2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

Distributed Multi-Agent Tracking and Estimation with Uncertain Agent Dynamics

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Abstract— **This paper addresses distributed target tracking and estimation using multiple mobile agents whose dynamics are subject to uncertainties and disturbances. A consensus-based distributed estimator is applied to estimate the motion of the target. To compensate for the uncertainties in the dynamics and prevent them from propagating into the communication network, a cascaded control structure is proposed, which uses the $\mathcal{L}_1$ adaptive controller to drive the real uncertain system to an ideal closed-loop system obtained from an existing flocking algorithm. Since the communication graph is induced by the position of the real agents, we cannot exactly implement the ideal flocking algorithm, which leads to coupling between the communication topology and the system dynamics. The guaranteed transient performance bounds of the $\mathcal{L}_1$ adaptive controller are essential towards resolving this coupling issue. Extensive simulation results demonstrate the capability of the proposed algorithms to recover the desired flocking behavior.**

I. Introduction

During the past decade there has been an increasing interest in the area of distributed control and estimation of multiple autonomous agents among the robotics and the control communities. This interest has been highly motivated by numerous applications such as distributed sensing, transportation, space exploration, etc.

An important topic in the multi-agent system research is the consensus algorithm, which aims to drive a team of agents to reach an agreement on a common value by negotiating with their neighbors. Originated from the area of parallel computation and distributed optimization, consensus algorithms have been extensively developed for various types of systems with different assumptions on the communication topology [1], [2], [3], [4].

In the area of distributed estimation, Stanković et al. proposed a consensus-based overlapping estimation framework, for both parameter and state estimation, in both continuous time [5] and discrete time [6]. The idea is to combine a local Kalman filter and the consensus algorithm for each agent. The framework has been applied to deep space formation control problems [7]. A distributed Kalman filter based on consensus algorithm has also been studied by Olfati-Saber [8], [9].

In [10], Olfati-Saber proposed a class of algorithms which lead to flocking of agents, velocity consensus, target following and obstacle avoidance. The algorithms are based on artificial potential, consensus algorithm and a navigational feedback term. In [11], the authors combine the flocking algorithm with a consensus based distributed Kalman filter to track a moving target.

However, most of the above mentioned references consider only ideal agent models, such as the ideal single/double integrator or the ideal nonholonomic unicycle (Dubin's car) model. Few results can be found in literature that explicitly consider uncertainties in the agent dynamics, which are inherent to real applications. In the presence of uncertainty, a consensus algorithm may fail to guarantee consensus or even diverge, possibly due to the propagation of uncertainties in the network, as is demonstrated in [12]. This motivates us to investigate flocking algorithms in the presence of uncertainties and disturbances in the agent dynamics.

This paper considers distributed target tracking and estimation using multiple autonomous agents with uncertain dynamics, which is different from [11]. Each agent implements a consensus based estimator from [5]. We propose a cascaded control structure which generate a reference signal based on the ideal flocking algorithm and use the $\mathcal{L}_1$ adaptive control structure [13] to compensate for the system uncertainties. The guaranteed transient performance of the $\mathcal{L}_1$ adaptive control architecture plays the key role to resolve the coupling issue introduced by the cascaded control structure.

This paper is organized as follows. Section II presents the problem formulation and some preliminaries. Section III applies the consensus-based distributed Kalman filter to estimate the motion of the target using multiple agents. Section IV proposes a cascaded control structure for a fleet of agents with uncertain dynamics to track a moving target. Section V verifies the proposed estimation and formation control algorithms by numerical simulations.

II. Problem Formulation

Consider a group of N mobile agents (UAVs or ground robots) tasked to track a moving target in an n-dimensional space, where $n = 2, 3$. The dynamics of each agent is described by

$$\begin{aligned} \dot{q}_i(t) &= p_i(t)\,, \\ \dot{p}_i(t) &= \omega_i u_i(t) + \theta_i(t) p_i(t) + \sigma_i(t)\,, \\ q_i(0) &= q_{i0}\,, \quad p_i(0) = p_{i0}\,, \quad i = 1, 2, \ldots, N\,, \end{aligned} \tag{1}$$

where $q_i\,, p_i \in \mathbb{R}^n$ are the position and velocity of the agent, respectively, $u_i \in \mathbb{R}^n$ is the control input, $\omega_i \in \mathbb{R}^{n\times n}$ is the unknown constant input gain matrix, $\theta_i \in \mathbb{R}^{n\times n}$ is an unknown matrix of uncertain parameters, and $\sigma_i \in \mathbb{R}^n$ is the unknown disturbance vector. We assume the following conservative bounds for the system uncertainties:

Research is sponsored by ARO grant W911NF-10-1-0044.
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Assumption 1: The input gain matrix ω_i is assumed to be nonsingular. Also there exist known conservative sets $\Omega_i \subset \mathbb{R}^{n\times n}$, $\Theta_i \subset \mathbb{R}^{n\times n}$ and $\Sigma_i \in \mathbb{R}^n$ such that $\omega_i \in \Omega_i$, $\theta_i \in \Theta_i$ and $\sigma_i \in \Sigma_i$.

Assumption 2: Let $\theta_i(t)$ and $\sigma_i(t)$ be continuously differentiable with uniformly bounded derivatives: $\|\dot{\theta}_i(t)\| \le d_{\theta_i}$ and $\|\dot{\sigma}_i\| \le d_{\sigma_i}$.

