [2]. In other words, the control input is updated whenever some events on the output arise. In this sense, the digital control with Lebesgue sampling can be regarded as an event-based control [3], [4]. As other related studies, a comparison between periodic and Lebesgue sampling for one-dimensional systems can be found in [5]. In such literature, it is shown that some impulsive control based on the Lebesgue sampling may reduce the average

sampling frequency to achieve the almost same performance as the periodic sampling case.

The concept of the Lebesgue sampling-based control is very natural for systems with many digital sensors such as encoders, and also for networked-systems. Typically we can say cheaper sensors are desired from the cost viewpoint for marketed products such as cars. Such cheaper sensors, however, doesn't provide good resolution in general, and a typical controller cannot achieve good performance. Hence, there are several studies on observer-based control which considers the quantization effect of low-resolution sensors to recover good control performance [6], [7].

The systems via some networks are another example of Lebesgue sampling systems. The information via networks is not continuous but intermittent. The arrival intervals between previous and current information are also not fixed and varying. Hence, if a natural conception on the system via networks leads to the control law updated when the new information comes, i.e. event-based control. Montestruque and Antsaklis [8] addressed this issue and proposed an interesting model-based control for networked systems. Their and our previous methods in [7] have many points of similarity.

This paper is an extension of our previous method in [7]. Especially a cart-pendulum system, which is nonlinear, is used as a specific example. The discretization method with Lebesgue sampling for this type of nonlinear systems are discussed first of all. The control purpose of the pendulum system is to keep the rotational speed of the pendulum, and a control. Once the discrete system is obtained, a control law based on the model is derived to realize the purpose. The control law practically can be designed by the well-known linear servo control theory [9] because the pendulum system can be represented by a linear system by the proposed discretization with Lebesgue sampling. However, applying this control method to some real system, the implementation of the controller becomes a crucial problem. To overcome the problem, according to the analogy of [7], an impulsive Luenberger observer is introduced. The impulsive Luenberger observer requires the forward mapping from the current system state to the one-step ahead state. Hence we also describe a numerical forward mapping by well-known Runge-Kutta method. As the result, a cart-pendulum system with a quantizer, whose quantization interval is relatively large, can be controlled effectively. Numerical simulations are performed to verify the effectiveness of the proposed method.

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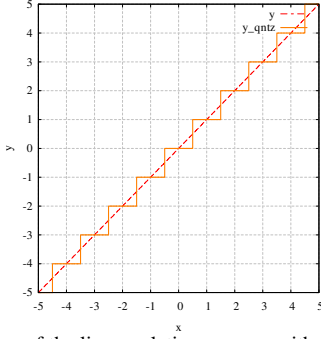


Fig. 1. Comparison of the linear relation, $y = x$, with the quantizer output, $q(y)$, in the case of $q_n = 1$.

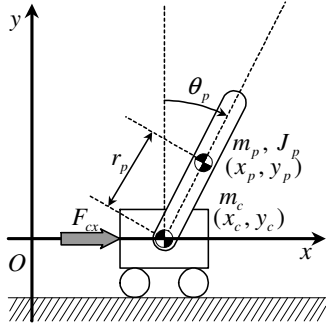


Fig. 2. The schematic figure of a cart-pendulum model. θ_p , (x_p, y_p) and (x_c, y_c) show the angle of the pendulum, the CoG of the pendulum, and the CoG of the cart, respectively.

TABLE I

PHYSICAL PARAMETERS AND VARIABLES OF THE CART-PENDULUM.		
Distance between hinge and CoG of pendulum	r_p	0.25 [m]
Principal moment of inertia of pendulum	J_p	0.2 [kg · m ²]
Mass of pendulum	m_p	0.75 [kg]
Mass of cart	m_c	1.5 [kg]
Angle of pendulum	θ_p	[rad]
Position of CoG of pendulum	(x_p, y_p)	[m]
Position of CoG of cart	(x_c, y_c)	[m]
Input force to cart	F_{cx}	[N]

