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Nonlinear Control of an Autonomous Underwater Vehicle: A RISE-Based Approach

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***Abstract*—Autonomous and remotely operated marine vehicles such as ships and submarines are becoming a key component in several aspects of maritime industry and defense. This paper explores the development of a nonlinear controller for a fully actuated autonomous underwater vehicle (AUV) using a robust integral of the sign of the error (RISE) feedback term with a neural network (NN) based feedforward term to achieve semi-global asymptotic tracking results in the presence of complete model uncertainty and unknown disturbances. A simulation is provided to demonstrate the proposed controller on an experimentally validated AUV model.**

I. Introduction

Autonomous and remotely operated marine vehicles such as ships and submarines are becoming a key component in several aspects of maritime industry and defense. Advances in sensing and control are enabling autonomous surface vehicles (ASV) and autonomous underwater vehicles (AUV) to become vital assets in search and recovery, exploration, surveillance, monitoring, and military applications [1]. Accurate and robust trajectory tracking control is crucial to the performance of these vehicles and to the advancement of autonomy in the maritime environment.

The development of controllers for AUVs is inhibited by the fact that the dynamics are time-varying, nonlinear, and include difficult to model effects such as hydrodynamic damping and the effects of external disturbances such as sea states and ocean currents. Some results in literature have been developed that assume exact knowledge of the dynamics, obtained from empirical studies [2]–[4]. While these controllers are able to achieve certain performance results, the empirical models are often inaccurate and extremely difficult to obtain, and the stability of the resulting controller when exact knowledge is not available is uncertain. Motivated to provide robustness to model uncertainty, adaptive controllers for AUVs were developed in results such as [5]–[7]. These results are based on the assumption that the unknown dynamics can be linearly parameterized. Results in [8] use traditional adaption methods and a discontinuous switching controller to compensate for nonlinearly parameterized terms. In comparison to these traditional adaptive control results, efforts in [9]–[14] exploit fuzzy logic or neural network (NN) based methods to approximate the uncertain dynamics including added disturbances (and without the linear in the parameters assumption); however, the presence of external disturbances and the inherent function approximation error result in a uniformly ultimately bounded tracking result. The results in [10], [11] use sliding mode control as an adjuvant to eliminate the steady state error, in a similar manner as the pure robust sliding mode control results in [15]–[17]; however, the resulting controllers are discontinuous.

Discontinuous controllers suffer from limitations such as the demand for infinite bandwidth and chatter, suggesting other composite black-box methods may provide better results. Motivated by fundamental problems with stand-alone NN solutions, [18] developed a continuous controller based on the robust integral of the sign error (RISE) approach incorporated with a NN-based feedforward term to achieve semi-global asymptotic tracking. Since both the NN and the RISE control structures are modular and model independent methods, the resulting controller is a universal controller [19] that can be used for general Euler-Lagrange dynamics; a fitting solution for a system with complex marine dynamics. Additionally, NN weights and thresholds are generated automatically on-line and require no offline training procedure. The contribution of this paper is to develop a continuous tracking controller for a general class of uncertainty for a coupled MIMO fully actuated underwater vehicle. A Lyapunov-based stability analysis is included to prove the continuous RISE augmented NN control method yields semi-global asymptotic tracking.

II. Kinematic and Dynamic Model Development

The AUV's position and orientation relative to the earth-fixed frame is given by the kinematic equation of motion [20]

$$\dot{\eta} = J(\eta)\,\nu, \tag{1}$$

where $\nu(t) \in \mathbb{R}^6$ is the linear and angular velocity vector with coordinates in the body-fixed frame, $\eta(t) \in \mathbb{R}^6$ is the position and orientation vector with coordinates in the earth-fixed frame, and $J(\eta) \in \mathbb{R}^{6\times 6}$ is a Jacobian transformation matrix relating the two frames, defined as

$$J \triangleq \begin{bmatrix} J_1(\eta) & 0_{3\times 3} \\ 0_{3\times 3} & J_2(\eta) \end{bmatrix}. \tag{2}$$

In (2), $J_1(\eta) \in \mathbb{R}^{3\times 3}$ and $J_2(\eta) \in \mathbb{R}^{3\times 3}$ are defined as

$$J_1(\eta) \triangleq \begin{bmatrix} c\psi c\theta & -s\psi c\phi + c\psi s\theta s\phi & s\psi s\phi + c\psi c\phi s\theta \\ s\psi c\theta & c\psi c\phi + s\phi s\theta s\psi & -c\psi s\phi + s\theta s\psi c\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix}$$