The target is moving with a constant velocity which is unknown to the agents. Some of the agents are able to obtain noisy measurements of the target's position, with different noise levels. Each agent is equipped with wireless communication device with a limited communication range. To avoid the "blind" agent from being lost in the beginning, we need the following assumption.

Assumption 3: At $t = 0$, each agent that cannot measure the target's coordinates is close enough to at least one agent that has the measurement of the target.

The objective is to design distributed control and estimation laws for each agent using only locally available information to track the target cooperatively.

A. Preliminaries

This section briefly introduces some basic concepts from algebraic graph theory [14]. Of particular importance are the proximity net, which is used to describe the position induced communication topology, and the collective potential function which is the basis of the flocking algorithm [10].

A graph $\mathcal{G}$ is defined as a pair $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ that consists of a set of vertices $\mathcal{V} = \{1, 2, \dots, N\}$ and a set of edges $\mathcal{E} = \{(i, j) : i, j \in \mathcal{V}, j \neq i\}$. A graph $\mathcal{G}$ is undirected if $(i, j) \in \mathcal{E} \Leftrightarrow (j, i) \in \mathcal{E}$.

The adjacency matrix of a graph $\mathcal{G}$ is defined as a matrix $A = [a_{ij}]$, where $a_{ij} \neq 0$ if and only if $(i, j) \in \mathcal{E}$. A graph is called unweighted if $a_{ij} \in \{0, 1\}$; otherwise it is called a weighted graph. The set of neighbors of vertex i is defined by $\mathcal{N}_i = \{j \in \mathcal{V} : a_{ij} \neq 0\} = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$.

For a group of N mobile agents, let $q = \text{col}(q_1, q_2, \dots, q_N)$, and let $r > 0$ be the interaction range between the two agents, i.e., agents i and j can sense and communicate with each other, only if $\|q_j - q_i\| < r$, where $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$.

A proximity net $\mathcal{G}^r(q) = (\mathcal{V}, \mathcal{E}^r(q))$ is a position-induced graph defined by the vertices set $\mathcal{V}$ and the set of edges

$$\mathcal{E}^r(q) = \{(i, j) \in \mathcal{V} \times \mathcal{V} : \|q_j - q_i\| < r, i \neq j\}.$$

Each vertex $i \in \mathcal{V}$ corresponds to an agent, and the collective position vector q is called the configuration of the proximity net. Given an interaction range r and a configuration q, the set of spacial neighbors of vertex (agent) i of the proximity net $\mathcal{G}^r(q)$ is given by

$$\mathcal{N}_i^r(q) = \{j \in \mathcal{V} : \|q_j - q_i\| < r, j \neq i\}.$$

The "σ-norm" of a vector $z \in \mathbb{R}^n$ is a map $\mathbb{R}^n \to \mathbb{R}^+$, defined as

$$\|z\|_\sigma = \frac{1}{\epsilon}\left[\sqrt{1 + \epsilon\|z\|^2} - 1\right],$$

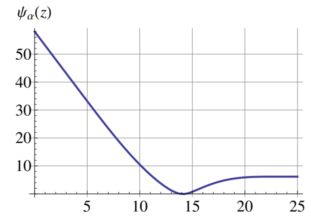


Fig. 1: Example of a pairwise potential function ψ_α

with a parameter $\epsilon > 0$ and its gradient given by $\sigma_\epsilon(z) \triangleq \nabla_z\|z\|_\sigma = \frac{z}{\sqrt{1+\epsilon\|z\|_2^2}} = \frac{z}{1+\epsilon\|z\|_\sigma}$, where $\mathbb{R}^+$ denotes the set of non-negative real numbers. Note that the "σ-norm" is not a norm. An important property is that the map $\|z\|_\sigma$ is differentiable everywhere, while the 2-norm $\|z\|$ is not differentiable at $z = 0$.

For a proximity net $\mathcal{G}^r(q)$, consider a spacial adjacency matrix $A(q) = [a_{ij}(q)]$, given by

$$a_{ij}(q) = \begin{cases} 0, & \text{if } j = i, \\ \rho_h(\|q_j - q_i\|_\sigma / \|r\|_\sigma), & \text{if } j \neq i, \end{cases}$$

where $\rho_h : \mathbb{R}^+ \to [0, 1]$, $h \in (0, 1)$ is a bump function

$$\rho_h(z) = \begin{cases} 1, & \text{if } z \in [0, h), \\ \frac{1}{2}\left[1 + \cos\left(\pi\frac{z-h}{1-h}\right)\right], & \text{if } z \in [h, 1], \\ 0, & \text{otherwise}. \end{cases}$$

The graph Laplacian associated with the proximity net $\mathcal{G}^r(q)$ is defined as $L(q) = \Delta(q) - A(q)$, where $\Delta(q) = \text{diag}\left(\sum_j a_{1j}(q), \sum_j a_{2j}(q), \dots, \sum_j a_{Nj}(q)\right)$ is called the degree matrix of $\mathcal{G}^r(q)$.