II. DISCRETIZATION OF CART-PENDULUM SYSTEM BY LEBESGUE SAMPLING

A quantizer, $q(\cdot)$, used in this study is defined as

$$q(\theta) := q_n \cdot \text{round}(\theta/q_n), \quad (1)$$

where θ is an input value to be quantized, and q_n is the quantization interval, and the function $\text{round}(\cdot)$ is the rounding function to the nearest integer. For example, Fig. 1 illustrates the quantization of $y = x$ by the quantizer, $q(x)$, with $q_n = 1$. Let us define a time when the quantizer output changes as t_k . In this paper we call the time, t_k , the *interrupt* time. Hence t_{k+1} shows the next interrupt time. Note that $t_{k+1} - t_k$ for all k is NOT constant. Introduce the notation to distinguish a quantized value, $\theta[k]$, from an original value, $\theta(t_k)$, at t_k by $\theta[k] = q(\theta(t_k))$.

A cart-pendulum system in Fig. 2 is used as a plant to be controlled in this paper. Its physical parameters and variables are shown in Table I. The equations of motion of the cart-pendulum system are given by

$$\begin{bmatrix} m_c + m_p & m_p r_p \cos \theta_p \\ m_p r_p \cos \theta_p & J_p + m_p r_p^2 \end{bmatrix} \begin{bmatrix} \ddot{x}_c \\ \ddot{\theta}_p \end{bmatrix} = \begin{bmatrix} F_{cx} + m_p r_p \sin \theta_p \dot{\theta}_p^2 \\ g m_p r_p \sin \theta_p \end{bmatrix}. \quad (2)$$

The following equation of motion of only the pendulum can be extracted from (2).

$$m_p r_p \cos \theta_p \ddot{x}_c + (J_p + m_p r_p^2) \ddot{\theta}_p = m_p g r_p \sin \theta_p. \quad (3)$$

The acceleration of the cart, \ddot{x}_c , is regarded as the input to the pendulum system (3).

$$(J_p + m_p r_p^2) \ddot{\theta}_p = m_p g r_p \sin \theta_p - m_p r_p \cos \theta_p \ddot{x}_c. \quad (4)$$

The following rearrangement of the angular acceleration, $\ddot{\theta}_p$, can hold in general.

$$\ddot{\theta}_p = \frac{d\dot{\theta}_p}{dt} = \frac{d\theta_p}{dt} \frac{d\dot{\theta}_p}{d\theta_p} = \dot{\theta}_p \frac{d\dot{\theta}_p}{d\theta_p} \quad (5)$$

Substituting (5) into (4) yields

$$(J_p + m_p r_p^2) \dot{\theta}_p \frac{d\dot{\theta}_p}{d\theta_p} = m_p g r_p \sin \theta_p - m_p r_p \cos \theta_p \ddot{x}_c. \quad (6)$$

Suppose the acceleration of the cart during the interval from t_k and t_{k+1} , $\ddot{x}_c[k]$, be *constant*. Integrating both term of (6),

$$\begin{aligned} & \int_{\dot{\theta}_p[k]}^{\dot{\theta}_p[k+1]} (J_p + m_p r_p^2) \dot{\theta}_p d\dot{\theta}_p \\ &= \int_{\theta_p[k]}^{\theta_p[k+1]} (m_p g r_p \sin \theta_p - m_p r_p \cos \theta_p \ddot{x}_c[k]) d\theta_p. \end{aligned} \quad (7)$$

Rearranging (7), we have

$$\begin{aligned} & \frac{1}{2} (J_p + m_p r_p^2) (\dot{\theta}_p^2[k+1] - \dot{\theta}_p^2[k]) \\ &= -m_p g r_p (\cos \theta_p[k+1] - \cos \theta_p[k]) \\ & \quad - m_p r_p (\sin \theta_p[k+1] - \sin \theta_p[k]) \ddot{x}_c[k], \end{aligned} \quad (8)$$

$$\begin{aligned} \dot{\theta}_p^2[k+1] &= \dot{\theta}_p^2[k] \\ & \quad + \alpha(\theta_p[k], \theta_p[k+1]) + \beta(\theta_p[k], \theta_p[k+1]) \ddot{x}_c[k], \end{aligned} \quad (9)$$

where

$$\begin{aligned} \alpha(\theta_p[k], \theta_p[k+1]) &= \frac{-2m_p g r_p (\cos \theta_p[k+1] - \cos \theta_p[k])}{J_p + m_p r_p^2}, \\ \beta(\theta_p[k], \theta_p[k+1]) &= \frac{-2m_p r_p (\sin \theta_p[k+1] - \sin \theta_p[k])}{J_p + m_p r_p^2}. \end{aligned}$$

