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Optimal Vehicle Stability Control Design based on Preview Game Theory Concept

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Abstract—**In this paper, vehicle stability is represented by a cooperative dynamic game such that its two agents (players), namely, the driver and the direct yaw controller (DYC), are working together to provide more stability to the vehicle system. While the driver provides the steering wheel control, the DYC control algorithm is obtained by the well-known Nash game theory to ensure optimal performance as well as robustness to disturbances. The common bicycle model is put into discrete form to develop the game equations of motion. To evaluate the control algorithm developed, a nonlinear vehicle model along with the combined-slip Pacejka tire model is used. The control algorithm is evaluated for a lane change maneuver, and the optimal set of steering angle and corrective yaw moment is calculated and fed to the test vehicle. The simulation results show that the optimal preview control algorithm can significantly reduce lateral velocity and yaw rate which all contribute to enhancing vehicle stability.**

I. Introduction

Vehicle stability control systems, VSC, help drivers maintain vehicle stability and avoid spinning out during emergency braking and steering maneuvers. These systems have been developed and recently commercialized by several companies. A comprehensive literature review conducted by Ferguson [1] reveals that VSC can effectively reduce single-vehicle crashes in cars and SUVs by 30-50%. Also, fatal rollover crashes are estimated to be about 70-90% lower with VSC regardless of vehicle type.

Among all vehicle stability enhancement strategies, direct yaw control (DYC) is one of the most effective methods of active chassis control which can considerably enhance the vehicle stability and controllability [2]. For vehicle control, the yaw moment control is a way to control the lateral motion of a vehicle during severe maneuvers using active steering, e.g., front and/or four wheel steering [3], or active differential braking using ABS [4]. However, it is reported that introducing the driver as part of the control algorithm will improve upon the performance of the vehicle stability control system. This can be accomplished through forming a common differential game.

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For decades, driver modeling has been an interesting issue for both traffic and vehicle control research [5,6,7]. Driver models are usually based on preview of the road ahead where the driver is represented as an optimal preview controller, constructing a path error functional by previewing the road over a known preview distance, and minimizing a weighted integral of squares of differences between the previewed path points and the corresponding estimated lateral position of the vehicle over the preview distance. Sharp et al. [6] introduced a new representation of optimal linear steering control where the standard lateral/yaw linear vehicle model was transformed into discrete-time formation that constructed a quadratic cost function consisting of terms describing path and attitude errors with respect to the road path. Based on this cost function, steering wheel angle control was minimized by linear quadratic regulator (LQR) control.

In 2001, Sharp et al. [6] proposed a simple linear vehicle model with absolute lateral position, lateral velocity, yaw angle, and yaw rate as the non-preview states in discrete form and the road sample inputs as the preview states.

$$x = \left(y, v_y, \psi, \dot{\psi}, y_{r0}, y_{r1}, y_{r2}, ..., y_{rN}\right)^T \tag{1}$$

with y_{r0} is the road reference position at one step before the current time, and y_{r1} is the current reference position, and $y_{r2}, ..., y_{rN}$ are road reference positions at $(N-1)$ steps ahead.

The main purpose of the controller is stable path following, the quality of which can be specified by the sum of the squares of the differences between the y-coordinate of the car's reference point and the corresponding value y_r for the lateral position of the road with respect to the fixed ground frame, O.

Similarly, the human driver is assumed to have the tendency to minimize the corresponding sum of squares of attitude angle differences and minimize the higher-order dynamics, typical terms that can be found analytically. These priorities are reflected into a quadratic cost function [9] by setting:

$$Q = N^T q N, \tag{2}$$

where the matrix N is defined as,

$$
N=\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & \cdots & 0\\
0 & 1 & 0 & 0 & 0 & 1/T_s & -1/T_s & 0 & \cdots & 0\\
0 & 0 & 1 & 0 & 0 & 1/v_xT_s & -1/v_xT_s & 0 & \cdots & 0\\
0 & 0 & 0 & 1 & 1/v_xT_s^2 & 0 & -1/v_xT_s^2 & 0 & \cdots & 0
\end{bmatrix} \tag{3}
$$

The relative importance attached to path errors, attitude errors and steer angle are set by choosing the diagonal values of q appropriately [6].