The design of the flocking algorithm involves a smooth collective potential function

$$V(q) = \frac{1}{2}\sum_i\sum_{j\neq i} V_{ij}(q) = \frac{1}{2}\sum_i\sum_{j\neq i}\psi_\alpha(\|q_j - q_i\|_\sigma), \quad (2)$$

where $\psi_\alpha : \mathbb{R}^+ \to \mathbb{R}^+$ (see Figure 1) is a nonnegative smooth attractive/repulsive pairwise potential function of the "distance" $\|q_i - q_j\|_\sigma$ between agents i and j. Here, $\psi_\alpha(z)$ reaches its maximum and global minimum at $z = 0$ and $z = \|d\|_\sigma$, respectively, and becomes constant for $z \ge \|r\|_\sigma$, where $0 < d < r$ is the desired distance between two agents. The detailed definition of ψ_α is given in [10].

III. Consensus-based Distributed Estimation

The dynamics of the target are given by

$$\dot{\xi}(t) = A\xi(t) + w(t), \quad (3)$$

where $\xi = [q_{\text{t}}^\top, p_{\text{t}}^\top]^\top$, $A = \begin{bmatrix} 0_{n\times n} & \mathbb{I}_n \\ 0_{n\times n} & 0_{n\times n} \end{bmatrix}$ and $q_{\text{t}}, p_{\text{t}} \in \mathbb{R}^n$ are the position and the velocity of the target, respectively. The subscript t denotes the target.

Each mobile agent (sensor) may or may not measure the position of the target. For each measuring agent i, the measurement equation is given by

$$y_i(t) = C_i\xi(t) + v_i(t),$$

where $C_i = [\mathbb{I}_n \,, 0_{n\times n}]$. In the aforementioned equations, $w(t)$ and $v_i(t)$ are zero-mean white Gaussian noise with covariances

$$\mathbf{E}\{w(t)w^\top(\tau)\} = Q(t)\delta(t-\tau)\,, \mathbf{E}\{v_i(t)w^\top(\tau)\} = 0\,,$$
$$\mathbf{E}\{v_i(t)v_i^\top(\tau)\} = R_i(t)\delta(t-\tau)\,, \mathbf{E}\{v_i(t)v_j^\top(\tau)\} = 0\,.$$

In [5], a consensus-based overlapping distributed state estimation algorithm is proposed for a general class of LTI systems. Since each measuring agent accesses the same part of the target dynamics, namely, the position, our estimation problem can be viewed as a special case of the problems covered by the algorithm of [5].

The consensus-based distributed estimator consists of a local filter f_{local_i} and a first order consensus law f_{con_i}

$$\dot{\hat{\xi}}_i(t) = f_{\text{local}_i} + f_{\text{con}_i}\,, \tag{4}$$

where $f_{\text{local}_i} = A\hat{\xi}_i(t) + L_i(y_i(t) - C_i\hat{\xi}_i(t))$ for agent i that can measure the target position and $f_{\text{local}_i} = 0$ for agent i that cannot measure, and $f_{\text{con}_i} = \sum_{j\in\bar{\mathcal{N}}_i^r(q)} K_{ij}(\tilde{\xi}_{ij}(t) - \hat{\xi}_i(t))$. In the above equations, L_i is the steady state Kalman gain given by $L_i = P_iC_i^\top R_i^{-1}$, P_i is the solution of the algebraic Riccati equation $AP_i + P_iA^\top - P_iC_i^\top R_i^{-1}C_iP_i + Q = 0$, $\bar{\mathcal{N}}_i^r(q)$ consists of agent i's neighbors that can measure the target's position, K_{ij} is the matrix of the consensus gains, and $\tilde{\xi}_{ij} = \hat{\xi}_j + w_{ij}$ is the estimate of the states of agent j received by agent i, where w_{ij} is a $2n$-dimensional zero-mean white communication noise between agents j and i with covariance $\mathbf{E}\{w_{ij}(t)w_{ij}^\top(\tau)\} = W_{ij}(t)\delta(t-\tau)$. For the selection of consensus matrix, please refer to [5].

IV. Flocking Algorithm for Uncertain Agent Dynamics

This section starts with a brief introduction to the tracking algorithm in [10] for ideal double-integrator agent dynamics. For uncertain agent dynamics, a cascaded control structure is proposed, which uses the ideal closed-loop system as a reference model and compensates for the uncertainty by an $\mathcal{L}_1$ adaptive controller. The guaranteed transient performance of the $\mathcal{L}_1$ adaptive control is the key to resolve the coupling between the communication topology and the collective dynamics introduced by the cascaded structure. In this section we assume that the target's motion is known to all agents.

A. *Flocking Algorithm for Ideal Agents*

Without uncertainties and disturbances, the ideal agent's dynamics are given by

$$\begin{aligned}\dot{q}_i^{\text{id}} &= p_i^{\text{id}}\,,\\ \dot{p}_i^{\text{id}} &= u_i^{\text{id}}\,,\end{aligned} \tag{5}$$

where $q_i^{\text{id}}, p_i^{\text{id}}, u_i^{\text{id}} \in \mathbb{R}^n$ are the position, velocity, and acceleration, respectively, of the ideal agent i, $i = 1, 2, \ldots, N$. Let $q^{\text{id}} = [(q_1^{\text{id}})^\top, \ldots, (q_N^{\text{id}})^\top]^\top$ and $p^{\text{id}} = [(p_1^{\text{id}})^\top, \ldots, (p_N^{\text{id}})^\top]^\top$ be the collective position and velocity, respectively.

