2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

Dynamic Quasi-Decentralized Control of Networked Process Systems with Limited Measurements

Yulei Sun and Nael H. El-Farra[†]
Department of Chemical Engineering & Materials Science
University of California, Davis, CA 95616 USA

Abstract— **This work presents a quasi-decentralized networked control structure with a dynamic communication logic for plants with limited state measurements and interconnected units that exchange information over a shared, resource-constrained communication network. Initially, an observer-based output feedback controller is synthesized for each unit, and Lyapunov techniques are used to explicitly characterize the closed-loop stability properties of each unit under continuous communication. This characterization is then used as the basis for developing a dynamic communication strategy that keeps the information transfer between the local control systems to a minimum without jeopardizing closed-loop stability. The key idea is to monitor the evolution of the observer-generated state estimates locally within each unit and suspend communication for as long as the expected stability threshold is met. During periods of communication suspension, each control system relies on a set of models that provide estimates of the states of the neighboring units. At times when the stability threshold is breached, communication is re-established and the neighboring units are prompted to send their data over the network to update the models. The stability threshold is determined using Lyapunov techniques and can be tightened or relaxed by proper controller and observer tuning. Finally, the stability and performance properties of the dynamic networked control structure are illustrated using a chemical plant example.**

I. Introduction

The development of systematic methods for control of large-scale dynamical systems composed of tightly interconnected subsystems is a fundamental problem that has been the subject of significant research work within process control over the past few decades (e.g., see [1], [2], [3], [4], [5], [6], [7], [8], [9] and the references therein). Traditionally, the controller synthesis problem for such plants has been addressed within either the centralized or decentralized control frameworks. An approach that provides a compromise between the complexity of centralized control schemes and the performance limitations of decentralized control approaches is quasi-decentralized control [10], [11], [12] which refers to a distributed control strategy in which most signals used for control are collected and processed locally, while some signals are transferred between the local units and controllers to adequately account for the interactions and minimize the propagation of process upsets. A key consideration in this strategy is to enforce the desired closed-loop stability and performance objectives of the plant with minimal cross communication between the component subsystems. This is an appealing objective particularly when the communication medium is resource-constrained (e.g., a wireless sensor network) and conserving network resources is key to prolonging the service life of the network and minimizing communication disruptions (the reader may refer to [13], [14] for surveys of results and references on networked control systems). Inspired by the ideas of model-based networked control [15], information transfer between the plant units under quasi-decentralized control is kept to a minimum by embedding within the local control systems dynamic models that provide the local controllers with estimates of the states of the neighboring units when communication is suspended, and updating the states of those models when communication is restored.

A key feature of the communication logic used in [10], [11], [12] is that it is static in the sense that the allowable communication rate is constant and can be computed off-line prior to plant operation. More recently, we developed in [16] a feedback-based communication policy in which the necessary communication rate can be determined and adjusted on-line (i.e., during plant operation) based on the evolution of the state of the plant. The key idea is to locally monitor the evolution of the state of each unit and request model updates from the rest of the plant only when a state-dependent stability bound is breached. This allows the plant to respond quickly in an adaptive fashion to a unit that requires immediate attention.

In most practical applications, however, direct measurements of the full-state are typically unavailable, and this introduces a number of challenges that need to be accounted for at the local control level, as well as at the plant-wide communication level. At the local control level, for example, the lack of full-state measurements necessitates the design of suitable state observers to generate estimates of the states of each unit from the available measurements. Each observer must be designed to enforce quick decay of the estimation error in the presence of interconnections between the plant units. This is critical given that the observer-generated estimates are needed to implement the local control action and must also be transmitted to the neighboring units to perform model updates at communication times. Observer estimation errors also require placing additional restrictions on the communication logic to account for these errors explicitly in the stability threshold and for the fact that only

[†] To whom correspondence should be addressed: E-mail: nhelfarra@ucdavis.edu. Financial support by NSF CAREER Award, CBET-0747954, is gratefully acknowledged.

the observer-generated estimates can be monitored and used to assess the stability properties of the individual subsystems.

Motivated by these considerations, we present in this work a methodology for the design of a quasi-decentralized networked control structure with a dynamic communication logic for plants with incomplete state measurements and interconnected subsystems that communicate over a shared, resource-constrained network. The structure integrates ideas from model-based control, high-gain observers, Lyapunov stability and feedback-based communication. Initially, an output feedback controller that enforces practical stability and ultimate boundedness in the absence of communication suspension is synthesized for each unit. To reduce information transfer over the network, a set of models are included within each local control system to provide estimates of the states of the neighboring units when communication is suspended. To determine when communication must be re-established, the evolution of the observer-generated state estimates is monitored locally within each unit such that if it begins to breach a pre-specified time-varying stability threshold at any time, the neighboring units are prompted to send their data over the network to update the corresponding models. Finally, the stability and performance properties of the dynamic networked control structure are illustrated using a chemical plant example.

