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Path Planning of a Dubins Vehicle for Sequential Target Observation with Ranged Sensors

Clarence Hanson, Jeremy Richardson and Anouck Girard

***Abstract*— We discuss time-optimal path planning for a Dubins vehicle operating in a planar environment required to investigate a series of static objects of interest. The objects are obscured such that they can be viewed only from a specified direction. The vehicle is equipped with an arbitrary number of sensors projecting circle sector viewing areas of arbitrary size and offset angle from the vehicle's heading, allowing for a range of vehicle orientations at which the objects can be viewed. Our objective is to minimize the total path distance the vehicle must travel while ensuring that the vehicle views all targets once from their respective required viewing direction. We discretize the sensors' area and examine the resulting possible solutions with an exhaustive search algorithm and show Monte Carlo simulation results of the exhaustive search algorithm compared to a standard Dubins algorithm.**

I. INTRODUCTION

A. Motivation

In the practical world, many objects of interest must be viewed from only a certain angle. These include doorways and windows along a street, dangerous objects that could be mistaken for harmless objects if seen from the wrong direction, and other objects whose identification depends on the angle at which they are viewed. Unmanned Aerial Vehicles (UAVs), which follow the kinematic constraints defined by the Dubins vehicle, equipped with conical (projected as circle sectors in a plane) field of view sensors, such as optical sensors, are often used to investigate these objects.

One can imagine that different sensor and vehicle properties could dramatically change the required travelling distance of the vehicle. For example, a vehicle with a nearly zero turning radius equipped with enough long-range sensors that it could see all 360 degrees would not need to travel nearly as far as a vehicle with a large turning radius required to go directly to the objects to see them. In this paper, we investigate the effects of these different properties as well as the effects of different properties of the objects of interest on the total travelling distance, or path length.

B. Literature Review

Dubins [1] found a geometric method to minimize the travelling distance of a vehicle restricted to a planar path with bounded curvature moving from one position and heading to another position and heading. The Dubins vehicle has been used extensively as a kinematic model in path planning algorithms for waypoint navigation, including the Traveling Salesperson Problem (TSP) [2] [3] [4]. These and similar works have been used to incorporate viewing objects of interest with ranged optical sensors [5]. In [8], the objects of interest must additionally be viewed from prescribed angles and the presence of a second fixed side-mounted camera has been taken into account.

The field-of-view of the sensors has been taken into account in [6] [7], though only for loitering on viewing one object of interest from any angle. This removes the difficulty of navigating between multiple waypoints and the difficulty of viewing the objects from a specific angle. To the authors knowledge, no literature exists that investigates the effects of both the viewing radius (range) of sensors and the viewing angle (field-of-view) of sensors for a Dubins vehicle tasked with viewing multiple objects of interest of prescribed viewing angles with multiple fixed sensors mounted with arbitrary orientations on the vehicle.

C. Original Contributions

1) In this paper we improve upon existing path planning techniques for single vehicle algorithms. We begin by analyzing the results if the vehicle is equipped with a camera and associated viewing angle.
2) In addition to looking for solutions at different angles, we also look for solutions that lie within the vieiwng area, rather than just the outer most solutions.
3) Further, we go on to look at the areas where the best improvements can be found.
4) We also investigate the savings assosciated with different positions of multiple cameras.

D. Organization

In Sec. II we explain the mathematical models used for both our brute-force method and for the standard Dubins method. Next in Sec. III we explain the problem investigated in this paper and then detail the two methods. Then in Sec. IV we show the numerical simulation results for the two methods, and conclude with Sec. V. Acknowledgments are given in Sec. VI with references following.