For the ideal agents (5) to track a target, Olfati-Saber proposed a flocking algorithm (see Algorithm-2 in [10]):

$$u_i^{\text{id}} = f_i^g + f_i^d + f_i^\gamma\,, \tag{6}$$

where $f_i^g = -\nabla_{q_i}V(q^{\text{id}})$ is a gradient-based force to regulate the distance between agent i and its neighbors, $f_i^d = -\sum_{j\in\mathcal{N}_i^r(q^{\text{id}})} a_{ij}(q^{\text{id}})(p_i^{\text{id}} - p_j^{\text{id}})$ is the velocity consensus term aligning the speed of each agent to its neighbors, and $f_i^\gamma = -c_1(q_i^{\text{id}} - q_{\text{t}}) - c_2(p_i^{\text{id}} - p_{\text{t}})$, $c_1, c_2 > 0$ is the navigational feedback due to the tracking objective of the group.

To analyze the algorithm (6), define $\tilde{q}_i^{\text{id}} = q_i^{\text{id}} - q_{\text{t}}$ and $\tilde{p}_i^{\text{id}} = p_i^{\text{id}} - p_{\text{t}}$ as the relative position and velocity between each ideal agent and the target, respectively. Notice that $q_i^{\text{id}} - q_j^{\text{id}} = \tilde{q}_i^{\text{id}} - \tilde{q}_j^{\text{id}}$, $p_i^{\text{id}} - p_j^{\text{id}} = \tilde{p}_i^{\text{id}} - \tilde{p}_j^{\text{id}}$, and $\nabla_{q_i^{\text{id}}}V(q^{\text{id}}) = \nabla_{\tilde{q}_i^{\text{id}}}V(q^{\text{id}})$. Thus, we have the following *ideal relative dynamics*:

$$\begin{aligned}\dot{\tilde{q}}_i^{\text{id}} =& \tilde{p}_i^{\text{id}}\,,\\ \dot{\tilde{p}}_i^{\text{id}} =& -c_1\tilde{q}_i^{\text{id}} - c_2\tilde{p}_i^{\text{id}} - \textstyle\sum_{j\in\mathcal{N}_i^r(q^{\text{id}})} a_{ij}(q)(\tilde{p}_i^{\text{id}} - \tilde{p}_j^{\text{id}})\\ & - \nabla_{\tilde{q}_i^{\text{id}}}V(q^{\text{id}})\,,\end{aligned} \tag{7}$$

which can be further rewritten in a compact form as

$$\dot{x}_i^{\text{id}}(t) = A_m x_i^{\text{id}}(t) + B_m r_i^{\text{id}}(t)\,, \quad x_i^{\text{id}}(0) = x_{i0}^{\text{id}} \tag{8}$$

where $x_i^{\text{id}} = [(\tilde{q}_i^{\text{id}})^\top, (\tilde{p}_i^{\text{id}})^\top]^\top$, $r_i^{\text{id}} = -\nabla_{\tilde{q}_i^{\text{id}}}V(q^{\text{id}}) + \sum_{j\in\mathcal{N}_i^r(q^{\text{id}})} a_{ij}(q^{\text{id}})(\tilde{p}_j^{\text{id}} - \tilde{p}_i^{\text{id}})$, $A_m = \begin{bmatrix} 0_{n\times n} & \mathbb{I}_n \\ -c_1\mathbb{I}_n & -c_2\mathbb{I}_n \end{bmatrix}$, and $B_m = [0_{n\times n} \;\; \mathbb{I}_n]^\top$.

In order for the ideal relative dynamics (8) to serve as a desired reference model, it is important to have $r_i^{\text{id}}(t)$ bounded. The following lemma is similar to the result in [15], in which only part of the agents have the knowledge of the target.

Lemma 1: Consider a system of N mobile agents, each with dynamics (5) and steered by the control protocol (6). Suppose that the initial energy $Q_0 \triangleq Q(q^{\text{id}}(0), p^{\text{id}}(0))$ is finite. Then

1) $\|q_i^{\text{id}}(t) - q_{\text{t}}(t)\|_2 \le \sqrt{2Q_0/c_1}$ for all $t > 0$ and i;
2) The velocities of all agents approach the target's velocity p_{t} asymptotically,

where $Q(q^{\text{id}}, p^{\text{id}}) = \frac{1}{2}\sum_{i=1}^N (U_i(q^{\text{id}}) + \|\tilde{p}_i^{\text{id}}\|_2^2)$ and $U_i(q^{\text{id}}) = \sum_{j=1}^N \psi_\alpha(\|q_i^{\text{id}} - q_j^{\text{id}}\|_\sigma) + c_1\|\tilde{q}_i^{\text{id}}\|_2^2$.

Proof: Let $\tilde{q}^{\text{id}} = [\tilde{q}_1^{\text{id}}, \ldots, \tilde{q}_N^{\text{id}}]^\top$ and $\tilde{p}^{\text{id}} = [\tilde{p}_1^{\text{id}}, \ldots, \tilde{p}_N^{\text{id}}]^\top$ be the collective relative position and velocity, respectively. Then (7) can be written as

$$\begin{aligned}\dot{\tilde{q}}^{\text{id}} &= \tilde{p}^{\text{id}}\,,\\ \dot{\tilde{p}}^{\text{id}} &= -\nabla_{\tilde{q}^{\text{id}}}V(q^{\text{id}}) - \hat{L}(q^{\text{id}})\tilde{p}^{\text{id}} - (c_1\tilde{q}^{\text{id}} + c_2\tilde{p}^{\text{id}})\,,\end{aligned} \tag{9}$$

where $\hat{L}(q^{\text{id}}) = L(q^{\text{id}}) \otimes \mathbb{I}_n$. By the definitions of $\tilde{q}^{\text{id}}$, $\tilde{p}^{\text{id}}$ and $V(q^{\text{id}})$, $Q(q^{\text{id}}, p^{\text{id}})$ can be written as

$$Q(q^{\text{id}}, p^{\text{id}}) = V(q^{\text{id}}) + \frac{1}{2}(\tilde{p}^{\text{id}})^\top\tilde{p}^{\text{id}} + \frac{1}{2}c_1(\tilde{q}^{\text{id}})^\top\tilde{q}^{\text{id}}\,.$$

