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Control Performance Monitoring of LP-MPC Cascade Systems

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***Abstract*— Traditional minimum variance control (MVC) based performance monitoring methods treats all controlled variables (CVs) the same (or with some preselected weights). However, due to the nature of soft CV constraint, CVs have priority in cascade systems of linear programming - model predictive control (LP-MPC). It is desired to reduce violations of constraints for CVs at their upper or lower bounds and to keep CVs under control. In this paper, we introduce block lower triangular interactor matrix, based on which conditional MVC and corresponding performance benchmark is developed. We state that conditional MVC first consider CVs with multiple level priority and a subset of CVs in each level of priority. A simulation example is given to compare proposed method with traditional MVC methods.**

I. Introduction

Research on control performance monitoring has been developing quickly these years in both academia and industry. Routine operation data are compared with some benchmark to determine the performance quality of the controller. Pioneer work by Harris [3] shows that MVC can be used as a SISO performance benchmark and be estimated from routing operation data. For MIMO systems, however, it is difficult to extract time delay structure known as the interactor without additional model knowledge beyond the input-output time delays, which plays an important role in determining feedback invariant terms [8]. Huang et al. [6] use unitary interactors to assess control performance based on the sum of output variances. Recently, Yu and Qin [13] propose left and right diagonal interactor to take advantage of diagonal delay structure to avoid the need for the first few Markov parameters.

The MPC objective includes not only an output penalty term but also input reference penalty and input move suppression term. Performance benchmark based on MVC may not be achievable by MPC. Huang and Shah [8] compare MPC with linear quadratic regulator (LQR) and propose the LQG benchmark. To fully utilize model and constraint information, some model based approaches, such as the design-case method [10] and the expectation-case method [14], are proposed. This type of methods evaluate MPC performance by comparing achieved MPC objective costs and designed/expected costs. However, these methods ignore the impact of constraints on the control performance. Recently, Harrison and Qin [5] propose a minimum variance map considering effects of constraints. Nevertheless, in the MPC practice, CVs have their priorities according to economics and safety. The ability of closely tracking setpoints does not necessarily mean more economic and safe operation.

Industrial MPC applications usually have a steady-state optimizer, mostly via linear programming (LP), above MPC to optimize MPC setpoints and achieve better economics [11]. Ying and Joseph [12] provide stability conditions for the LP-MPC system. Recent research by Nikandrov and Swartz [9] provides sensitivity analysis on constraints in LP-MPC. They point out that for MIMO systems disturbance on one CV may affect other CVs through interactions. The effects can be even amplified, thus cause large variations on CVs with small disturbances. For CVs at their bounds, such behavior is undesirable because the probability of violations of safety limits increases. Meanwhile, if variance of other less critical CVs decreases, the MVC based performance monitoring methods will still regard controller as improved, failing to identify potential performance degradations of the more critical CVs. Although we can add weights to unitary interactor, the CVs far away from the constraints cannot be excluded completely. Therefore, it calls for an MPC monitoring approach that considers CVs with priority.

In this paper, we propose a method to evaluate the minimum variance based on CV priority. In the MPC formulation, constraints on manipulated variables (MV) are always ensured, but due to disturbances CVs may exceed their bounds which are treated as soft constraints. Hence, CV constraints of both LP and MPC actually back off from their safety limits. The conditional MVC with group priority in the proposed method enables one to reduce violations of safety limits, and the performance benchmark evaluates the potential reduction of violations. Moreover, improved performance constraints of LP-MPC may be moved closer to safety limits, which generally leads to better economics.