II. MODELING

A. Dubins Path Solutions

In 1957 Dubins developed a method for determining the minimum path length for a vehicle with a constant forward velocity, travelling from one position and heading to another position and heading. It was shown that the optimal solution always consists of three line segments. Each line segment

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Clarence Hanson is with the Aero. Engr. Dept., U. of Michigan, 1320 Beal Ave, Ann Arbor, MI 48109, USA, cbhanson@umich.edu

Jeremy Richardson is with the Aero. Engr. Dept., U. of Michigan, 1320 Beal Ave, Ann Arbor, MI 48109, USA, richarjs@umich.edu

is either a straight line or a turn of the minimum turning radius. All solutions of this type begin and end with turns of minimum turning radius. If we denote a straight line with S, L as a left turn and R as a right turn we can list all the possible solution types $\{LSL, LSR, LRL, RSR, RSL, RLR\}$. When we refer to a Dubins solution we are referring to the solution from that set with the minimum path length, shown by Dubins to be optimal for that situation [1]. Dubins path navigation techniques are commonly used in situations where objects are obscured or can only be identified from certain viewing angles, as well landing and take-off situations. An object of interest, coupled with a sensor's viewing range is sufficient to create a waypoint and orientation needed to view said object, and it is these values that are then plugged into the Dubins algorithm.

B. Vehicle

We consider a single vehicle travelling in a two-dimensional static plane defined by $\mathbb{R}^2$, so that aerial vehicles remain at a fixed altitude and ground vehicles are on a relatively level terrain. In addition to its position in the two dimensional space, a heading angle defined as between the velocity vector and x-axis of the inertial space, is used to adequately define the vehicle state. At the start of all missions we place the vehicle at a position of (0,0) and a zero heading angle, without loss of generality. The kinematics are described using the unicycle model.

$$\begin{aligned} \dot{X}_v &= V \cos \psi_v \\ \dot{Y}_v &= V \sin \psi_v \\ \dot{\psi}_v &= \omega \end{aligned}$$

The velocity of the vehicle is set to be constant throughout, and always in the same direction with respect to the body fixed axis of the vehicle. The maximum turn rate is also preset, while no minimum is specified. Defining a maximum turning speed is equivalent to defining a minimum turning radius, such that

$$\mathrm{R}_v \geq \frac{V}{\omega_{max}}$$

C. Object Model

An object of interest is something that occupies the planar environment defined above. Each object is required to be sensed from a specified direction in order for a mission to be successfully completed. Objects are defined in a predetermined order prior to being run through our system. Given W objects we define M to be the ordered set such as

$$M = \{O_1, \{O_1, O_2\}, .., \{O_1, ..., O_W\}\}$$

Each object is described using a position and orientation in the two dimensional plane, similar to that of the vehicle,

$$O_i = \{X_i, Y_i, \psi_{desired_i}\}$$

A success set for each object, S, can be created using both object and sensor information. This set describes all possible vehicle orientations and positions that constitute successfully viewing an object of interest. The success set size is determined by N, the number of discretizations performed on the sensor. This is the point at which our system differs from that of a standard Dubins algorithm. In the standard algorithm, this success set would be comprised of a single vehicle state. S can be defined as

$$S_i = \{[X_{v_{i1}}, Y_{v_{i1}}, \psi_{v_{i1}}], [X_{v_{i2}}, Y_{v_{i2}}, \psi_{v_{i2}}], ..., [X_{v_{iN}}, Y_{v_{iN}}, \psi_{v_{iN}}]\}$$

D. Sensor Model

The sensor model used throughout the paper is that of a three dimensional cone, projected onto a two dimensional plane perpendicular to the base of the cone. The origin of this shape is at a fixed location on the vehicle. Each sensor is described by three parameters, the viewing range, viewing angle, and the center of the sensor's cone offset from the velocity vector. These values can be seen in relation to the vehicle in Figure 1.