Taking the time derivative of Q and considering the collective relative dynamics (9), we have

$$\begin{aligned}\dot{Q} &= (\dot{\tilde{q}}^{\text{id}})^\top\nabla_{\tilde{q}^{\text{id}}}V(q^{\text{id}}) + c_1(\tilde{q}^{\text{id}})^\top\dot{\tilde{q}}^{\text{id}} + (\tilde{p}^{\text{id}})^\top\dot{\tilde{p}}^{\text{id}}\\ &= -(\tilde{p}^{\text{id}})^\top(\hat{L}(q^{\text{id}}) + c_2I_{2N})\tilde{p}^{\text{id}} \le 0\,,\end{aligned} \tag{10}$$

which implies $Q(q^{\text{id}}, p^{\text{id}}) \le Q_0$ for all $t \ge 0$. Hence, the distance between each agent and the target verifies $\|\tilde{q}_i(t)\| \le \sqrt{(\tilde{q}^{\text{id}}(t))^\top\tilde{q}^{\text{id}}(t)} \le \sqrt{2Q_0/c_1}$ for all $t \ge 0$. From LaSalle's invariance principle, all the solutions of (9), starting from $\Psi \triangleq \{(\tilde{q}^{\rm id}, \tilde{p}^{\rm id}) : Q(\tilde{q}^{\rm id}, \tilde{p}^{\rm id}) \le Q_0\}$, converge to the largest invariant set $\{(\tilde{q}^{\rm id}, \tilde{p}^{\rm id}) \in \Psi : \dot{Q} = 0\}$, which, according to (10), leads to $\tilde{p}_i^{\rm id} = 0$ for all i. ■

The ideal double-integrator agents, if the conjectures in [10] hold, will form a flock asymptotically with all the agents' velocity converging to the target's velocity asymptotically. However, in the presence of uncertainties, if the algorithm (6) is applied blindly, these properties may not hold any more, as demonstrated in Section V-A. This motivates the design of a cascaded control structure (similar to [12]) for compensation of the uncertainties locally and preventing the propagation of those into the network, as is presented in the next section.

B. Cascaded Control Structure

Similar to the ideal agent case, let $\tilde{q}_i = q_i - q_{\rm t}$ and $\tilde{p}_i = p_i - p_{\rm t}$ be the relative position and velocity between each real agent and the target, respectively. Letting $x_i = [\tilde{q}_i^\top, \tilde{p}_i^\top]^\top$, the relative dynamics between the real agent i and the target can be written as

$$\dot{x}_i(t) = A_m x_i(t) + B_m \left(\omega_i u_i + \vartheta(t) x_i(t) + \sigma_i(t)\right) , \quad (11)$$

where $x_i(0) = x_{i0} \triangleq [(q_i(0) - q_{\rm t}(0))^\top, (p_i(0) - p_{\rm t}(0))^\top]^\top$, and $\vartheta(t) = [c_1\mathbb{I}_n \quad c_2\mathbb{I}_n + \theta(t)]$. From Assumption 1, there exists $\Theta_1 \subset \mathbb{R}^{n\times 2n}$, such that $\vartheta \in \Theta_1$.

The basic idea of the cascaded control structure is to make the real system behave like the ideal system, while avoiding propagation of the uncertainties into the communication network. To achieve this, each agent i implements the double integrator model (5) of the "virtual ideal agent", using its own initial conditions for initialization, i.e., setting $q_i^{\rm id}(0) = q_i(0)$ and $p_i^{\rm id}(0) = p_i(0)$. As the system evolves, each agent exchanges its virtual ideal agent's states with its neighbors and calculates the ideal control input $u_i^{\rm id}$, which yields the closed-loop ideal relative system (8) with the initial condition $x_{i0}^{\rm id} = x_{i0}$. Next, a local $\mathcal{L}_1$ adaptive controller is designed for each agent, using the closed-loop ideal system as the reference model, to compensate for the uncertainties and disturbances of the real agent.

Figure 2 shows the block diagram of the cascaded control structure. Note that instead of broadcasting the position and velocity of the real states, each agent broadcasts the states of the virtual ideal dynamics.

