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Collaborative Target Tracking using Distributed Kalman Filtering on Mobile Sensor Networks

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***Abstract*— In this paper, we introduce a theoretical framework for coupled distributed estimation and motion control of mobile sensor networks for collaborative target tracking. We use a Fisher Information theoretic metric for quality of sensed data. The mobile sensing agents seek to improve the information value of their sensed data while maintaining a safe-distance from other neighboring agents (i.e. perform information-driven flocking). We provide a formal stability analysis of continuous Kalman-Consensus filtering (KCF) algorithm on a mobile sensor network with a flocking-based mobility control model. The discrete-time counterpart of this coupled estimation and control algorithm is successfully applied to tracking of two types of targets with stochastic linear and nonlinear dynamics.**

***Index Terms*— mobile sensor networks, distributed Kalman filtering, target tracking, collaborative localization, information-driven control**

I. Introduction

Collaborative tracking of multiple targets (or events) in an environment arise in a variety of surveillance and security applications and intelligent transportation. Most of the past research on target tracking has been focused on the use of centralized algorithms that run on static multi-sensor platforms [1]. Centralized Kalman filtering plays a crucial role in such target tracking algorithms.

The existing *distributed algorithms* for target tracking using mobile sensor networks are extremely limited to a few instances [7], [10], [2]. In [9] the KCF algorithm of the first author is successfully used for multi-target tracking using a camera network.

In this paper, we present a systematic analysis framework for mobile sensor networks with a flocking-based mobility control model that run a novel distributed Kalman filtering algorithm [8] for collaborative tracking of a single target.

The sensors in our framework have an *information value function* $I_i = f(\rho_i)$ where ρ_i denotes the target range and defined as the distance between the agent and the predicted position of target γ. In addition, $f(\rho)$ is a decreasing function of the target range. According to this model of quality of sensed data, the information value of a sensor increases as the sensor comes closer to the target. This notion of the information value that was also used in [7] is the same as the trace of the *Fisher Information Matrix (FIM)* of sensed data for target tracking applications [3], [4].

We propose a solution to the problem of collision-free tracking of a mobile target via mobile sensor networks using a combination of the flocking and Kalman-Consensus Filtering algorithms [6], [8] of the first author.

The major challenge in analysis of the resulting coupled estimation and control algorithm for mobile sensor networks that we call *information-driven flocking* is that each sensing agent α_i has its own dedicated γ-agent called $\hat{\gamma}_i$ (See [5], for the definition of α- and γ-agent). The state of $\hat{\gamma}_i$ is the estimate of the state of target γ by agent i and the n different estimates $\hat{\gamma}_i$ of the target are distinct. In the flocking algorithms presented in [5], all n γ-agents are the same. This change results in a perturbed structural dynamics of the flock where the perturbation terms depend on the estimation errors.

Our *main result* is to establish that the coupled distributed estimation and control algorithm for a mobile sensor network has a combined cost (Lyapunov function) that is monotonically decreasing in time and guarantees reaching a consensus on estimates of the state of the target by all mobile sensors. We also introduce a cascade nonlinear normal form and stability analysis for structural dynamics of mobile sensor networks performing information-driven flocking.

The outline of the paper is as follows. Some basic notations and problem setup are discussed in Section II. Our main theoretical results on distributed target tracking algorithms for mobile sensor networks are provided in Section III. Our experimental results are presented in Section V. Finally, concluding remarks are made in Section VI.

II. Preliminaries: Notations and Problem Setup

Consider n mobile sensors α_i with the dynamics

$$\begin{cases} \dot{q}_i = p_i \\ \dot{p}_i = u_i \end{cases} \tag{1}$$

where $q_i, p_i, u_i \in \mathbb{R}^d$ and the goal to track the state of a mobile target γ with dynamics

$$\dot{x} = Ax + Bw; \quad x \in \mathbb{R}^m \tag{2}$$

The sensing agents make the following partial-state noisy measurements of the state of γ

$$z_i = H_i x + v_i, \quad i = 1, 2, \dots, n; z_i \in \mathbb{R}^l \tag{3}$$

where the matrices A, B, and H_i are generally time-varying and of appropriate dimensions and w and v_i are zero-mean Gaussian noise.