Introducing a new input, $u_p[k]$, the present input to (9), the cart acceleration $\ddot{x}_c[k]$, can be rewritten by

$$\ddot{x}_c[k] = \frac{u_p[k] - \alpha(\theta_p[k], \theta_p[k+1])}{\beta(\theta_p[k], \theta_p[k+1])}. \quad (10)$$

Hence (9) comes to

$$\dot{\theta}_p^2[k+1] = \dot{\theta}_p^2[k] + u_p[k]. \quad (11)$$

In (11), define the state as $x[k] = \dot{\theta}_p^2[k]$ and the system matrices as $\Phi = 1$ and $\Gamma = 1$. A discrete system of the cart-pendulum system derived by Lebesgue sampling is finally given by

$$x[k+1] = \Phi x[k] + \Gamma u_p[k]. \quad (12)$$

We're interested in a class of nonlinear systems which can be shown by the discrete system representation (12)

or a time-varying discrete system representation with the same structure with (12). Unfortunately, at the moment, we cannot describe such class clearly yet. But a piston-crank model, which can present combustion engine dynamics, can be classified into this class. We also try to extend the discretization by Lebesgue sampling with multivariable case although this study just think the case only a single variable, θ_p , is quantized. Those issues are our ongoing works.

III. CONTROL SYSTEM DESIGN

This paper considers a control task to realize constant-speed rotational motion for the pendulum of a cart-pendulum system. This task can be formulated by a servo control design to keep the state $x[k] = \dot{\theta}_p^2[k]$ of (12) be a constant desired value.

To derive the following control system, we assume that the angular velocity of the pendulum, $\dot{\theta}_p^2[k]$, can be known at each Lebesgue sampling. This implies

$$y[k] = Cx[k] = x[k], \quad (13)$$

with $C = 1$. Of course, this assumption is not valid for the real system. Hence, this issue will be discussed later, and can be solved by combination of some numerical integration method and impulsive Luenberger observer, which is an extension of our previous method proposed by an author [7].

Basically the control input, $u_p[k]$, in (12) is designed by the well-known optimal type-1 servo design [9]. Considering a quadratic cost function under the discrete system (12);

$$J = \sum_{i=0}^{\infty} (x[i]^T Q x[i] + u[i]^T R u[i]), \quad (14)$$

the optimal state feedback control law, $u[k]$, is given by

$$u[k] = -fx[k] \text{ with } f = (R + \Gamma^T P \Gamma)^{-1} \Gamma^T P \Phi, \quad (15)$$

where P is the positive symmetric matrix as the solution of the following discrete-time Riccati equation;

$$P = Q + \Phi^T P \Phi - \Phi^T P \Gamma (R + \Gamma^T P \Gamma)^{-1} \Gamma^T P \Phi. \quad (16)$$

Here, define a reference value as y_r , and consider an augmented system as follows:

$$\begin{bmatrix} x[k+1] \\ z[k+1] \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 1 \end{bmatrix} \begin{bmatrix} x[k] \\ z[k] \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_p[k] + \begin{bmatrix} 0 \\ y_r \end{bmatrix}. \quad (17)$$

A state feedback control law for (17),

$$u_p[k] = -[h \quad -k] \begin{bmatrix} x[k] \\ z[k] \end{bmatrix}, \quad (18)$$

leads to the optimal type-1 servo controller for the closed-loop system. The feedback gain is given by

$$[h \quad k] = [f\Phi \quad f\Gamma + I] \begin{bmatrix} \Phi - I & \Gamma \\ C & 0 \end{bmatrix}^{-1}, \quad (19)$$

where f is the optimal feedback gain in (15).

