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Hierarchical Supervisory Control of Fuzzy Discrete Event Systems

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***Abstract*— Hierarchical structuring of supervisory control which represents the vertical modularity, has been discussed in discrete event systems (DES) to resolve the control complexity of large-scaled systems. Recently, fuzzy discrete event systems (FDES) has been introduced as an extension to the crisp DES in order to better represent the uncertainties and imprecisions of asynchronous event driven dynamical systems. In this paper, we investigate the hierarchical supervisory control problem of FDES with partial observation for modeling large-scale systems with associated uncertainties in their states and event transitions. Some important definitions are introduced and a hierarchical supervisory control theory for FDES is established.**

I. Introduction

Systems which are event driven, dynamical and asynchronous in nature can be modeled using discrete event systems (DES) which describe their operations using formal methods. Most of them use the well established Ramadge and Wonham supervisory control framework and their extensions to synthesize supervisors for achieving desired behaviors [1], [2]. Many systems which have been developed under DES framework such as communication systems, network systems, manufacturing systems, etc. have proved its success in system modeling.

To decompose the control complexities of large-scale compound event driven systems in a vertical modular fashion, hierarchical supervisory control of DES has been discussed [3]–[5]. In this approach, multi level control hierarchies are designed with detailed low-level and abstract high-level models of the plant. The control decisions are made according to the high-level abstraction and the corresponding low-level commands are generated while preserving the consistency [6]. The effect of partial observation in hierarchical supervisory control of DES is studied in [7] for the systems with unobservable events.

Control of DES with uncertainties in model representation is addressed under hierarchical supervision in [8]. However, most systems suffer from uncertainty and vagueness when defining events and state transitions due to imprecision of sensors. As a result, the crisp state specifications may not accurately represent the exact condition of the system at a given time. Hence, the representation of events and states using possibility distributions is more appropriate for such scenarios. Extension of crisp DES theory to fuzzy DES (FDES) provides the flexibility to integrate associated uncertainties into events and state transitions [9]. Later on, the (centralized) supervisory control of FDES has been studied in [10], [11]. The decentralized supervisory control of FDES is presented in [12], [13], and decentralized modular control of FDES is discussed in [14] for concurrently operating multiple interacting FDES modules. The applications of FDES theory such as AIDS treatments [15], behavior-based robot control [16], [17], etc. have proved its validity.

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In order to successfully model and control the large-scaled systems with inherited event and state uncertainties, in this paper we discuss the hierarchical supervisory control problem of FDES with partial observation. Each fuzzy event is associated with a degree of controllability and a degree of observability as in [13], which represents a more general setting. The notions of *output-control-consistency* and *strictly-output-control-consistency* are introduced for fuzzy languages of FDES. Also, the property of *H-observability* in crisp DES [7] is extended to fuzzy domain as *H-fuzzy-observability*, to preserve the hierarchical consistency under partial observation of low-level fuzzy events. Moreover, an algorithm is proposed to compute the supremal fuzzy controllable sub language of the low-level specification. Furthermore, a hierarchical supervisory control theory of FDES is presented for achieving desired high-level behavior by controlling the low-level FDES. Some examples are also discussed to clarify the theoretical developments.

The rest of the paper is organized as: Section II discusses some preliminaries of FDES theory. Section III establishes a hierarchical supervisory control theory of FDES with some associated definitions. Section IV concludes the paper.

II. Preliminaries

The fuzzy finite automaton is defined by the quadruple as $\tilde{G} = (\tilde{Q}, \tilde{\Sigma}, \tilde{\delta}, \tilde{q}_0)$ where $\tilde{Q}$ is set of fuzzy states, $\tilde{\Sigma}$ represents the set of fuzzy events, $\tilde{\delta}$ represents the transition mapping, $\tilde{\delta}: \ \tilde{Q} \times \tilde{\Sigma} \to \tilde{Q}$ and $\tilde{q}_0$ is initial fuzzy state of the system. Then the following properties hold. [10], [11]:

$$\tilde{L}_{\tilde{M}} \subseteq \tilde{L}_{\tilde{G}} \Longleftrightarrow \tilde{L}_{\tilde{M}}(\tilde{s}) \leq \tilde{L}_{\tilde{G}}(\tilde{s}) \qquad \tilde{s} \in \tilde{\Sigma}^*$$
$$\tilde{L}_{\tilde{G}}(\varepsilon) = 1,\ \tilde{L}_{\tilde{G}}(\tilde{\alpha}) \geq \tilde{L}_{\tilde{G}}(\tilde{\alpha}\tilde{\beta}) \qquad \tilde{\alpha}, \tilde{\beta} \in \tilde{\Sigma}^*$$

$\tilde{L}_{\tilde{G}}$ is the fuzzy language generated by the system, $\tilde{L}_{\tilde{M}}$ is a fuzzy sub language of the same system. ε represents the null-event. $\tilde{\Sigma}^*$ is the Kleene-closure of $\tilde{\Sigma}$. Note that $\tilde{s}$ is a fuzzy string which is made by a continuation of fuzzy events. The possibility of fuzzy string $\tilde{s}$ be in $\tilde{L}_{\tilde{G}}$ is represented by $\tilde{L}_{\tilde{G}}(\tilde{s})$ and it is also the physical possibility of occurring $\tilde{s}$. (Note that the possibility of a string of fuzzy events to be in any set or any fuzzy language is bounded by $[0,\ 1]$). The transition mapping $\tilde{\delta}$ is defined as: $\tilde{\delta}(\tilde{q}, \tilde{\sigma}) = \tilde{q} \circ \tilde{\sigma}$. Here "$\circ$" represents either Max-Min or Max-Product operation, which describes that $\tilde{G}$ is modeled by either Max-Min automata or Max-Product automata respectively.

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A fuzzy language, which is generated by the corresponding fuzzy automaton, is characterized by continuous occurrence of fuzzy events in the event set and it can be represented by Zadeh's notation. E.g: $\tilde{L}=\frac{0.4}{\tilde{\alpha}\tilde{\zeta}}+\frac{0.3}{\tilde{\alpha}\tilde{\beta}\tilde{\gamma}}+\frac{0.4}{\tilde{\alpha}\tilde{\beta}\tilde{\delta}}$.

Its Prefix-closure $\bar{\tilde{L}}$ is another fuzzy language which shows how the fuzzy events are continued. For example:
$\bar{\tilde{L}}=\frac{1}{\varepsilon}+\frac{0.8}{\tilde{\alpha}}+\frac{0.5}{\tilde{\alpha}\tilde{\beta}}+\frac{0.4}{\tilde{\alpha}\tilde{\zeta}}+\frac{0.3}{\tilde{\alpha}\tilde{\beta}\tilde{\gamma}}+\frac{0.4}{\tilde{\alpha}\tilde{\beta}\tilde{\delta}}$.

Here $\varepsilon,\tilde{\alpha},\tilde{\beta},\tilde{\gamma},\tilde{\delta},\tilde{\zeta}\in\tilde{\Sigma}$ and $\frac{0.8}{\tilde{\alpha}}$ means the possibility of occurrence of $\tilde{\alpha}$ is 0.8. Note that $\tilde{L}\subseteq\bar{\tilde{L}}$. By definition $\tilde{L}_{\tilde{G}}$ is a prefix-closed fuzzy language (i.e. $\tilde{L}_{\tilde{G}}=\bar{\tilde{L}}_{\tilde{G}}$). Assume a fuzzy sub language $\tilde{L}_{\tilde{M}}=\frac{0.2}{\tilde{\alpha}\tilde{\beta}}+\frac{0.4}{\tilde{\alpha}\tilde{\zeta}}$ and $\tilde{L}_{\tilde{M}}\subseteq\tilde{L}$. Its prefix-closure can be shown as: $\bar{\tilde{L}}_{\tilde{M}}=\frac{1}{\varepsilon}+\frac{0.5}{\tilde{\alpha}}+\frac{0.2}{\tilde{\alpha}\tilde{\beta}}+\frac{0.4}{\tilde{\alpha}\tilde{\zeta}}$.