II. Preliminaries and problem formulation

We consider a large-scale distributed plant composed of n interconnected processing units, each of which is described by a continuous-time uncertain nonlinear system, and represented by the following state-space description:

$$\begin{array}{lll} \dot{x}_1 = f_1(\mathbf{x}) + G_1(\mathbf{x})u_1 + W_1(\mathbf{x})\theta_1(t), & & y_1 = h_1(x_1) \\ \dot{x}_2 = f_2(\mathbf{x}) + G_2(\mathbf{x})u_2 + W_2(\mathbf{x})\theta_2(t), & & y_2 = h_2(x_2) \\ \quad\vdots & & \quad\vdots \\ \dot{x}_n = f_n(\mathbf{x}) + G_n(\mathbf{x})u_n + W_n(\mathbf{x})\theta_n(t), & & y_n = h_n(x_n) \end{array} \tag{1}$$

where $x_i := [x_i^{(1)} \; x_i^{(2)} \; \cdots \; x_i^{(p_i)}]' \in \mathbb{R}^{p_i}$ denotes the vector of process state variables associated with the i-th processing unit, x' denotes the transpose of a vector x, $\mathbf{x} = [x_1' \; x_2' \; \cdots \; x_n']'$, $y_i := [y_i^{(1)} \; y_i^{(2)} \; \cdots \; y_i^{(q_i)}]' \in \mathbb{R}^{q_i}$ and $u_i := [u_i^{(1)} \; u_i^{(2)} \; \cdots \; u_i^{(r_i)}]' \in \mathbb{R}^{r_i}$ denote the vector of measured outputs and manipulated inputs associated with the i-th processing unit, respectively, $\theta_i := [\theta_i^{(1)} \; \theta_i^{(2)} \; \cdots \; \theta_i^{(s_i)}]' \in \mathbb{R}^{s_i}$ denotes the vector of uncertain (possibly time-varying), but bounded, variables which takes values in a nonempty compact convex subset of $\mathbb{R}^{s_i}$, and satisfies $\| \theta_i \| \leq \theta_{b,i}$, for $i = 1, \cdots, n$, where $\theta_{b,i}$ is a positive real number and $\| \cdot \|$ denotes the standard Euclidean norm of a vector. The uncertain variables may describe time-varying parametric uncertainty and/or exogenous disturbances. The functions $f_i(\cdot)$, $G_i(\cdot)$, $W_i(\cdot)$, and $h_i(\cdot)$ are sufficiently smooth nonlinear functions. Without loss of generality, it is assumed that the origin is an equilibrium point of the nominal uncontrolled plant (i.e., $f_i(0) = 0$ for $i = 1, \cdots, n$). Referring to (1), we consider the problem of designing a distributed, networked control strategy that robustly stabilizes the individual units near the origin, while simultaneously accounting for the constrained resources of the plant-wide communication network and the lack of complete state measurements within each unit.

III. Robust quasi-decentralized networked output feedback control structure

A. Robust output feedback controller synthesis

To realize the desired robust quasi-decentralized networked control structure, the first step is to synthesize for each unit an output feedback controller that enforces robust closed-loop stability and an arbitrary degree of asymptotic attenuation of the effect of the uncertainty on the closed-loop system in the absence of communication suspension (i.e., when the sensors of each unit transmit their data continuously to the control systems of the other plant units). To illustrate the main idea, we consider as an example the following Lyapunov-based controller (e.g., [17], [18], [19], [20]) (note that other controller design methods can be used as well):

$$u_i = k_i(\mathbf{x}, \theta_{b,i}, \rho_i, \chi_i, \phi_i), \; i = 1, 2, \cdots, n$$
$$= -\left(\frac{L_{f_i}^* V_i + \sqrt{(L_{f_i}^{**} V_i)^2 + \| (L_{G_i} V_i)' \|^4}}{\| (L_{G_i} V_i)' \|^2} \right) (L_{G_i} V_i)'(\mathbf{x}) \tag{2}$$

when $\| (L_{G_i} V_i)' \| \neq 0$, and $u_i = 0$ when $\| (L_{G_i} V_i)' \| = 0$, where $V_i(x_i)$ is a robust control Lyapunov function for the i-th unit,

$$L_{f_i}^{**} V_i = L_{f_i} V_i + \rho_i \| x_i \| + \chi_i \| (L_{W_i} V_i)' \| \theta_{b,i} \tag{3}$$

$$L_{f_i}^* V_i = L_{f_i} V_i + \left(L_{f_i}^{**} V_i - L_{f_i} V_i \right) \left(\frac{\| x_i \|}{\| x_i \| + \phi_i'} \right) \tag{4}$$

and $L_{f_i} V_i = (\partial V_i / \partial x_i) f_i(\mathbf{x})$, $L_{G_i} V_i = [L_{g_{i,1}} V_i \; \cdots \; L_{g_{i,r_i}} V_i]$, $L_{g_{i,j}} V_i = (\partial V_i / \partial x_i) g_{i,j}(\mathbf{x})$, $g_{i,j}(\mathbf{x})$ is the j-th column of $G_i(\mathbf{x})$, $L_{W_i} V_i = [L_{w_{i,1}} V_i \; \cdots \; L_{w_{i,s_i}} V_i]$, $L_{w_{i,j}} V_i = (\partial V_i / \partial x_i) w_{i,j}(\mathbf{x})$, $w_{i,j}(\mathbf{x})$ is the j-th column of $W_i(\mathbf{x})$, and $\rho_i > 0$, $\chi_i > 1$, $\phi_i' > 0$ are tunable parameters.