II. Revisit of MIMO interactor and minimum variance benchmark

Let the MIMO process take the following form:

$$y_t = G(q)u_t + N(q)a_t \tag{1}$$

where $G(q)$ and $N(q)$ are process and disturbance transfer functions which are proper and rational; y_t, u_t and a_t are process output, input and noise vectors. It is further assumed that a_t is Gaussian with zero mean and covariance of Σ_a. $N(q)$ is then realized in a form such that it is monic. For $n \times n$

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transfer function matrix $G(q)$, there exists a non-unique, non-singular $n \times n$ polynomial matrix D such that $\det(D) = q^r$ and

$$\lim_{q^{-1}\to 0} DG(q) = \lim_{q^{-1}\to 0} \tilde{G}(q) = K \quad (2)$$

where K is a non-zero and finite matrix with full rank, the integer r is defined as the number of infinite zeros of $G(q)$, and $\tilde{G}$ is the delay-free transfer function matrix corresponding to $G(q)$. The interactor matrix, D, can be written as

$$D = D_0 q^d + D_1 q^{d-1} + \cdots + D_v q^{d-v} \quad (3)$$

where d denotes the order of interactor matrix D, v is defined as the relative degree of D [7], and D_i $(i = 0, \ldots, v)$ are coefficient matrices. The interactor matrix D can be categorized into three forms. The simplest form is $D = q^d I$, called simple interactor matrix. A more complicated interactor matrix can be $D = \text{diag}(q^{d_1}, q^{d_2}, \ldots, q^{d_n})$, which is called diagonal interactor matrix. Otherwise, for some special processes, interactor matrix can have off-diagonal elements and is called general interactor matrix. The common forms of general interactor matrix include lower triangular, nilpotent and unitary matrix. The unitary interactor matrix satisfies

$$D^T(q^{-1})D(q) = I. \quad (4)$$

For unitary interactor, by introducing interactor filtered output [8], [4]:

$$\begin{aligned}\tilde{y}_t &= q^{-d} D y_t \\ &= \tilde{F}_0 a_t + \tilde{F}_1 a_{t-1} + \cdots + \tilde{F}_{d-1} a_{t-(d-1)} + \tilde{F}_d a_{t-d} + \cdots,\end{aligned}$$

the sum of the variances of original output can be minimized as

$$\min \mathbb{E}(y_t^T y_t) = \mathbb{E}(\tilde{y}_t^T \tilde{y}_t) = \text{tr}[\text{cov}(\tilde{y}_t)] \quad (5)$$

where the first equality is due to unitary interactor matrix. The MVC benchmark can be evaluated as

$$\eta = \frac{\mathbb{E}(y_t^T y_t)_{\min}}{\mathbb{E}(y_t^T y_t)}. \quad (6)$$

For a lower triangular interactor matrix, the minimum variance is conditional [1]. The minimum variance control law derived from interactor filtered output minimizes:

1) the variance of first output;
2) the variance of second output on the condition that the variance of first output is minimized;
3) the variance of third output on the condition that the variance of first output is minimized first and then the variance of second output is minimized; and so on.

III. Introduction of Block Lower Triangular Interactor Matrix

The lower triangular interactor matrix introduced in the previous section can be extended to block lower triangular interactor matrix.

Definition 1: For any $n \times n$ proper and rational transfer function matrix $G(q)$, $D(q)$ is a block lower triangular interactor matrix if it satisfies

$$G(q) = D(q)^{-1}\tilde{G}(q) \quad (7)$$

$$\lim_{q^{-1}\to 0} \tilde{G}(q) = \lim_{q^{-1}\to 0} D(q)G(q) = K \quad (8)$$

and

$$D(q) = \begin{pmatrix} D_{11}(q) & 0 & \cdots & 0 \\ D_{21}(q) & D_{22}(q) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ D_{m,1}(q) & D_{m,2}(q) & \cdots & D_{m,m}(q) \end{pmatrix} \quad (9)$$

where K denotes a full rank finite and non-zero matrix and $D_{i,i}$ $(i = 1, \ldots, m)$ are unitary interactor matrices satisfying (4).