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Let $G = G(q)$ be the proximity graph (network) of the mobile sensors. The set of vertices of G is $V = \{1, 2, \ldots, n\}$. Let $r > 0$ be the interaction range of every sensor. Then, the set of edges of G is a time-varying set defined as

$$E(q) = \{(i, j) \in E : \|q_j - q_i\| < r\} \tag{4}$$

and the set of neighbors N_i of sensor i on this proximity network is given by

$$N_i = \{j \in V : \|q_j - q_i\| < r\}.$$

The *main problem* of interest is to design distributed motion control and estimation algorithms that achieve two objectives: i) the group of sensing agents improve their *collective information* value $\sum_i I_i$ and ii) avoid collisions during tracking of target γ. We refer to this problem as "information-driven flocking." We propose a solution to this problem using a combination of flocking and Kalman-Consensus Filtering algorithms [8].

III. Distributed Tracking with Mobile Sensors

The Kalman-Consensus filtering algorithm (or Algorithm 1) relies on reaching a consensus on estimates obtained by local Kalman filters rather than distributed averaging-based Kalman filtering. Algorithm 1 is the discrete-time analog of the continuous-time Kalman-Consensus filter described in the following.

Theorem 1. *(Kalman-Consensus Filter [6]) Consider a sensor network with a continuous-time linear sensing model in* (3)*. Suppose each node applies the following distributed estimation algorithm*

$$\begin{aligned} \dot{\hat{x}}_i &= A\hat{x}_i + K_i(z_i - H_i\hat{x}_i) + \mu P_i \sum_{j\in N_i} (\hat{x}_j - \hat{x}_i) \\ K_i &= P_i H_i^T R_i^{-1},\ \mu > 0 \\ \dot{P}_i &= AP_i + P_i A^T + BQB^T - K_i R_i K_i^T \end{aligned} \tag{5}$$

with a Kalman-Consensus estimator and initial conditions $P_i(0) = P_0$ *and* $\hat{x}_i(0) = x(0)$*. Then, the collective dynamics of the estimation errors* $\eta_i = x - \hat{x}_i$ *(without noise) is a stable linear system with a Lyapunov function* $V(\eta) = \sum_{i=1}^n \eta_i^T P_i^{-1} \eta_i$*. Moreover,* $\dot{V} \le -2\mu\Psi_G(\eta) \le 0$ *where*

$$\Psi_G(\hat{\mathbf{x}}) = \hat{\mathbf{x}}^T \hat{L} \hat{\mathbf{x}} = \frac{1}{2} \sum_{(i,j)\in E} \|\hat{x}_j - \hat{x}_i\|^2$$

and $\hat{L} = L \otimes I_m$ *is the* m*-dimensional Laplacian of the network. Furthermore, all estimators asymptotically reach a consensus, i.e.* $\hat{x}_1 = \cdots = \hat{x}_n = x$*.*

The following flocking algorithm is a modified form of Algorithm 2 in [5].

Algorithm 2: (flocking with n distinct γ-agents) Let $\hat{x}_i = \mathrm{col}(\hat{q}_{i,\gamma}, \hat{p}_{i,\gamma})$ be the estimate of the state of target γ by mobile sensor i obtained via Kalman-Consensus filtering. Then, each sensing agent α_i with dynamics in (1) applies the following distributed control to interact with its neighboring sensors on $G(q)$:

$$u_i = \sum_{j\in N_i} \phi_\alpha(\|q_j - q_i\|_\sigma)\mathbf{n}_{ij} + \sum_{j\in N_i} a_{ij}(q)(p_j - p_i) + f_i^\gamma \tag{6}$$

where f_i^γ is a linear feedback for tracking particle $\hat{\gamma}_i$ with state $\hat{x}_i$:

$$f_i^\gamma = -c_1(q_i - \hat{q}_{i,\gamma}) - c_2(p_i - \hat{p}_{i,\gamma}); \quad c_1, c_2 > 0 \tag{7}$$

where $\mathbf{n}_{ij} = (q_j - q_i)/\sqrt{1 + \epsilon\|q_j - q_i\|^2}$ is a subnormal vector connecting agent i to agent j. Please, refer to [5] for the definitions of ϕ_α, the σ-norm $\|\cdot\|_\sigma$, and smooth adjacency elements $a_{ij}(q)$.

Remark 1. According to the flocking framework in [5], there exists a smooth potential function in explicit form

$$U_\lambda(q) = \sum_{j\neq i} \psi_\alpha(\|q_j - q_i\|_\sigma) + \frac{\lambda}{2} \sum \|q_i - q_c\|^2 \tag{8}$$

with $q_c = 1/n \sum_{i=1}^n q_i$ such that u_i can be stated as a distributed gradient-based control:

$$u_i = -\nabla_{q_i} U_\lambda(q) + \sum_{j\in N_i} a_{ij}(q)(p_j - p_i) + f_i^\gamma. \tag{9}$$

Algorithm 1 Kalman-Consensus Filter [8] (message-passing during one cycle at time index k for node i)

Given P_i, $\bar{x}_i$, and messages $m_j = \{w_j, W_j, \bar{x}_j\}, \forall j \in J_i = N_i \cup \{i\}$,

1: Obtain measurement z_i with covariance R_i.

