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PD+ Based Output Feedback Attitude Control of Rigid Bodies with Improved Performance

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***Abstract*— We address the problem of output feedback attitude control of a rigid body in quaternion coordinate space through a modified PD+ based tracking controller. The control law ensures faster convergence to the desired operating point during attitude maneuver, while keeping the gains small for station keeping, thus being less sensitive to measurement noise. The angular velocity is estimated with a similar technique, thus keeping the property of lower sensitivity to measurement noise. A direct consequence is a drop in energy consumption and more accurate estimation results when affected by sensor noise. More precisely, we show uniform practical asymptotic stability of the equilibrium point for the closed loop system in the presence of unknown, bounded input disturbances. Simulation results illustrate the performance improvement with respect to PD+ based output feedback control with static gains.**

I. INTRODUCTION

Attitude control on the rotational sphere is an interesting theoretical problem since, due to the parametrization of the attitude for the unit quaternion, the mapping from S^3 to $SO(3)$ is two-to-one on the manifold. From a more practical viewpoint, besides achieving stability in some sense, control of a rigid body demands fast and accurate settling using minimal effort. Thus, a wide number of controllers have been developed during the past years, by focusing on the enhancement of performance while guaranteeing robust stability and minimizing the control effort.

Attitude tracking control naturally lies on a bulk of literature on tracking control of robot manipulators –*cf.* [1]. A classic in robot control literature is the PD+ controller of Paden and Panja –*cf.* [2] which, together with the Slotine and Li controller –[3], was the first algorithm for which global asymptotic stability was demonstrated. A PD+ based controller for spacecraft was presented in [4], called model-dependent control, and more recently for leader-follower spacecraft formation in [5].

An angular velocity observer for rigid body motion was presented in [6], while a passivity approach was considered in [7] where the passivity properties were exploited in a nonlinear controller to ensure asymptotic stability. In [8] two different schemes were presented based on results for output control of robot manipulators (*cf.* [9]). A different approach was presented in the recent paper [10] working on $SO(3)$ directly where the associated singularities are worked around by restricting the initial values based on gain conditions, and furthermore, an uniform bound is guaranteed on the resulting control torque.

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In this paper we use a modified PD+ based quaternion feedback controller without angular velocity measurements which, roughly speaking, includes nonlinear gains of exponential growth. That is, the controller ensures fast convergence of attitude and estimation error; on the other hand, the control effort is reduced exponentially in a neighborhood of the reference operating point. Consequently, very little control effort is used in station-keeping tasks, especially in the presence of sensor noise. Strictly speaking, we show that the origin of the closed-loop system is uniformly practically asymptotically stable with respect to perturbations. Our theoretical findings are validated in simulation for an Earth orbiting spacecraft.

II. PRELIMINARIES

The cross product operator $\times$ between two vectors $\mathbf{a}$ and $\mathbf{b}$ is written as $\mathbf{S}(\mathbf{a})\mathbf{b}$ where $\mathbf{S}$ is skew-symmetric. The symbol $\boldsymbol{\omega}^c_{b,a}$ denotes angular velocity of frame a relative to frame b, expressed in the frame c; $\mathbf{R}^b_a$ is the rotation matrix from frame a to frame b; $\|\cdot\|$ denotes the Euclidean norm of a vector and the induced $\mathcal{L}_2$ norm of a matrix. Coordinate reference frames are denoted by $\mathcal{F}^{(\cdot)}$ where the superscript denotes the frame in question; i denotes the inertial frame while b denotes the body frame, and we denote $\mathbb{R}_+ = \{\alpha \in \mathbb{R} : \alpha \in (0,\infty)\}$ as the set of all positive numbers. When the context is sufficiently explicit, we omit the arguments of functions.

A. *Quaternions*

The attitude of a rigid body is represented by a rotation matrix $\mathbf{R} \in SO(3)$ fulfilling

$$SO(3) = \{\mathbf{R} \in \mathbb{R}^{3\times 3} \ : \ \mathbf{R}^\top \mathbf{R} = \mathbf{I}, \ \det \mathbf{R} = 1\}, \qquad (1)$$

which is the special orthogonal group of order three. Quaternions are used to parameterize members of $SO(3)$ where the unit quaternion is defined as $\mathbf{q} = [\eta, \ \boldsymbol{\epsilon}^\top]^\top \in S^3 = \{\mathbf{x} \in \mathbb{R}^4 \ : \ \mathbf{x}^\top\mathbf{x} = 1\}$, where $\eta \in \mathbb{R}$ is the scalar part and $\boldsymbol{\epsilon} \in \mathbb{R}^3$ is the vector part. The rotation matrix may be described by [11]

$$\mathbf{R} = \mathbf{I} + 2\eta\mathbf{S}(\boldsymbol{\epsilon}) + 2\mathbf{S}^2(\boldsymbol{\epsilon}). \qquad (2)$$

The inverse rotation can be performed by using the inverse conjugated of $\mathbf{q}$ as $\bar{\mathbf{q}} = [\eta, \ -\boldsymbol{\epsilon}^\top]^\top$. The set S^3 forms a group with quaternion multiplication, which is distributive and associative, but not commutative, and the quaternion product of two arbitrary quaternions $\mathbf{q}_1$ and $\mathbf{q}_2$ is defined as [11]

$$\mathbf{q}_1 \otimes \mathbf{q}_2 = \begin{bmatrix} \eta_1\eta_2 - \boldsymbol{\epsilon}_1^\top\boldsymbol{\epsilon}_2 \\ \eta_1\boldsymbol{\epsilon}_2 + \eta_2\boldsymbol{\epsilon}_1 + \mathbf{S}(\boldsymbol{\epsilon}_1)\boldsymbol{\epsilon}_2 \end{bmatrix}. \qquad (3)$$