Consider now the i-th subsystem of the nonlinear plant of (1) under the control law of (2)-(4). Evaluating the time-derivative of the i-th Lyapunov function along the closed-loop trajectories, it can be verified that $\dot{V}_i$ satisfies the following bound:

$$\dot{V}_i \leq -\rho_i \frac{\| x_i \|^2}{\| x_i \| + \phi_i'} < 0 \;\; \forall \, \| x_i \| \geq \phi_i' (\chi_i - 1)^{-1}, \tag{5}$$

for $i = 1, 2, \cdots, n$, which implies that the state of the i-th closed-loop unit remains bounded and converges in finite time to a terminal neighborhood of the origin that can be made arbitrarily small by appropriate selection of the controller tuning parameters ϕ_i' and χ_i. It can also be verified that the inequality in (5) implies that $\dot{V}_i$ satisfies:

$$\dot{V}_i(x) \quad \leq \quad -\alpha_i(\|x_i\|) + \gamma_i(\phi_i) \tag{6}$$

for some class $\mathscr{K}$ functions, $\alpha_i(\cdot)$ and $\gamma_i(\cdot)$, where $\phi_i = \phi_i'(\chi_i - 1)^{-1}$.

Note that the implementation of the control law of (2)-(4) as written requires the availability of full-state measurements both from the local subsystem being controlled, x_i, and from the other units, x_j, which are seldom available in practice. To compensate for the lack of full-state measurements, a suitable state observer needs to be designed for each local control system to generate estimates of the local state variables from the measured outputs, and combined with the state feedback controller of (2)-(4) to enforce closed-loop stability. Specifically, we consider an observer-based output feedback controller of the following general form:

$$\begin{aligned} \dot{w}_i &= \psi_i(\mathbf{w}, y_i, u_i, \mu_i) \\ u_i &= k_i(\mathbf{w}, \theta_{b,i}, \rho_i, \chi_i, \phi_i') \end{aligned} \tag{7}$$

where w_i is the observer-generated estimate of the state of the i-th subsystem, $\mathbf{w} = [w_1' \ w_2' \ \cdots \ w_n']'$, $\mu_i > 0$ is an observer design parameter, and $\psi_i(\cdot)$ is a smooth vector function. We consider that an appropriate set of state observers have been synthesized for the different plant units. The necessary requirements for such observer designs are stated in the following assumption and discussed thereafter.

Assumption 1: Refereing to the closed-loop system of (1), (2)-(4) and (7), given any set of positive real numbers $\{\delta_{b,i}, \theta_{b,i}, \delta_{d,i}\}$, there exists $\phi_i^* > 0$, and for each $\phi_i \in (0, \phi_i^*]$, there exists $\mu_i^* > 0$, such that if $\phi_i \leq \phi_i^*$, $\mu_i \leq \mu_i^*$, $\| x_i(0) \| \leq \delta_{b,i}$, $\| w_i(0) \| \leq \delta_{b,i}$, and $\| \theta_i \| \leq \theta_{b,i}$, the trajectories of the closed-loop system are bounded and satisfy $\limsup_{t\to\infty} \| x_i(t) \| \leq \delta_{d,i}$. Furthermore, given any $T_i^b > 0$, there exists $\tilde{\mu}_i \leq \mu_i^*$ such that if $0 < \mu_i \leq \tilde{\mu}_i$, $\| x_i(t) - w_i(t) \| \leq K_i\mu_i$ for all $t \geq T_i^b$, for some $K_i > 0$, $i = 1, \cdots, n$.

Remark 1: Assumption 1 requires that the observer designed for each unit be able to (a) ensure that the closed-loop system under the output feedback controller of (7) is stable with an ultimate bound on the closed-loop state that can be tuned via proper selection of the observer design parameter, and (b) enforce an arbitrarily fast convergence of the observer-generated estimate to the actual state by proper selection of the observer design parameter. As will be shown later, these requirements will facilitate the design and implementation of the dynamic communication policy under output feedback control. In principle, any observer satisfying these requirements can be used. Typical examples include high-gain observers (e.g., see [21], [20]) where μ_i scales inversely with the observer gain. Note that convergence of the estimation error below some desired level is ensured only after a short period of time T_i^b which can be made arbitrarily small by proper selection of μ_i.

The following proposition provides an explicit characterization of the stability properties of the closed-loop system under output feedback control.

Proposition 1: Consider the closed-loop system of (1)-(4) and (7) for which Assumption 1 holds with $\phi_i \leq \phi_i^*$, $\mu_i \leq \tilde{\mu}_i$, $\| x_i(0) \| \leq \delta_{b,i}$, $\| w_i(0) \| \leq \delta_{b,i}$, and $\| \theta_i \| \leq \theta_{b,i}$ for all $i = 1, \cdots, n$. Then, there exists a class $\mathscr{K}$ function $\epsilon_i(\cdot)$ such that:

$$\dot{V}_i(x_i(t)) \leq -\alpha_i(\|x_i(t)\|) + \gamma_i(\phi_i) + \epsilon_i(\mu_o) \tag{8}$$

for all $t \geq T^b := \max_{j=1,\cdots,n} \{T_j^b\} > 0$, where $\mu_o = \max_{j=1,\cdots,n} \{\mu_j\}$.