2: Compute information vector and matrix of node i

$$w_i = H_i^T R_i^{-1} z_i$$
$$W_i = H_i^T R_i^{-1} H_i$$

3: Broadcast message $m_i = (u_i, U_i, \bar{x}_i)$ to neighbors.

4: Receive messages from all neighbors.

5: Fuse information matrices and vectors

$$y_i = \sum_{j\in J_i} w_j, S_i = \sum_{j\in J_i} W_j.$$

6: Compute the Kalman-Consensus state estimate

$$\begin{aligned} M_i &= \left(P_i^{-1} + S_i\right)^{-1}, \\ \hat{x}_i &= \bar{x}_i + M_i(y_i - S_i\bar{x}_i) + \mu F_i G_i \sum_{j\in N_i} (\bar{x}_j - \bar{x}_i), \\ \mu &= \epsilon/(1 + \|F_i G_i\|),\ \|X\| = \mathrm{tr}(X^T X)^{\frac{1}{2}} \\ F_i &= I - M_i S_i, \\ G_i &= AM_i A^T + BQB^T + P_i S_i P_i \end{aligned}$$

7: Update the state of the Microfilter (x^+ is the updated x)

$$P_i^+ = AM_i A^T + BQB^T$$
$$\bar{x}_i^+ = A\hat{x}_i$$

∇_{q_i} denotes the partial derivative with respect to q_i.

Note that the state estimates generated by Algorithm 1 is directly used in equation (7) of Algorithm 2 for distributed mobility-control of the sensors. We refer to the combined Algorithms 1 and 2 as the *cascade distributed estimation and control algorithm* for collision-free distributed tracking of a mobile target γ. The analysis of the this discrete-time coupled estimation and control algorithm is tremendously challenging and is one of our future research objectives.

In this paper, we seek to provide the *stability analysis of the continuous-time version of this coupled distributed estimation and control algorithm.*

IV. Stability Analysis: Coupled Distributed Estimation and Control Algorithms

The formulation of our main analytical result as well as the following assumptions are inspired by our experimental observations and consistent collective behavior of a group of mobile sensors tracking two types of mobile targets: 1) a linear target and 2) a maneuverable nonlinear target called particle-in-the-box. Both models of the motion of targets will be discussed in detail in Section V. The notions of flocks, structural stability, and cohesion of flocks are used in the following proposition and defined in [5].

A *flock* is a connected network of dynamic agents. *Flocking* is the collective behavior of a network of dynamic agents with the objective to self-assemble and maintain a connected network in a collision-free manner.

Assumption 1. Assume there exists a finite time $T_1 > 0$ such that the proximity graph $G(q(t))$ becomes connected for all $t \geq T_1$, i.e. after some finite time a single flock of sensors forms.

The following definition clarifies that the Laplacian and algebraic connectivity of the networks used in flocking and KCF algorithms are *not* the same.

Definition 1. (Laplacian and λ_2 of the proximity networks in flocking vs. KCF) Let $a_{ij}(q)$ be the smooth adjacency elements of the proximity network of mobile agents with configuration $q = \text{col}(q_1, \ldots, q_n)$. We represent the adjacency matrix of flocking with $A_f(q) = [a_{ij}(q)]$ and its Laplacian and algebraic connectivity with L_f and $\lambda_2^f = \lambda_2(L_f)$, respectively. The adjacency matrix $A_e = [a_{ij}^e(q)]$ of networked filters in KCF has 0-1 elements, i.e. $a_{ij}^e = 1$ if $a_{ij}(q) > 0$ and $a_{ij}^e = 0$, otherwise. Similarly, we denote the Laplacian and algebraic connectivity of the networked filters with $L_e(q)$ and $\lambda_2^e = \lambda_2(L_e)$, respectively.

Assumption 2. Assume there exist constant thresholds $\epsilon_1, \epsilon_2 \in (0, 1)$ such that the algebraic connectivity functions $\lambda_2^f(t) = \lambda_2(L_f(q(t)))$ and $\lambda_2^e(t) = \lambda_2(L_e(q(t)))$ along the trajectory of mobile agents cross the levels ϵ_1 and ϵ_2, respectively, at time $T_2 = T_2(\epsilon_1, \epsilon_2) > T_1$ and remain above those threshold values thereafter, i.e. $\lambda_2^f(t) \geq \epsilon, \lambda_2^e(t) \geq \epsilon$ for all $t \geq T_2$.