Note that $\tilde{L}_M\subseteq\tilde{L}\Rightarrow\bar{\tilde{L}}_M\subseteq\bar{\tilde{L}}$ and $\bar{\tilde{L}}_M(\tilde{s})\leq\bar{\tilde{L}}(\tilde{s})$.

Assume $\tilde{\Sigma}_c$ and $\tilde{\Sigma}_{uc}$ as fuzzy controllable event set and fuzzy uncontrollable event set respectively. Also $\tilde{\Sigma}_o$ and $\tilde{\Sigma}_{uo}$ as fuzzy observable event set and fuzzy unobservable event set respectively. All four of these are subsets of $\tilde{\Sigma}$.

We also assume that any fuzzy event $\tilde{\sigma}$, $(\tilde{\sigma}\in\tilde{\Sigma})$ is associated with a degree of being controllable $(\tilde{\Sigma}_c(\tilde{\sigma}))$ and a degree of being observable $(\tilde{\Sigma}_o(\tilde{\sigma}))$ as in [13].

$\tilde{\Sigma}_c(\tilde{\sigma})+\tilde{\Sigma}_{uc}(\tilde{\sigma})=1$ and $\tilde{\Sigma}_o(\tilde{\sigma})+\tilde{\Sigma}_{uo}(\tilde{\sigma})=1$.

Hereafter, we do not distinguish fuzzy controllable events and fuzzy uncontrollable events separately. Each fuzzy event is associated with a degree of controllability and a degree of uncontrollability. Also each fuzzy event is associated with a degree of observability and a degree of unobservability. Note that hereafter, $\tilde{\sigma}\in\tilde{\Sigma}_c$ implies $\tilde{\Sigma}_c(\tilde{\sigma})>0$.

Let $\tilde{S}/\tilde{G}$ represents the fuzzy supervisor $\tilde{S}$ controlling the FDES $\tilde{G}$ and $\tilde{L}_{\tilde{S}/\tilde{G}}$ is its corresponding fuzzy language. By extending the language generated by supervisor controlling the system for crisp DES presented in [2], we can define $\tilde{L}_{\tilde{S}/\tilde{G}}$ for FDES and achieve it recursively as follows:

1.$\tilde{L}_{\tilde{S}/\tilde{G}}(\varepsilon)=1$

2.$\tilde{L}_{\tilde{S}/\tilde{G}}(\tilde{s}\tilde{\sigma})=\tilde{L}_{\tilde{S}/\tilde{G}}(\tilde{s})\tilde{\cap}\tilde{S}_{\tilde{s}}(\tilde{\sigma})\tilde{\cap}\tilde{L}_{\tilde{G}}(\tilde{s}\tilde{\sigma})$

where, $\tilde{S}_{\tilde{s}}(\tilde{\sigma})$ is the possibility of fuzzy event $\tilde{\sigma}$ being enabled by the fuzzy supervisor $\tilde{S}$, after observing the fuzzy string $\tilde{s}$. Here $\tilde{L}_{\tilde{S}/\tilde{G}}\subseteq\tilde{L}_{\tilde{G}}$ and it is prefixed-closed. Also $\tilde{\cap}$ represents the fuzzy-And operation (taking minimum or product).

Note that contrast to the crisp languages in DES, hereafter we specify the fuzzy languages in FDES in their prefix-closed forms which show the way that how they are evolved over the time.

Let $\bar{\tilde{L}}_{\tilde{G},m}$ be the prefix-closure of the fuzzy language of marked fuzzy strings of $\tilde{L}_{\tilde{G}}$, which is used to represent the successfully completed operations of the system. A new fuzzy language $\bar{\tilde{L}}_{\tilde{S}/\tilde{G},m}$, which is a sub language of $\bar{\tilde{L}}_{\tilde{G},m}$ and contains marked fuzzy strings which survives under $\tilde{S}/\tilde{G}$, can be achieved as: $\bar{\tilde{L}}_{\tilde{S}/\tilde{G},m}=\tilde{L}_{\tilde{S}/\tilde{G}}\cap\bar{\tilde{L}}_{\tilde{G},m}$

The possibility of fuzzy string $\tilde{s}$ $(\tilde{s}\in\tilde{\Sigma}^*)$ be in $\bar{\tilde{L}}_{\tilde{S}/\tilde{G},m}$ can be defined as:

$\bar{\tilde{L}}_{\tilde{S}/\tilde{G},m}(\tilde{s})=\tilde{L}_{\tilde{S}/\tilde{G}}(\tilde{s})\tilde{\cap}\bar{\tilde{L}}_{\tilde{G},m}(\tilde{s})\Rightarrow\bar{\tilde{L}}_{\tilde{S}/\tilde{G},m}(\tilde{s})\leq\bar{\tilde{L}}_{\tilde{G},m}(\tilde{s})$

The fuzzy language $\tilde{L}_{\tilde{S}/\tilde{G}}$ called "non-blocking" if it is exactly same to the prefix-closure of $\tilde{L}_{\tilde{S}/\tilde{G},m}$ (i.e. $\tilde{L}_{\tilde{S}/\tilde{G}}=\bar{\tilde{L}}_{\tilde{S}/\tilde{G},m}$). Which means: $\forall\tilde{s}\in\tilde{\Sigma}^*$: $\tilde{L}_{\tilde{S}/\tilde{G}}(\tilde{s})=\bar{\tilde{L}}_{\tilde{S}/\tilde{G},m}(\tilde{s})$

Assume a fuzzy language specification $\bar{\tilde{k}}$ is given. If $\bar{\tilde{k}}(\tilde{s})\leq\bar{\tilde{L}}_{\tilde{G},m}(\tilde{s})$, then $\bar{\tilde{k}}$ is said to be $\tilde{L}_{\tilde{G},m}$-closed.

Neglecting the observability issues of fuzzy events, the fuzzy controllability condition is given below [18].

Let $\bar{\tilde{k}}$ be the prefix-closure of the fuzzy language $\tilde{k}$ (note that $\bar{\tilde{k}}=\tilde{k}$) and $\bar{\tilde{k}}\subseteq\tilde{L}_{\tilde{G}}$. The physical possibility of fuzzy string $\tilde{s}\tilde{\sigma}$ is given by $\tilde{L}_{\tilde{G}}(\tilde{s}\tilde{\sigma})$. Then $\bar{\tilde{k}}$ is said to be satisfying fuzzy controllability condition with respect to $\tilde{L}_{\tilde{G}}$ and $\tilde{\Sigma}_{uc}$, if following inequality holds for any $\tilde{s}\in\tilde{\Sigma}^*$ and $\tilde{\sigma}\in\tilde{\Sigma}$:

$\bar{\tilde{k}}(\tilde{s})\tilde{\cap}\tilde{\Sigma}_{uc}(\tilde{\sigma})\tilde{\cap}\tilde{L}_{\tilde{G}}(\tilde{s}\tilde{\sigma})\leq\bar{\tilde{k}}(\tilde{s}\tilde{\sigma})$

Definition 1: The natural projection of $\tilde{\sigma}$ is defined as:

$\tilde{P}(\tilde{\sigma})=\left[\tilde{\Sigma}_{uo}(\tilde{\sigma})\cdot\varepsilon+\tilde{\Sigma}_o(\tilde{\sigma})\cdot\tilde{\sigma}\right]$

Intuitively, this means that the matrix representing natural projection of $\tilde{\sigma}$ can be achieved by multiplying each element of "ε" (an identity matrix) by $\tilde{\Sigma}_{uo}(\tilde{\sigma})$ and add them together with the corresponding elements of the matrix which is made by multiplying each element of "$\tilde{\sigma}$" (the event matrix) by $\tilde{\Sigma}_o(\tilde{\sigma})$. It can be easily seen that when the unobservability of a fuzzy event $\tilde{\sigma}$ increases $\tilde{P}(\tilde{\sigma})$ reaches to ε. Then the supervisor of the closed loop system be likely unobserve $\tilde{\sigma}$. Also when the observability of the fuzzy event $\tilde{\sigma}$ increases the supervisor tends to observe $\tilde{\sigma}$.