Tamaddoni et al. [10,11] used Nash strategy to develop an optimal control strategy which takes into account the driver of the vehicle as an inherent part of the controller. In the presented method, the driver, commanding the steering wheel angle, and the vehicle stability controller, applying the corrective yaw moment, are defined as dynamic players in a 2-player differential linear quadratic game. They found out that the cooperative game theory method brings more optimal performance by setting the driver's steering angle and the controller's corrective yaw moment compared to the independently optimized set of steering angle and corrective yaw moment obtained through linear quadratic regulation approach.

Following Tamaddoni et al. [10,11], the game theory framework is adapted and modified in this paper to include time-previewed driver model in collaboration with the VSC system. To do so, the equations of motion are put into discrete form and a preview system is defined. Using the Riccati equation for discrete difference game, the optimal set of the steering angle and the corrective yaw moment can be obtained.

II. VEHICLE MODELS

A detailed standard nonlinear vehicle model is used in numerical simulations to analyze the response of the controlled vehicle. The model includes nonlinear tire models according to combined sideslip theory [12], nonlinear spring model, nonlinear front steering system, and incorporates the major kinematics and compliance effects in the suspension and steering systems including differential load transfer for each wheel. However, to design the controller, a widely used simplified linear single track vehicle model is considered which captures the essential vehicle steering dynamics. In this respect, the tire forces are assumed to be linear functions of tire slip angle.

A. Evaluation Model

In order to study the handling and roll dynamic responses of the vehicle, a nonlinear model of a vehicle is derived which includes longitudinal and lateral translational motions, and roll and yaw motions with rotational dynamics of each of the four wheels [12]. It must be mentioned that the roll dynamics and the suspension compliance properties play an important role in providing a more realistic simulation environment for evaluating the control algorithm that will be developed in this paper.

Based on the vehicle coordinate system, parameters, and external forces depicted in Fig. 1, the nonlinear vehicle model is derived by writing the translational and rotational equations in the vehicle fixed coordinate frame

$$
\begin{aligned}
m\left(\dot{v}_x - v_y\dot{\psi}\right) - m_\phi h\left(\phi\ddot{\psi} + 2\dot{\psi}\dot{\phi}\right) &= \left(F_{xFR}+F_{xFL}\right)\cos\delta_F \\
&\quad -\left(F_{yFR}+F_{yFL}\right)\sin\delta_F + \left(F_{xBR}+F_{xBL}\right),
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
I_z\ddot{\psi} + (I_{z,\phi}\theta_R - I_{xz,\phi})\ddot{\phi} - m_\phi\left(h\dot{v}_x\phi - v_y\phi\dot{\psi}\right) &= \\
l_F\left(F_{xFR}+F_{xFL}\right)\sin\delta_F &+ l_F\left(F_{yFR}+F_{yFL}\right)\cos\delta_F \\
-l_B\left(F_{yBR}+F_{yBL}\right)& \\
+s_L\left(F_{xBL}+F_{xFL}\cos\delta_F - F_{yFL}\sin\delta_F\right)& \\
-s_R\left(F_{xBR}+F_{xFR}\cos\delta_F - F_{yFR}\sin\delta_F\right)&,
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
m\left(\dot{v}_y + v_x\dot{\psi}\right) + m_\phi h\left(\ddot{\phi} - \phi\dot{\psi}^2\right) &= \left(F_{xFR}+F_{xFL}\right)\sin\delta_F \\
&\quad +\left(F_{yFR}+F_{yFL}\right)\cos\delta_F + \left(F_{yBR}+F_{yBL}\right),
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
&\left(I_{x,\phi}+m_\phi h^2\right)\ddot{\phi} + m_\phi h\left(\dot{v}_y + v_x\dot{\psi}\right) + C_\phi\dot{\phi} + \left(I_z\theta - I_{xz}\right)\ddot{\psi} \\
&\quad -\left(m_\phi h^2 + I_{y,\phi} - I_{z,\phi}\right)\phi\dot{\psi}^2 + \left(K_\phi - m_\phi g h\right)\phi = 0.
\end{aligned} \tag{7}
$$