As an example, CVs are partitioned to two groups according to LP results. We aim at minimizing variance of CVs in Group 1 unconditionally and then minimize the variance of CVs in Group 2 under the condition that CVs in Group 1 have minimum variance. Hence, we focus on the case $m = 2$. The results can be easily extended to other cases with $m > 2$ in a similar fashion.

Based on LP results, $G(q)$ is partitioned as

$$G(q) = \begin{pmatrix} G_1(q) \\ G_2(q) \end{pmatrix} \quad (10)$$

where $G_1(q)$ and $G_2(q)$ are $n_1 \times n$ and $n_2 \times n$ transfer function matrices, respectively. Using algorithms discussed by Huang and Shah [8], one can obtain unitary interactors $D_{11}(q)$ and $D_{22}(q)$ for $G_1(q)$ and $G_2(q)$. According to property of unitary interactor, there exists full rank constant matrices K_1 and K_2 such that

$$K_1 = \lim_{q\to\infty} D_{11}(q)G_1(q)$$
$$K_2 = \lim_{q\to\infty} D_{22}(q)G_1(q)$$

where $K_1 \in \mathbb{R}^{n_1 \times n}$ and $K_2 \in \mathbb{R}^{n_2 \times n}$. If matrix

$$K = \begin{pmatrix} K_1 \\ K_2 \end{pmatrix} \quad (11)$$

is non-singular, then

$$D(q) = \begin{pmatrix} D_{11}(q) & 0 \\ 0 & D_{22}(q) \end{pmatrix} \quad (12)$$

is the block lower triangular interactor matrix for $G(q)$.

Otherwise, there must be non-zero off-diagonal block in D matrix. Since K is non-singular, the left null space U^T of matrix $(K_2^T \quad K_1^T)^T$ can be factorized in reduced row echelon form.

$$U^T \begin{pmatrix} K_2 \\ K1 \end{pmatrix} = 0 \quad (13)$$

where $U \in \mathbb{R}^{n_u \times n}$

1) whose rows have the form $(0 \ \cdots \ 0 \ 1 \ * \ \cdots \ *)$;

2) the number of leading zeros in each row is strictly greater than number of leading zeros in the row above it; and
3) the column containing leading one is the only nonzero component in its column.

Since both K_1 and K_2 are full row rank, the leading zeros in last row of U is less than n_2. This means that given U some rows of K_2 can be represented by linear combination of K_1 and K_2. Let the leading one in the i-th row of U be in column n_i and $U = (u_{i,j})$.

$$K_2(n_i,:) = \sum_{j=n_i+1}^{n_2} -u_{i,j}\, K_2(j,:) + \sum_{j=n_2+1}^{n_1+n_2} -u_{i,j}\, K_1(j,:), \quad i = 1,\ldots,n_u \qquad (14)$$

where $K_1(j,:)$ and $K_2(j,:)$ denote j-th row of K_1 and K_2, respectively, by borrowing notations in MATLAB. Hence, delays in $G_2(q)$ can be increased in a similar fashion as the method constructing lower triangular interactor matrix [2]. Let D_2^0 be $(0 \quad D_{22})^T$ which is the second row of $D(q)$. Set

$$D_2^1(n_i,:) = q^{d^1_{2,n_i}} \left[D_2^0(n_i,:) + \sum_{j=n_i+1}^{n_2} u_{i,j}\, D_2(j,:) + \sum_{j=n_2+1}^{n_1+n_2} u_{i,j}\, D_1(j,:) \right], \quad i = 1,\ldots,n_u \qquad (15)$$

where d^1_{2,n_i} is a unique integer such that $\lim_{q\to\infty} D_2^1(n_i,:)G(q) = K_2^1(n_i,:)$ is finite and nonzero. If $K_2^1(n_i,:)$ linearly independent of rows of K_1 and $K_2(1 : n_i - 1,:)$, then n_i-th row of D_2 is set to be $D_2^1(n_i,:)$. If not, delay of n_i-th row of D_2 can be increased in the same way. The approach will eventually terminate since $\det G(q) \neq 0$ and $D_{11}(q), D_{22}(q)$ have at least delay of 1.