Assumption 3. The parameters $c_1, c_2 > 0$ in the tracking feedback f_i^γ of the flocking algorithm satisfy $c_1 < c_2 < 1$ and $c_2 > 1 - \epsilon_1$ where ϵ_1 is defined in Assumption 2.

Here is our main analytical result:

Proposition 1. *Consider a network of n mobile sensing agents with dynamics* (1)*, the sensing model in* (3)*, and the proximity graph $G(q)$ with the set of edges* (4)*. Suppose that the agents apply the Kalman-Consensus filter in* (5) *to obtain n estimates $\hat{x}_i$ of the state of a mobile target γ with dynamics* (2)*. These state estimates of the target determine the states of n γ-agents $\hat{\gamma}_i$. Suppose that every sensing agent i tracks its associated γ-agent $\hat{\gamma}_i$ by applying the flocking algorithm in* (6)*. Let Σ_e and Σ_c be the collective dynamics of the n networked estimators and mobility-controlled agents, respectively, and denote their cascade with Σ. Then, the following statements hold:*

(i) *Σ can be separated into three subsystems that consist of the structural and translational dynamics of the group of mobile sensors in cascade with the error dynamics of the Kalman-Consensus filter.*
(ii) *Given Assumption 1, the agents form a cohesive flock in finite time.*
(iii) *Suppose that Assumptions 1 through 3 hold. Then, the solutions of the structural dynamics of the flock of mobile sensors are asymptotically stable.*
(iv) *Given the assumptions in part (iii), all estimators asymptotically reach a consensus on the state estimates of the target $\hat{x}_1 = \cdots = \hat{x}_n$ (for the error dynamics of KCF with zero noise).*

The proof of proposition 1 is relatively lengthy; therefore, we present the proof in separate parts.

A. Proof of Part (i):

Let us first determine the error dynamics of the Kalman-Consensus filter in (5). The estimation error of sensor i is defined as $\eta_i = x - \hat{x}_i$, thus error dynamics of (5) (without noise) is in the form:

$$\dot{\eta}_i = F_i \eta_i + \mu P_i \sum_{j \in N_i} (\eta_j - \eta_i)$$

with $F_i = A - K_i H_i$. Defining block diagonal matrices $F = \text{diag}[F_i]$ and $P = \text{diag}[P_i]$ and $\eta = \text{col}\{\eta_i\}$, one can rewrite the last equation as

$$\dot{\eta} = F\eta - \mu P \hat{L}_e \eta = F_e \eta \qquad (10)$$

where $F_e = F - \mu P \hat{L}_e$. According to Theorem 1, the error dynamics $\dot{\eta} = F_e \eta$ is stable and has a quadratic Lyapunov function $V(\eta) = \eta^T P^{-1} \eta = \sum_i \eta_i^T P_i^{-1} \eta_i$.

The flocking dynamics of the agents can be written as

$$\begin{cases} \dot{q}_i = p_i \\ \dot{p}_i = -\nabla_{q_i} U_\lambda(q) + \displaystyle\sum_{j \in N_i} a_{ij}(q)(p_i - p_i) \\ \qquad - c_1(q_i - \hat{q}_{i,\gamma}) - c_2(p_i - \hat{p}_{i,\gamma}) \end{cases} \qquad (11)$$

or

$$\begin{cases} \dot{q}_i = p_i \\ \dot{p}_i = -\nabla_{q_i} U_\lambda(q) + \sum_{j\in N_i} a_{ij}(p_i - p_i) \\ \quad - c_1(q_i - q_\gamma + q_\gamma - \hat{q}_{i,\gamma}) - c_2(p_i - p_\gamma + p_\gamma - \hat{p}_{i,\gamma}) \end{cases}$$

After defining the block matrix $C = [c_1 I_m \; c_2 I_m]$, one can express the last equation in a form with an input η_i:

$$\begin{cases} \dot{q}_i = p_i \\ \dot{p}_i = -\nabla_{q_i} U_\lambda(q) + \sum_{j\in N_i} a_{ij}(q)(p_i - p_i) + f_i^\gamma - C\eta_i \end{cases}$$

with a linear tracking feedback

$$f_i^\gamma = -c_1(q_i - q_\gamma) - c_2(p_i - p_\gamma).$$

This enables us to express the dynamics of Σ as the cascade of its estimation and control subsystems Σ_e and Σ_c:

$$\begin{aligned} \Sigma_c : & \begin{cases} \dot{q}_i = p_i \\ \dot{p}_i = -\nabla U_\lambda(q) - D(q)p + f^\gamma - \hat{C}\eta \end{cases} \\ \Sigma_e : & \; \dot{\eta} = F_e \eta \end{aligned} \tag{12}$$

where $D(q) = c_2 I + \hat{L}_f(q)$ is a positive definite damping matrix, $f = \text{col}\{f_i^\gamma\}$, and $\hat{C} = C \otimes I_n$ is a constant matrix. System (12) is the *cascade normal form of estimation and control subsystems* of a mobile sensor network in which its sensing agents apply the flocking algorithm for mobility control and the Kalman-Consensus filter for distributed tracking.