Steering system is modeled as a second order system.

$$
\ddot{\delta}_F + b_{st}\dot{\delta}_F + k_{st}\left(\delta_F - \frac{\delta_{SW}}{r_{st}}\right) = 0 \tag{8}
$$

The equation of motion that governs wheel dynamics is given by

$$
I_w\dot{\omega} = -F_xR_w + T_{Driving} - T_{Braking} \tag{9}
$$

Tire velocity for each wheel are approximated as

$$
\begin{aligned}
v_{wFR} &= \begin{pmatrix} v_x - s_R\dot{\psi} \\ v_y + l_F\dot{\psi} \end{pmatrix}, \quad v_{wFL} = \begin{pmatrix} v_x + s_L\dot{\psi} \\ v_y + l_F\dot{\psi} \end{pmatrix} \\
v_{wBR} &= \begin{pmatrix} v_x - s_R\dot{\psi} \\ v_y - l_B\dot{\psi} \end{pmatrix}, \quad v_{wBL} = \begin{pmatrix} v_x + s_L\dot{\psi} \\ v_y - l_B\dot{\psi} \end{pmatrix}
\end{aligned} \tag{10}
$$

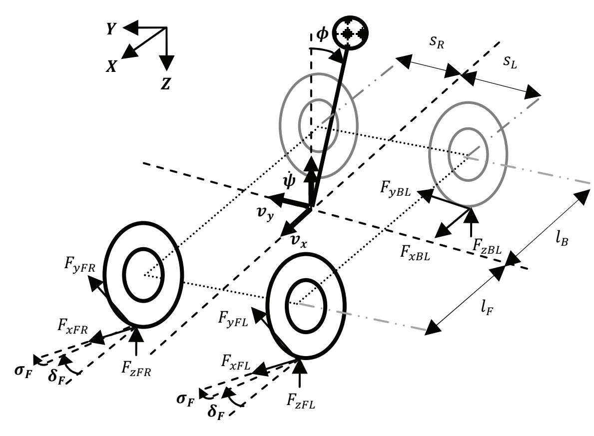


Fig. 1. Vehicle evaluation model

Tire slip angle is defined as the angular difference between the treads in the contact patch and the direction the wheel is turned.

$$\sigma_{Fi}\dot{\alpha}_{Fi} + v_{wFi}^{x}\cdot\left(\alpha_{Fi} - \delta_F + \tan^{-1}\left(\frac{v_{wFi}^{y}}{v_{wFi}^{x}}\right)\right) = 0,$$
$$\sigma_{Bi}\dot{\alpha}_{Bi} + v_{wBi}^{x}\cdot\left(\alpha_{Bi} + \tan^{-1}\left(\frac{v_{wBi}^{y}}{v_{wBi}^{x}}\right)\right) = 0, \qquad i\in\{R,L\} \tag{11}$$

Longitudinal slip of the tire is defined as the difference between the tire tangential speed and the speed of the axle relative to the road. For each wheel represented by index $i\in\{FR,FL,BR,BL\}$, tire slip is calculated as

$$\kappa_i = \begin{cases}\left(\dfrac{R_w\omega_i - v_{wi}^{x}}{\max\{R_w\omega_i, v_{wi}^{x}\}}\right).\text{sign}(|v_{wi}^{x}| - |R_w\omega_i|), & v_{wi}^{x} \neq R_w\omega_i \\ 0 & , v_{wi}^{x} = R_w\omega_i\end{cases} \tag{12}$$

Tire forces and moments are calculated based on a widely used semi-empirical tire model based on trigonometric functions known as Magic Formula [9]. Magic-Formula tire models are considered the state-of-the-art for modeling tire-road interaction in vehicle dynamics applications.