Remark 1: Although the lower triangular interactor is unique [2], the block lower triangular interactor is non-unique because of unitary diagonal blocks. For any unitary interactor matrix $D(q)$ and unitary real matrix Γ, $\bar{D}(q) = \Gamma D(q)$ is also a unitary interactor matrix.

Example 1: Assume

$$G(q) = \begin{pmatrix} \frac{4q^{-1}}{1-0.4q^{-1}} & \frac{q^{-2}}{1-0.1q^{-1}} & \frac{q^{-2}}{1-0.2q^{-1}} & \frac{q^{-2}}{1-0.3q^{-1}} \\ \frac{3q^{-1}}{1-0.1q^{-1}} & \frac{q^{-2}}{1-0.8q^{-1}} & \frac{q^{-2}}{1-0.3q^{-1}} & \frac{2q^{-2}}{1-0.5q^{-1}} \\ \frac{q^{-1}}{1-0.5q^{-1}} & \frac{q^{-1}}{1-0.6q^{-1}} & \frac{q^{-1}}{1-0.8q^{-1}} & \frac{q^{-1}}{1-0.7q^{-1}} \\ \frac{-4q^{-1}}{1-0.7q^{-1}} & \frac{q^{-1}}{1-0.5q^{-1}} & \frac{q^{-1}}{1-0.4q^{-1}} & \frac{q^{-1}}{1-0.6q^{-1}} \end{pmatrix} = \begin{pmatrix} G_1(q) \\ G_2(q) \end{pmatrix}$$

in which the first two CVs are of primary interest. It can be easily verified that the interactors for $G_1(q)$ and $G_2(q)$ are

$$D_{11}(q) = \begin{pmatrix} -0.8q & -0.6q \\ -0.6q^2 & 0.8q^2 \end{pmatrix}, \qquad (16)$$

$$D_{22}(q) = \begin{pmatrix} q & 0 \\ 0 & q \end{pmatrix} \qquad (17)$$

respectively. The corresponding

$$K = \lim_{q\to\infty} \begin{pmatrix} D_{11}(q) & 0 \\ 0 & D_{22}(q) \end{pmatrix} \begin{pmatrix} G_1(q) \\ G_2(q) \end{pmatrix} = \begin{pmatrix} -5 & 0 & 0 & 0 \\ -0.72 & 0.2 & 0.2 & 1 \\ 1 & 1 & 1 & 1 \\ -4 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} K_1 \\ K_2 \end{pmatrix}$$

Then, the left null space of $(K_2 \quad K_1)^T$ in reduced row echelon form is found to be

$$U^T = \begin{pmatrix} 1 & -1 & 1 & 0 \end{pmatrix}$$

which indicates that $D_2(1,:) + D_1(1,:) - D_2(2,:) = 0$. Therefore, we need to increase the delay for this row.

$$\begin{aligned} D_2^1(1,:) &= q^{d^1_{2,1}} \left(D_2^0(1,:) + D_1(1,:) - D_2(2,:) \right) \\ &= q^{d^1_{2,1}} [(0 \quad 0 \quad q \quad 0) + (-0.8q \quad -0.6q \quad 0 \quad 0) \\ &\quad - (0 \quad 0 \quad 0 \quad q)] \\ &= q^{d^1_{2,1}} (-0.8q \quad -0.6q \quad q \quad -q) \end{aligned}$$

where $d^1_{2,1} = 1$ leads to a non-singular and finite $K = \lim_{q\to\infty} DG(q)$:

$$K = \begin{pmatrix} -5 & 0 & 0 & 0 \\ -0.72 & 0.2 & 0.2 & 1 \\ 0 & -1.3 & -1 & -1.9 \\ -4 & 1 & 1 & 1 \end{pmatrix}.$$