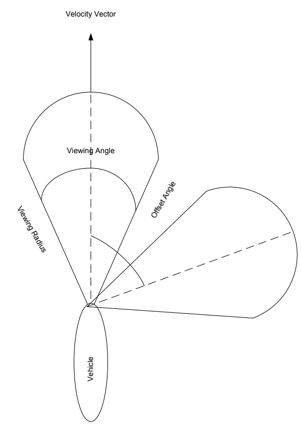

Fig. 1. Parameters used to define a sensor

Everything within the area of the sensor is considered visible, however this set needs to be discretized in order to be computationally executable. From a vehicle's position, orientation and sensor information, a discretized visualization set, I, can be created. This set is described as a list of objects, which may or may not be objects of interest.

$$I = \{O_{I_1}, O_{I_2}, ..., O_{I_N}\}$$

The direction in which an object is sensed, $\psi_{desired}$, within the sensor cone is simply the angle made between the x-axis and the line connecting the object with the origin of the cone. A typical discretized sensor can be seen in Figure 2.

III. PROBLEM FORMULATION

We consider a single vehicle tasked with viewing N objects contained within $\mathbb{R}^2$. Each object is defined by its position {X,Y} and desired heading angle, $\psi_{desired}$, from which to be viewed. The vehicle is similarly defined with a position and current heading angle $\{X_v, Y_v, \psi_v\}$. The vehicle is given an initial position and orientation, and travels at a constant speed in a body-fixed forward direction. The inclusion of wind and sideslip are not considered. The vehicle is outfitted with a number of sensors of arbitrary orientation with respect to the vehicle's body fixed axis. Each sensor is modeled as a 3-dimensional cone projected onto a two-dimensional plane, forming a circle sector. Each sensor is defined by a viewing

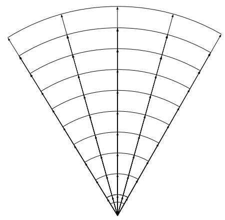

Fig. 2. Discretizations preformed on a typical sensor

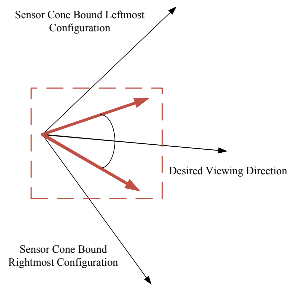

Fig. 3. Geometry associated with angular solution set. Region in the dashed box represents all the different vehicle heading angles with the given sensor configuration that satisfactorily view the object of interest.

angle, viewing radius, and offset of the center of the sensor area from that of the heading angle.

A. Creating Discrete Solution Sets

To create the discretized solution, the sensor radius, viewing angle, sensor offset, object position and number of discretizations performed on the sensor need to be accounted for. To do so, we first look for the solutions as far from the vehicle as possible and the effect of the viewing angle on the results, geometrically, as shown in Figure 3.

Figure 4 shows the resulting solution family from simply accounting for the viewing radius.

In addition to the angular discretization, a radial discretization is performed. Figure 5 shows the solution family from only discretizing in the radial direction. It is interesting to note that all solution positions are in a line originating at the object of interest, with length determined by the viewing radius and direction identical to $\psi_{desired}$.

These discretizations form possible solution sets of a size determined by the number of discretizations performed on the sensor, which is where our algorithm differs from that of the standard Dubins algorithm, for in the standard algorithm each solution set would contain only a single vehicle state.

B. Method for Determining Best Solution

To determine the best solution an exhaustive search is performed on the different possible solutions. If there are W objects in a given mission, with N discretizations of the sensor, there are a total of $N^{(2W-1)}$ possible ways to complete a mission. Recall that an object of interest has a success set, S, that describes all the vehicle orientations and positions, P, that allow for the object to be viewed.

$$S_i = \{[P_{v_{i1}}],[P_{v_{i2}}],....,[P_{v_{iN}}]\}$$

We can say that paths that satisfactorily complete a mission, Q, consist of at least one state from each of the success