According to [5], since f_i^γ is a linear feedback, the flocking dynamics Σ_c can be further decomposed as the cascade of structural and translational dynamics of particles. The position and velocity of the center of mass (CM) of the particles is given by

$$q_c = \frac{1}{n}\sum_i q_i, \; p_c = \frac{1}{n}\sum_i p_i.$$

Consider a moving frame centered at q_c. Then, the position and velocity of agent i can be written as $x_i = q_i - q_c$ and $v_i = p_i - p_c$. We refer to the dynamics of the motion of the group of agents in the moving frame coordinates as *structural dynamics*. The structural and translational dynamics of Σ_c can be written as

$$\Sigma_s : \begin{cases} \dot{x} = v \\ \dot{v} = -\nabla U_\lambda(x) - D(x)v + \delta - \bar{\delta} \otimes \mathbf{1}_n \end{cases}$$

with $\mathbf{1}_n \in \mathbb{R}^n$ representing the column vector of ones and

$$\Sigma_t : \begin{cases} \dot{q}_c = p_c \\ \dot{p}_c = -c_1(q_c - q_\gamma) - c_2(p_c - p_\gamma) + \bar{\delta} \end{cases}$$

where the perturbation terms $\delta = \text{col}\{\delta_i\}$ and $\bar{\delta}$ depend on the target estimation errors by the sensors and are defined as

$$\begin{aligned} \delta_i &= -c_1(q_\gamma - \hat{q}_{i,\gamma}) - c_2(p_\gamma - \hat{p}_{i,\gamma}) = -C\eta_i \\ \bar{\delta} &= \frac{1}{n}\sum_i \delta_i = -C\bar{\eta}; \quad \bar{\eta} = \frac{1}{n}\sum_i \eta_i = \frac{1}{n}(\mathbf{1}_n^T \eta) \end{aligned}$$

The normal form of Σ can be written as follows

$$\begin{aligned} \Sigma_s : & \begin{cases} \dot{x} = v \\ \dot{v} = -\nabla U_\lambda(x) - D(x)v - \hat{C}\eta + C(\mathbf{1}_n^T\eta) \otimes \mathbf{1}_n \end{cases} \\ \Sigma_t : & \begin{cases} \dot{q}_c = p_c \\ \dot{p}_c = -c_1(q_c - q_\gamma) - c_2(p_c - p_\gamma) - C(\mathbf{1}_n^T\eta) \end{cases} \\ \Sigma_e : & \; \dot{\eta} = F_e\eta \end{aligned}$$

B. Proof of parts (ii) to (iv)

The solutions of the structural dynamics in cascade with Σ_e is called *cohesive* for all $t \geq 0$ if the position of all agents remains in a ball of radius R_0 for $t \geq 0$. Note that this cascade nonlinear system is globally Lipschitz and all of its solutions are bounded for arbitrary initial conditions. The global Lipschitz property is a byproduct of the design of the smooth potential function $U_\lambda(q)$ which has a globally bounded gradient. This implies that over the interval $[0, T]$ the solutions of the cascade system and therefore the position of all agent remain bounded. For all $t \geq T$, the proximity graph $G(q(t))$ is connected and thus has a finite diameter $d(t) \leq (n-1)$ at any time t. Define the diameter of the flock as

$$d_{max}(t) = \max_{j\neq i} \|q_j(t) - q_i(t)\|, \quad t \geq T$$

Then, $d_{max} = d(t)r \leq (n-1)r$ and by setting $R_0 = (n-1)r/2$ the position of the agents remain cohesive for all $t \geq T$ inside a ball of radius R_0.