IV. IMPLEMENTATION

During an interval from t_k and t_{k+1} , $u_p[k]$ is constant and given by (18). The corresponding cart acceleration, $\ddot{x}_c[k]$, is calculated by (10). Note that $\theta_p[k+1]$ is given as a prior information, and is available for calculation of (10) because $\theta_p[k+1]$ is an output of the quantizer, (1), and the quantization interval of (1) is known preliminarily.

From (2), the cart acceleration, \ddot{x}_c , can be represented by

$$\ddot{x}_c = \frac{(J_p + m_p r_p^2) (F_{cx} + m_p r_p \sin \theta_p \dot{\theta}_p^2) - m_p^2 r_p^2 g \cos \theta_p \sin \theta_p}{(m_c + m_p) (J_p + m_p r_p^2) - (m_p r_p \cos \theta_p)^2}. \quad (20)$$

Therefore, once $\ddot{x}_c[k]$ is obtained from $u_p[k]$, the horizontal force applied to the real cart, F_{cx} , is derived by

$$F_{cx} = (m_c + m_p - (m_p r_p \cos \theta_p)^2) \left(\frac{u_p[k] - \alpha(\theta_p[k], \theta_p[k+1])}{\beta(\theta_p[k], \theta_p[k+1])} \right) - m_p r_p \sin \theta_p \dot{\theta}_p^2 + \frac{m_p^2 r_p^2 g \cos \theta_p \sin \theta_p}{J_p + m_p r_p^2}. \quad (21)$$

Note that F_{cx} in (21) is continuous with respect to time, and varying even though $u_p[k]$, $\theta_p[k]$ and $\theta_p[k+1]$ are constant during the Lebesgue sampling interval, because (21) requires continuous values of θ_p and $\dot{\theta}_p$.

As aforementioned, new measurement data, $\dot{\theta}_p[k]$, is obtained only at the interrupt time i_k , i.e. only when the quantizer output changes. In this sense, $\theta_p[k]$ and $\theta_p[k+1]$ are known a priori because $\theta_p[k]$ is the quantized value of the original θ_p . During the interrupt times, the original signals, θ_p and $\dot{\theta}_p$ cannot be measured. So (21) cannot be applied and implemented to the system directly. Hence in the following section, we propose a numerical method to solve the problem.

Here we also give a remark to control the rotational direction of the pendulum. The rotational direction depends on whether define $\theta_p[k+1] = \theta_p[k] + q_n$ or $\theta_p[k+1] = \theta_p[k] - q_n$ with the quantizer interval q_n .

A. Numerical integration of nonlinear system by Runge-Kutta method in SDC form

To overcome the addressed issue on the implementation, the key is to introduce an impulsive Luenberger observer. This scheme is a kind of analogy in [7] and [8]. Such impulsive Luenberger observer, however, requires the mapping of the current system state to the one-step ahead state. That means a discrete system of the target nonlinear system is required. However, it is a difficult problem to obtain such discrete time system. We regard the difficulties is caused by the fact the input of the system affects to the system matrices in the case of the discretization for nonlinear systems. That is, the input must be known a priori over the interval for discretization. This requirements causes a circular reference problem in control system design because the system matrices are required a prior to design the input. In our case, on the other hand, the discrete system of the target nonlinear system is used only when the state estimation is updated posteriori, i.e. after each interrupt. The diagram of the proposed controller is shown in Fig. 3. In the following

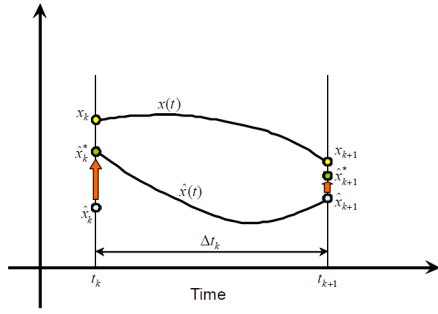


Fig. 3. The diagram of the impulsive Luenberger observer-based controller for Lebesgue sampled systems.

part, the detail is derived by using the cart-pendulum system as the example.