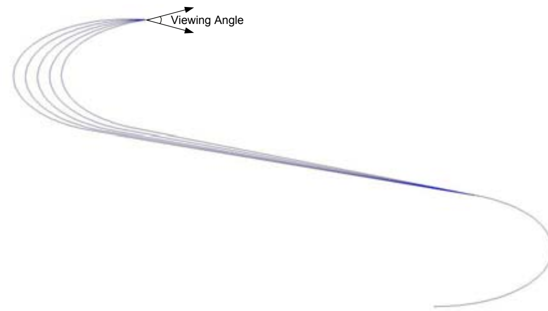

Fig. 4. Effects of angular discretization on possible solutions

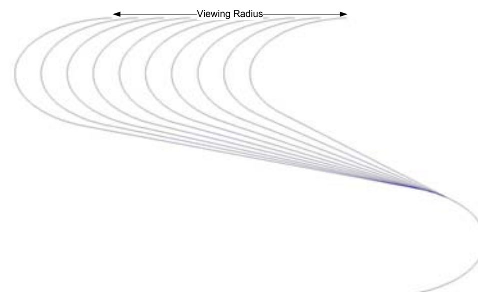

Fig. 5. Effects of radial discretization on possible solutions

sets, in the specified order defined. The goal of this algorithm is to find the solution with the minimum path length. This type of situation is often referred to as one-in-a-set path planning. Although every possible path from one object of interest to another needs be calculated, not every complete path needs be calculated to ensure the optimal solution is found.

$$Q = \{\{W_{v_{11}},..,W_{v_{1N}}\},..,\{W_{v_{M1}},..,W_{v_{MN}}\}\}$$

C. Quantification of Results

To determine the effectiveness of a particular solution, we always compare the resulting path length with the path length of the solution if Dubins path planning were applied under identical conditions. For example, if we model a vehicle as having a minimum turning radius of 5 meters, and a sensor of viewing radius 10 meters, we would build our Dubins comparison solution around those parameters as well. The same object of interest locations and orientations are also used. In addition the ordering of these objects remains the same, so if a TSP solver is employed the resulting ordering is also used in the Dubins comparison.

IV. SIMULATION RESULTS

Many Monte Carlo simulations were run to determine the effectiveness of the system as compared to the standard Dubins solution, each examining different parameters. In order of analysis, the simulations examine the effect of the distance between objects of interest, the effect of the turning radius, field size, and object ordering, and the effect of multiple sensors.

A. Distance between Objects of Interest

To determine the effect of the distance between objects of interest, we begin by running simulations keeping all parameters static except the distance between objects of interest. For each static configuration of the vehicle, we run a total of 500 scenarios. The simulations use Monte Carlo simulation methods, with both random heading and direction from previous object. Shown in Figure 6 we fix the viewing radius and angle to be 1m and 30°. We averaged the 20 different simulation results for each distance, the data for which is shown in Figure 6.

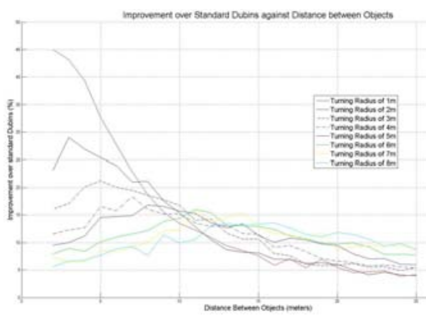

Fig. 6. Effectiveness of system against Distance between Objects of Interest

It is clear from the figures that the effectiveness of the system is quite dependent upon the conditions of the simulation. Under certain conditions, results as much as 45% shorter, on average, paths can be achieved, while that same vehicle and sensor configuration can result in only 5% savings in other areas. As the proximity of the objects increases, a more significant improvement over standard Dubins path planning can be expected, to a point. The expected improvement reaches a point, at which placing the objects any closer will degrade in effectiveness. After a certain distance between objects, the averaged improvements seen for larger turning radii are actually greater than that of the shorter radii. This is due to the fact that we expect no contributions from the introduction of the viewing angle as the ratio between the distance between objects of interest and turning radius increases. This is due to the fact that as this ratio approaches infinity, the vehicle is able to head straight for the object, or the time spent in the straight portion of the Dubins curve is much larger than that of the turning portions combined. Having a zero turning radius yields almost identical results to that of a 360° sensor while the objects are sufficiently far apart. Thus, we should expect the improvement over Dubins to peak at a certain point then for greater distance between objects of interest all curves should curve towards a zero percent improvement.