Proof: Consider the i-th subsystem of the nonlinear plant of (1) under the controller of (2)-(4) and (7). Evaluating the time-derivative of V_i along the trajectories of the closed-loop system yields:

$$\begin{aligned} \dot{V}_i(x_i) &= L_{f_i}V_i(\mathbf{x}) + L_{G_i}V_i(\mathbf{x})k_i(\mathbf{x}) + L_{W_i}V_i(\mathbf{x})\theta_i \\ &+ L_{G_i}V_i(\mathbf{x})[k_i(\mathbf{w}) - k_i(\mathbf{x})] \\ &\leq -\alpha_i(\|x_i\|) + \gamma_i(\phi_i) \\ &+ \| L_{G_i}V_i(\mathbf{x}) \| \| k_i(\mathbf{w}) - k_i(\mathbf{x}) \| \end{aligned} \tag{9}$$

where we have used the fact that $L_{f_i}V_i(\mathbf{x}) + L_{G_i}V_i(\mathbf{x})k_i(\mathbf{x}) + L_{W_i}V_i(\mathbf{x})\theta_i \leq -\alpha_i(\|x_i\|) + \gamma_i(\phi_i)$ for $\phi_i \leq \phi_i^*$ from (6). Since the state of the closed-loop system is bounded (from Assumption 1), we have that there exist positive real numbers M_i and L_{k_i}, such that $\| L_{G_i}V_i(\mathbf{x}) \| \leq M_i$ and $\| k_i(\mathbf{w}) - k_i(\mathbf{x}) \| \leq L_{k_i} \| \mathbf{w} - \mathbf{x} \|$. Substituting these estimates into (9) yields:

$$\begin{aligned} \dot{V}_i(x_i) &\leq -\alpha_i(\|x_i\|) + \gamma_i(\phi_i) + M_iL_{k_i} \| \mathbf{w} - \mathbf{x} \| \\ &\leq -\alpha_i(\|x_i\|) + \gamma_i(\phi_i) + M_iL_{k_i} \textstyle\sum_{j=1}^{n} \| w_j - x_j \| \end{aligned} \tag{10}$$

From the definitions of T_b and μ_o in Proposition 1, as well as the bound on the estimation error given in Assumption 1, it follows that for all $t \geq T_b$ and $\mu_j \leq \tilde{\mu}_j$, $\| w_j(t) - x_j(t) \| \leq K_j\mu_j \leq K_j\mu_o$. Substituting this estimate into (10) yields:

$$\begin{aligned} \dot{V}_i(x_i(t)) &\leq -\alpha_i(\|x_i(t)\|) + \gamma_i(\phi_i) + M_iL_{k_i} \textstyle\sum_{j=1}^{n} K_j\mu_o \\ &= -\alpha_i(\|x_i(t)\|) + \gamma_i(\phi_i) + \epsilon_i(\mu_o), \ \forall \, t \geq T^b \end{aligned} \tag{11}$$

where $\epsilon_i(\mu_o) := M_iL_{k_i} \sum_{j=1}^{n} K_j\mu_o$. ■

Remark 2: The bound in (8) implies that the closed-loop state under output feedback control is bounded and converges in finite time to a neighborhood of the origin whose size depends on the choices of both the controller and observer design parameters. When comparing this bound with its counterpart in (6) under full-state feedback control, it can be observed that the effect of the state estimation error is to essentially increase the ultimate bound obtained under state feedback control (i.e., the terminal set) by a certain amount that can be made arbitrarily small by appropriate selection of the observer design parameter. In the limit as μ_o goes to zero (e.g., sufficiently high observer gains are used for all the subsystems), the state feedback bound can be recovered.

B. Model-based quasi-decentralized control

The implementation of each control law in (7) requires the availability of state estimates generated by the observers both within the local subsystem being controlled and within the units connected to it. Unlike the local observer estimates which are available continuously through a dedicated network, the observer-generated estimates from the neighboring units are available only through the shared plant-wide network. To reduce the transfer of information between the local control systems as much as possible without sacrificing stability, a set of dynamic models of the interconnected plant units are embedded in the local control system of each unit to provide it with an estimate of the evolution of the states of the neighboring units when state estimates are not transmitted over the network. The use of models allows the neighboring units to send their data at discrete time instants since the models can provide an approximation of the plant's dynamics. Feedback from one unit to another is performed by updating the state of each model using the observer estimates of the corresponding unit whenever communication between the plant units is allowed. Under this architecture, the local control law for each unit is implemented as follows:

$$\begin{aligned} u_i(t) &= k_i(w_i(t), \underline{\widehat{\mathbf{x}}}^i(t)), \ i = 1, 2, \cdots, n \\ \dot{w}_i(t) &= \psi_i(w_i(t), \underline{\widehat{\mathbf{x}}}^i(t), y_i(t), u_i(t), \mu_i) \\ \dot{\widehat{x}}^i_j(t) &= \widehat{f}_j(w_i(t), \underline{\widehat{\mathbf{x}}}^i(t)) + \widehat{G}_j(w_i(t), \underline{\widehat{\mathbf{x}}}^i(t))\widehat{u}^i_j(t) \\ \widehat{u}^i_j(t) &= k_j(w_i(t), \underline{\widehat{\mathbf{x}}}^i(t)), \ t \in (t^i_k, t^i_{k+1}) \\ \widehat{x}^i_j(t^i_k) &= w_j(t^i_k), \ \ j = 1, \cdots, n, \ , j \neq i, \ k = 0, 1, 2, \cdots \end{aligned} \tag{12}$$

where $\widehat{x}^i_j$ is an estimate of x_j, used by the local control system of the i-th unit, $\underline{\widehat{\mathbf{x}}}^i$ is a vector containing the model estimates of the states of all the plant units except for the i-th unit, i.e., $\underline{\widehat{\mathbf{x}}}^i = [\widehat{x}^{i'}_1, \cdots, \widehat{x}^{i'}_{i-1}, \widehat{x}^{i'}_{i+1}, \cdots, \widehat{x}^{i'}_n]'$, $\widehat{f}_j(\cdot)$ and $\widehat{G}_j(\cdot)$ are nonlinear functions that model the dynamics of the j-th unit. The notation t^i_k is used to indicate the k-th time instance that the states of the models embedded in i-th control system are updated using the observer generated state estimates transmitted from the rest of the plant.