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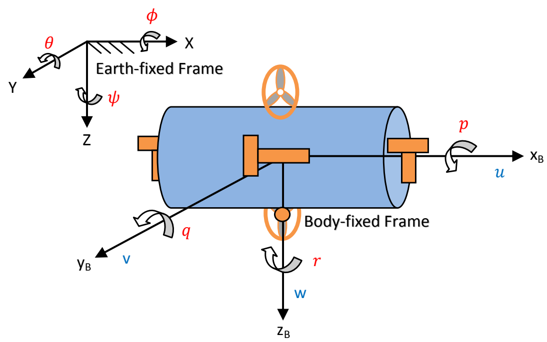

Figure 1. AUV reference frames and associated state vector directions.

$$J_2(\eta) \triangleq \begin{bmatrix} 1 & s\phi t\theta & c\phi t\theta \\ 0 & c\phi & -s\phi \\ 0 & s\phi/c\theta & c\phi/c\theta \end{bmatrix},$$

where $s\cdot, c\cdot, t\cdot$ are shorthand notation for $\sin(\cdot)$, $\cos(\cdot)$, and $\tan(\cdot)$, respectively. From [20], the state vectors of the AUV are illustrated in Fig. 1 and are defined as

$$\begin{aligned} \eta &\triangleq \begin{bmatrix} x & y & z & \phi & \theta & \psi \end{bmatrix}^T \\ \nu &\triangleq \begin{bmatrix} u & v & w & p & q & r \end{bmatrix}^T, \end{aligned}$$

where x, y, and z represent the Cartesian position of the center of mass, ϕ, θ, and ψ are represent the orientation (roll, pitch and yaw), u, v, and w represent the surge, sway and heave velocities, and p, q, and r represent the angular velocities.

Under the assumptions that i) the body-fixed frame coincides with the center of gravity of the AUV, ii) accelerations of a point on the surface of the earth can be neglected (i.e. reference frame XYZ is considered to be inertial), and iii) added mass is constant (independent of wave frequency), the dynamic motion of the AUV can be described by a body-fixed vector representation as [20]

$$M(\nu)\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) + \tau_d(t) = \tau_b \tag{3}$$

where $M(\nu) \in \mathbb{R}^{6\times6}$ is the inertia matrix (including added mass), $C(\nu) \in \mathbb{R}^{6\times6}$ is the matrix of Coriolis and centripetal effects, $D(\nu) \in \mathbb{R}^{6\times6}$ is the friction and hydrodynamic damping matrix, $g(\eta) \in \mathbb{R}^6$ is the vector of gravitational and buoyancy forces and moments, $\tau_d(t) \in \mathbb{R}^6$ is a vector of nonlinear disturbances (e.g., current, waves, etc), and $\tau_b(t) \in \mathbb{R}^6$ is a vector of external forces and moments about the center of mass in the body-fixed frame. An earth-fixed representation of the dynamics [20] can be generated by applying kinematic transformations in (1) to (3), assuming $J(\eta)$ is nonsingular, yielding:

$$\bar{M}(\eta,\dot{\eta})\dot{\nu} + \bar{C}(\eta,\dot{\eta})\nu + \bar{D}(\eta,\dot{\eta})\nu + \bar{g}(\eta) + \tau_d(t) = \tau. \tag{4}$$

The dynamic model can also be expressed using desired trajectories where each term is subscripted as $(\cdot)_d$ and the arguments of each term, $\eta_d, \dot{\eta}_d(t) \in \mathbb{R}^6$, denote a time-varying desired trajectory and its derivative. The subsequent development is based on the assumptions that $\eta(t)$ and $\nu(t)$ are measurable (using sensors discussed in [21]) and that $M(\nu)$, $C(\nu)$, $D(\nu)$, $g(\eta)$, and $\tau_d(t)$ are unknown. Additionally, the following properties and assumptions will be used throughout the paper.

Assumption 1: The Jacobian and its inverse exist and are bounded by a known positive constant $\bar{J} \in \mathbb{R}$ such that

$$\|J(\eta)\|, \left\|J^{-1}(\eta)\right\| \leq \bar{J}.$$

Remark 1. During the subsequent control development, we assume that the minimum singular value of $J(\eta)$ is greater than a known small positive constant $\delta > 0$, such that $max\left\{\left\|J^{-1}(\eta)\right\|\right\}$ is known *a priori*, and hence, all kinematic singularities are always avoided.

Assumption 2: The disturbance term and its first two time derivatives are bounded, i.e., $\tau_d(t), \dot{\tau}_d(t), \ddot{\tau}_d(t) \in \mathcal{L}_\infty$.

Property 1: The inertia matrix $\bar{M}(\eta,\dot{\eta})$ is symmetric, positive definite and satisfies the following inequality $\forall y(t) \in \mathbb{R}^n$:

$$\underline{m}\|y\|^2 \leq y^T\bar{M}(\eta,\dot{\eta})y \leq \bar{m}(\eta,\dot{\eta})\|y\|^2,$$

where $\underline{m} \in \mathbb{R}$ is a known positive constant, and $\bar{m}(\nu) \in \mathbb{R}$ is a known positive function.