B. Various Single Sensor Configurations

In these simulations, the effects of different viewing radii and viewing angles are studied. For completeness, we also study how these results change with varying turning radii, varying field size, and varying number of objects. Varying the field size varies the maximum distance possible between objects. Figures 7 and 8 are the averaged results of 25 runs with randomly ordered and randomly oriented objects of interest. Though not visible in the figures, simulations show that the improvements reach an asymptote when the viewing angle reaches 360°.

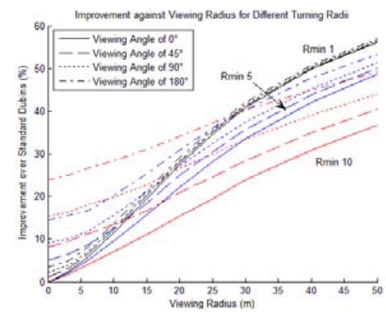

Fig. 7. Effects of turning radii on viewing radii and viewing angles

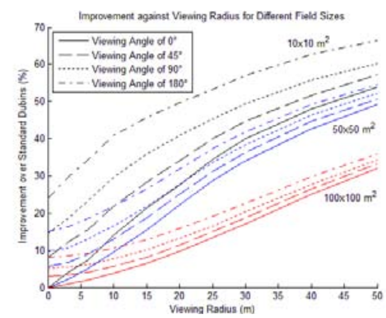

Fig. 8. Effects of field sizes on viewing radii and viewing angles

In Fig. 7, the field size is set to be 50m by 50m and the number of objects of interest is set to be 25. This system yields the best improvements when the vehicle has a low minimum turning radius, high viewing radius, and high viewing angle. However, the exact dependence on minimum turning radius depends on the viewing radius and viewing angle. When the viewing angle is 45°, the system yields better results for higher minimum turning radius when the viewing radius is less than about 12 m and yields better results for lower minimum turning radius when the viewing radius is greater than the 12m threshold.

This threshold increases with viewing angle - when the viewing angle is 180°, the threshold is 25 m. It also increases with field size - when the field size is 100m by 100m and the viewing angle is 45°, the threshold is at about 22 m as opposed to 12 m when the field size is 50m by 50m.

It is important to note that the lower the minimum turning radius, the less important the sensor's viewing angle and the more important the sensor's viewing radius. Conversely, for large minimum turning radii, the sensor's viewing angle is extremely important and the sensor's viewing radius is less important.

Some important design considerations can be taken from this analysis. Clearly, a vehicle with a sharp turning radius would be best equipped with long-range sensors. On the other hand, a vehicle with a large turning radius would benefit greatly from many wide sensors and, to a lesser extent, from long-range sensors.

In Fig. 8, the vehicle has a turning radius of 5 meters and the number of objects of interest is set to be 25. This system yields the best improvements when the objects of interest are placed in a small field, and when the viewing radius and viewing angle are high. Note that the viewing radius is

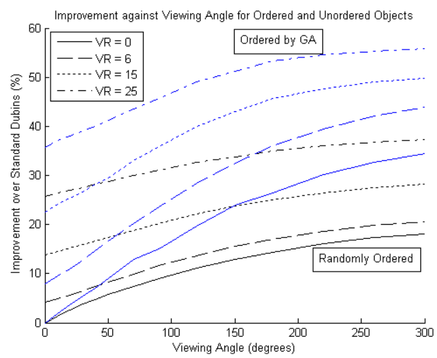

Fig. 9. Effects of GA ordering

important for all field sizes, especially for small field sizes, but that the viewing angle has more of an effect for small field sizes than for large field sizes.