To establish stability of the flock, we need to construct an energy-type Lyapunov function φ for the cascade of Σ_s and Σ_e. Let $H_\lambda(x, v) = U_\lambda(x) + \frac{1}{2}\|v\|^2$ be the Hamiltonian of the unperturbed structural dynamics Σ_s and $V(\eta) = \eta^T P^{-1}\eta$ be the Lyapunov function of Σ_e. We propose the following Lyapunov function for the cascade nonlinear system (Σ_s, Σ_e):

$$\varphi(x, v, \eta) = H_\lambda(x, v) + \frac{k}{2\mu}V(\eta) \tag{13}$$

Before computing $\dot{\varphi}$, let us state a simple inequality. For an $n \times m$ matrix M and two vectors $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$, the following inequality holds:

$$|x^T My| \leq \frac{1}{2}(\|x\|^2 + \|My\|^2) \leq \frac{1}{2}(\|x\|^2 + \sigma_{max}^2(M)\|y\|^2)$$

In the special case of $M = C = [c_1 I_m \; c_2 I_m]$, we have

$$|x^T Cy| \leq \frac{1}{2}(\|x\|^2 + c_3^2\|y\|^2).$$

where $c_3 = \max(c_1, c_2)$. By direct differentiation, we obtain

$$\dot{\varphi} = \dot{H}_\lambda + \frac{k}{2\mu}\dot{V}(\eta).$$

From Theorem 1 and Assumptions 1 and 2, for all $t \geq T_2$, one gets

$$\dot{V}(\eta) \leq -2\mu(\eta^T \hat{L}_e \eta) \leq -2\mu\bar{\lambda}_2^e\|\eta\|^2$$

where $\bar{\lambda}_2^e = \min_{t\geq T_2} \lambda_2(L_e(q(t))$ always exists based on Assumption 2.

Now, let us compute $\dot{H}_\lambda(x, v, \eta)$. We have

$$\dot{H}_\lambda = -v^T \hat{L}_f(x)v - c_2\|v\|^2 - \sum_i (v_i^T C\eta_i + v_i^T \bar{\delta}).$$

Note that $|v_i^T C\eta_i| \leq \frac{1}{2}(\|v_i\|^2 + c_3^2\|\eta_i\|^2)$ thus

$$\sum_i |v_i^T C\eta_i| \leq \frac{1}{2}(\|v\|^2 + c_3^2\|\eta\|^2).$$

In addition, $v_i^T \bar{\delta} = \frac{1}{n}\sum_j v_i^T C\eta_j$. Hence

$$|v_i^T \bar{\delta}| \leq \frac{1}{2}\sum_j (\|v_i\|^2 + c_3^2\|\eta_j\|^2) = \frac{n}{2}\|v_i\|^2 + \frac{1}{2}c_3^2\|\eta\|^2$$

and

$$\sum_i |v_i^T \bar{\delta}| \leq \frac{1}{2}(\|v\|^2 + c_3^2\|\eta\|^2).$$

Based on the above upper bounds, we get

$$\dot{H}_\lambda \leq -v^T \hat{L}_f(x)v - c_2\|v\|^2 + \|v\|^2 + c_3^2\|\eta\|^2.$$

Given the fact that

$$v^T \hat{L}_f(x)v \geq \lambda_2(L(x))\|v\|^2$$

and setting $\bar{\lambda}_2^f = \min_{t \geq T_2} \lambda_2(L_f(x(t)))$, one concludes

$$\dot{\varphi} \leq (1 - c_2 - \bar{\lambda}_2^f)\|v\|^2 + (c_3^2 - k\bar{\lambda}_2^e)\|\eta\|^2 < 0, \ \forall (v, \eta) \neq 0$$

if the following two conditions hold:

$$\begin{cases} \bar{\lambda}_2^f > 1 - c_2 \\ \bar{\lambda}_2^e > c_3^2/k \end{cases} \tag{14}$$

Given the definition of $\bar{\lambda}_2^f$ and $\bar{\lambda}_2^e$ and Assumption 2, we have $\bar{\lambda}_2^f = \epsilon_1$ and $\bar{\lambda}_2^e = \epsilon_2$. By choosing $k \geq 1/\epsilon_2$ and $c_2 > 1 - \epsilon_1$ (as in Assumption 3) both conditions will be satisfied. Thus

$$\dot{\varphi}(x, v, \eta) < 0, \ \forall (v, \eta) \neq 0$$

Based on LaSalle's invariance principle, for any set of initial conditions, the solutions of the cascade system (Σ_s, Σ_e) asymptotically converge to the largest invariant set in

$$\mathcal{E} = \{(x, v, \eta) : \nabla U_\lambda(x) = 0, v = 0, \eta = 0\} = \mathcal{E}_s \times \{0\}$$

where $\mathcal{E}_s$ is the equilibria of the unperturbed structural dynamics. From the equilibria in $\mathcal{E}_s$, only the local minima of $U_\lambda(x)$ are asymptotically stable.