Property 2: If $\eta(t), \nu(t) \in \mathcal{L}_\infty$, then $C(\nu)$, $D(v)$, $g(\eta) \in \mathcal{L}_\infty$.

III. Control Objective

The objective is to design a controller that enables the six-degree of freedom (DOF) position and orientation of the AUV to track a desired, time-varying trajectory despite uncertainties in the dynamic model. To quantify this objective, a position tracking error $e_1(t) \in \mathbb{R}^6$ is defined as

$$e_1 \triangleq \eta_d - \eta, \tag{5}$$

where the desired trajectory and its derivatives exist and are bounded such that

$$\|\eta_d\|, \|\dot{\eta}_d\|, \|\ddot{\eta}_d\|, \|\dddot{\eta}_d\| \leq \zeta_1, \tag{6}$$

where $\zeta_1 \in \mathbb{R}^6$ is a known positive constant. Taking the time derivative of (5), inserting a virtual control $\nu_d(t) \in \mathbb{R}^6$, and using (1) yields

$$\dot{e}_1 = \dot{\eta}_d + Je_2 - J\nu_d, \tag{7}$$

where $e_2(t) \in \mathbb{R}^6$ is a backstepping error that quantifies the mismatch between the actual and virtual control inputs and is defined as

$$e_2 \triangleq \nu_d - \nu. \tag{8}$$

Based on (7), $\nu_d(e_1, t)$ is designed as

$$\nu_d = J^{-1}(k_1 e_1 + \dot{\eta}_d), \tag{9}$$

where $k_1 \in \mathbb{R}^{6\times6}$ is a positive-definite gain matrix. Substituting (9) into (7) yields

$$\dot{e}_1 = Je_2 - k_1 e_1. \tag{10}$$

To facilitate the subsequent stability analysis, a filtered tracking error, $r(e_2, t) \in \mathbb{R}^6$, is defined as

$$r \triangleq \dot{e}_2 + \alpha e_2, \tag{11}$$

where $\alpha \in \mathbb{R}$ is a positive gain scalar. The filtered tracking error defined in (11) is not measurable since it is dependent on acceleration, $\dot{\nu}(t)$.

Property 3: The ideal NN weights are assumed to exist and be bounded by known positive values such that

$$\|V\|_F^2 \leq V_B \qquad \|W\|_F^2 \leq W_B, \tag{12}$$

where $\|\cdot\|_F$ is the Frobenius norm of a matrix.

IV. Control Development

A. Open Loop Error System

After premultiplying (11) by $\bar{M}(\eta, \dot{\eta})$ from (4), and using (1), (3), (5), (8) and (10), the open loop error system for $r(t)$ can be expressed as

$$\bar{M}r = f_d + S + \tau_d - \tau, \tag{13}$$

where the auxiliary function $f_d(\eta_d, \dot{\eta}_d, \ddot{\eta}_d) \in \mathbb{R}^6$ is defined as

$$\begin{aligned} f_d \;=\;& \bar{M}_d \dot{J}_d^{-1} \dot{\eta}_d + \bar{M}_d J_d^{-1} \ddot{\eta}_d \\ & + \bar{C}_d J_d^{-1} \dot{\eta}_d + \bar{D}_d J_d^{-1} \dot{\eta}_d + \bar{g}_d, \end{aligned} \tag{14}$$

and the auxiliary function $S(\eta, \dot{\eta}, e_1, \dot{e}_1, t) \in \mathbb{R}^6$ is defined as

$$\begin{aligned} S \;=\;& \bar{M}\dot{J}^{-1} k_1 e_1 + \left(\bar{M}\dot{J}^{-1} - \bar{M}_d \dot{J}_d^{-1}\right) \dot{\eta}_d \\ & + \bar{M} J^{-1} k_1 \dot{e}_1 + \left(\bar{M} J^{-1} - \bar{M}_d J_d^{-1}\right) \ddot{\eta}_d \\ & + \bar{M}\alpha J^{-1} k_1 e_1 + \bar{M}\alpha J^{-1} \dot{e}_1 + \bar{C} J^{-1} \dot{\eta} - \bar{C}_d J_d^{-1} \dot{\eta}_d \\ & + \bar{D} J^{-1} \dot{\eta} - \bar{D}_d J_d^{-1} \dot{\eta}_d + \bar{g} - \bar{g}_d. \end{aligned}$$