One cannot generally choose the configuration of the objects, but Fig. 8 shows what types of sensors are effective in different configurations. If the objects are restricted to a small field, multiple wide range sensors would be more effective than a single long range sensor. However, increasing the range of the sensors would also help, though to a lesser extent. On the other hand, if the objects are restricted to a large field, increasing the range of the sensors is by far the better option.

Additional simulations were run to evaluate the effects of ordering the objects of interest with a TSP heuristic solver. In this case, a fixed start open genetic algorithm[1] [9] run with 10000 generations was used to order 25 objects of interest oriented randomly in a 50 by 50 field. It is important to note that neither the vehicle parameters nor the sensor parameters were taken into account in this genetic algorithm - it is a standard fixed start open genetic algorithm. 10000 generations brings the solution to within a few percent of optimal for a standard TSP. The vehicle has a turning radius of 5 meters. Fig. 9 shows the averaged results of 50 different random configurations of objects of interest.

The total possible percent savings are higher with the genetic algorithm TSP solver. Additionally, the improvements increase significantly more with increasing viewing angle when using the genetic algorithm, and marginally more for increasing viewing radius.

This figure clearly shows that this system is even more effective in ordered scenarios, which are often characteristic of realistic urban settings. It also shows that in such scenarios, increasing the viewing angle of a sensor, or adding more sensors, as well as increasing the viewing range of the sensor will often significantly increase the improvements even though this is less often the case in randomly ordered scenarios.

After these simulations were run for differing numbers of objects, we found that this system is as effective for a few objects as it is for many. Of course, total time increases with an increasing number of objects.

[1] An iterative approach to finding a near-optimal solution to a TSP. "Fixed start" means that the starting point is specified, "open" means that the vehicle does not return to the starting point, as is appropriate for our scenarios.

The trends explored in the analysis of Fig. 7, 8, and 9 all hold together. When the minimum turning radius of the vehicle is large or the field size is small, the viewing angle becomes very important. The viewing radius also remains important though to a lesser extent than the viewing angle. When the minimum turning radius of the vehicle is small or the field size is large, the viewing angle has very little effect and thus the viewing radius becomes important. For example, improvements of nearly 78% on average can be obtained when the field size is 10x10, the viewing radius 50 meters, the viewing angle 180°, the turning radius 1 meter, and the objects ordered are by a genetic algorithm TSP solver.

C. Multiple Sensor Configurations

We investigated the case with a random waypoint distribution with a set distance between each waypoint against several different dual sensor configurations. In this simulation, we created 100 different scenarios, each having a distance between objects of interest of 20 meters, and 25 objects of interest. The system assumed a minimum turning radius of 3m. In the case of dual sensors, one was placed straight forward, with a viewing radius of 6 and viewing angle of 30°, while the second sensor, whose offset is being varied, had a viewing radius of 6m and viewing angle of 15°. The average results of the multiple sensor configuration can be seen in Figure 10. For completely stochastic missions there is in fact an optimal location for the second camera.

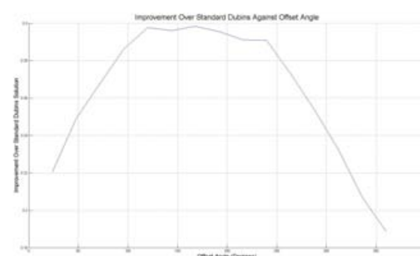

Fig. 10. Effectiveness of various placements of a second visual sensor

It is important to note that these results come from the waypoints made with completely random approach angles. In urban scenarios or situations where the objects of interest are arranged such that common angles are formed between direct paths and the desired viewing angle the optimal second camera placement is not necessarily at 180° from the first camera. An example of a mission where the optimal placement is perpendicular to the forward mounted camera is shown in Figure 11.