C. A dynamic strategy for terminating and restoring communication

Our aim in this section is to devise a communication policy that allows each local control system to determine, and adaptively adjust (based on the conditions of the local subsystem), the rate at which it requests updates from the rest of the plant. The main idea is to use the Lyapunov stability condition derived in Section III-A as a guide for establishing and suspending communication. Specifically, consider the nonlinear plant of (1) subject to the model-based networked controller of (12). Evaluating the time-derivative of V_i along the trajectories of the networked closed-loop subsystem for $t \in [t^i_k, t^i_{k+1})$, where $t^i_k > T^b$, yields:

$$\begin{aligned} \dot{V}_i(x_i) &= L_{f_i}V_i(\mathbf{x}) + L_{G_i}V_i(\mathbf{x})k_i(w_i, \underline{\widehat{\mathbf{x}}}^i) + L_{W_i}V_i(\mathbf{x})\theta_i \\ &= L_{f_i}V_i(\mathbf{x}) + L_{G_i}V_i(\mathbf{x})k_i(\mathbf{w}) + L_{W_i}V_i(\mathbf{x})\theta_i \\ &+ L_{G_i}V_i(\mathbf{x})[k_i(w_i, \underline{\widehat{\mathbf{x}}}^i) - k_i(\mathbf{w})] \\ &\leq -\alpha_i(\|x_i\|) + \gamma_i(\phi_i) + \epsilon_i(\mu_o) \\ &+ L_{G_i}V_i(\mathbf{x})[k_i(w_i, \underline{\widehat{\mathbf{x}}}^i) - k_i(\mathbf{w})] \end{aligned} \tag{13}$$

where we have used the bound in (8) to derive the inequality in (13). Examining this inequality and comparing it with the inequality in (8) obtained for the non-networked plant (i.e., under continuous communication) reveals the perturbation introduced by suspending the transfer of state estimates from the rest of the plant to the i-th unit and the reliance on the model estimates instead. This perturbation potentially alters the rate at which V_i decays, especially as the model estimation error grows, and could become large enough to cause instability. Before this happens, communication with the rest of the plant must be re-established to allow updating the states of the models embedded in the local control system so that the plant-model mismatch can be corrected in time.

However, since full-state measurements are not available, the bound in (8) cannot be used directly as the criterion for verifying when communication should be terminated or re-established. To address this problem, we need to derive an alternative bound that can be checked by monitoring only the state estimate generated by the local observer. The following proposition provides such a bound and characterizes the evolution of the time-derivative of V_i in terms of w_i (instead of x_i) under continuous communication by exploiting the error convergence properties of the local observers.

Proposition 2: Consider the closed-loop system of (1)-(4) and (7) for which Assumption 1 holds with $\phi_i \leq \phi^*_i$, $\mu_i \leq \tilde{\mu}_i$, $\| x_i(0) \| \leq \delta_{b,i}$, $\| w_i(0) \| \leq \delta_{b,i}$, and $\| \theta_i \| \leq \theta_{b,i}$ for all $i = 1, \cdots, n$. Then, there exists a class $\mathscr{K}$ function $\Theta_i(\cdot)$ such that:

$$\dot{V}_i(w_i(t)) \leq -\alpha_i(\|w_i(t)\|) + \gamma_i(\phi_i) + \Theta_i(\mu_o) \tag{14}$$

for all $t \geq T^b$, with T_b and μ_o as defined in Proposition 1.

Proof: From the continuity of $\alpha_i(\cdot)$ and $\dot{V}_i(\cdot)$ in x_i, it follows that for any $\mu_i > 0$ there exist class $\mathscr{K}$ functions $\Delta_i(\cdot)$ and $\Gamma_i(\cdot)$, such that if $\| x_i - w_i \| \leq K_i\mu_i$ for some K_i, the following estimates hold:

$$\dot{V}_i(w_i) - \dot{V}_i(x_i) \leq \Gamma_i(\mu_i), \ \alpha_i(\|x_i\|) \geq \alpha_i(\|w_i\|) - \Delta_i(\mu_i)$$

Substituting these relations into (8) yields (14) where $\Theta_i(\cdot) := \Delta_i(\cdot) + \Gamma_i(\cdot) + \epsilon_i(\cdot)$. ∎

Having characterized the expected behavior of $V_i(w_i)$ under continuous communication, we are now in a position to state the main result of this section. The following theorem describes a strategy for suspending and re-establishing communication on the basis of the evolution of the local observer-generated state estimate.