B. Non-linear system discretization by the Runge-Kutta method

Assume that input force u from t_k to t_{k+1} is constant. Let a state vector of the system be $x = [x_c, \theta_p, \dot{x}_c, \dot{\theta}_p]^T$. Motion equations of the system (2) can be changed the formula

$$\dot{x} = f(x, t), \quad (22)$$

$$= A(x)x + B(x)u. \quad (23)$$

(22) is approximated by using the Runge-Kutta method as follows

$$x[k+1] = x[k] + \frac{\Delta}{6} (k_1 + 2k_2 + 2k_3 + k_4), \quad (24)$$

where

$$\begin{aligned} k_1 &= f(x[k], t_k), \\ k_2 &= f\left(x[k] + \frac{\Delta}{2}k_1, t_k + \frac{\Delta}{2}\right), \\ k_3 &= f\left(x[k] + \frac{\Delta}{2}k_2, t_k + \frac{\Delta}{2}\right), \\ k_4 &= f(x[k] + \Delta k_3, t_k + \Delta). \end{aligned}$$

Rearranging (23) into the approximated formula, k_1 is obtained as follows

$$k_1 = A(x[k])x[k] + B(x[k])u.$$

For simplicity let $A(x[k]) = A_1$, $B(x[k]) = B_1$

$$k_1 = A_1 x[k] + B_1 u. \quad (25)$$

In a similar way k_2 is obtained as follows

$$\begin{aligned} k_2 &= A\left(x[k] + \frac{\Delta}{2}k_1\right)\left(x[k] + \frac{\Delta}{2}k_1\right) \\ &\quad + B\left(x[k] + \frac{\Delta}{2}k_1\right)u. \end{aligned}$$

For simplicity let $A\left(x[k] + \frac{\Delta}{2}k_1\right) = \bar{A}_2$, $B\left(x[k] + \frac{\Delta}{2}k_1\right) = \bar{B}_2$

$$k_2 = \bar{A}_2\left(x[k] + \frac{\Delta}{2}k_1\right) + \bar{B}_2 u. \quad (26)$$

Rearranging (25) into (26), k_2 is obtained as follows

$$\begin{aligned} k_2 &= \bar{A}_2\left(x[k] + \frac{\Delta}{2}(A_1 x[k] + B_1 u)\right) + \bar{B}_2 u \\ &= \bar{A}_2\left(I + \frac{\Delta}{2}A_1\right)x[k] + \left(\frac{\Delta}{2}\bar{A}_2 B_1 + \bar{B}_2\right)u. \end{aligned} \quad (27)$$

For simplicity let $\bar{A}_2\left(I + \frac{\Delta}{2}A_1\right) = A_2$, $\frac{\Delta}{2}\bar{A}_2 B_1 + \bar{B}_2 = B_2$

$$k_2 = A_2 x[k] + B_2 u. \quad (28)$$

In a similar way k_3 , k_4 are obtained as follows

$$k_3 = A_3 x[k] + B_3 u, \quad (29)$$

$$A_3 = \bar{A}_3\left(I + \frac{\Delta}{2}A_2\right),$$

$$\bar{A}_3 = A\left(x[k] + \frac{\Delta}{2}k_2\right),$$

$$k_4 = A_4 x[k] + B_4 u, \quad (30)$$

$$A_4 = \bar{A}_4\left(I + \Delta A_3\right),$$

$$\bar{A}_4 = A\left(x[k] + \Delta k_3\right).$$

Thus a discretized formula of (23) can be described as follows

$$x[k+1] = \Phi(x[k], \Delta)x[k] + \Gamma(x[k], \Delta)u, \quad (31)$$

$$\Phi(x[k], \Delta) = I + \frac{\Delta}{6}(A_1 + 2A_2 + 2A_3 + A_4),$$

$$\Gamma(x[k], \Delta) = \frac{\Delta}{6}(B_1 + 2B_2 + 2B_3 + B_4).$$

C. Impulsive Luenberger Observer

In the system if the angular velocity $\dot{\theta}_p$ can not measure then $\dot{\theta}_p$ need to be estimated. Consequently $\dot{\theta}_p$ is estimated by using ILO which consider the quantization of the pendulum angle. ILO is defined as follows