B. Control Model

The commonly used single track bicycle model is considered in this paper [12]. This model captures the needed dynamic information for yaw as well as lateral degrees of freedom. In order to derive the equations, it is assumed that vehicle motion is represented by its global lateral position and velocity, and the yaw angle and yaw rate at the vehicle center of mass as shown in Fig. 2. The state variable vector becomes

$$x = \begin{pmatrix} y & \dot{y} & \psi & \dot{\psi}\end{pmatrix}^T \tag{13}$$

where, y is the global lateral position of the vehicle CG, $\dot{y}$ is the global lateral velocity in Y direction with respect to a fixed ground coordinate, and $\psi, \dot{\psi}$ are yaw angle and yaw rate, respectively.

For the sake of simplicity, the mathematical model is linearized around the operating conditions

$$x^* = 0_{4\times1},\ \delta_{SW}^* = 0,\ M_{zc}^* = 0$$

Thus, the equation of motion for a constant forward speed is given by:

$$\dot{x}_c(t) = A_c x_c(t) + B_{1c}\delta_{SW}(t) + B_{2c}M_{zc}(t) \tag{14}$$

with

$$A_c = \begin{bmatrix} 0 & 1 & v_x & 0 \\ 0 & -\dfrac{C_{\alpha F}+C_{\alpha B}}{mv_x} & 0 & -v_x - \dfrac{l_F C_{\alpha F} - l_B C_{\alpha B}}{mv_x} \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{l_F C_{\alpha F} - l_B C_{\alpha B}}{I_z v_x} & 0 & -\dfrac{l_F^2 C_{\alpha F} + l_B^2 C_{\alpha B}}{I_z v_x}\end{bmatrix}$$

$$B_{1c} = \begin{bmatrix} 0 & \dfrac{C_{\alpha F}}{r_{st} m} & 0 & \dfrac{l_F C_{\alpha F}}{r_{st} I_z}\end{bmatrix}^T, \quad B_{2c} = \begin{bmatrix} 0 & 0 & 0 & \dfrac{1}{I_z}\end{bmatrix}^T$$

where u_1 is the steering wheel angle (δ_{SW}), u_2 is the compensated yaw moment (M_{zc}), and $C_{\alpha F}, C_{\alpha B}$ denote the front and rear tire cornering stiffness, respectively, relating linear tire forces to their corresponding linear sideslip angles.

The relevant linear vehicle model is translated to the discrete-time difference equation as,

$$x_d\{k+1\} = A_d x_d\{k\} + B_{1d}u_1\{k\} + B_{2d}u_2\{k\} \tag{15}$$

where A_d, B_{1d}, B_{2d} are obtained by discretizing the corresponding continuous-time matrices of A_c, B_{1c}, B_{2c}.

C. Preview Model

The idea of a multi-point preview model of path-following steering control used by Sharp et al. originates from linear discrete-time preview control of active suspension [6]. The inputs to the model are effectively the previewed path error and the lateral and yaw velocities of the vehicle, and the output of the model is the steering angle. Applying optimization theory he showed that the feedback gains depended on the weights applied to path and heading errors and steering control action, and that the preview gains reflect the vehicle dynamics.

The relevant linear vehicle model is translated to the discrete-time form of Eq. (15), and the lateral profile of the road is considered in discrete sample value form, with sample values from past observations of the road ahead being stored as states of the full vehicle/road system. As the system moves forward in time, a new road sample value is read in and the oldest stored value is discarded, corresponding to the vehicle having passed the point on the road to which this oldest value refers. All the other road sample values are shifted through the time step, nearer to the vehicle. The dynamics of this shift register process are represented mathematically by

$$y_r\{k+1\} = A_r y_r\{k\} + B_r y_{ri}\{k\} \tag{16}$$

where A_r and B_r are of the form

$$A_r = \begin{bmatrix} 0 & 1 & 0 & \dots & \dots & 0 \\ 0 & 0 & 1 & \dots & \dots & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 0\end{bmatrix}, \quad B_r = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dots \\ 1\end{bmatrix}$$