Assume $\tilde{s}=\tilde{\sigma}_1\tilde{\sigma}_2\ldots\tilde{\sigma}_n$. Let $\tilde{P}(\tilde{s})=\tilde{l}$ be the natural projection of $\tilde{s}$. The following is obtained by considering the natural projection of each fuzzy event individually.

$$\tilde{P}(\tilde{s})=\tilde{l}=\tilde{P}\left(\tilde{\sigma}_1\tilde{\sigma}_2\ldots\tilde{\sigma}_n\right)\Rightarrow\tilde{P}\left(\tilde{\sigma}_1\right)\tilde{P}\left(\tilde{\sigma}_2\right)\ldots\tilde{P}\left(\tilde{\sigma}_n\right)$$

The fuzzy admissibility condition in [11], is extended by introducing partially observation supervisory control:

$\tilde{\Sigma}_{uc}(\tilde{\sigma})\tilde{\cap}\tilde{L}_{\tilde{G}}(\tilde{s}\tilde{\sigma})\leq\tilde{S}^{\tilde{P}}_{\tilde{t}}(\tilde{\sigma})$

Here $\tilde{S}^{\tilde{P}}$ is the fuzzy partially observation supervisor, $\tilde{P}(\tilde{s})=\tilde{t}$ and $\tilde{S}^{\tilde{P}}_{\tilde{t}}(\tilde{\sigma})$ is the possibility of $\tilde{\sigma}$ being enabled by $\tilde{S}^{\tilde{P}}$ after observing fuzzy string $\tilde{t}$. The following can be derived from above:

$\tilde{L}_{\tilde{S}^{\tilde{P}}/\tilde{G}}(\tilde{s}\tilde{\sigma})=\tilde{L}_{\tilde{S}^{\tilde{P}}/\tilde{G}}(\tilde{s})\tilde{\cap}\tilde{S}^{\tilde{P}}_{\tilde{t}}(\tilde{\sigma})\tilde{\cap}\tilde{L}_{\tilde{G}}(\tilde{s}\tilde{\sigma})$

Where $\tilde{L}_{\tilde{S}^{\tilde{P}}/\tilde{G}}$ is the fuzzy language generated by the fuzzy partially observation supervisor $\tilde{S}^{\tilde{P}}$, controlling the system $\tilde{G}$.

Assume $\tilde{P}^{-1}\left(\tilde{P}(\tilde{s})\right)$ as a subset which gives the possibility of the natural projection of a fuzzy string, to be observed as same as the natural projection of $\tilde{s}$. (e.g. $\tilde{P}^{-1}\left(\tilde{P}(\tilde{s})\right)(\tilde{s})=1$). This new subset defines the "likelihood" of a fuzzy string to be observed as a different one.

Extending the fuzzy observability defined in [18] we can derive the following definition for fuzzy observability.

Definition 2: Let $\bar{\tilde{k}}\subseteq\tilde{L}_{\tilde{G}}$, $\tilde{s}'\tilde{\sigma}\in\bar{\tilde{k}}$ and $\tilde{s}\in\tilde{P}^{-1}\left(\tilde{P}(\tilde{s}')\right)$. For any $\tilde{s}\in\tilde{\Sigma}^*$ and $\tilde{\sigma}\in\tilde{\Sigma}$, $\tilde{k}$ is said to be satisfying fuzzy observability condition with respect to $\tilde{L}_{\tilde{G}}$, $\tilde{P}$ and $\tilde{\Sigma}_c$, if following inequality holds:

$$\bar{\tilde{k}}(\tilde{s})\tilde{\cap}\tilde{L}_{\tilde{G}}(\tilde{s}\tilde{\sigma})\tilde{\cap}\bar{\tilde{k}}(\tilde{s}'\tilde{\sigma})\tilde{\cap}\tilde{P}^{-1}\left(\tilde{P}(\tilde{s}')\right)(\tilde{s})\tilde{\cap}\tilde{\Sigma}_c(\tilde{\sigma})\leq\bar{\tilde{k}}(\tilde{s}\tilde{\sigma})$$

Intuitively, the possibility of fuzzy string $\tilde{s}\tilde{\sigma}$ belongs to $\bar{\tilde{k}}$ is greater than or equal to the minimum (or product) of followings:

1. Possibility of $\tilde{s}$ belongs to $\bar{\tilde{k}}$

2. Physical possibility of $\tilde{s}\tilde{\sigma}$
3. Possibility of $\tilde{s}'\tilde{\sigma}$ belongs to $\bar{\tilde{k}}$
4. Possibility of $\tilde{P}(\tilde{s})$ to be observed as same as $\tilde{P}(\tilde{s}')$
5. The degree of $\tilde{\sigma}$ being controllable.

This definition of fuzzy observability is important as it supports the "likelihood" of a fuzzy string to be seen differently which adds an extra dimension for fuzzy observability.

Similarly, assuming $\tilde{P}(\tilde{s}) = \tilde{t}$ and $\tilde{s}'\tilde{\sigma} \in \bar{\tilde{k}}$, an observable fuzzy supervisor can be defined as below for all $\tilde{\sigma} \in \tilde{\Sigma}$:

$\tilde{S}_{\tilde{t}}^{\tilde{P}}(\tilde{\sigma}) \geq \tilde{L}_{\tilde{G}}(\tilde{s}\tilde{\sigma}) \tilde{\cap} \bar{\tilde{k}}(\tilde{s}'\tilde{\sigma}) \tilde{\cap} \tilde{P}^{-1}\left(\tilde{P}(\tilde{s}')\right)(\tilde{s}) \tilde{\cap} \tilde{\Sigma}_c(\tilde{\sigma})$

The terms have been described earlier.

Definition 3: Combining fuzzy admissibility condition and the above inequality for partial observation supervisory control, we can define the possibility of $\tilde{\sigma}$ ($\tilde{\sigma} \in \tilde{\Sigma}$) being enabled by $\tilde{S}^{\tilde{P}}$ after observing $\tilde{t}$ (where $\tilde{s}$ has been occurred in the system and $\tilde{P}(\tilde{s}) = \tilde{t}$), as follows:

Let $\mu_1 = \tilde{\Sigma}_{uc}(\tilde{\sigma}) \tilde{\cap} \tilde{L}_{\tilde{G}}(\tilde{s}\tilde{\sigma})$, $\tilde{s}'\tilde{\sigma} \in \bar{\tilde{k}}$ and

$$\mu_2 = \tilde{L}_{\tilde{G}}(\tilde{s}\tilde{\sigma}) \tilde{\cap} \bar{\tilde{k}}(\tilde{s}'\tilde{\sigma}) \tilde{\cap} \tilde{P}^{-1}\left(\tilde{P}(\tilde{s}')\right)(\tilde{s}) \tilde{\cap} \tilde{\Sigma}_c(\tilde{\sigma}),$$

For any $\tilde{\sigma} \in \tilde{\Sigma}$:

$$\tilde{S}_{\tilde{t}}^{\tilde{P}}(\tilde{\sigma}) = \begin{cases} \mu_1, & \text{if } \mu_1 \geq \mu_2 \text{ and } \mu_1 \geq \bar{\tilde{k}}(\tilde{s}\tilde{\sigma}) \\ \mu_2, & \text{if } \mu_2 > \mu_1 \text{ and } \mu_2 \geq \bar{\tilde{k}}(\tilde{s}\tilde{\sigma}) \\ \bar{\tilde{k}}(\tilde{s}\tilde{\sigma}), & \text{otherwise.} \end{cases}$$

Intuitively, this explains the possibility of a fuzzy event $\tilde{\sigma}$ being enabled by the partially observation supervisor $\tilde{S}^{\tilde{P}}$ (after observing $\tilde{t}$), according to the possibility of $\tilde{\sigma}$ being uncontrollable and the possibility of $\tilde{\sigma}$ being controllable.