Theorem 1: Consider the nonlinear plant of (1), for which the Lyapunov functions V_i, $i = 1, \cdots, n$, satisfy (8) when the state estimates generated by the local observers of the corresponding units are exchanged continuously between the plant units. Consider also the i-th plant unit subject to the model-based networked controller of (12) with $\widehat{x}^i_j(t) = w_j(t)$ for all $0 \leq t \leq T^b$, for all j and all i. Let $t^{i-}_k > T^b$ be the earliest time such that:

$$\dot{V}_i(w_i(t^{i-}_k)) > -\alpha_i(\|w_i(t^{i-}_k)\|) + \gamma_i(\phi_i) + \Theta_i(\mu_o) \tag{15}$$

where $w_i(t^{i-}_k) = \lim_{t \to t^{i-}_k} w_i(t)$, then the update law given by $\widehat{x}^i_j(t^i_k) = w_j(t^i_k)$ for all $j \neq i$ ensures that $\dot{V}_i(x_i(t^i_k)) \leq -\alpha_i(\|x_i(t^i_k)\|) + \gamma_i(\phi_i) + \Psi_i(\mu_o)$, for some class $\mathscr{K}$ function $\Psi_i(\cdot) \geq \Theta_i(\cdot)$.

Proof: In light of (8), (13) and (14), we have that for all $t \geq T^b$, the evolution of the local state estimate, w_i, under the model-based networked controller of (12) satisfies:

$$\begin{aligned} \dot{V}_i(w_i(t)) &\leq -\alpha_i(\|w_i(t)\|) + \gamma_i(\phi_i) + \Theta_i(\mu_o) \\ &\quad + L_{G_i}V_i(\mathbf{x}(t))[k_i(w_i(t), \underline{\widehat{\mathbf{x}}}^i(t)) - k_i(\mathbf{w}(t))] \end{aligned} \tag{16}$$

Therefore, if at any time $t^{i-}_k > T^b$ (15) is satisfied, we conclude that $L_{G_i}V_i(\mathbf{x}(t^{i-}_k))[k_i(w_i(t^{i-}_k), \underline{\widehat{\mathbf{x}}}^i(t^{i-}_k)) - k_i(\mathbf{w}(t^{i-}_k))] > 0$. By re-setting the states of the models embedded in the i-th control system such that $\widehat{x}^i_j(t^i_k) = w_j(t^i_k)$, for $j \neq i$, we have $k_i(\underline{\widehat{\mathbf{x}}}^i(t^i_k), w_i(t^i_k)) - k_i(\mathbf{w}(t^i_k)) = 0$ which when substituted into (16) restores the original bound obtained under continuous communication: $\dot{V}_i(w_i(t^i_k)) \leq -\alpha_i(\|w_i(t^i_k)\|) + \gamma_i(\phi_i) + \Theta_i(\mu_o)$. This, together with the $O(\mu_o)$ closeness between x_i and w_i for all $t \geq T^b$ and the continuity of $\dot{V}_i(\cdot)$ and $\alpha_i(\cdot)$ with respect to w_i, implies the existence of a class $\mathscr{K}$ function $\Psi_i(\mu_o) \geq \Theta_i(\mu_o)$ such that $\dot{V}_i(x_i(t^i_k)) \leq -\alpha_i(\|x_i(t^i_k)\|) + \gamma_i(\phi_i) + \Psi_i(\mu_o)$. ∎

Remark 3: Notice that, unlike the case under full-state feedback, continuous communication is initially needed for a short period of time to ensure that for the given choice of the observer design parameter, the state estimation error has decreased to a sufficiently small value such that, from that point in time onwards, the position of the local state can be inferred reliably from the local state estimate. Recall that the bounds in (8) and (14) which constitute the basis for turning on and off the communication are valid only after the observer estimation error has become sufficiently small.

IV. Simulation Study: Application to Chemical Reactors with Recycle

We consider a plant composed of two cascaded non-isothermal continuous stirred-tank reactors (CSTRs) with recycle (see the plant model in [10]). The output of CSTR 2 is passed through a separator that removes the products and recycles the unreacted material to CSTR 1. The reactant species A is consumed in each reactor by three parallel irreversible exothermic reactions; and a jacket is used to remove/provide heat to each reactor. The control objective is to stabilize the plant at the (open-loop) unstable steady-state with the rates of heat input, denoted by Q_1 and Q_2, chosen as the manipulated variables for the two reactors. Only the temperatures of the two reactors are assumed to be available as measurements. The control objective is to be achieved with minimal data exchange between the local control systems of the reactors over a shared communication network. Following the methodology presented in Section III, the plant is cast in the form of (1) with $n = 2$, where x_i and u_i are the (dimensionless) state and manipulated input vectors for the i-th unit, respectively, and θ_i represents the vector of parametric uncertainties in the enthalpies of the three reactions. A Lyapunov-based controller of the form of (2)-(4) with $V_i = x_i' P_i x_i$, $i = 1, 2$, $P_1 = P_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}$, is then designed for each reactor to enforce robust stability and uncertainty attenuation in the absence of communication suspensions. A state observer of the following form is also designed for each reactor: $\dot{w}_i = f_i(w_1, w_2) + g_i u_i + L_i(y_i - C_i w_i)$, where w_i is an estimate of x_i, L_i is the observer gain (chosen as $L_1 = [79.4 \quad -94.8]'$ and $L_2 = [72.3 \quad 352.5]'$ in the simulations). The controller and observer design parameters were chosen to ensure that the closed-loop state of each unit converges in finite time to a small neighborhood of the desired steady-state. It was verified that, when w_1 and w_2 are communicated continuously between the two units, the controllers successfully stabilize the closed-loop state of the plant near the desired steady state.