Combining vehicle and road equations together, we obtain the full dynamic system description

$$\begin{bmatrix} x_d\{k+1\} \\ y_r\{k+1\}\end{bmatrix} = \begin{bmatrix} A_d & 0 \\ 0 & A_r\end{bmatrix}\begin{bmatrix} x_d\{k\} \\ y_r\{k\}\end{bmatrix} + \begin{bmatrix} B_{1d} \\ 0\end{bmatrix}\delta_{SW}\{k\} + \begin{bmatrix} B_{2d} \\ 0\end{bmatrix}M_{zc}\{k\} + \begin{bmatrix} 0 \\ B_r\end{bmatrix}y_{ri}\{k\} \tag{17}$$

The complete problem is now in a standard form,

$$z\{k+1\} = Az\{k\} + B_1u_1\{k\} + B_2u_2\{k\} + Ey_{ri}\{k\} \tag{18}$$

where $u_1 = \delta_{SW}$ is the discrete-time steering wheel angle signal, and $u_2 = M_{zc}$ is the discrete corrective yaw moment around the vehicle center of mass.

If the reference sample y_{ri} is a white-noise sample from a random sequence, the state-preview system of Eq. (17) is controllable if the state system of Eq. (14) is controllable, i.e., if (A_c, B_c) is controllable.

III. Control Algorithm Design

Consider the discrete-time infinite linear quadratic difference game with 2 players as formed in Eq. (17), and each player $i=1,2$ has a control function $u_i=(\delta_{SW}, M_{zc})$ at his disposal which is dynamically coupled by a system of difference rewritten as,

$$z^+ = Az + B_1u_1 + B_2u_2 \tag{19}$$

where the time step, k, is omitted for the sake of simplicity, and the white-noise reference signal y_{ri} is neglected in the difference game.

For every player $i=1,2$, a quadratic cost function is defined as

$$\varphi_i = \frac{1}{2}\sum_{k=0}^{\infty} L_i = \frac{1}{2}\sum_{k=0}^{\infty}\left(z^T Q_i z + \sum_{j=1}^{2} u_j^T R_{ij} u_j\right) \tag{20}$$

where all weighting matrices are constant, Q_i is symmetric, $R_{ii} > 0$.

The Nash equilibrium is defined such that it has the property that there is no incentive for any unilateral deviation by any one of the players. In the other words, at Nash equilibrium with u_i^*, the player who chooses to change his/her strategy cannot improve his/her payoff. Therefore, the Nash equilibrium is constituted by the N-tuple of strategies u_i^* if the following inequalities are satisfied for all admissible strategies:

$$\begin{cases}\varphi_1^* \triangleq \varphi_1\left(u_1, u_2^*\right) \ge \varphi_1\left(u_1^*, u_2^*\right) \\ \varphi_2^* \triangleq \varphi_2\left(u_1^*, u_2\right) \ge \varphi_2\left(u_1^*, u_2^*\right)\end{cases} \tag{21}$$

Restricting the controller to the linear feedback control approach, the optimal solution becomes,

$$u_i^* = -R_{ii}^{-1} B_i^T P_i^+ z^+, \quad i=1,2 \tag{22}$$

where P_i is found from the Riccati equation for discrete linear quadratic games as,

$$\begin{aligned} P_i &= Q_i - G_1^T R_{i1} G_1 - G_2^T R_{i2} G_2 \\ &\quad + \left(A + B_1G_1 + B_2G_2\right)^T P_i^+ \left(A + B_1G_1 + B_2G_2\right) \end{aligned} \tag{23}$$

and G_i satisfies,

$$G_i = -\left(R_{ii} + B_i^T P_i^+ B_i\right)^{-1} B_i^T P_i^+ \left(A + B_{\hat{i}} G_{\hat{i}}\right) \tag{24}$$

with $i=(1,2)$ and $\hat{i}$ is the counter-coalition, i.e. the player counter-acting to the player with index i.