Theorem 1: Fuzzy controllability and fuzzy observability Theorem:

There exists a non-blocking fuzzy partially observation supervisor $\tilde{S}^{\tilde{P}}$ for the system $\tilde{G}$ such that $\bar{\tilde{k}}(\tilde{s}) = \bar{\tilde{L}}_{\tilde{S}^{\tilde{P}}/\tilde{G},m}(\tilde{s})$ and $\tilde{L}_{\tilde{S}^{\tilde{P}}/\tilde{G}}(\tilde{s}) = \bar{\tilde{k}}(\tilde{s})$ if and only if following conditions are hold:

1. $\tilde{k}$ is fuzzy controllable with respect to $\tilde{L}_{\tilde{G}}$ and $\tilde{\Sigma}_{uc}$.
2. $\tilde{k}$ is fuzzy observable with respect to $\tilde{L}_{\tilde{G}}$, $\tilde{P}$ and $\tilde{\Sigma}_c$.
3. $\tilde{k}$ is $\tilde{L}_{\tilde{G},m}$-closed.

Proof: Omitted due to space limitations.

III. Hierarchical supervisory control of Fuzzy Discrete Event Systems

Figure 1 shows the basic architecture of a two-level control hierarchy adopted from [3]. The actual plant to be controlled is modeled by the low-level FDES $\tilde{G}_{lo}$ and supervised by low-level controller $\tilde{S}_{lo}$. Assume $\tilde{G}_{hi}$ as a high-level FDES which represents the abstract simplified specification of $\tilde{G}_{lo}$. Control is virtually exercised on $\tilde{G}_{hi}$ by the high-level supervisor $\tilde{S}_{hi}$ through the channel Con_{hi}. Corresponding commands are generated and passed to $\tilde{S}_{lo}$ from $\tilde{S}_{hi}$ through the command channel Com_{hi-lo}, to control $\tilde{G}_{lo}$ appropriately. The low-level control is exerted on $\tilde{G}_{lo}$ through the channel Con_{lo} and the results are informed to $\tilde{S}_{lo}$ via Inf_{lo}. The high-level abstraction $\tilde{G}_{hi}$ is entirely driven by $\tilde{G}_{lo}$ and updated via the information channel Inf_{lo-hi} accordingly. Finally, the summary of the implemented control actions are reported to $\tilde{S}_{hi}$ via Inf_{hi}.

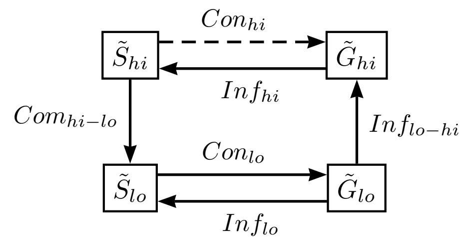


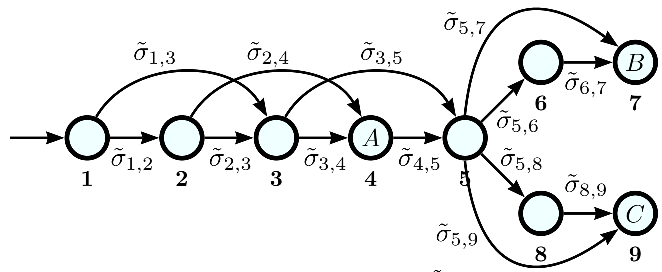
Fig. 1. A two-level hierarchical supervisory control system

Assume $\tilde{\Sigma}$ and $\tilde{T}$ are the sets of fuzzy events of low-level and high-level FDES modules respectively. Let the fuzzy languages generated from low-level FDES $\tilde{G}_{lo}$ (i.e. behavior of $\tilde{G}_{lo}$) and high-level FDES $\tilde{G}_{hi}$ are given by $\tilde{L}_{lo}$ and $\tilde{L}_{hi}$ respectively. To model the low-level to high-level information flow Inf_{lo-hi}, we can define the *prefix-preserving* map $\theta : \tilde{L}_{lo} \rightarrow \tilde{T}^*$ with following properties as in crisp DES [3]:

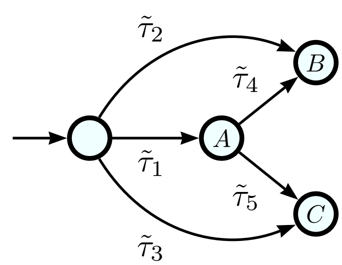
1) $\theta(\varepsilon) = \varepsilon$,
2) For $\tilde{s} \in \tilde{L}_{lo}$ and $\tilde{\sigma} \in \tilde{\Sigma}$:

$$\theta(\tilde{s}\tilde{\sigma}) = \begin{cases} \text{either } \theta(\tilde{s}) \\ \text{or } \theta(\tilde{s})\tilde{\tau}, \text{ for some } \tilde{\tau} \in \tilde{T} \end{cases}$$

Consider the low-level FDES $\tilde{G}_{lo}$ and its high-level abstraction $\tilde{G}_{hi}$ in figure 2.



(a) Low-level FDES $\tilde{G}_{lo}$



(b) High-level FDES abstraction $\tilde{G}_{hi}$

Fig. 2. A low-level FDES $\tilde{G}_{lo}$ and its high-level abstraction $\tilde{G}_{hi}$

A fuzzy state in $\tilde{G}_{lo}$ which generates an alphabet, is defined as a *vocal node*. Otherwise it is a *silent node*. The fuzzy states 4, 7, and 9 of low-level FDES $\tilde{G}_{lo}$ shown in figure 2(a) are *vocal nodes* and others are *silent*. There exist multiple paths in $\tilde{G}_{lo}$ to achieve fuzzy states 4 (A), 7 (B) and 9 (C). As a result, we can generate several possible high-level fuzzy events to indicate the transitions to above fuzzy states as shown in figure 2(b).

Each high-level fuzzy event is defined by low-level fuzzy strings which occurred between two consecutive vocal nodes or between the initial node and a vocal node. For example in figure 2(a) the high-level fuzzy event $\tilde{\tau}_1$ can be generated from three possible low-level fuzzy strings: $\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,3}\tilde{\sigma}_{3,4}$, $\sigma_{1,2}\tilde{\sigma}_{2,4}$ or $\tilde{\sigma}_{1,3}\tilde{\sigma}_{3,4}$. Hence, Definition 4 is given.

Definition 4: We define the *main-path*, $\tilde{L}_{lo}^{MP} \subseteq \tilde{L}_{lo}$, of $\tilde{G}_{lo}$, such that it consists of low-level fuzzy strings that achieve each high-level fuzzy string $\tilde{t} \in \tilde{L}_{hi}$ with the mapping θ, and compute it as described in Algorithm 1.

input : $\tilde{L}_{hi}, \tilde{\Sigma}, \tilde{L}_{lo}$
output: $\tilde{L}_{lo}^{MP}$
for $\forall\ \tilde{t} \in \tilde{L}_{hi}$ **do**
  Let $\theta^{-1}(\tilde{t}) = \tilde{s}_1, ..., \tilde{s}_n$
  for $i \in (1, ..., n)$ **do**
    Let $\tilde{s}_i = \tilde{\sigma}_{i,1}...\tilde{\sigma}_{i,m}$ // m depends on $|\tilde{s}_i|$
    Define $\tilde{L}_{lo,uc}$ such that
    $\tilde{L}_{lo,uc}(\tilde{s}_i) = \min\{\tilde{\Sigma}_{uc}(\tilde{\sigma}_{i,1}), ..., \tilde{\Sigma}_{uc}(\tilde{\sigma}_{i,m})\}$
  end
  Select all $\tilde{s}_k$, $(1 \le k \le n)$ such that $\forall\ j \in (1, ..., n)$:
  $\tilde{L}_{lo,uc}(\tilde{s}_k)\tilde{\cap}\tilde{L}_{lo}(\tilde{s}_k) \ge \tilde{L}_{lo,uc}(\tilde{s}_j)\tilde{\cap}\tilde{L}_{lo}(\tilde{s}_j)$
  Define $\tilde{M}_{\tilde{s}}^{\tilde{t}}$ such that: $\tilde{M}_{\tilde{s}}^{\tilde{t}} \leftarrow \tilde{s}_k$
  Let $|\tilde{M}_{\tilde{s}}^{\tilde{t}}| = p$, $p \ge 1$ // $\tilde{M}_{\tilde{s}}^{\tilde{t}}$ has p number of strings
  Reassign strings in $\tilde{M}_{\tilde{s}}^{\tilde{t}}$ such that:
  $\forall\ l \in (1, ..., p)$, $\tilde{s}_l \in \tilde{M}_{\tilde{s}}^{\tilde{t}}$
  Select $\tilde{s}_q$, $(1 \le q \le p)$ such that $\forall\ l \in (1, ..., p)$:
  $\tilde{L}_{lo}(\tilde{s}_q) \ge \tilde{L}_{lo}(\tilde{s}_l)$
  $\tilde{L}_{lo}^{MP} \leftarrow \tilde{s}_q$
end

Algorithm 1: The computation of *main-path*, $\tilde{L}_{lo}^{MP}$

Note that each low-level fuzzy string in $\tilde{L}_{lo}^{MP}$ contributes to generate the high-level specification with the mapping θ.