The universal approximation theorem can be used to approximate the uncertain auxiliary function in (14) by a three-layer NN as

$$f_d = W^T \sigma\left(V^T \gamma_d\right) + \varepsilon(\gamma_d), \tag{15}$$

where $V(t) \in \mathbb{R}^{(18+1)\times N_2}$ and $W(t) \in \mathbb{R}^{(N_2+1)\times 6}$ are bounded constant ideal weights for the first-to-second and second-to-third layers respectively, N_2 is the number of neurons in the hidden layer, $\sigma(\cdot) \in \mathbb{R}^{N_2+1}$ is an activation function, and $\gamma_d(t) \in \mathbb{R}^{19}$ denotes the input to the NN defined on a compact set containing the known bounded desired trajectories as

$$\gamma_d = \left[1, \eta_d^T, \dot{\eta}_d^T, \ddot{\eta}_d^T\right]^T. \tag{16}$$

From (6), the following inequalities hold

$$\begin{aligned} \|\varepsilon(\gamma_d)\| \leq \varepsilon_{b_1}, \qquad & \|\dot{\varepsilon}(\gamma_d, \dot{\gamma}_d)\| \leq \varepsilon_{b_2}, \\ \|\ddot{\varepsilon}(\gamma_d, \dot{\gamma}_d, \ddot{\gamma}_d)\| \leq & \varepsilon_{b_3}, \end{aligned} \tag{17}$$

where $\varepsilon_{b_1}, \varepsilon_{b_2}, \varepsilon_{b_3} \in \mathbb{R}$ are known positive constants.

B. Control Design

From (13), the controller is designed using a three-layer NN feedforward term augmented by a RISE feedback term as

$$\tau = \hat{f}_d + \mu. \tag{18}$$

The RISE feedback term $\mu(e_2, t) \in \mathbb{R}^6$ is defined as [22], [23]

$$\mu = (k_s + 1) e_2 - (k_s + 1) e_2(0) + \upsilon \tag{19}$$

where $\upsilon(e_2, t) \in \mathbb{R}^6$ is the generalized solution to

$$\dot{\upsilon} = (k_s + 1)\alpha e_2 + \beta sgn(e_2), \;\; \upsilon(0) = 0, \tag{20}$$

and $k_s \in \mathbb{R}$ and $\beta \in \mathbb{R}$ are positive, constant control gains. The NN feedforward term $\hat{f}_d(t) \in \mathbb{R}^6$ in (18) is designed as

$$\hat{f}_d = \hat{W}^T \sigma\left(\hat{V}^T \gamma_d\right), \tag{21}$$

where $\hat{V}(t) \in \mathbb{R}^{19\times N_2}$ and $\hat{W}(t) \in \mathbb{R}^{(N_2+1)\times 6}$ are estimates of the ideal weights, and $\gamma_d(t)$ is defined in (16). The estimates for the NN weights in (21) are generated on-line as

$$\dot{\hat{W}} \;=\; proj\left(\Gamma_1 \hat{\sigma}' \hat{V}^T \dot{\gamma}_d e_2^T\right) \tag{22}$$

$$\dot{\hat{V}} \;=\; proj\left(\Gamma_2 \dot{\gamma}_d \left(\hat{\sigma}'^T \hat{W} e_2\right)^T\right), \tag{23}$$

where $\Gamma_1 \in \mathbb{R}^{(N_2+1)\times(N_2+1)}$ and $\Gamma_2 \in \mathbb{R}^{19\times 19}$ are positive-definite, constant symmetric control gain matrices, and $\hat{\sigma}'(\cdot) \in \mathbb{R}^{N_2+1}$ denotes the partial derivative of $\hat{\sigma} = \sigma\left(\hat{V}^T \gamma_d\right)$.

C. Closed Loop Error System

Substituting the controller in (18) into the open loop tracking error in (13) yields the closed loop tracking error system

$$\bar{M}r = f_d - \hat{f}_d + S + \tau_d - \mu. \tag{24}$$

Estimate mismatches for the ideal weights are defined as $\tilde{V}(t) = V(t) - \hat{V}(t)$ and $\tilde{W}(t) = W(t) - \hat{W}(t)$, where $\tilde{V}(t) \in \mathbb{R}^{19\times N_2}$ and $\tilde{W}(t) \in \mathbb{R}^{(N_2+1)\times 6}$. To facilitate the subsequent RISE based stability analysis, the time derivative of (24) is determined by using (15) and (21), and adding and subtracting $W^T \hat{\sigma}' \hat{V}^T \dot{\gamma}_d + \hat{W}^T \hat{\sigma}' \tilde{V}^T \dot{\gamma}_d$ to the resulting expression as

$$\begin{aligned} \bar{M}\dot{r} \;=\;& -\dot{\bar{M}}r + \hat{W}^T \hat{\sigma}' \tilde{V}^T \dot{\gamma}_d + \tilde{W}^T \hat{\sigma}' \hat{V}^T \dot{\gamma}_d \\ & + W^T \sigma' V^T \dot{\gamma}_d - W^T \hat{\sigma}' \hat{V}^T \dot{\gamma}_d \\ & - \hat{W}^T \hat{\sigma}' \tilde{V}^T \dot{\gamma}_d + \dot{S} - \dot{\hat{W}}^T \hat{\sigma} \\ & - \hat{W}^T \hat{\sigma}' \dot{\hat{V}}^T \gamma_d + \dot{\varepsilon} + \dot{\tau}_d - \dot{\mu}, \end{aligned} \tag{25}$$