For the case when the observer estimates can be received only through the shared network, and in order to reduce utilization of network resources, the following models are used during periods of communication suspension to generate estimates of the state of the neighboring unit: $\widehat{x}_i = \widehat{f}_i(w_i, \widehat{x}_j) + \widehat{g}_i u_i(w_i, \widehat{x}_j)$, where for simplicity, $\widehat{f}_i = f_i$ and $\widehat{g}_i = g_i$. Using the models' estimates, the control laws are implemented as in (12) where the estimates are used by the local controller so long as no data from the neighboring unit are transmitted over the network, but are updated using the observer estimates provided by the local state observer of the other reactor whenever they become available from the network. The solid profiles in Figs.1(a)-(b) depict the resulting evolution of the closed-loop state profiles when the plant is operated using the dynamic communication policy presented in Section III. In obtaining these plots, models with parametric uncertainty of $\theta_i = [-0.1 \ -0.1 \ -0.1]'$, $i = 1, 2$, were used, and the following values were chosen for the controller tuning parameters: $\chi_i = 1.1$, $\rho_i = 0.0001$, $\phi_i' = 0.0001$. In this case, the evolution of $V_i(w_i)$ is monitored locally within each unit, and an update is requested and received only when either (1) $V_i(w_i)$ is on the verge of increasing while w_i is outside the terminal set, or (2) w_i is on the verge of escaping the terminal set while previously inside. It can be seen from the figures that the plant can be successfully stabilized near the desired steady-state.

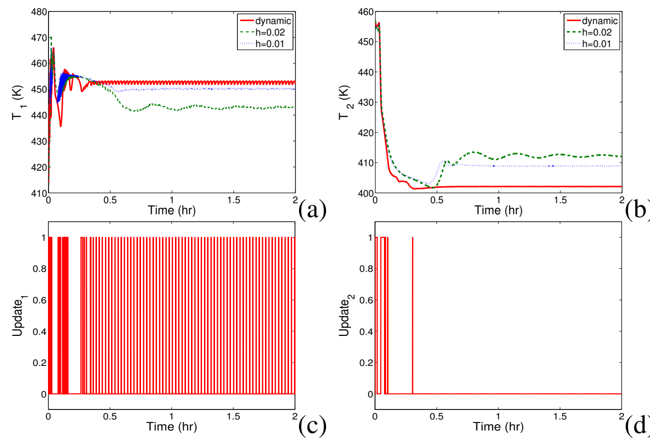

Fig. 1. Plots (a)-(b): Closed-loop temperature profiles under the dynamic communication policy between the reactors. Plots (c)-(d): Update times of the models embedded in the local control systems of the two reactors.

Figs.1(c)-(d) show the time instances at which the models embedded in the local control systems of the two reactors are updated. The variable "Update_i" takes a value of 1 when the local control system for the i-th reactor requests and receives an update from its neighbor to reset the state of the model embedded within it, and takes a value of zero when no updates are needed. It can be seen from the two plots that continuous communication between the two local control systems is needed only initially and over a short period of time. As the closed-loop plant state settles close to the desired operating point, no further communication from the first unit to the second is required; however, periodic updates for CSTR 1 (although less frequent than initially required), are still needed due to a tighter ultimate bound requirement. Notice that, compared with the state feedback case, communication between the two reactors is needed more frequently under output feedback because of the uncertainty in the models (as well as the observer estimation errors). However, it can be shown that in the case of perfect models, we can practically recover the same communication pattern obtained under state feedback control.

For comparison, we also implemented a static communication policy in which the two reactors communicate with each other over the network periodically, and each unit transmits its observer estimates at a constant rate (the same for both units) to update the model embedded in the local controller of its neighboring unit. This policy assumes that the sensors of all the units are given access to the network and can successfully transmit their data simultaneously. The dashed and dotted profiles in Figs.1(a)-(b) depict the evolution of the closed-loop state profiles of the two reactors under this policy. It can be verified that the maximum allowable update period that guarantees closed-loop stability is $h = 0.02$ hr, and that the closed-loop plant becomes unstable for $h > 0.02$ hr. However, operating at the maximum allowable update period leads to poor performance (see dashed profiles), especially when compared with the performance obtained under the dynamic policy. In order to achieve an overall closed-loop response comparable to the one obtained under the dynamic communication policy, a smaller update period of $h = 0.01$ hr must be used (see dotted profiles) which requires further increase in communication.

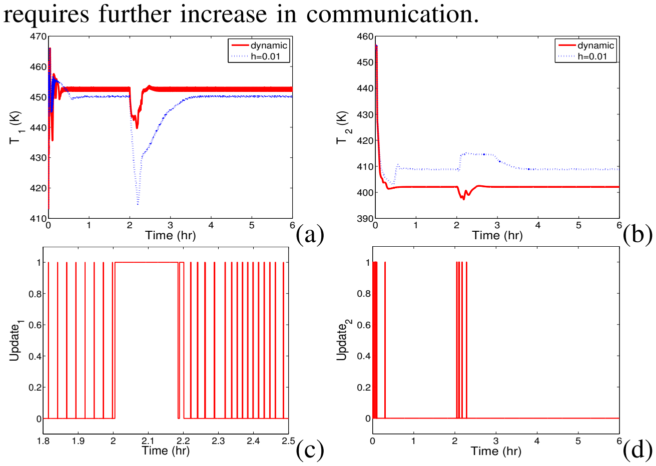

Fig. 2. Plots (a)-(b): Closed-loop state profiles under dynamic and static communication policies between the reactors when the closed-loop system is subject to unexpected external disturbance in the flow rate of the fresh feed stream to CSTR 1. Plots (c)-(d): Update times of the models embedded in the local controllers under the dynamic communication policy.