C. $\mathcal{L}_1$ Adaptive Controller Design

The $\mathcal{L}_1$ adaptive controller for the system in (11) consists of three components [13]:

- State Predictor:

$$\dot{\hat{x}}_i(t) = A_m \hat{x}_i(t) + B_m \left(\hat{\omega}_i u_i(t) + \hat{\vartheta}(t) x_i(t) + \hat{\sigma}(t)\right) ,$$
$$\hat{x}_i(0) = x_{i0} \quad (12)$$

- Adaptive Law:

$$\begin{aligned} \dot{\hat{\omega}}_i(t) &= \Gamma \mathbf{Proj}(\hat{\omega}_i(t), -(\tilde{x}_i^\top(t) P B_m)^\top u_i^\top) , \\ \dot{\hat{\vartheta}}_i(t) &= \Gamma \mathbf{Proj}(\hat{\vartheta}_i(t), -(\tilde{x}_i^\top(t) P B_m)^\top x_i^\top(t)) \\ \dot{\hat{\sigma}}_i(t) &= \Gamma \mathbf{Proj}(\hat{\sigma}_i(t), -(\tilde{x}_i^\top(t) P B_m)^\top) \end{aligned} \quad (13)$$

- Control Law:

$$u_i(t) = -K_i D_i(s) \left(\hat{\eta}_i(s) - r_i^{\rm id}(s)\right) , \quad (14)$$

where $r_i^{\rm id}(s)$ and $\eta_i(s)$ are the Laplace transforms of $r_i^{\rm id}(t)$ and $\hat{\eta}_i(t) \triangleq \hat{\omega}_i(t) u_i(t) + \hat{\vartheta}_i(t) x_i(t) + \hat{\sigma}_i(t)$, respectively.

In the above definitions, $\tilde{x}_i(t) = \hat{x}_i(t) - x_i(t)$, $P = P^\top > 0$ is the solution to the algebraic Lyapunov equation $A_m^\top P + P A_m = -Q$, $Q > 0$, $\Gamma > 0$ is the adaptation gain, and $\mathbf{Proj}(\cdot, \cdot)$ denotes the projection operator [16].

The design of the $\mathcal{L}_1$ adaptive controller involves a strictly proper transfer matrix $D_i(s)$ and a gain matrix $K \in \mathbb{R}^{n\times n}$, which lead to a strictly proper stable low-pass filter

$$C(s) \triangleq \omega K(\mathbb{I}_n + D(s)\omega K)^{-1} D(s) \quad (15)$$

with DC gain $C(0) = \mathbb{I}_n$.

The $\mathcal{L}_1$ adaptive controller is defined via (12), (13), (14), subject to the following $\mathcal{L}_1$ norm condition:

$$\|G(s)\|_{\mathcal{L}_1} L < 1 , \quad (16)$$

where $G(s) \triangleq (s\mathbb{I} - A_m)^{-1} B_m (\mathbb{I} - C(s))$ and $L \triangleq \max_{\vartheta\in\Theta_1} \|\vartheta\|_1$.

An important property of the $\mathcal{L}_1$ adaptive controller is that $x_i(t)$ and $u_i(t)$ of the uncertain system (11) can be rendered arbitrarily close to $x_{\rm ref}(t)$ and $u_{\rm ref}(t)$ of a closed-loop reference system (see Section 2.2 of [13]), given according to

$$\begin{aligned} \dot{x}_{\rm ref}(t) &= A_m x_{\rm ref}(t) + B_m(\omega u_{\rm ref}(t) + \theta(t) x_{\rm ref}(t) + \sigma(t)) , \\ u_{\rm ref}(s) &= \omega^{-1} C(s)(r_i^{\rm id}(s) - \eta_{\rm ref}(s)) , \ x_{\rm ref}(0) = x_{i0} \end{aligned} \quad (17)$$

by increasing the adaptation gain Γ and the bandwidth of the filter $C(s)$, where $\eta_{\rm ref}(s)$ is the Laplace transform of $\eta_{\rm ref}(t) = \theta(t) x_{\rm ref}(t) + \sigma(t)$. We have the following proposition.

Proposition 1: The transient error between $x_i(t)$ and $x_i^{\rm id}(t)$, measured by $\|x_i - x_i^{\rm id}\|_{\mathcal{L}_\infty}$ can be rendered arbitrarily small by increasing the adaptation gain Γ and the bandwidth of the low-pass filter $C(s)$.

Proof: The closed-loop reference system (17) can be written in frequency domain as $x_{\rm ref}(s) = G(s)\eta_{\rm ref}(s) + (s\mathbb{I} - A_m)^{-1} B_m C(s) r_i^{\rm id}(s) + (s\mathbb{I} - A_m)^{-1} x_{\rm ref}(0)$. Similarly, the ideal system (8) can be written as $x_i^{\rm id}(s) = (s\mathbb{I} - A_m)^{-1} B_m C(s) r_i^{\rm id}(s) + (s\mathbb{I} - A_m)^{-1} x^{\rm id}(0)$. Since $x_{\rm ref}(0) = x_i^{\rm id}(0) = x_i(0)$, we have $x_{\rm ref}(s) - x_i^{\rm id}(s) = G(s)\eta_{\rm ref}(s)$.