$$\begin{cases} \dot{\hat{x}}(t) = f(\hat{x}(t), t, u) & \forall t \notin \{t_k\}_{k=0}^{\infty} \\ \hat{x}(t) \leftarrow \hat{x}(t) + L_k C(\bar{x}(t) - \hat{x}(t)) & \forall t \in \{t_k\}_{k=0}^{\infty} \end{cases} \quad (32)$$

where \hat{x} is estimated state, and $\bar{x} = [x_c, q(\theta_p), \dot{x}_c, \dot{\theta}_p]^T$. Assume that the pendulum angle $q(\theta_p)$ and the cart position x_c can be measured, coefficient matrix of the output equation is as follows

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

The observer gain L_k can be obtained so that all eigenvalues of $\Phi(x[k], \Delta) - L_k C \Phi(x[k], \Delta)$ are inside the unit disc.

V. NUMERICAL SIMULATION

In the following simulation the control input and the estimated states can only be updated at the quantizer transition time. The quantization width $q_n = 5\pi/180[\text{deg}]$. Setting the constant reference $y_r = (2\pi)^2$ in the system (17), the angular velocity of the system is controlled to the constant velocity $\dot{\theta}_p = 2\pi[\text{rad/sec}]$.

At first simulation results with the measurement velocity $\dot{\theta}_p$ by using the servo control law are shown.

Next simulation results with only the cart position x_c and the pendulum angle θ_p are shown. In this results the other states is estimated by using ILO (32) with the discrete system (31).

A. Angular velocity control with the measurement $\dot{\theta}_p$

In this simulation integral calculation function is `rkf45()` with *MaTX Windows9x/ME/NT/2000/XP(Visual C++ 2005) version 5.3.37*. Step size for `rkf45()` is 10^{-5} . Initial condition of the pendulum angle is $25\pi/180$ [rad], the pendulum velocity is $\dot{\theta}_p = 0.75$ [rad/sec], otherwise 0. Weight matrix are set to $Q = [1]$ and $R = [60]$ for $x[k]$ and $u[k]$.

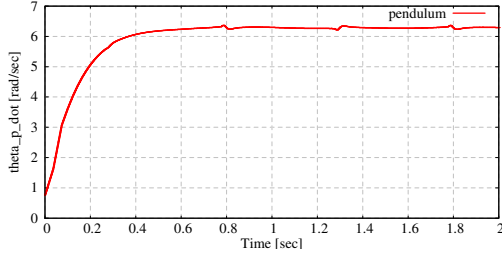


Fig. 4. The pendulum angular velocity of the cart pendulum.

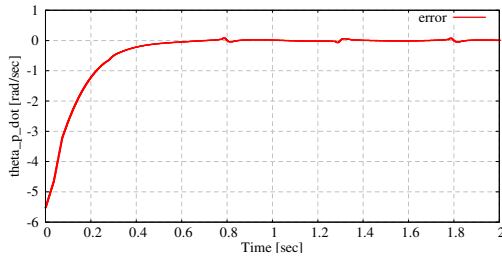


Fig. 5. The pendulum angular velocity error from constant reference 2π [rad/sec].

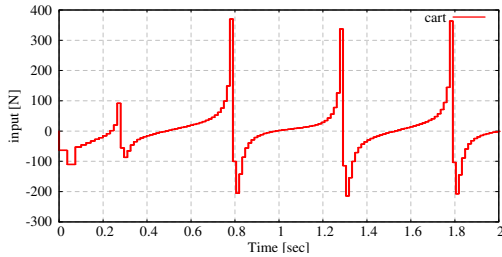


Fig. 6. The input to the cart.

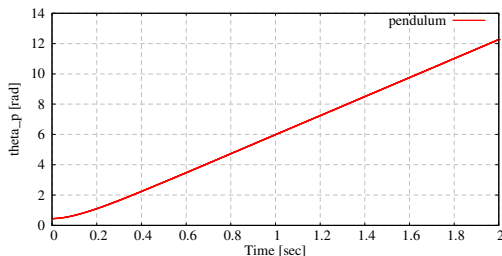


Fig. 7. The pendulum angle of the car pendulum.