Example 1: With the Algorithm 1, the computed *main-path* $\tilde{L}_{lo}^{MP}$ of the low-level FDES in figure 2(a), is shown at the bottom of the page.

The physical possibility of a high-level fuzzy string $\tilde{t}$ can be defined according to that of low-level fuzzy strings $\tilde{s}$ in $\tilde{L}_{lo}^{MP}$, which generate $\tilde{t}$ with the mapping θ.

$\tilde{L}_{hi}(\varepsilon) = 1$.

$\tilde{L}_{hi}(\tilde{t}) = \tilde{L}_{lo}(\tilde{s})$, such that $\tilde{s} \in \tilde{L}_{lo}^{MP}$ and $\theta(\tilde{s}) = \tilde{t}$.

Example 2: Refer to low-level FDES $\tilde{G}_{lo}$ in figure 3(a). Let $\tilde{\tau}_1, \tilde{\tau}_2, \tilde{\tau}_3, \tilde{\tau}_1\tilde{\tau}_4, \tilde{\tau}_1\tilde{\tau}_5 \in \tilde{L}_{hi}$. Assume the degrees of being uncontrollable of low-level fuzzy events, and the physical possibilities of low-level fuzzy strings which generate corresponding high-level fuzzy strings are given as at the bottom of the page. With the above definition, the physical possibilities of high-level fuzzy strings are: $\tilde{L}_{hi}(\tilde{\tau}_1) = 0.9$, $\tilde{L}_{hi}(\tilde{\tau}_2) = 0.8$, $\tilde{L}_{hi}(\tilde{\tau}_3) = 0.4$, $\tilde{L}_{hi}(\tilde{\tau}_1\tilde{\tau}_4) = 0.7$, $\tilde{L}_{hi}(\tilde{\tau}_1\tilde{\tau}_5) = 0.8$.

As each fuzzy event in $\tilde{G}_{lo}$ generally associated with a degree of being uncontrollable, $\tilde{\tau} \in \tilde{T}$ also posses this property in high-level. This leads to define the *output-control-consistency* condition for FDES .

Definition 5: Consider a hierarchical system with low-level FDES $\tilde{G}_{lo}$ and its high-level FDES $\tilde{G}_{hi}$. Assume the set of fuzzy events of $\tilde{G}_{lo}$ and $\tilde{G}_{hi}$ are as $\tilde{\Sigma}$ and $\tilde{T}$ respectively. Let the fuzzy languages generated by $\tilde{G}_{lo}$ and $\tilde{G}_{hi}$ are given as $\tilde{L}_{lo}$ and $\tilde{L}_{hi}$. Assume the high-level fuzzy event $\tilde{\tau}$ is generated by the low-level fuzzy string $\tilde{s}$ through the main path with mapping $\tilde{\theta}$ (i.e $\theta(\tilde{s}) = \tilde{\tau}$), where $\tilde{s}$ is defined between two consecutive vocal nodes. Let $\tilde{s} = \tilde{\sigma}_1...\tilde{\sigma}_n$, Then the FDES $\tilde{G}_{lo}$ is said to be *output-control-consistent* if:

$\tilde{T}_{uc}(\tilde{\tau}) = \min\{\tilde{\Sigma}_{uc}(\tilde{\sigma}_1), ..., \tilde{\Sigma}_{uc}(\tilde{\sigma}_n)\}$

Example 3: Refer to low-level FDES $\tilde{G}_{lo}$ in Fig 2(a). Assuming FDES $\tilde{G}_{lo}$ is *output-control-consistent*, the degrees of being uncontrollable of high-level fuzzy events $\tilde{\tau}_i \in \tilde{T}$ $(i = 1, 2, 3, 4, 5)$ can be calculated as mentioned at the bottom of the page.

$$\tilde{L}_{lo}^{MP} = \{(\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,3}\tilde{\sigma}_{3,4}), (\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,3}\tilde{\sigma}_{3,5}\tilde{\sigma}_{5,6}\tilde{\sigma}_{6,7}), (\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,3}\tilde{\sigma}_{3,5}\tilde{\sigma}_{5,9}), (\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,3}\tilde{\sigma}_{3,4}\tilde{\sigma}_{4,5}\tilde{\sigma}_{5,6}\tilde{\sigma}_{6,7}), (\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,3}\tilde{\sigma}_{3,4}\tilde{\sigma}_{4,5}\tilde{\sigma}_{5,8}\tilde{\sigma}_{8,9})\}$$

$\tilde{\Sigma}_{uc}(\tilde{\sigma}_{1,2}) = 0.2$,$\tilde{\Sigma}_{uc}(\tilde{\sigma}_{2,3}) = 0.3$, $\tilde{\Sigma}_{uc}(\tilde{\sigma}_{3,4}) = 0.4$, $\tilde{\Sigma}_{uc}(\tilde{\sigma}_{4,5}) = 0.1$, $\tilde{\Sigma}_{uc}(\tilde{\sigma}_{5,6}) = 0.3$, $\tilde{\Sigma}_{uc}(\tilde{\sigma}_{6,7}) = 0.5$, $\tilde{\Sigma}_{uc}(\tilde{\sigma}_{5,8}) = 0.1$, $\tilde{\Sigma}_{uc}(\tilde{\sigma}_{8,9}) = 0.4$, $\tilde{\Sigma}_{uc}(\tilde{\sigma}_{1,3}) = 0.1$, $\tilde{\Sigma}_{uc}(\tilde{\sigma}_{2,4}) = 0.3$, $\tilde{\Sigma}_{uc}(\tilde{\sigma}_{3,5}) = 0.4$, $\tilde{\Sigma}_{uc}(\tilde{\sigma}_{5,7}) = 0.4$, and $\tilde{\Sigma}_{uc}(\tilde{\sigma}_{5,9}) = 0.3$.