Therefore, the block lower triangular interactor matrix for $G(q)$ is

$$D(q) = \begin{pmatrix} D_1(q) & D_2(q) \end{pmatrix}^T = \begin{pmatrix} -0.8q & -0.6q & 0 & 0 \\ -0.6q^2 & 0.8q^2 & 0 & 0 \\ -0.8q^2 & -0.6q^2 & q^2 & -q^2 \\ 0 & 0 & 0 & q \end{pmatrix}$$

IV. LP-BASED MPC CONTROL PERFORMANCE MONITORING

A. LP problem and active CV constraints

Assume that the setpoint of MPC controlling the process described by (1) is obtained from the following LP problem:

$$\begin{aligned} \min_{\bar{y},\bar{u}} \quad & \alpha^T \bar{y} + \beta^T \bar{u} \\ \text{s.t.} \quad & \bar{y} = G^{ss}\bar{u} + d \\ & \bar{y}^{\min} \leq \bar{y} \leq \bar{y}^{\max} \\ & \bar{u}^{\min} \leq \bar{u} \leq \bar{u}^{\max} \end{aligned} \qquad (18)$$

where $\bar{y} = (\bar{y}_1,\ldots,\bar{y}_n)^T$ and $\bar{u} = (\bar{u}_1,\ldots,\bar{u}_m)^T$. Note that the constraints on $\bar{y}$ are soft constraints in the MPC optimization problem. For simplicity, the solution to (18) is denoted as $\bar{y}, \bar{u}$. The active constraints can be inferred from sensitivity analysis or the dual solution.

Based on the solution to (18), CVs can be separated into two groups: CVs with active (binding or more important) constraints and CVs with inactive (non-binding or less important) constraints. The CVs in first group are of our primary interest, whose variances along normal direction of active constraints will be minimized. The CVs in second group are of less importance, but one may still keep them under control.

Remark 2: In the case of multiple solutions to the LP problem (18), it is recommended to choose the center of solution subspace as setpoint of MPC. The reason is that the number of active constraints may be reduced. For example, let $(\bar{y}^1, \bar{u}^1)$ and $(\bar{y}^2, \bar{u}^2)$ be two optimal vertices, and $\bar{y}_1 = \bar{y}_1^{\max}$ and $\bar{y}_1 = \bar{y}_1^{\min}$ are the active constraints for $(\bar{y}^1, \bar{u}^1)$ and $(\bar{y}^2, \bar{u}^2)$ respectively. Then, $(\bar{y}, \bar{u}) = [(\bar{y}^1, \bar{u}^1) + (\bar{y}^2, \bar{u}^2)]/2$ is still optimal but none of $\bar{y}_1$ constraint is active.

Remark 3: Although the LP problem optimizing setpoint of MPC usually takes the form (18) whose constraints are upper and lower bounds, there may exists general constraint $A_1\bar{y} + A_2\bar{u} \leq b$ in LP constraints. Under this circumstance, active constraints can involve more than one CV. However, it can be converted to upper and lower bound types of constraints by introducing variable transformation.

B. Conditional MVC law and performance benchmark under block lower triangular interactor

With CVs divided to two groups, the conditional MVC law can be further derived. According to the CV priority, y_t is partitioned as

$$y_t = \begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix}$$

where $y_{1,t} \in \mathbb{R}^{n_1}$ is the vector of CVs with high priority and $y_{2,t} \in \mathbb{R}^{n_2}$ is the vector of other CVs. The corresponding block lower triangular interactor matrix is

$$D = \begin{pmatrix} D_{11} & 0 \\ D_{21} & D_{22} \end{pmatrix}.$$

Then, the process (1) becomes

$$y_t = D^{-1}\tilde{G}u_t + Na_t. \tag{19}$$

Defining interactor filtered output $\tilde{y}_t = q^{-d}D(q)y_t$, we obtain

$$\tilde{y}_t = \begin{pmatrix} \tilde{y}_{1,t} \\ \tilde{y}_{2,t} \end{pmatrix} = q^{-d} \begin{pmatrix} D_{11} & 0 \\ D_{21} & D_{22} \end{pmatrix} \begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} \tag{20}$$