Since the reference system is stable [13], $\eta_{\rm ref}$ is bounded. By the inequality $\|x_{\rm ref} - x_i^{\rm id}\|_{\mathcal{L}_\infty} \le \|G(s)\|_{\mathcal{L}_1} \|\eta_{\rm ref}\|_{\mathcal{L}_\infty}$ and definition of $G(s)$, which is a low-pass filter $(s\mathbb{I} - A_m)^{-1} B_m$ cascaded with a high pass filter $\mathbb{I} - C(s)$, $\|x_{\rm ref} - x_i^{\rm id}\|_{\mathcal{L}_\infty}$ can be rendered arbitrarily small by increasing the bandwidth of $C(s)$. The conclusion of the proposition follows from the triangle inequality. ■

D. Coupling between dynamics and topology

Notice that in Section IV-A, the ideal control input is generated based on the proximity net $\mathcal{G}^r(q^{\rm id})$ induced by ideal configuration $q^{\rm id}$. However, to run the "virtual ideal agent" dynamics and generate the corresponding $r_i^{\rm id}$, each

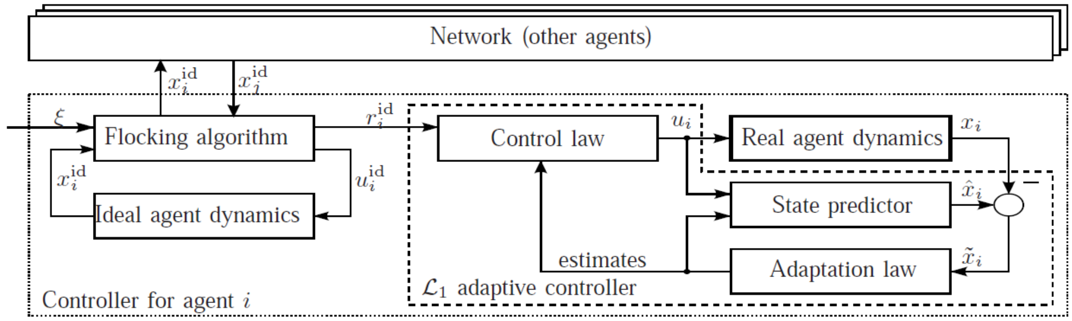


Fig. 2: Flocking control for uncertain agent dynamics

real agent identifies its neighbor based on the real configuration q. This discrepancy may invalidate the results of Lemma 1, because some terms in (10) may not be canceled. An example is shown in Figure 3, in which the real agents i and j cannot communicate, but the vertices i and j in $\mathcal{G}^r(q^{\text{id}})$ are neighbors.

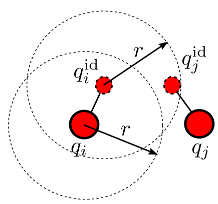


Fig. 3: Coupling Between Dynamics and Topology

To solve this issue, we select an interaction range $\bar{r} < r$ for the ideal agents such that $(i, j) \in \mathcal{E}^r(q) \Rightarrow (i, j) \in \mathcal{E}^{\bar{r}}(q^{\text{id}})$. This is possible because the $\mathcal{L}_1$ adaptive controller guarantees that $\|q_i - q_i^{\text{id}}\|$ can be arbitrarily small. In this case, whenever the ideal agents i and j are neighbors, the corresponding real agents are also neighbors, but not vice versa.

It is worthwhile to mention that the coupling between the topology and the dynamics is due to the fact that the flocking algorithm from [10] does not impose any artificial assumption on the communication network, which is induced naturally by the motion of the agents. This makes it crucial for the adaptive controller to have *guaranteed transient performance* instead of having only asymptotic convergence results; because otherwise, the real agents may deviate from their corresponding "virtual ideal agents" before the asymptotic convergence could happen finally, which will reduce the connectivity of the network without an option of recovering.

V. Simulation Results

In this section we present simulation results of the proposed estimation and tracking algorithms. In Sec V-A we demonstrate the case when the tracking law (6) is applied directly to the uncertain agent dynamics, and each agent broadcasts the its real states to its neighbors. Then in Section V-B we show the results, when the cascaded control structure is used. The following parameters remain fixed throughout all simulations.

- In the ideal flocking algorithm: $d = 5$, $r = 1.4d$, $\epsilon = 0.1$, $h = 0.2$, $c_1 = c_2 = 0.5$ for the bump function.
- In $\mathcal{L}_1$ adaptive controller: $\Gamma_i = 10^4$, $K_i = 20$, $D_i(s) = \frac{1}{s}\mathbb{I}_n$, for each i.

A. Failure of the Flocking Algorithm for Dynamics with Uncertainties

To clearly demonstrate the effects of the uncertainties on the flocking algorithm, we use only 4-agents, each following the dynamics (1). The initial position and velocity of each agent are assigned randomly from the boxes $[-20\,, 20] \times [-20\,, 20]$ and $[-1\,, 1] \times [-1\,, 1]$, respectively. The target is initialized at position $[80\,, 80]^\top$ and velocity $[3\,, 3]^\top$.

The unknown parameters ω_i, $\theta_i(t)$ and the disturbances $\sigma_i(t)$ are given by $\omega_1 = \begin{bmatrix} 0.7 & 2 \\ 0 & 1.2 \end{bmatrix}$, $\omega_2 = \begin{bmatrix} 3 & 2.5 \\ 1.5 & 1.6 \end{bmatrix}$, $\omega_3 = \begin{bmatrix} 0.6 & 0.4 \\ 0.5 & 0.8 \end{bmatrix}$, $\omega_4 = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$; $\theta_1 = \begin{bmatrix} -1 & 0.9 \\ 1.7 & -1.8 \end{bmatrix}$, $\theta_2 = \begin{bmatrix} -0.2 & 0.1 \\ 0.1 & -0.8 \end{bmatrix}$, $\theta_3 = \begin{bmatrix} -1 & 1.5 \\ 0.5 & -0.8 \end{bmatrix}$, $\theta_4 = \begin{bmatrix} -2 & 2.5 \\ 1.3 & -1.8 \end{bmatrix}$, $\sigma_1 = [3 + 2\sin(2t + 0.9)\,, 4 + 3\sin(0.5t - 0.5)]^\top$, $\sigma_2 = [2 + 1\sin(1.8t + 2)\,, 1.5\sin(3.2t)]^\top$,$\sigma_3 = [1 + 4\sin(t + 1)\,, 1.5 + 0.5\sin(3t + 1.5)]^\top$,$\sigma_4 = [0.5 + 3\sin(4t + 0.5)\,, 3 + 2\sin(2t - 3)]^\top$.