$\tilde{\tau}_1$: $\tilde{L}_{lo}(\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,3}\tilde{\sigma}_{3,4}) = 0.9$, $\tilde{L}_{lo}(\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,4}) = 0.5$, $\tilde{L}_{lo}(\tilde{\sigma}_{1,3}\tilde{\sigma}_{3,4}) = 0.7$.
$\tilde{\tau}_2$: $\tilde{L}_{lo}(\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,3}\tilde{\sigma}_{3,5}\tilde{\sigma}_{5,6}\tilde{\sigma}_{6,7}) = 0.8$, $\tilde{L}_{lo}(\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,3}\tilde{\sigma}_{3,5}\tilde{\sigma}_{5,7}) = 0.6$, $\tilde{L}_{lo}(\tilde{\sigma}_{1,3}\tilde{\sigma}_{3,5}\tilde{\sigma}_{5,6}\tilde{\sigma}_{6,7}) = 0.4$, $\tilde{L}_{lo}(\tilde{\sigma}_{1,3}\tilde{\sigma}_{3,5}\tilde{\sigma}_{5,7}) = 0.5$.
$\tilde{\tau}_3$: $\tilde{L}_{lo}(\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,3}\tilde{\sigma}_{3,5}\tilde{\sigma}_{5,8}\tilde{\sigma}_{8,9}) = 0.7$, $\tilde{L}_{lo}(\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,3}\tilde{\sigma}_{3,5}\tilde{\sigma}_{5,9}) = 0.4$, $\tilde{L}_{lo}(\tilde{\sigma}_{1,3}\tilde{\sigma}_{3,5}\tilde{\sigma}_{5,8}\tilde{\sigma}_{8,9}) = 0.8$, $\tilde{L}_{lo}(\tilde{\sigma}_{1,3}\tilde{\sigma}_{3,5}\tilde{\sigma}_{5,9}) = 0.7$.
$\tilde{\tau}_1\tilde{\tau}_4$: $\tilde{L}_{lo}(\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,3}\tilde{\sigma}_{3,4}\tilde{\sigma}_{4,5}\tilde{\sigma}_{5,6}\tilde{\sigma}_{6,7}) = 0.7$, $\tilde{L}_{lo}(\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,3}\tilde{\sigma}_{3,4}\tilde{\sigma}_{4,5}\tilde{\sigma}_{5,7}) = 0.6$, $\tilde{L}_{lo}(\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,4}\tilde{\sigma}_{4,5}\tilde{\sigma}_{5,6}\tilde{\sigma}_{6,7}) = 0.4$, $\tilde{L}_{lo}(\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,4}\tilde{\sigma}_{4,5}\tilde{\sigma}_{5,7}) = 0.3$, $\tilde{L}_{lo}(\tilde{\sigma}_{1,3}\tilde{\sigma}_{3,4}\tilde{\sigma}_{4,5}\tilde{\sigma}_{5,6}\tilde{\sigma}_{6,7}) = 0.5$, $\tilde{L}_{lo}(\tilde{\sigma}_{1,3}\tilde{\sigma}_{3,4}\tilde{\sigma}_{4,5}\tilde{\sigma}_{5,7}) = 0.4$.
$\tilde{\tau}_1\tilde{\tau}_5$: $\tilde{L}_{lo}(\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,3}\tilde{\sigma}_{3,4}\tilde{\sigma}_{4,5}\tilde{\sigma}_{5,8}\tilde{\sigma}_{8,9}) = 0.8$, $\tilde{L}_{lo}(\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,3}\tilde{\sigma}_{3,4}\tilde{\sigma}_{4,5}\tilde{\sigma}_{5,9}) = 0.6$, $\tilde{L}_{lo}(\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,4}\tilde{\sigma}_{4,5}\tilde{\sigma}_{5,8}\tilde{\sigma}_{8,9}) = 0.5$, $\tilde{L}_{lo}(\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,4}\tilde{\sigma}_{4,5}\tilde{\sigma}_{5,9}) = 0.4$, $\tilde{L}_{lo}(\tilde{\sigma}_{1,3}\tilde{\sigma}_{3,4}\tilde{\sigma}_{4,5}\tilde{\sigma}_{5,8}\tilde{\sigma}_{8,9}) = 0.5$, $\tilde{L}_{lo}(\tilde{\sigma}_{1,3}\tilde{\sigma}_{3,4}\tilde{\sigma}_{4,5}\tilde{\sigma}_{5,9}) = 0.6$.

$$\tilde{T}_{uc}(\tilde{\tau}_1) = \min\left\{\tilde{\Sigma}_{uc}(\tilde{\sigma}_{1,2}), \tilde{\Sigma}_{uc}(\tilde{\sigma}_{2,3}), \tilde{\Sigma}_{uc}(\tilde{\sigma}_{3,4})\right\} = 0.2,$$
$$\tilde{T}_{uc}(\tilde{\tau}_2) = \min\left\{\tilde{\Sigma}_{uc}(\tilde{\sigma}_{1,2}), \tilde{\Sigma}_{uc}(\tilde{\sigma}_{2,3}), \tilde{\Sigma}_{uc}(\tilde{\sigma}_{3,5}), \tilde{\Sigma}_{uc}(\tilde{\sigma}_{5,6}), \tilde{\Sigma}_{uc}(\tilde{\sigma}_{6,7})\right\} = 0.2$$
$$\tilde{T}_{uc}(\tilde{\tau}_3) = \min\left\{\tilde{\Sigma}_{uc}(\tilde{\sigma}_{1,2}), \tilde{\Sigma}_{uc}(\tilde{\sigma}_{2,3}), \tilde{\Sigma}_{uc}(\tilde{\sigma}_{3,5}), \tilde{\Sigma}_{uc}(\tilde{\sigma}_{5,9})\right\} = 0.2$$
$$\tilde{T}_{uc}(\tilde{\tau}_4) = \min\left\{\tilde{\Sigma}_{uc}(\tilde{\sigma}_{4,5}), \tilde{\Sigma}_{uc}(\tilde{\sigma}_{5,6}), \tilde{\Sigma}_{uc}(\tilde{\sigma}_{6,7})\right\} = 0.1,\ \tilde{T}_{uc}(\tilde{\tau}_5) = \min\left\{\tilde{\Sigma}_{uc}(\tilde{\sigma}_{4,5}), \tilde{\Sigma}_{uc}(\tilde{\sigma}_{5,8}), \tilde{\Sigma}_{uc}(\tilde{\sigma}_{8,9})\right\} = 0.1$$

Note that because of these degrees of being uncontrollable, any high-level fuzzy event in $\tilde{G}_{hi}$ cannot be completely disabled by low-level supervisory control of $\tilde{G}_{lo}$. Only partial disablement of high-level fuzzy events is possible. Assume the low-level FDES $\tilde{G}_{lo}$ is output control consistent. Let the high-level specification language of $\tilde{G}_{hi}$ is given by $\bar{\tilde{k}}_{hi} \subseteq \tilde{L}_{hi}$ which is prefix-closed and fuzzy controllable with respect to $\tilde{L}_{hi}$. The inverse map $\theta^{-1}(\bar{\tilde{k}}_{hi})$ generates the corresponding prefix-closed low-level fuzzy specification language $\bar{\tilde{k}}_{lo} \subseteq \tilde{L}_{lo}$ of $\tilde{G}_{lo}$.

Note that same as in crisp DES case, in FDES the derived low-level specification $\bar{\tilde{k}}_{lo}$ for $\tilde{G}_{lo}$, is not necessarily fuzzy controllable. Let $\theta^{-1}(\tilde{\tau})_m$ represent the language of *marked* low-level fuzzy strings which generate $\tilde{\tau}$ in high-level with mapping θ. A language $\tilde{m}_{lo}$ can be constructed as follows.

Let $\tilde{t} \in \bar{\tilde{k}}_{hi}$, $\forall\ \tilde{\tau} \in \tilde{T}$ where, $\tilde{t}\tilde{\tau} \in \bar{\tilde{k}}_{hi}$: $\theta^{-1}(\tilde{\tau})_m \subseteq \tilde{m}_{lo}$.

Assume $\bar{\tilde{k}}^{\uparrow}_{lo}$ represents the supremal fuzzy controllable prefix-closed sub language of $\bar{\tilde{k}}_{lo}$. The computation of $\bar{\tilde{k}}^{\uparrow}_{lo}$ can be performed using following steps.