From Fig. 4 and Fig. 5, the pendulum velocity $\dot{\theta}_p$ achieve the constant reference 2π [rad/sec]. From Fig. 7, the pendulum angle monotonic increase by the pendulum velocity which achieve the constant reference. From Fig. 6, the input force to the cart updates at the quantizer transition time. From Fig. 8 and Fig. 9, the cart position and speed change with

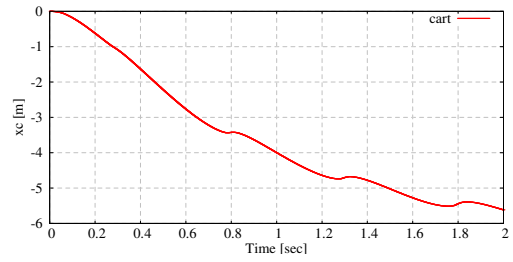


Fig. 8. The cart position of the car pendulum.

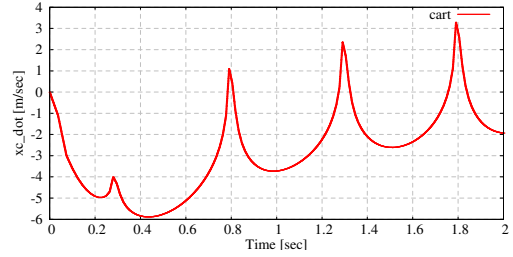


Fig. 9. The cart velocity of the car pendulum.

the input force which designed the above.

B. Angular velocity control with ILO

In this simulation integral calculation function is `rkf45()` with *MaTX Windows9x/ME/NT/2000/XP(Visual C++ 2005) version 5.3.37*. Step size for `rkf45()` is 10^{-5} . Initial condition of the pendulum angle is $25\pi/180$ [rad], the pendulum velocity is $\dot{\theta}_p = 0.75$ [rad/sec], otherwise 0. Weight matrix are set to $Q = [1]$ and $R = [60]$ for $x[k]$ and $u[k]$. The observer gain L_k is designed discrete-time linear quadratic regulator with controllable pair $(\Phi(x[k], \Delta)^T, (C\Phi(x[k], \Delta))^T)$. Weight matrix are set to $Q_o = \text{diag}(1, 100, 1, 10000)$ and $R_o = \text{diag}(1, 1)$ for $x(t_k)$ and the correction term.

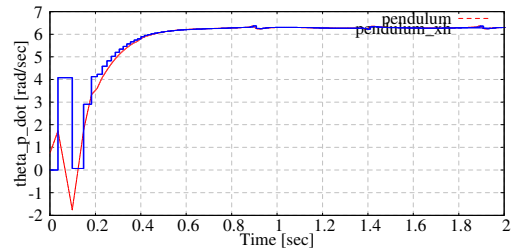


Fig. 10. The pendulum angular velocity of the cart pendulum with ILO. Red solid line is the pendulum angular velocity $\dot{\theta}_p$. Orange solid line is the estimated pendulum angular velocity.

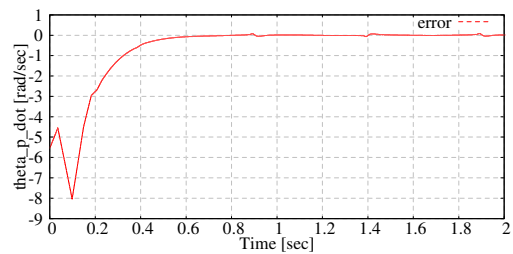


Fig. 11. The pendulum angular velocity error from constant reference 2π [rad/sec] with ILO.

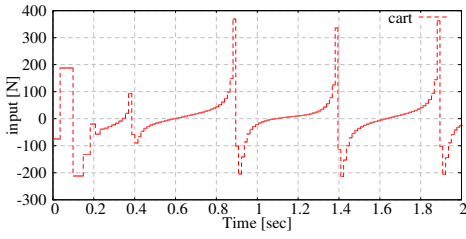


Fig. 12. The input to the cart with ILO.