$$\tilde{y}_{1,t} = q^{-d}D_{11}y_{1,t} \tag{21}$$

$$\tilde{y}_{2,t} = q^{-d}(D_{21}y_{1,t} + D_{22}y_{2,t}). \tag{22}$$

Equation (19) can be simplified as

$$\tilde{y}_{t+d} = \tilde{G}u_t + DNa_t. \tag{23}$$

Since $N(q)$ is a rational transfer function matrix of disturbance dynamics without delay, DN has some factors with positive power of q. It can be factorized as

$$DN = \underbrace{F_d q^d + F_{d-1}q^{d-1} + \cdots + F_1 q}_{F(q)} + R(q).$$

Equation (23) becomes

$$\tilde{y}_{t+d} = \tilde{G}u_t + Fa_t + Ra_t. \tag{24}$$

The MVC law is achieved by

$$u_t = -\tilde{G}^{-1}Ra_t \tag{25}$$

since $\tilde{G}$ is invertible. We have the following lemma for MVC law (25) under block lower triangular interactor.

Lemma 1: The control law (25) minimizes

1) the sum of variances of first n_1 CVs; and
2) the sum of variances of last n_2 CVs when 1) is achieved.

Proof:

Consider first n_1 CVs. Multiplying $e_1^T = (\mathbf{I}_{n_1} \quad 0)$ to both sides of (23) or (24), the first block row of (23) is obtained:

$$\tilde{y}_{1,t+d} = e_1^T\tilde{G}u_t + e_1^T Fa_t + e_1^T Ra_t. \tag{26}$$

Because $\tilde{y}_t$ is stationary and a_t is white noise,

$$\begin{aligned} \text{tr}[\text{cov}(\tilde{y}_{1,t})] &= \text{tr}[\text{cov}(\tilde{y}_{1,t+d})] \\ &= \text{tr}[\text{cov}(e_1^T\tilde{G}u_t + e_1^T Ra_t) + \text{cov}(e_1^T Fa_t)] \\ &\geq \text{tr}[\text{cov}(e_1^T Fa_t)] = \text{tr}(e_1^T F\Sigma_a F^T e_1) \end{aligned} \tag{27}$$

On the other hand, substituting control low (25) leads to

$$\tilde{y}_{1,t+d} = e_1^T Fa_t. \tag{28}$$

which means the equality in (27) holds. Recalling D_{11} is unitary, (21) implies $\text{cov}(\tilde{y}_{1,t}) = \text{cov}(y_{1,t})$. Therefore, the sum of variances of first n_1 CVs is minimized.

Next, consider the last n_2 CVs. Similarly, by multiplying $e_2^T = (0 \quad \mathbf{I}_{n_2})$ to both sides of (23), we obtain

$$\tilde{y}_{2,t+d} = e_2^T\tilde{G}u_t + e_2^T Fa_t + e_2^T Ra_t. \tag{29}$$

Further, substitute (22) into (29),

$$\begin{aligned} D_{21}y_{1,t} + D_{22}y_{2,t} &= e_2^T\tilde{G}u_t + e_2^T Fa_t + e_2^T Ra_t \\ D_{22}y_{2,t} &= -D_{21}D_{11}^T\tilde{y}_{1,t} + e_2^T\tilde{G}u_t + e_2^T Fa_t + e_2^T Ra_t \end{aligned} \tag{30}$$

where $D_{11}^T = D_{11}^T(q^{-1})$ is the conjugate transpose. Since the control law (25) yields (28), (30) becomes

$$\begin{aligned} D_{22}y_{2,t} &= -D_{21}D_{11}^T e_1^T Fa_t + e_2^T\tilde{G}u_t + e_2^T Fa_t + e_2^T Ra_t \\ &= e_2^T\tilde{G}u_t + e_2^T Ra_t + (e_2^T - D_{21}D_{11}^T e_1^T)Fa_t. \end{aligned} \tag{31}$$