The matrices P_i, Q_i can be also rewritten in the following form to simplify the Riccati equations,

$$Q_i = \begin{bmatrix} Q_{id} & Q_{im} \\ Q_{im}^T & Q_{ir} \end{bmatrix}, \; P_i = \begin{bmatrix} P_{id} & P_{im} \\ P_{im}^T & P_{ir} \end{bmatrix}, \qquad i=1,2 \tag{25}$$

where the sub-matrices are in appropriate sizes.

Hence, the optimal preview linear feedback control (22) becomes,

$$u_i^* = R_{ii}^{-1} B_{id}^T \begin{bmatrix} P_{id}^+ & P_{im}^+ \end{bmatrix} z^+, \qquad i=1,2 \tag{26}$$

The mathematical steps to derive the preview-time linear-quadratic linear-feedback Nash optimal controller for the vehicle system (18) are introduced in [13]. The optimal controller is obtained by first solving Eq. (28), and then feeding the resulting time-independent P_{id} matrices into Equation (29) to find the matrices P_{im}. Substituting the resulting P_{id}, P_{im} into Eq. (26) yields the optimal preview feedback control u_i^* that guarantees Nash equilibrium.

$$\begin{aligned} &P_{id}^+ = Q_{id} \\ &-A_d^T \left(\Delta_1 B_{1d}^T P_{1d} + \Delta_2 B_{2d}^T P_{2d}\right)^T R_{i1} \left(\Delta_1 B_{1d}^T P_{1d} + \Delta_2 B_{2d}^T P_{2d}\right) A_d \\ &-A_d^T \left(\Delta_3 B_{1d}^T P_{1d} + \Delta_4 B_{2d}^T P_{2d}\right)^T R_{i2} \left(\Delta_3 B_{1d}^T P_{1d} + \Delta_4 B_{2d}^T P_{2d}\right) A_d \\ &+A_d^T \left(I + \left(B_{1d}\Delta_1 + B_{2d}\Delta_3\right) B_{1d}^T P_{1d} + \left(B_{1d}\Delta_2 + B_{2d}\Delta_4\right) B_{2d}^T P_{2d}\right)^T \\ &P_{id} \left(I + \left(B_{1d}\Delta_1 + B_{2d}\Delta_3\right) B_{1d}^T P_{1d} + \left(B_{1d}\Delta_2 + B_{2d}\Delta_4\right) B_{2d}^T P_{2d}\right) A_d \end{aligned} \tag{28}$$

and

$$\begin{aligned} &P_{im}^+ = Q_{im} \\ &-A_d^T \left(\Delta_1 B_{1d}^T P_{1d} + \Delta_2 B_{2d}^T P_{2d}\right)^T R_{i1} \left(\Delta_1 B_{1d}^T P_{1m} + \Delta_2 B_{2d}^T P_{2m}\right) A_r \\ &-A_d^T \left(\Delta_3 B_{1d}^T P_{1d} + \Delta_4 B_{2d}^T P_{2d}\right)^T R_{i2} \left(\Delta_3 B_{1d}^T P_{1m} + \Delta_4 B_{2d}^T P_{2m}\right) A_r \\ &-A_d^T \left(I + \left(B_{1d}\Delta_1 + B_{2d}\Delta_3\right) B_{1d}^T P_{1d} + \left(B_{1d}\Delta_2 + B_{2d}\Delta_4\right) B_{2d}^T P_{2d}\right)^T \\ &P_{id} \left(\left(B_{1d}\Delta_1 + B_{2d}\Delta_3\right) B_{1d}^T P_{1m} + \left(B_{1d}\Delta_2 + B_{2d}\Delta_4\right) B_{2d}^T P_{2m}\right) A_r \end{aligned} \tag{29}$$

IV. Simulation and Results

It is known that handling stability is guaranteed provided that the controller can keep the vehicle yaw rate close to the desired value that can be dynamically calculated based on the driver's steering input and vehicle forward speed:

$$\dot{\psi}_{desired} = \frac{v_x}{\left(l_F + l_B\right)\left(1 + k_{us} v_x^2\right)} \delta_F \tag{30}$$

where k_{us}, or so-called understeering coefficient, is a positive constant.