where $\hat{\sigma}'(\cdot)$ is introduced in (22) and (23), and the time derivative of (19) is given by $\dot{\mu}(e_2, r, t) \in \mathbb{R}^6$,

$$\dot{\mu}(t) = (k_s + 1) r + \beta sgn(e_2). \tag{26}$$

Through the strategic grouping of terms, (25) can be rewritten as

$$\bar{M}\dot{r} = -\frac{1}{2}\dot{\bar{M}}r + \tilde{N} + N - e_2 - (k_s + 1)\, r - \beta sgn(e_2), \quad (27)$$

where and $\tilde{N}\left(\hat{W}, \hat{V}, \gamma_d, \dot{\gamma}_d, e_1, e_2, r, t\right) \in \mathbb{R}^6$ and $N\left(\hat{W}, \hat{V}, \gamma_d, \dot{\gamma}_d, t\right) \in \mathbb{R}^6$ are defined as

$$\tilde{N} \triangleq -\frac{1}{2}\dot{\bar{M}}r - proj\left(\Gamma_1 \hat{\sigma}' \hat{V}^T \dot{\gamma}_d e_2^T\right)\hat{\sigma} - \hat{W}^T \hat{\sigma}' proj\left(\Gamma_2 \dot{\gamma}_d \left(\hat{\sigma}'^T \hat{W} e_2\right)^T\right)\gamma_d + \dot{S} + e_2, \quad (28)$$

$$N \triangleq N_d + N_B. \quad (29)$$

In (29), $N_d(\gamma_d, \dot{\gamma}_d, t) \in \mathbb{R}^6$ is defined as

$$N_d \triangleq W^T \sigma' V^T \dot{\gamma}_d + \dot{\varepsilon} + \dot{\tau}_d$$

and $N_B\left(\hat{W}, \hat{V}, \gamma_d, \dot{\gamma}_d, t\right) \in \mathbb{R}^6$ is separated such that

$$N_B \triangleq N_{B_1} + N_{B_2}, \quad (30)$$

where $N_{B_1}\left(\hat{W}, \hat{V}, \gamma_d, \dot{\gamma}_d, t\right), N_{B_2}\left(\hat{W}, \hat{V}, \gamma_d, \dot{\gamma}_d, t\right) \in \mathbb{R}^6$ are defined as

$$N_{B_1} \triangleq -W^T \hat{\sigma}' \hat{V}^T \dot{\gamma}_d - \hat{W}^T \hat{\sigma}' \tilde{V}^T \dot{\gamma}_d \quad (31)$$

$$N_{B_2} \triangleq \hat{W}^T \hat{\sigma}' \tilde{V}^T \dot{\gamma}_d + \tilde{W}^T \hat{\sigma}' \hat{V}^T \dot{\gamma}_d. \quad (32)$$

The motivation for separating the terms in (29) is motivated by the fact that the different components in (29) have different bounds. Segregating the terms in (29)-(32) introduces the development of the NN weight update laws and the subsequent stability analysis [18]. Using the Mean Value Theorem, the following upper bound for (28) can be determined as

$$\tilde{N} \leq \rho(\|\psi\|)\|\psi\|, \quad (33)$$

where $\psi(e_1, e_2, r) \in \mathbb{R}^{18}$ is defined as

$$\psi \triangleq \begin{bmatrix} e_1^T & e_2^T & r^T \end{bmatrix}^T \quad (34)$$

and $\rho(\cdot) \in \mathbb{R}$ is a positive globally invertible nondecreasing function. From (12), (17), and (30)-(32), the following inequalities can be developed:

$$\|N_d\| \leq \zeta_1, \quad \|N_B\| \leq \zeta_2, \quad \left\|\dot{N}_d\right\| \leq \zeta_3.$$

From (22) and (23), the time derivative of (30) can be upper bounded as

$$\left\|\dot{N}_B\right\| \leq \zeta_4 + \zeta_5 \|e_2\|,$$

where $\zeta_i \in \mathbb{R}$, $(i = 1, ..., 5)$ are known positive constants.