D. Computational Complexity and Scalability

Computational complexity scales linearly with the number of objects, viewing angles, viewing radii, turning radii, field sizes, sensors, and iterations but scales with the square of the number of discretizations of the sensor in both the radial and angular directions. The standard Dubins solution also scales linearly with the number of objects. The algorithms work very well with multi-core processing, this is due to the number of computations that can be performed in parallel in the one-in-a-set path planning techniques.

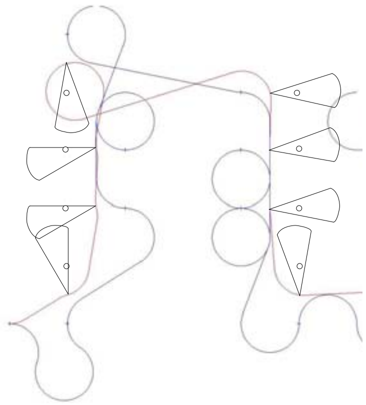

Fig. 11. Sample mission segment with side mounted sensor. The longer path with loops represents the Dubins solution. The sensor footprints are used to illustrate where the side mounted camera visualizes each object and to show the size of the sensor. The circles represent the objects of interest.

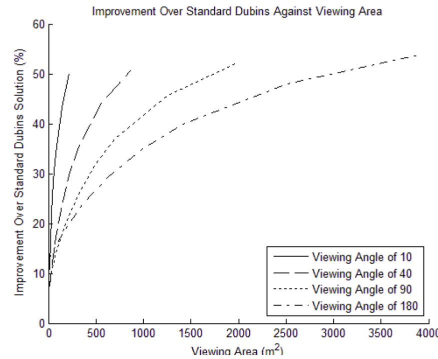

Fig. 12. Improvement over Dubins against viewing area of a sensor

V. CONCLUSIONS

From the simulation results it is clear that accounting for the entire viewing area encompassed by the sensor we choose to model can achieve significant improvements to the solution time when compared to standard the Dubins path planning technique. The results are strongly correlated to all the aspects of the vehicle, the turning and viewing radius, viewing angle, and the average distance between objects of interest. The best results can be expected when objects of interest are relatively close together, in relation to the turning and viewing radius. It is also clear that the effectiveness of a sensor is not based primarily on the viewing angle of a particular sensor. It is shown that sensors are much more effective when the viewing radius is maximized. The placement of a second sensor was also shown, in a stochastically created environment, to be optimal when placed directly behind the vehicle.

A. Sensor Design Considerations

Based on the simulation results, design considerations pertaining to the sensor configuration can be made. It is difficult to make design considerations for the vehicle as these are much more difficult to change based on mission specifications. This does not mean that information pertaining to varying the vehicle parameters are without use, they should be used to determine the average savings expected given mission parameters and determine if running the algorithm is indeed worth the time taken to compute a solution.

Sensor information clearly shows that the most effective attribute of a sensor is the viewing radius. When comparing the average savings against area, maximizing the radius is still the most beneficial, as can be seen in Figure 12.

B. Future Work

The future work related to the optimal path planning accounting for the full range of a sensor will focus on a variety of things. First, the environment will be three-dimensional. The sensor will have an optional property pertaining to a gimbaled mount. Additionally, in order to couple the viewing angle and radius the focal length of the camera, which pertains to optical zoom properties, will be analyzed. With the addition of the altitude, gimbaled mount and the optical zoom, the sensor footprint size, shape and orientation with respect to the vehicle would need to be dynamic.

Location of secondary or tertiary sensors will also be investigated in more detail. An algorithm that looks at angles formed between lines of sight between objects of interest (this is the approximate approach angle) and desired viewing angle to determine optimal placement of extra sensors.

Finally using the information pertaining to all aspects of the mission, developing a genetic algorithm specifically tailored to find the optimal ordering of objects of interest prior to running through our algorithm.

VI. ACKNOWLEDGMENTS

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