The control objective is to guarantee handling performance in a single lane change maneuver using the following desired states:

$$x_{desired} = \begin{pmatrix} y_{desired} & 0 & 0 & \dot{\psi}_{desired} \end{pmatrix}^T, \tag{31}$$

where $y_{desired} = 4\,m$.

The optimal strategies defined in equations (30) are computed for the CarSim's D-class sedan at a nominal speed $v_x = 20\,\text{m/s}$ and the following values of other involved parameters:

$$\begin{aligned} &m = 1450 \text{ kg}, \\ &I_z = 4192 \text{ kg.m}^2, \\ &r_{st} = \delta_{SW} / \delta_F = 17.8, \\ &l_F = 1.11 \text{ m}, \qquad l_B = 1.67 \text{ m}, \\ &C_{\alpha F} = C_{\alpha B} \approx 80000 \text{ N/rad}. \end{aligned} \tag{32}$$

To discretize the above system, the MATLAB function '*c2d*' (continuous to discrete) is used in this paper. The sampling frequency of $100\,Hz$ that corresponds to the sampling time of $T_s = 0.01$ sec is assumed.

These priorities of the cost function are reflected by arbitrarily setting:

$$Q_1 = N^T Q_{1d} N, \qquad Q_2 = \begin{bmatrix} Q_{2d} & 0 \\ 0 & 0 \end{bmatrix}$$

$$Q_{1d} = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.01 \end{bmatrix}, Q_{2d} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad (33)$$

$$R_{11} = 1, \quad R_{12} = 0,$$

$$R_{21} = 10, \quad R_{22} = 10^{-7},$$

where the matrix N is defined in Eq. (3).

To reserve solution accuracy, the solving frequency in MATLAB was set to $1000\,Hz$, while using “down sample” block in Simulink, the control frequency was reduced to $100\,Hz$.

Following the aforementioned procedure in Section 3.3, the optimal feedback gains are obtained for three different number of preview windows, namely, no preview time $(T_p = 0 \text{ s})$, short preview time $(T_p = 30\,T_s = 0.3 \text{ s})$, and long preview time $(T_p = 100\,T_s = 1 \text{ s})$.

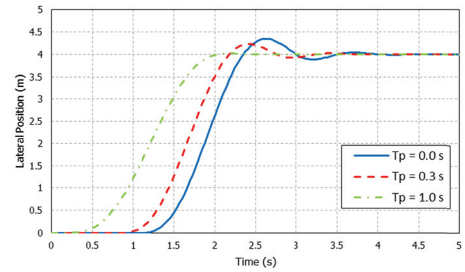
The optimal steering angle and corrective yaw moment are obtained for the three cases: non-previewed, short-previewed, and long-previewed.

Fig. 2 shows the simulation results of the vehicle states, including lateral position and velocity, yaw and roll, angles and rates.

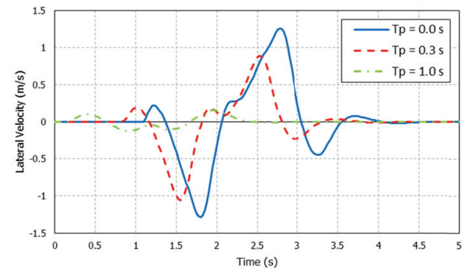
Figure 2(a) shows that all three drivers successfully steers the vehicle through a single lane change maneuver of four meters; however, the vehicle with the driver of larger preview time ability kicks off the lane changing sooner. As the preview time decreases, the vehicle exhibits slower response and more overshoot.