Recall that D_{22} is unitary,

$$\begin{aligned} \text{tr}[\text{cov}(y_{2,t})] = \text{tr}[&\text{cov}(e_2^T\tilde{G}u_t + e_2^T Ra_t) + \\ &\text{cov}((e_2^T - D_{21}D_{11}^T e_1^T)Fa_t)] \\ \geq \text{tr}[&\text{cov}[(e_2^T - D_{21}D_{11}^T e_1^T)Fa_t]] \end{aligned} \tag{32}$$

where the equality holds if and only if the control law (25) is applied. Therefore, the sum of variances of last n_2 CVs is minimized by (25) when 1) is achieved. ■

Remark 4: Block diagonal interactor is a special case of block lower triangular interactor. In this case, minimum variance of the CV groups are independent from one another.

Based on the MVC law (25), the conditional minimum variance benchmark is given by

$$J_{\text{MV},1} = \text{tr}(e_1^T F\Sigma_a F^T e_1) \tag{33}$$

for CVs with higher priority and

$$J_{\mathrm{MV},2} = \mathrm{tr}[(e_2^T - D_{21}D_{11}^T e_1^T)F\Sigma_a F^T(e_2 - e_1 D_{11} D_{21}^T)] \tag{34}$$

for CVs with lower priority when J_1 is minimized.

V. Simulation Results

In this section, an example is demonstrated.

Example 2: The following process is taken from [13].

$$G(q) = \begin{pmatrix} q^{-1} & 0 \\ q^{-1} & q^{-2} \end{pmatrix}$$

and

$$N(q) = \begin{pmatrix} \frac{1}{1-q^{-1}} & \\ & \frac{1}{1-q^{-1}} \end{pmatrix}$$

where $\Sigma_a = 0.01$. Suppose the LP problem is

$$\begin{aligned} \max_{\bar{y},\bar{u}} \quad & \bar{y}_2 \\ \text{s.t.} \quad & \bar{y} = G^{ss}\bar{u} + d \\ & 0 \le \bar{y}_1, \bar{y}_2 \le 2 \\ & 0 \le \bar{u}_1, \bar{u}_2 \le 5 \end{aligned}$$

where d is integrating white noise subject to (2) such that it does not affect active set. There are two alternative solutions $(\bar{y}_1, \bar{y}_2)^T = (0, 2)^T$ and $(\bar{y}_1, \bar{y}_2) = (0, 2)^T$. The middle point of them $(1, 2)^T$ is selected as MPC setpoint. The only active constraint is $\bar{y}_2 = 2$. Reversing y_1 and y_2 to get y_2 prioritized, we obtain

$$G'(q) = \begin{pmatrix} q^{-1} & q^{-2} \\ q^{-1} & 0 \end{pmatrix}$$

and

$$N'(q) = \begin{pmatrix} \frac{1}{1-q^{-1}} & \\ & \frac{1}{1-q^{-1}} \end{pmatrix}$$

The unitary interactor of $G'(q)$ is

$$D'_u = \begin{pmatrix} -q^2/\sqrt{2} & -q^2/\sqrt{2} \\ -q/\sqrt{2} & -q/\sqrt{2} \end{pmatrix}$$

and the corresponding feedback invariant term is

$$F'_u = \begin{pmatrix} -(1+q^{-1})/\sqrt{2} & (1+q^{-1})/\sqrt{2} \\ -q^{-1}/\sqrt{2} & -q^{-1}/\sqrt{2} \end{pmatrix}.$$

The variance of y_2 using MVC based on unitary interactor is

$$\mathrm{var}(y_2) = \mathrm{var}(y'_1) = e_1^T F'_u \Sigma_a {F'}_u^T e_1 = 0.02.$$