V. Stability Analysis

Theorem 2. *The controller developed in (18) ensures that all signals are bounded under closed-loop control and that the position tracking error is regulated in the sense that* $\|e_1(t)\| \to 0$ *as* $t \to \infty$ *provided the control gain* k_s *from (19) is selected sufficiently large, and* β_1, β_2 *and* α *are selected according to the following sufficient conditions:*

$$\beta_1 > \zeta_1 + \zeta_2 + \frac{1}{\alpha}\zeta_3 + \frac{1}{\alpha}\zeta_4, \qquad \beta_2 > \zeta_5,$$

$$\alpha > \beta_2 - \frac{\bar{J}^2}{4k_B}. \quad (35)$$

Proof: Let $y(t) \in \mathbb{R}^{18+2}$ be defined as

$$y \triangleq \begin{bmatrix} \psi^T & \sqrt{P} & \sqrt{Q} \end{bmatrix}^T. \quad (36)$$

The auxiliary function $P(t) \in \mathbb{R}$ is the generalized solution to the following differential equation

$$\dot{P} = -L \quad (37)$$

$$P(0) = \beta_1 \sum_{i=1}^{n} |e_{2i}(0)| - e_2(0)^T N(0),$$

where $L(t) \in \mathbb{R}$ is defined as

$$L \triangleq r^T \left(N_{B_1} + N_d - \beta_1 sgn(e_2)\right) - \dot{e}_2^T N_{B_2} + \beta_2 \|e_2\|^2$$

and $\beta_1, \beta_2 \in \mathbb{R}$ are chosen according to the sufficient conditions in (35). Provided the gain conditions in (35) are satisfied, $P(t)$ is a positive function (see [18] for details). The positive auxiliary function $Q(t) \in \mathbb{R}$ in (36) is defined as

$$Q \triangleq \frac{\alpha}{2} tr\left(\tilde{W}^T \Gamma_1^{-1} \tilde{W}\right) + \frac{\alpha}{2} tr\left(\tilde{V}^T \Gamma_2^{-1} \tilde{V}\right), \quad (38)$$

where $\alpha > 0$. Let $V(y, t) : \mathcal{D} \times [0, \infty) \to \mathbb{R}$ be a Lipschitz continuous regular positive definite function defined as

$$V = \frac{1}{2}e_1^T e_1 + \frac{1}{2}e_2^T e_2 + \frac{1}{2}r^T \bar{M} r + P + Q \quad (39)$$

which satisfies the following inequalities:

$$U_1(y) \leq V(y, t) \leq U_2(y), \quad (40)$$

where $U_1(y)$, $U_2(y) \in \mathbb{R}$ are positive definite functions defined as $U_1 \triangleq \frac{1}{2} min\{1, \underline{m}\}\|y\|^2$, $U_2 \triangleq max\left\{\frac{1}{2}\bar{m}(\eta, \dot{\eta}), 1\right\}\|y\|^2$.

The differential equations of the closed loop dynamics given in (27) are continuous except in the set $\{y | e_2 = 0\}$. Using Filippov's differential inclusion [24]–[27], the existence of solutions can be established for $\dot{y} = f(y)$, where $f(y) \in \mathbb{R}^{18+2}$ denotes the right-hand side of the closed-loop error signals. Under Filippov's framework, a generalized Lyapunov stability theory can be used to establish strong stability of the closed-loop system. The generalized time derivative of (39) exists almost everywhere (a.e.), and $\dot{V}(y) \in^{a.e.} \dot{\tilde{V}}(y)$ where

$$\dot{\tilde{V}} = \bigcap_{\xi \in \partial V(y)} \xi^T K \begin{bmatrix} \dot{e}_1^T & \dot{e}_2^T & \dot{r}^T & \frac{1}{2}P^{-\frac{1}{2}}\dot{P} & \frac{1}{2}Q^{-\frac{1}{2}}\dot{Q} & 1 \end{bmatrix}^T,$$

∂V is the generalized gradient of $V(y)$ [28], and $K[\cdot]$ is defined as [29], [30] as

$$K[f] \triangleq \bigcap_{\delta>0} \bigcap_{\mu\Upsilon=0} \overline{co} f\left(B(x,\delta)-\Upsilon\right),$$

where $\bigcap_{\mu\Upsilon=0}$ denotes the intersection of all sets Υ of Lebesgue measure zero, $\overline{co}$ denotes convex closure, and $B(y,\delta) = \left\{ \upsilon \in \mathbb{R}^{18+2} | \ \|y-\upsilon\| < \delta \right\}$. Since $V(y)$ is a Lipschitz continuous regular function

$$\begin{aligned}
\dot{\tilde{V}} &= \nabla V^T K\left[\begin{array}{cccccc} \dot{e}_1^T & \dot{e}_2^T & \dot{r}^T & \frac{1}{2}P^{-\frac{1}{2}}\dot{P} & \frac{1}{2}Q^{-\frac{1}{2}}\dot{Q} & 1 \end{array}\right]^T \\
&\subset \left[\begin{array}{cccccc} e_1^T & e_2^T & r^T M & 2P^{\frac{1}{2}} & 2Q^{\frac{1}{2}} & \frac{1}{2} r^T \dot{M} r \end{array}\right] K \qquad (41) \\
&\quad \left[\begin{array}{cccccc} \dot{e}_1^T & \dot{e}_2^T & \dot{r}^T & \frac{1}{2}P^{-\frac{1}{2}}\dot{P} & \frac{1}{2}Q^{-\frac{1}{2}}\dot{Q} & 1 \end{array}\right]^T.
\end{aligned}$$