In addition to closed-loop performance and network utilization considerations, we have also investigated the disturbance-handling capabilities of both the static and dynamic communication policies in order to compare their robustness with respect to unanticipated disturbances during plant operation. To this end, a 50% step disturbance was introduced in the flow rate of the fresh feed stream in CSTR 1, F_0, at time $t = 2$ hr (i.e., after the plant has reached its steady-state), and this disturbance lasts for 0.2 hrs. Fig.2 panels (a)-(b) depict the resulting closed-loop temperature profiles subject to the unexpected external disturbance. It can be seen that while the control systems under the dynamic communication policy can successfully recover from the disturbance and force the plant to return to its steady-state (see the solid profiles), the closed-loop performance deteriorates under the static communication policy (in this example we used a constant update period $h = 0.01$ hr) where the states move away from the desired steady-state significantly in the presence of the disturbance (see the dotted profiles). Fig.2 panels (c)-(d) show the update times of the models embedded in the local control systems of the two reactors when the dynamic communication policy is implemented. These two plots highlight the adaptive nature of the dynamic communication policy which is the reason for its ability to overcome the influence of the disturbance on the closed-loop plant. Specifically, it can be seen that the dynamic policy responds to the external disturbance by increasing the frequency of communication between the two control systems following the onset of the disturbance. This in turn allows the plant states to return to the desired steady-state by the end of the disturbance following which only less frequent communication is needed to maintain stability.

References

[1] D. D. Siljak, *Decentralized Control of Complex Systems*. London: Academic Press, 1991.

[2] J. Lunze, *Feedback Control of Large Scale Systems*. U.K.: Prentice-Hall, 1992.

[3] E. Camponogara, D. Jia, B. H. Krogh, and S. Talukdar, "Distributed model predictive control," *IEEE Contr. Syst. Mag.*, vol. 22, pp. 44–52, 2002.

[4] A. N. Venkat, J. B. Rawlings, and S. J. Wright, "Stability and optimality of distributed model predictive control," in *Proceedings of 44th IEEE Conference on Decision and Control*, Seville, Spain, 2005, pp. 6680–6685.

[5] M. Baldea, P. Daoutidis, and A. Kumar, "Dynamics and control of integrated networks with purge streams," *AIChE J.*, vol. 52, pp. 1460–1472, 2006.

[6] K. Jillson and B. Ydstie, "Process networks with decentralized inventory and flow control," *J. Proc. Contr.*, vol. 17, pp. 399–413, 2007.

[7] M. D. Tetiker, A. Artel, F. Teymour, and A. Cinar, "Control of grade transitions in distributed chemical reactor networks: An agent-based approach," *Comp. & Chem. Eng.*, vol. 32, pp. 1984–1994, 2008.

[8] J. Liu, D. M. de la Pena, and P. D. Christofides, "Distributed model predictive control of nonlinear systems subject to asynchronous and delayed measurements," *Automatica*, vol. 46, pp. 52–61, 2009.

[9] J. M. Maestre, D. M. de la Pena, and E. F. Camacho, "Distributed model predictive control based on a cooperative game," *Optimal Control Applications and Methods,* in press.

[10] Y. Sun and N. H. El-Farra, "Quasi-decentralized model-based networked control of process systems," *Comp. & Chem. Eng.*, vol. 32, pp. 2016–2029, 2008.

[11] ——, "Resource-aware quasi-decentralized control of nonlinear plants over communication networks," in *Proceedings of American Control Conference*, St. Louis, MO, 2009, pp. 154–159.

[12] ——, "A quasi-decentralized approach for networked state estimation and control of process systems," *Ind. & Eng. Chem. Res.*, vol. 49, pp. 7957–7971, 2010.

[13] T. C. Yang, "Networked control systems: a brief survey," *IEE Procedings-Control Theory and Applications*, vol. 152, pp. 403–412, 2006.

[14] J. P. Hespanha, P. Naghshtabrizi, and Y. Xu, "A survey of recent results in networked control systems," *Proceedings of the IEEE*, vol. 95, pp. 138–162, 2007.

[15] L. A. Montestruque and P. J. Antsaklis, "On the model-based control of networked systems," *Automatica*, vol. 39, pp. 1837–1843, 2003.

[16] Y. Sun and N. H. El-Farra, "Quasi-decentralized networked process control using an adaptive communication policy," in *Proceedings of American Control Conference*, Baltimore, MD, 2010, pp. 2841 – 2846.

[17] E. D. Sontag, "Smooth stabilization implies coprime factorization," *IEEE Trans. Automat. Contr.*, vol. 34, pp. 435–443, 1989.

[18] R. A. Freeman and P. V. Kokotovic, *Robust Nonlinear Control Design: State-Space and Lyapunov Techniques*. Boston: Birkhauser, 1996.

[19] M. Krstic and H. Deng, *Stabilization of Nonlinear Uncertain Systems*, 1st ed. Berlin, Germany: Springer, 1998.

[20] P. D. Christofides and N. H. El-Farra, *Control of Nonlinear and Hybrid Process Systems: Designs for Uncertainty, Constraints and Time-Delays*. Berlin, Germany: Springer-Verlag, 2005.

[21] H. Khalil, "Robust servomechanism output feedback controller for feedback linearizable systems," *Automatica*, vol. 30, pp. 1587–1599, 1994.