The proof of part (iv) is a byproduct of the above stability analysis: the estimation errors η_i asymptotically vanish for all sensors and therefore all state estimates become the same.

Remark 2. If in addition to Assumptions 1 through 3, Conjectures 1 and 2 in [5] hold, then almost every solution of the structural dynamics of the flock asymptotically converges to a *quasi α-lattice*. In all of our experimental results, we have observed finite-time self-assembly of quasi α-lattices.

V. EXPERIMENTAL RESULTS

In this section, we apply our coupled distributed estimation and control algorithm—namely, KCF plus flocking—to two types of targets: 1) a target with a linear model which is a particle moving in R^2 and 2) a maneuvering target with nonlinear dynamics. The later target remains in a rectangular region (box) for all time $t \geq 0$.

A. Linear Target

Consider a particle in $\mathbb{R}^2$ with a linear dynamics

$$x(k+1) = Ax(k) + Bw(k)$$

with

$$A = \begin{bmatrix} I_2 & \epsilon I_2 \\ 0 & I_2 \end{bmatrix}, \ B = \begin{bmatrix} (\epsilon^2/2)I_2 \\ \epsilon I_2 \end{bmatrix}.$$

where $\epsilon = 0.01$ is the discretization step-size. The sensor makes noisy measurements of the position of the target, i.e.

$$z_i(k) = H_i(k)x(k) + v_i(k); \ H_i = [I_2 \ 0].$$

The noise statistics for zero-mean Gaussian signals $w(k)$ and $v_i(k)$ are

$$E[w(k)w(l)^T] = Q_k\delta_{kl}, \ E[v_i(k)v_j(l)^T] = R_i(k)\delta_{kl}\delta_{ij}.$$

where $\delta_{kl} = 1$ if $k = l$ and $\delta_{kl} = 0$, otherwise. According to the model of information value in [7], the measurement error covariance matrix of sensor i is $R_i = \frac{2}{f(\rho_i)}I_2$ where $f(\rho_i)$ is the information value function

$$I_i = f(\rho_i) = 2I_0(a + b + (a - b)\frac{\rho_i - l}{\sqrt{1 + (\rho_i - l)^2}})^{-1} \tag{15}$$

where $\rho_i = \|H_i\bar{x}_i - q_i\|$, $I_0 = 0.1$, and $a > b > 0$. In our experiment, we use a mobile sensor network with $n = 20$ agents. The parameters of R_i are $a = 8b$, $b = 1$, and $l = 10d$. The interaction range of the agents in the flock is $r = 1.2d$ and their desired inter-agent distance is $d = 7$. For the KCF algorithm, $P_0 = 100I_4$, $x_0 \sim \mathcal{N}(0, \sigma^2 I_4)$ with $\sigma = 60$, and $Q = 100I_2$.

Fig. 1 shows the MSE of tracking error over 10 random runs, the average information value, and the algebraic connectivity plots during tracking. From Fig. 1 (c), one can readily verify that Assumptions 1 through 2 hold.

B. Maneuvering Nonlinear Target: Particle-in-the-Box

We also consider a maneuvering target with the following nonlinear dynamics:

$$x(k+1) = A(x(k))x(k) + Bw(k) \tag{16}$$

where $x(k) = (q_1(k), p_1(k), q_2(k), p_2(k))^T$ denotes the state of the target at time k. The target moves inside and outside of a square field $[-l, l]^2$. Matrix $A(x)$ is defined as

$$A(x) = M(x) \otimes F_1 + (I_2 - M(x)) \otimes F_2$$

$$F_1 = \begin{bmatrix} 1 & \epsilon \\ 0 & 1 \end{bmatrix}, \ F_2 = \begin{bmatrix} 1 & \epsilon \\ -\epsilon c_1 & 1 - \epsilon c_2 \end{bmatrix},$$