Using (5), (8), (11), (26), (27), and (37), the expression in (41) becomes

$$\begin{aligned}
\dot{\tilde{V}} \subset\ & e_1^T J e_2 - k_1\|e_1\|^2 + e_2^T r - \alpha\|e_2\|^2 + r^T\tilde{N} + r^T N \\
& - r^T e_2 - (k_s+1)\|r\|^2 - r^T N_{B_1} - r^T N_d \\
& - \dot{e}_2^T N_{B_2} + \beta_2\|e_2\|^2 + \dot{Q}, \qquad (42)
\end{aligned}$$

where the fact that $(r^T - r^T)_i SGN(e_{2_i}) = 0$ is used (the subscript i denotes the i^{th} element), where $K[sgn(e_2)] = SGN(e_2)$ [30] such that $SGN(e_{2_i}) = 1$ if $e_{2_i} > 0$, $[-1, 1]$ if $e_{2_i} = 0$, and -1 if $e_{2_i} < 0$. Substituting for (29)-(32) and utilizing (22), (23), (33), and the time derivative of (38), (42) becomes

$$\begin{aligned}
\dot{\tilde{V}} \leq & -(k_A+k_B)\|e_1\|^2 + \|J\|\|e_1\|\|e_2\| - (\alpha-\beta_2)\|e_2\|^2 \\
& + \rho(\|\psi\|)\|r\|\|\psi\| - (k_s+1)\|r\|^2, \qquad (43)
\end{aligned}$$

where $k_1 = k_A + k_B$. By completing the squares for e_1, the expression in (43) can be reduced to

$$\dot{\tilde{V}} \leq -\lambda_1\|\psi\|^2 - k_s\|r\|^2 + \rho(\|\psi\|)\|r\|\|\psi\|, \qquad (44)$$

where $\lambda_1 = \min\left\{k_A, \alpha-\beta_2-\frac{\bar{J}^2}{4k_B}, 1\right\}$ and ψ was defined in (34). Provided the sufficient conditions in (35) are satisfied, and after completing the squares for r in (44), the following expression is obtained:

$$\dot{\tilde{V}} \leq -\lambda_1\|\psi\|^2 + \frac{\rho^2(\psi)\|\psi\|^2}{4k_s} \leq -U(y), \qquad (45)$$

where $U(y) = \lambda_2\|\psi\|^2$, for some positive constant $\lambda_2 \in \mathbb{R}$, is a continuous positive semi-definite function such that

$$\mathcal{D} \triangleq \left\{ y \in \mathbb{R}^{18+2} \mid \|y\| \leq \rho^{-1}\left(2\sqrt{\lambda_1 k_s}\right)\right\}.$$

The size of the domain $\mathcal{D}$ can be increased by increasing the gain k_s. The inequalities in (40) and (45) can be used to show that $V(y,t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, thus, $e_1(t), e_2(t), r(t), P(t), Q(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Given that $e_1(t), e_2(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, standard linear analysis can be used to show that $\dot{e}_1(t), \dot{e}_2(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$ from (10), (11) and Assumption 1. Since $e_1(t), e_2(t), r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, and $\eta_d, \dot{\eta}_d, \ddot{\eta}_d$ exist and are bounded in (6), (5), (8), (9) can be used to show that $\eta(t), \nu(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Property 2, Assumption 2, and (3) can be used to show that $\tau \in \mathcal{L}_\infty$ in $\mathcal{D}$. Given that $e_2(t), \dot{e}_2(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, (26) can be used to show that $\dot{\mu}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. From Property 1, Property 2, and (1), $\bar{M}(\eta,\dot{\eta}), \bar{C}(\eta,\dot{\eta}), \bar{D}(\eta,\dot{\eta}), \bar{g}(\eta) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Property 2, (17), and (25), and the fact that $\dot{\mu}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, can be used to show that $\dot{r}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Since $\dot{e}_1(t), \dot{e}_2(t), \dot{r}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, $U(y)$ is uniformly continuous in $\mathcal{D}$ based on the definition for $U(y)$ and $\psi(t)$. Let $\mathcal{S} \subset \mathcal{D}$ denote the set defined as:

$$\mathcal{S} \triangleq \left\{ y \subset \mathcal{D} \mid U_2 < \frac{1}{2} min\{1, \underline{m}\}\left(\rho^{-1}\left(2\sqrt{\lambda_1 k_s}\right)\right)^2\right\}.$$