When we apply (6) to the above agents directly, assuming each agent knows the position and the velocity of the target, the flocking algorithm fails in a sense that the velocities of all agents do not converge and the system is not self-assembled. Figure 4 shows the time history of the agents' velocity. Figure 5 shows some snapshots of the target positions (denoted by squares), agent positions (denoted by circles), agent velocity directions (denoted by arrows) and communication links (denoted by solid lines) at different times. Notice in Figure 5 that a formed link breaks as the system evolves.

B. Flocking with Cascaded Control Structure

To demonstrate the benefits of the cascaded control structure, we first implement the control algorithms proposed in Sections IV-B-IV-C to the same uncertain agent dynamics given above. Figure 6 shows that the velocity of all the agents converge to the target's velocity, and the group of agents form a flock.

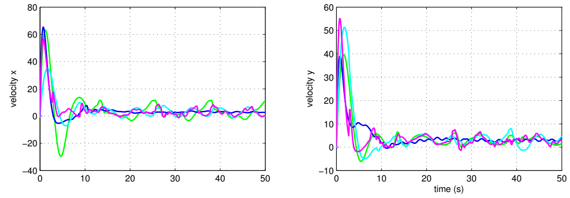
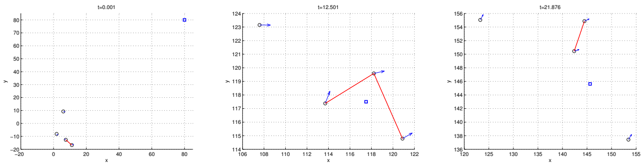


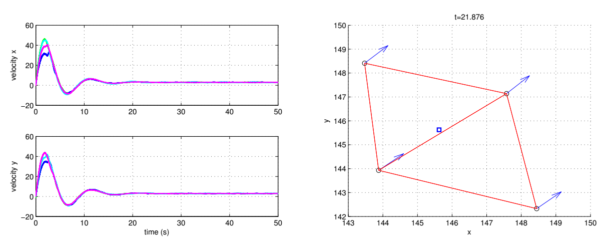
Fig. 4: Velocities of 4 agents without adaptive control



(a) $t = 0.001$ (b) $t = 12.5$ (c) $t = 21.9$

Fig. 5: Flocking for 4 agents without adaptive control

Next, we implement the proposed algorithm to a larger group with 100 agents, 10 of which cannot measure the target and are initialized close to some measuring agents. The initial position and velocity of each measuring agent are assigned randomly from the boxes $[-50, 50] \times [-50, 50]$ and $[-2, 1] \times [-2, 2]$, and the target is initialized with position $[100\,, 0]^\top$ and velocity $[5\,, 0]^\top$. The time history of the velocities and the snapshots of the agents are shown in Figure 7 and 8, respectively.



(a) velocity (b) agents at $t = 21.9$

Fig. 6: Flocking for 4 agents with $\mathcal{L}_1$ adaptive control

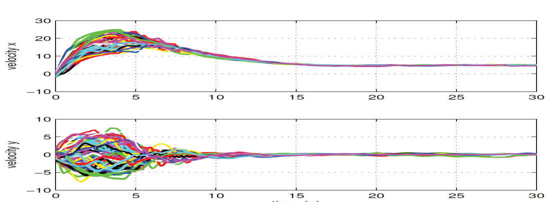
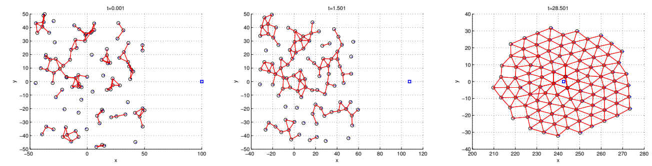


Fig. 7: Velocities of 100 agents with $\mathcal{L}_1$ adaptive control

VI. Conclusions

In this paper we address distributed target tracking and estimation using multiple mobile agents whose dynamics are subject to uncertainties and disturbances. By investigating an existing flocking algorithm, we find that the uncertainties may lead to undesired behavior of the system. We propose a cascaded control structure, in which each agent implements



(a) $t = 0.001$ (b) $t = 1.5$ (c) $t = 28.5$

Fig. 8: Flocking for 100 agents with $\mathcal{L}_1$ adaptive control

a "virtual ideal agent model" and exchanges the states of the ideal agents instead of the real agents. The resulting closed-loop ideal system is then used as a reference model of the real uncertain system, for which the $\mathcal{L}_1$ adaptive control structure is applied to compensate for the uncertainties.The guaranteed performance bounds of the $\mathcal{L}_1$ adaptive controller can be used to resolve the coupling between the communication topology and the system dynamics by slightly modifying the ideal flocking algorithm.

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