1) If $\bar{\tilde{k}}_{lo}$ contains a fuzzy string $\tilde{s} = \tilde{\sigma}_1..\tilde{\sigma}_k$, such that for some i $(1 \leq i \leq k)$, $\tilde{\Sigma}_c(\tilde{\sigma}_i) = 1$ and $\tilde{s} \in (\tilde{L}_{lo} \backslash \bar{\tilde{k}}_{lo})/\tilde{\Sigma}^*_{uc}$ (refer to the quotient operation in [2]) then, any string in $\bar{\tilde{k}}_{lo}$ which contains $\tilde{s}$ as a prefix must be removed by complete disablement of $\tilde{\sigma}_i$ (which is same as in crisp DES).
2) With the remaining fuzzy strings, $\bar{\tilde{k}}^{\uparrow}_{lo}$ can be achieved as described in Algorithm 2.

input : $\tilde{T}, \tilde{L}_{hi}, \bar{\tilde{k}}_{hi}, \tilde{\Sigma}, \tilde{L}_{lo}, \tilde{m}_{lo}$
output: $\bar{\tilde{k}}^{\uparrow}_{lo}$
for $\forall\ \tilde{\tau} \in \tilde{T}$ **do**
  Let $\tilde{t} \in \bar{\tilde{k}}_{hi}$ and $\eta = \tilde{T}_{uc}(\tilde{\tau}) \tilde{\cap} \tilde{L}_{hi}(\tilde{t}\tilde{\tau})$. Compute the degree of $\tilde{\tau}$ being enabled by $\tilde{S}_{hi}$ as follows:

$$\tilde{S}_{hi,\tilde{t}}(\tilde{\tau}) = \begin{cases} \eta, & \text{if } \eta \geq \bar{\tilde{k}}_{hi}(\tilde{t}\tilde{\tau}) \\ \bar{\tilde{k}}_{hi}(\tilde{t}\tilde{\tau}), & \text{otherwise.} \end{cases}$$

  Assign $\tilde{m}_{lo}\left(\theta^{-1}(\tilde{\tau})_m\right) = \tilde{S}_{hi,\tilde{t}}(\tilde{\tau})$
end
Let $|\tilde{m}_{lo}| = n$ `//` $\tilde{m}_{lo}$ `has` n `number of strings`
Let $\tilde{r} \in \tilde{L}^{MP}_{lo}$ such that $\theta(\tilde{r}) = \tilde{t}$ `//` $\tilde{r}$ `is in the main path`
for $\tilde{s}_i \in \tilde{m}_{lo}, i \in (1, ..., n)$ **do**
  Let $\tilde{s}_i = \tilde{\sigma}_0 ... \tilde{\sigma}_k$, $\tilde{\sigma}_0 = \varepsilon$, and $\bar{\tilde{k}}^{\uparrow}_{lo}(\varepsilon) = 1$
  for $\tilde{\sigma}_j, j \in (1, ..., k)$ **do**
    Let $\zeta = \tilde{\Sigma}_{uc}(\tilde{\sigma}_j) \tilde{\cap} \tilde{L}_{lo}(\tilde{r}\tilde{\sigma}_j)$
    Calculate the feasibility of $\tilde{\sigma}_j$ being available in $\tilde{s}_i$, $\tilde{F}_{\tilde{s}_i}(\tilde{\sigma}_j)$ as follows:

$$\tilde{F}_{\tilde{s}_i}(\tilde{\sigma}_j) = \begin{cases} \zeta, & \text{if } \zeta \geq \tilde{m}_{lo}(\tilde{s}_i) \\ \tilde{m}_{lo}(\tilde{s}_i), & \text{otherwise.} \end{cases}$$

$$\bar{\tilde{k}}^{\uparrow}_{lo}(\tilde{r}\tilde{\sigma}_j) = \bar{\tilde{k}}^{\uparrow}_{lo}(\tilde{r}\tilde{\sigma}_{j-1}) \tilde{\cap} \tilde{F}_{\tilde{s}_i}(\tilde{\sigma}_j) \tilde{\cap} \tilde{L}_{lo}(\tilde{r}\tilde{\sigma}_j)$$

  end
end

Algorithm 2: The computation of $\tilde{k}^{\uparrow}_{lo}$

Example 4: Assume we want to enforce the occurrence of high-level fuzzy event $\tilde{\tau}_1 \in \bar{\tilde{k}}_{hi}$ in figure 2(b) while keeping the possibilities occurring of other high-level fuzzy events (or strings) at their minimum levels (which is equal to their degrees of being uncontrollable). There exist three different low-level fuzzy string paths for achieving $\tilde{\tau}_1$.

$$\theta^{-1}(\tilde{\tau}_1) = \{\varepsilon, \tilde{\sigma}_{1,2}, \tilde{\sigma}_{1,2}\tilde{\sigma}_{2,3}, \tilde{\sigma}_{1,2}\tilde{\sigma}_{2,3}\tilde{\sigma}_{3,4}, \tilde{\sigma}_{1,2}\tilde{\sigma}_{2,4}, \tilde{\sigma}_{1,3}, \tilde{\sigma}_{1,3}\tilde{\sigma}_{3,4}\}.$$

$$\tilde{\theta}^{-1}(\tilde{\tau}_1)_m = \{\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,3}\tilde{\sigma}_{3,4}, \tilde{\sigma}_{1,2}\tilde{\sigma}_{2,4}, \tilde{\sigma}_{1,3}\tilde{\sigma}_{3,4}\} \subseteq \tilde{m}_{lo}$$

Assume $\tilde{L}_{lo}(\tilde{\sigma}_{1,2}) = \tilde{L}_{lo}(\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,3}) = \tilde{L}_{lo}(\tilde{\sigma}_{1,3}) = 1$. Note that $\tilde{L}_{lo}(\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,3}\tilde{\sigma}_{3,4}) = 0.9$, $\tilde{L}_{lo}(\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,4}) = 0.5$, and $\tilde{L}_{lo}(\tilde{\sigma}_{1,3}\tilde{\sigma}_{3,4}) = 0.7$. Let $\bar{\tilde{k}}_{hi}(\tilde{\tau}_1) = 0.7 \rightarrow \tilde{S}_{hi,\varepsilon}(\tilde{\tau}_1) = 0.7$.

Then $\tilde{m}_{lo}(\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,3}\tilde{\sigma}_{3,4}, \tilde{\sigma}_{1,2}\tilde{\sigma}_{2,4}, \tilde{\sigma}_{1,3}\tilde{\sigma}_{3,4}) = 0.7$ implies that: $\tilde{k}^{\uparrow}_{lo}(\tilde{\sigma}_{1,2}) = \tilde{k}^{\uparrow}_{lo}(\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,3}) = \bar{\tilde{k}}^{\uparrow}_{lo}(\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,3}\tilde{\sigma}_{3,4}) = 0.7$, $\tilde{k}^{\uparrow}_{lo}(\tilde{\sigma}_{1,2}\tilde{\sigma}_{2,4}) = 0.5$, and $\tilde{k}^{\uparrow}_{lo}(\tilde{\sigma}_{1,3}) = \tilde{k}^{\uparrow}_{lo}(\tilde{\sigma}_{1,3}\tilde{\sigma}_{3,4}) = 0.7$.

It is true that $\bar{\tilde{k}}^{\uparrow}_{lo} \subseteq \bar{\tilde{k}}_{lo}$ and $\bar{\tilde{k}}^{\uparrow}_{lo}(\tilde{s}) \leq \bar{\tilde{k}}_{lo}(\tilde{s})$. As a result, $\theta(\bar{\tilde{k}}^{\uparrow}_{lo}) \subseteq \bar{\tilde{k}}_{hi}$ and $\theta(\bar{\tilde{k}}^{\uparrow}_{lo})(\tilde{t}) \leq \bar{\tilde{k}}_{hi}(\tilde{t})$. Hence, same as in crisp DES case, only a sub set of high-level specification can be achieved by low-level supervisory control of FDES.

An attempt to enhance the possibility of the desired fuzzy string $\tilde{\tau}_1\tilde{\tau}_5$ in high-level indeed requires to increase the possibility of occurring of the low-level fuzzy event $\tilde{\sigma}_{4,5}$ to a necessary higher degree. This in turn increases the possibilities of low-level fuzzy strings $\tilde{\sigma}_{4,5}\tilde{\sigma}_{5,7}$ and $\tilde{\sigma}_{4,5}\tilde{\sigma}_{5,6}\tilde{\sigma}_{6,7}$ occurring in $\tilde{G}_{lo}$, since their degrees of being uncontrollable are depend on that of $\tilde{\sigma}_{4,5}$ (i.e. $\tilde{\Sigma}_{uc}(\tilde{\sigma}_{4,5}\tilde{\sigma}_{5,7}) = \tilde{\Sigma}_{uc}(\tilde{\sigma}_{4,5}\tilde{\sigma}_{5,6}\tilde{\sigma}_{6,7}) = \tilde{\Sigma}_{uc}(\tilde{\sigma}_{4,5})$). As a result, This will eventually increases the possibility of occurring of the undesired high-level fuzzy string $\tilde{\tau}_1\tilde{\tau}_4$ in $\tilde{G}_{hi}$. This leads to define a *strictly-output-control-consistency* condition for FDES as given below.