The region of attraction in $\mathcal{S}$ can be made arbitrarily large to include any initial conditions by increasing the control gain k_s (i.e. semi-global type of stability) and hence $e_1(t) \to 0$ as $t \to \infty\, \forall y(0) \in \mathcal{S}$. ■

VI. Simulation Results

Simulation results demonstrate successful performance of the controller described in Section IV for a tracking application using a nonlinear AUV model developed in [31]. This vehicle was selected for simulation due to the experimental validation provided in [32]. The vehicle's actuation is represented by 3 independent forces and 3 independent moments about the center of mass of the vehicle. A thruster mapping algorithm, such as the one described in [33], can be used to map the controlled forces and moments onto a custom thruster configuration for the AUV. The external disturbance for the simulation is represented by

$$\tau_d = \left[\begin{array}{cccccc} usin\left(\frac{t}{2}\right) & 0.5vsin\left(\frac{t}{4}\right) & 0.2w\mathrm{rand}(\cdot) & 0 & 0 & 0 \end{array}\right]^T$$

where $\mathrm{rand}(\cdot) \in \mathbb{R}_{[-1,1]}$ is a random function generator and u, v and w are the linear velocities. The following helical reference trajectory is selected:

$$\eta_d = \left[\begin{array}{cccccc} 2sin\left(\frac{t}{10}\right) & 2cos\left(\frac{t}{10}\right) & \frac{t}{20} & 0 & 0 & -\frac{t}{10} \end{array}\right]^T. \qquad (46)$$

The initial conditions for the system were selected as $\eta(0) = \left[\begin{array}{cccccc} 0 & 2 & 0 & 0 & 0 & 0 \end{array}\right]^T$. The initial parameters for the NN ideal weight matrices are selected as

$$\hat{V}_{init} = rand(19,\ N_2), \quad \hat{W}_{init} = zeros(N_2+1,\ 6),$$

where number of neurons in the hidden layer is chosen as $N_2 = 5$. The control gains for the RISE feedback term are selected as diagonal matrices constructed from the following vectors:

$$\begin{aligned}
k_1 &= \left[\begin{array}{cccccc} 1 & 1 & 0.5 & 3 & 3 & 5 \end{array}\right]^T \\
k_s &= 10^3 \cdot \left[\begin{array}{cccccc} 4 & 4 & 7 & 8 & 8 & 7 \end{array}\right]^T \\
\beta &= \left[\begin{array}{cccccc} 0.3 & 0.3 & 0.3 & 0.3 & 0.3 & 0.1 \end{array}\right]^T \\
\alpha &= \left[\begin{array}{cccccc} 0.2 & 0.2 & 0.1 & 0.1 & 0.1 & 0.2 \end{array}\right]^T.
\end{aligned}$$

The control gains for the NN feedforward term are selected as $\Gamma_1 = 2000 \cdot I_{6\times 6}$, $\Gamma_2 = 500 \cdot I_{6\times 6}$, where $I_{i\times i}$ denotes the identity matrix of size i by i.

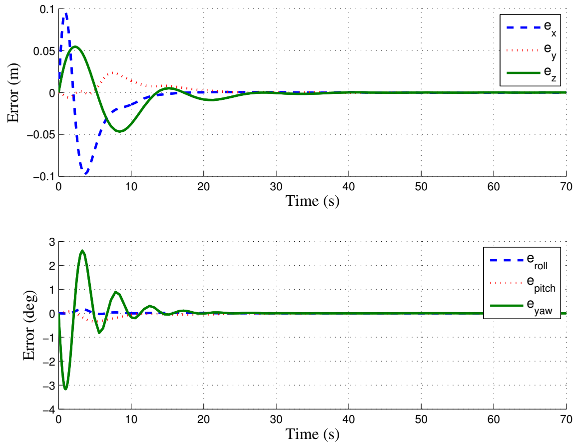

Figure 2. Tracking errors for surge (x), sway (y), and heave (z) and roll (ϕ), pitch (θ), and yaw (ψ) versus time.

Fig. 2 shows the tracking errors for surge, sway and heave and roll, pitch and yaw for the trajectory described by (46). The controller is able to reject the disturbances in the system quickly and continuously achieve tracking performance.

VII. Conclusion

A six-DOF AUV controller is shown to asymptotically track a desired trajectory by adaptively estimating uncertainties in the plant dynamics on-line and by rejecting unknown disturbances. A RISE control scheme incorporated with a multi-layer neural network allows the AUV to track a continuous, inertial trajectory. The stability of the controller is validated by means of a Lyapunov stability theorem. Simulation results demonstrate the performance using an experimentally validated AUV model in the presence of unknown disturbances without requiring any model knowledge. Experimental validation of the AUV controller and a saturated version of the control law are future goals of this work.

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