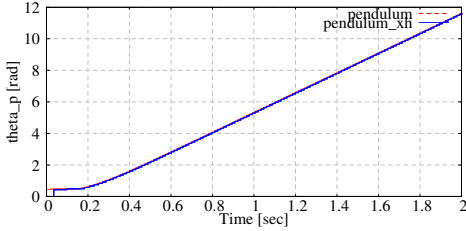


Fig. 13. The pendulum angle of the car pendulum with ILO.

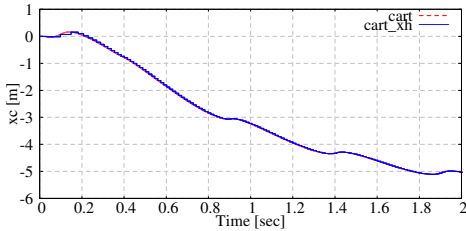


Fig. 14. The cart position of the car pendulum with ILO.

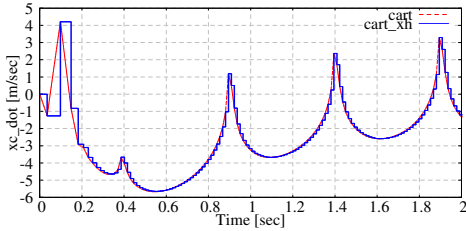


Fig. 15. The cart velocity of the car pendulum with ILO.

From Fig. 10 and Fig. 11, the pendulum velocity $\dot{\theta}_p$ achieve the constant reference 2π [rad/sec]. From Fig. 10 to Fig. 15, the input force, estimated states are updated at the quantizer transition time. The estimated states achieve the real states.

VI. DISCUSSION

In the control system design, we assume that measurement states are changed when the quantizer output changed. The other states are estimated by using the measured states at the same time. Inputs to the system are determined by the measured states and the estimated states.

The above cart-pendulum model don't consider viscous friction at the joint of the pendulum. In order to consider effect of the viscous friction to discretization by Lebesgue sampling, we derive cart-pendulum model that includes the viscous friction force $-C_p\dot{\theta}_p$ Nm at the joint of the pendulum. The equation of motion of the pedulum with the viscous friction force is given by

$$m_p r_p \cos \theta_p \ddot{x}_c + (J_p + m_p r_p^2) \ddot{\theta}_p = m_p g r_p \sin \theta_p - C_p \dot{\theta}_p. \quad (33)$$

Now we focus on right term of (33), $C_p \dot{\theta}_p$. In order to discretize by Lebesgue sampling, both term of (33) are integrated same way as (7). However the integrated right term of (33), $\int_{\theta_p[k]}^{\theta_p[k+1]} C_p \dot{\theta}_p d\theta_p$, can NOT be solved analytically because $\dot{\theta}_p$ is NOT constant. Therefore the discretization of system by Lebesgue sampling can be performed for the system can be solved analytically same way as (7).

VII. CONCLUSION

In this paper, first of all, a discretization with Lebesgue sampling has been considered for a type of nonlinear system such as a cart-pendulum system. For example the cart-pendulum system was converted into the corresponding discrete system (12), which was a time-invariant linear or time-varying linear system, by the proposed method. Hence many current control design schemes can be applied to the discrete system. On the other hand, the implementation of the controller was a crucial problem, and to overcome this problem, in this paper, an impulsive Luenberger observer was introduced. This observer require basically the mapping of the current state to the one-step ahead state of the nonlinear system, and then a numerical integration method based on Runge-Kutta was also derived to give such mapping. Hence the controller implemented was given by the combination of the impulsive Luenberger observer and the numerical integration method. With the controller, the output of the closed-loop system was controlled to be a desired value even though the controller worked only when the quantization occurred. The numerical simulation showed the effectiveness of the proposed system. As future works, we're now interested in the class of nonlinear systems to which the proposed system can be applied. A combustion engine piston-crank model might be an example.

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