Definition 6: Consider a hierarchical system with a low-level FDES $\tilde{G}_{lo}$ and its high-level FDES $\tilde{G}_{hi}$. If $\tilde{G}_{lo}$ is *output-control consistent*, and increasing the possibility of occurring of a desired high-level fuzzy event by controlling its corresponding low-level fuzzy string does not increase the possibility of occurring of an undesired high-level fuzzy event (more than its degree of being uncontrollable), then FDES $\tilde{G}_{lo}$ is said to be *strictly-output-control-consistent*.

Note that the property of *strictly-output-control-consistency* of FDES implies that the low-level system allows to enable/disable each high-level fuzzy event individually, according to its degree of being controllable.

When the low-level FDES system satisfies the *strictly-output-control-consistency* we say the low-level FDES is *hierarchically-consistent* with its high-level abstraction (i.e. The high-level specification can be exactly achieved by low-level supervisory control). Note that in this case $\theta(\bar{\tilde{k}}^{\uparrow}_{lo}) = \bar{\tilde{k}}_{hi}$ and $\theta(\bar{\tilde{k}}^{\uparrow}_{lo})(\tilde{t}) = \bar{\tilde{k}}_{hi}(\tilde{t})$ for $\tilde{t} \in \tilde{T}^*$. If the low-level FDES system is not *strictly-output-control-consistent*, then $\theta(\bar{\tilde{k}}^{\uparrow}_{lo})(\tilde{t}) \neq \bar{\tilde{k}}_{hi}(\tilde{t})$.

When unobservability is associated with low-level fuzzy events of $\tilde{G}_{lo}$, the high-level specification language $\tilde{\tilde{k}}_{hi}$ of $\tilde{G}_{hi}$ has to incorporate the resulting high-level fuzzy events and strings with their relevant degrees. This leads to extend the *H-observability* of crisp DES in [7] to FDES as mentioned in Definition 7.

Definition 7: Consider a hierarchical FDES system with a high-level FDES $\tilde{G}_{hi}$, its specification language $\tilde{\tilde{k}}_{hi} \subseteq \tilde{L}_{hi}$, a low-level FDES $\tilde{G}_{lo}$ and its fuzzy controllable and fuzzy observable specification $\tilde{\tilde{k}}'_{lo} \subseteq \tilde{L}_{lo}$. Let $\tilde{s}, \tilde{s}' \in \tilde{\tilde{k}}'_{lo}$ where $\tilde{s}' \in \tilde{P}^{-1}\left(\tilde{P}(\tilde{s})\right)$. Also $\tilde{\theta}(\tilde{s}) = \tilde{t}, \tilde{\theta}(\tilde{s}') = \tilde{t}'$ where $\tilde{t}, \tilde{t}' \in \tilde{\tilde{k}}_{hi}$. Then the high-level specification language $\tilde{\tilde{k}}_{hi}$ is said to be *H-fuzzy-observable* with respect to $\tilde{\tilde{k}}'_{lo}$ and $\tilde{G}_{hi}$ for all $\tilde{\tau} \in \tilde{T}$, if the inequality at the bottom of the page holds.

Intuitively, this means that the possibility of fuzzy string $\tilde{t}'\tilde{\tau}$ belonging to prefix-closed high-level specification language $\tilde{\tilde{k}}_{hi}$ is greater than or equal to the minimum of the possibility of $\tilde{t}\tilde{\tau}$ belonging to $\tilde{\tilde{k}}_{hi}$ and possibility of $\tilde{t}'$ belonging to $\tilde{\tilde{k}}_{hi}$ together with physical possibility of $\tilde{t}'\tilde{\tau}$ being occurred in high-level, the degree of high-level fuzzy event $\tilde{\tau}$ being controllable and the possibility of $\tilde{P}(\tilde{s}')$ to be observed as same as $\tilde{P}(\tilde{s})$.

Remark 1: Extending from crisp DES to FDES, we can provide following remark.

Let the low-level FDES $\tilde{G}_{lo}$ is *output-control-consistent*, the high-level specification language be $\tilde{\tilde{k}}_{hi} \subseteq \tilde{L}_{hi}$ and the low-level specification be $\tilde{\tilde{k}}_{lo}$ where $\tilde{\tilde{k}}_{lo} = \theta^{-1}(\tilde{\tilde{k}}_{hi})$. If $\tilde{\tilde{k}}_{lo}$ is fuzzy controllable with respect to $\tilde{L}_{lo}$ then $\tilde{\tilde{k}}_{hi}$ is also fuzzy controllable with respect to $\tilde{L}_{hi}$.

Theorem 2: Hierarchical supervisory control theory of FDES with partial observation.

Let low-level FDES $\tilde{G}_{lo}$ be *output-control consistent* with respect to high-level FDES $\tilde{G}_{hi}$. Assume $\tilde{\tilde{k}}_{hi}$ as the prefix-closed fuzzy controllable high-level specification of $\tilde{G}_{hi}$. Also, let $\tilde{\tilde{k}}_{lo} = \theta^{-1}(\tilde{\tilde{k}}_{hi})$ be the corresponding prefix-closed low-level fuzzy specification language of $\tilde{G}_{lo}$ and $\tilde{\tilde{k}}^{\uparrow}_{lo}$ represent the supremal prefix-closed fuzzy sub language of $\tilde{\tilde{k}}_{lo}$ which is fuzzy controllable. Then under the foregoing assumptions there exist a low-level supervisor $\tilde{S}_{lo}$ for $\tilde{G}_{lo}$ such that $\tilde{L}_{\tilde{S}_{lo}/\tilde{G}_{lo}}(\tilde{s}) = \tilde{\tilde{k}}^{\uparrow}_{lo}(\tilde{s})$ and $\theta(\tilde{\tilde{k}}^{\uparrow}_{lo}) = \tilde{\tilde{k}}_{hi}$ if following conditions are hold.

1. $\tilde{G}_{lo}$ is *strictly-output-control-consistent*.
2. $\tilde{\tilde{k}}^{\uparrow}_{lo}$ is fuzzy observable with respect to $\tilde{L}_{lo}, \tilde{P}$ and $\tilde{\Sigma}_c$.
3. $\tilde{\tilde{k}}_{hi}$ is *H-fuzzy-observable* with respect to $\tilde{\tilde{k}}^{\uparrow}_{lo}$ and $\tilde{G}_{hi}$.

Proof: Omitted.

IV. Conclusion

In this paper, we have discussed the hierarchical supervisory control of FDES in a general setting, in which each fuzzy event is associated with a degree of controllability and a degree of observability. Some important properties of crisp DES are extended and redefined for FDES to maintain the *hierarchical-consistency* between low-level and high-level FDES modules. Finally, a hierarchical supervisory control theory of FDES is established.

Studies on hierarchical control of decentralized FDES will be an interesting research area.

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$$\tilde{\tilde{k}}_{hi}(\tilde{t}'\tilde{\tau}) \geq \tilde{\tilde{k}}_{hi}(\tilde{t}\tilde{\tau}) \tilde{\cap} \tilde{\tilde{k}}_{hi}(\tilde{t}') \tilde{\cap} \tilde{L}_{hi}(\tilde{t}'\tilde{\tau}) \tilde{\cap} \tilde{T}_c(\tilde{\tau}) \tilde{\cap} \tilde{P}^{-1}\left(\tilde{P}(\tilde{s})\right)(\tilde{s}')$$