After partition $G'(q)$ according to LP results, the (block) lower triangular interactor is

$$D'_{\mathrm{tri}} = \begin{pmatrix} q & 0 \\ q^2 & q^{-2} \end{pmatrix}$$

and feedback invariant term becomes

$$F'_{\mathrm{tri}} = \begin{pmatrix} q & 0 \\ q^2 + q & -q^2 - q \end{pmatrix}.$$

The minimum variance of y_2 based on block lower triangular interactor is

$$\mathrm{var}(y_2) = \mathrm{var}(y'_1) = e_1^T F'_{\mathrm{tri}} \Sigma_a {F'}_{\mathrm{tri}}^T e_1 = 0.01.$$

In this case, variance of y_2 is reduced at the expense of variance of y_1, as can be observed from second row of F'_{tri}.

Control results of both controllers are illustrated in Fig. 1. Since y_2 upper limit constraint is active, MVC based on block-lower triangular interactor minimizes the variance of y_2 first and then minimizes the variance of y_1. From Fig. 1, we can see that the variance of y_2 controlled by block-lower triangular interactor based MVC is the smallest. This verifies the conditional minimum variance property of the proposed method. If the safety limit of y_2 is chosen as $y_2^{\max} = 2.3$, the violation rate of conditional MVC and traditional unitary interactor based MVC are 0.2% and 1%, respectively.

Next, performance monitoring will be conducted on an MPC. Since LP results are known, one may want to emphasize more on y_2. The weights for CV are chosen to be $(1, 4)$, and the weights for MV are $(0.5, 0.5)$. Prediction and control horizon are 5 and 2 respectively. Results are also shown in Fig. 1. The variance of y_2 is

$$J_{y_2} = 0.0150$$

as compared to

$$J_{\mathrm{MV},y_2} = 0.01.$$

The ratio is $J_{\mathrm{MV},y_2}/J_{y_2} = 0.67$. It is concluded that this MPC has the potential to improve in terms of y_2 variance or y_2 upper limit violations.

VI. Conclusions

For the LP-MPC system, we prioritize CVs based on importance of their constraints for performance assessment. Since CV constraints are generally soft constraints, smaller variance along normal direction of these constraints results in less violations of the constraint limits. To achieve this, we introduce block lower triangular interactor to construct the conditional MVC law, whose output variance is minimized with priority and used as the performance benchmark. The method is illustrated by simulation results showing that traditional unitary interactor based method is unable to distinguish the importance of CVs and that the proposed method is.

VII. Acknowledgments

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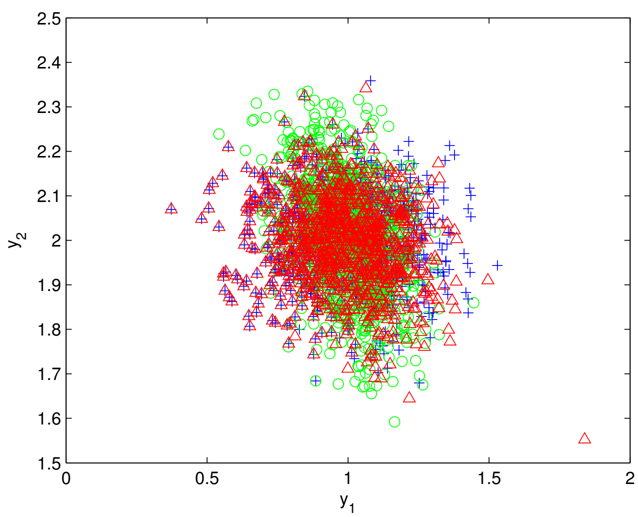

Fig. 1. Control results of Example 2 in y_1-y_2 plane. Green markers are results by MVC using unitary interactor; blue markers are results by conditional MVC using block lower triangular interactor; and red markers are the dynamic control results by an MPC.