Figure 2(b) show that the vehicle lateral motion is more stable as the preview time increases. In the other words, the preview time gives the vehicle more time to cope with the dynamics of the previewed maneuver.

Figure 2(d) indicates that the vehicle yaw rate is more close to its corresponding desired yaw rate as the preview time increases; thus, it is concluded that the handling performance is best guaranteed for the driver with higher preview time ability.



(a)



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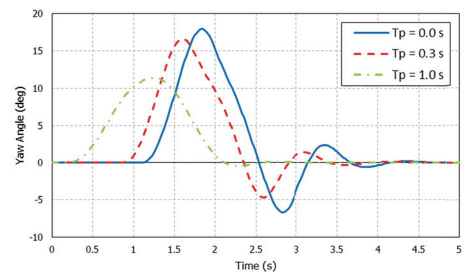
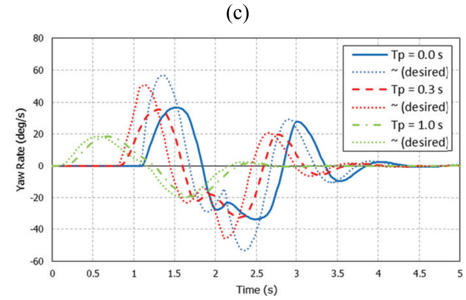


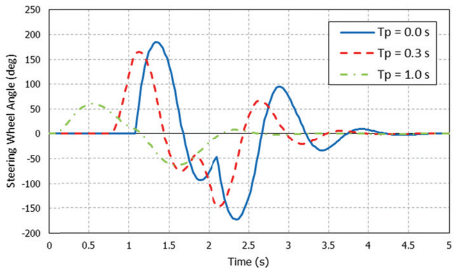
(c)



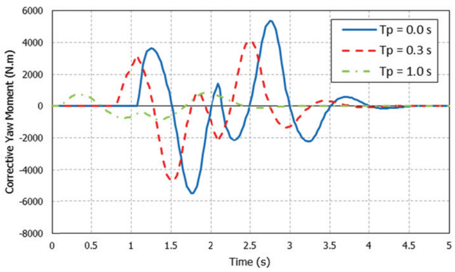
(d)

Fig. 2. Time history of vehicle states in a lane change maneuver of 4m: (a) lateral position, (b) lateral velocity, (c) yaw angle, (d) yaw rate.

Fig. 3 shows the driver’s steering angle and the corrective yaw moment. It can be seen from Fig. 3 that the driver and the controller get involved sooner to follow the direction change as the preview time increases. Hence, as the preview time increases, the steering wheel angle is extended more in time, but the peak value drops. It is also shown that the required corrective yaw moment is similarly extended more in time as the preview time increases, however the peak value drops with preview time.



(a)



(b)

Fig. 3. Time history of control inputs: (a) steering wheel angle, and wheel brake torque: (b) corrective yaw moment

V. Conclusion

A new structure for optimal linear car steering and yaw control has been devised based on the game theory concept. Using the definition of a linear quadratic difference game, the driver's steering input and the controller's corrective yaw moment are defined as two dynamic players of the game "vehicle stability", and their corresponding control efforts are optimized through the Nash optimal strategy. The game theory framework provides an optimal set of the steering wheel angle and the corrective yaw moment that needs to be applied by the driver and the vehicle controller, respectively. Hence, if a player deviates from his/her optimal strategy, his/her payoffs cannot improve.

Results show that in all cases, the game theory approach resulted in different sets of feedback gains to form the driver's steering wheel angle and the vehicle stability control system's corrective yaw moment. The final control system successfully maneuvered the vehicle through the desired lane change. It is concluded that the look-ahead preview information brings more time to the driver to cope with the desired path, and reduces the instability in lateral and yaw motions due the sudden direction change. Compared to the previewed control cases, the vehicle with no preview of ahead road reference experiences larger steering angle peaks, and consequently, more severe yaw dynamics. Similarly, the wheel braking torque is extended more through time and its peak value is lowered as the preview time increases.

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