$$M(x) = \begin{bmatrix} \mu(x_1) & 0 \\ 0 & \mu(x_3) \end{bmatrix}.$$

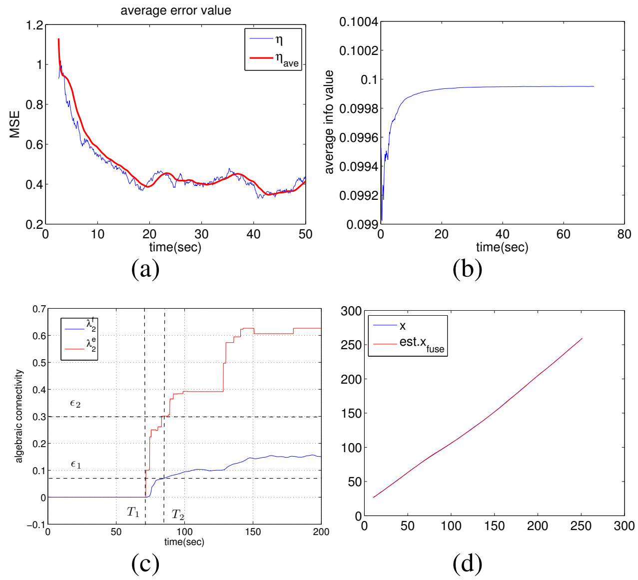

Fig. 1. Experimental results for the linear target: (a) MSE for distributed target tracking, (b) average information value, (c) λ_2 plots for flocking (smooth blue curve) and Kalman-Consensus filtering (piecewise constant red curve), and (d) a target trajectory and fused estimates of 20 sensors.

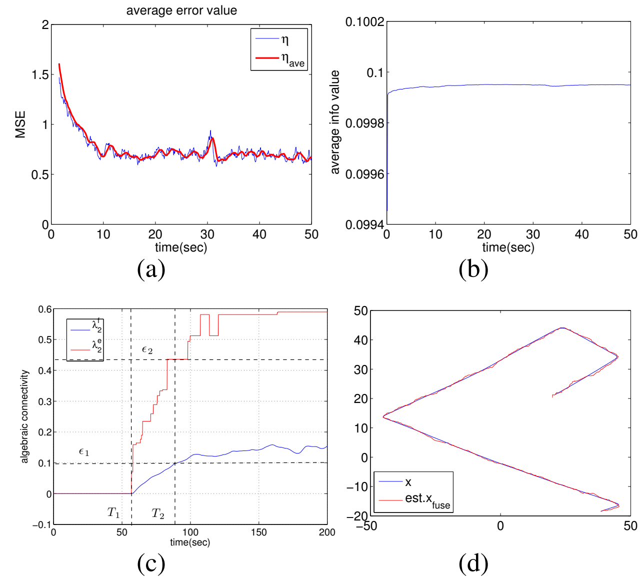

Fig. 2. Experimental results for the nonlinear target: (a) MSE for distributed target tracking, (b) average information value, (c) λ_2 plots for flocking (smooth blue curve) and Kalman-Consensus filtering (piecewise constant red curve), and (d) a target trajectory and fused estimates of 30 sensors.

where F_1 and F_2 determine the dynamics of the target inside and outside of the region, respectively, and $\mu(z)$ is a switching function taking 0-1 values defined by

$$\mu(z) = \frac{\sigma(a+z) + \sigma(a-z)}{2}$$

$$\sigma(z) = \begin{cases} 1, & z \geq 0; \\ -1, & z < 0 \end{cases}$$

In addition, matrix B is given by

$$B = I_2 \otimes G, \; G = \begin{bmatrix} \epsilon^2 \sigma_0 / 2 \\ \epsilon \sigma_0 \end{bmatrix}.$$

where $\epsilon = 0.03$ is the step-size, $\sigma_0 = 2$, $a = 45$, $l = 50$, $c_1 = 7.5$ and $c_2 = 10$ are the parameters of a PD controller, and the elements of $w(k)$ are normal zero-mean Gaussian noise with $Q = 100 I_2$. The initial condition of the target is $x_0 \sim \mathcal{N}(0, \sigma^2 I_4$ with $\sigma = 2$ and $P_0 = 100 I_2$. The parameters of the information value function in (15) are $I + 0 = 0.1$, $a = 10b$, $b = 1$, $l = 10d$ and $d = 7$. We consider a mobile sensor network with $n = 30$ nodes with a linear sensing model and

$$H_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

Fig. 2 illustrates the tracking estimation error, average information value, and the algebraic connectivity plots for the nonlinear target. Similarly, Assumptions 1 and 2 hold based on Fig. 2 (c).

VI. Conclusions

We introduced a theoretical framework for coupled distributed estimation and flocking-based control of mobile sensor networks for collaborative target tracking. The mobile sensing agents seek to improve the information value of their sensed data while avoiding inter-agent collisions. We demonstrated that the coupled dynamics of the combined distributed estimation and control algorithm has a separable cascade nonlinear normal form. Then, we provided the stability analysis of the structural dynamics of a flock with n dedicated γ-agents in cascade with the error dynamics of the continuous-time KCF. Based on out experimental results, the discrete-time counterpart of the information-driven flocking algorithm is effectively applicable to tracking both a linear and a nonlinear maneuverable target.

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