B. Kinematics and Dynamics

The time derivative of (2) can be written as [11]

$$\dot{\mathbf{R}}_b^a = \mathbf{S}\left(\boldsymbol{\omega}_{a,b}^a\right)\mathbf{R}_b^a = \mathbf{R}_b^a\mathbf{S}\left(\boldsymbol{\omega}_{a,b}^b\right), \tag{4}$$

and the kinematic differential equations can be expressed as [11]

$$\dot{\mathbf{q}}_{i,b} = \mathbf{T}(\mathbf{q}_{i,b})\boldsymbol{\omega}_{i,b}^b, \tag{5}$$

where

$$\mathbf{T}(\mathbf{q}_{i,b}) = \frac{1}{2}\begin{bmatrix} -\boldsymbol{\epsilon}_{i,b}^\top \\ \eta_{i,b}\mathbf{I} + \mathbf{S}(\boldsymbol{\epsilon}_{i,b}) \end{bmatrix} \in \mathbb{R}^{4\times 3}. \tag{6}$$

The dynamical model of a rigid body can be described by a differential equation for angular velocity, and is deduced from Euler's moment equation. This equation describes the relationship between applied torque and angular momentum on a rigid body as [12]

$$\mathbf{J}\dot{\boldsymbol{\omega}}_{i,b}^b = -\mathbf{S}(\boldsymbol{\omega}_{i,b}^b)\mathbf{J}\boldsymbol{\omega}_{i,b}^b + \boldsymbol{\tau}^b \tag{7}$$

$$\tag{8}$$

where where $\boldsymbol{\tau}^b \in \mathbb{R}^3$ is the total torque working on the body frame, and $\mathbf{J} \in \mathbb{R}^{3\times 3}$ is the spacecraft inertia matrix. The torque working on the body is expressed as $\boldsymbol{\tau}^b = \boldsymbol{\tau}_a^b + \boldsymbol{\tau}_d^b$, where $\boldsymbol{\tau}_d^b$ is the disturbance torque, and $\boldsymbol{\tau}_a^b$ is the actuator (control) torque.

III. CONTROL OF RIGID BODY

A. Problem Formulation

The control problem is to steer the state $\mathbf{q}_{i,b}(t)$ towards a given reference trajectory $\mathbf{q}_{i,d}(t)$ satisfying the kinematic equation

$$\dot{\mathbf{q}}_{i,d} = \mathbf{T}(\mathbf{q}_{i,d})\boldsymbol{\omega}_{i,d}^b. \tag{9}$$

The tracking error in quaternion coordinates, $\tilde{\mathbf{q}} = [\tilde{\eta},\ \tilde{\boldsymbol{\epsilon}}^\top]^\top$ is given by

$$\tilde{\mathbf{q}} := \bar{\mathbf{q}}_{i,d} \otimes \mathbf{q}_{i,b} = \begin{bmatrix} \eta_{i,d}\eta_{i,b} + \boldsymbol{\epsilon}_{i,d}\boldsymbol{\epsilon}_{i,b} \\ \eta_{i,d}\boldsymbol{\epsilon}_{i,b} - \eta_{i,b}\boldsymbol{\epsilon}_{i,d} - \mathbf{S}(\boldsymbol{\epsilon}_{i,d})\boldsymbol{\epsilon}_{i,b} \end{bmatrix}, \tag{10}$$

and the quaternion velocities may be expressed as (*cf.* [13])

$$\dot{\tilde{\mathbf{q}}} = \mathbf{T}(\tilde{\mathbf{q}})\left(\boldsymbol{\omega}_{i,b}^b - \boldsymbol{\omega}_{i,d}^b\right). \tag{11}$$

For the purpose of establishing meaningful stability properties we define the error functions

$$\mathbf{e}_{q\pm} = [1 \mp \tilde{\eta},\ \tilde{\boldsymbol{\epsilon}}^\top]^\top, \quad \mathbf{e}_\omega = \boldsymbol{\omega}_{i,b}^b - \boldsymbol{\omega}_{i,d}^b. \tag{12}$$

Moreover, we have

$$\dot{\mathbf{e}}_{q\pm} = \mathbf{T}_e(\mathbf{e}_{q\pm})\mathbf{e}_\omega, \tag{13}$$

where

$$\mathbf{T}_e(\mathbf{e}_{q\pm}) = \frac{1}{2}\begin{bmatrix} \pm\tilde{\boldsymbol{\epsilon}}^\top \\ \tilde{\eta}\mathbf{I} + \mathbf{S}(\tilde{\boldsymbol{\epsilon}}) \end{bmatrix}. \tag{14}$$

Since measurements of the angular velocity is not available we define an estimation error defined as $\mathbf{e}_{e\omega} = \boldsymbol{\omega}_{i,b}^b - \boldsymbol{\omega}_{i,e}^b$, where super-/sub-script e denotes the estimated frame, together with an attitude estimation error defined as $\mathbf{q}_{e,b} = [\eta_{e,b},\ \boldsymbol{\epsilon}_{e,b}^\top]^\top = \bar{\mathbf{q}}_{i,e} \otimes \mathbf{q}_{i,b}$, thus the error function is defined as $\mathbf{e}_{eq} = [1 \mp \eta_{e,b},\ \boldsymbol{\epsilon}_{e,b}^\top]^\top$ with $\mathbf{T}_{eq}$ in accordance.

Note that due to the redundancy implicit to the quaternion representation, $\tilde{\mathbf{q}}$ and $-\tilde{\mathbf{q}}$ represent the *same* physical attitude but correspond to different equilibria. That is; the two attitude positions differ by a rotation of 2π rad about an arbitrary axis. Furthermore, when quaternion coordinates are considered it is not appropriate to speak of *global* stability properties since the adjective *global* pertains to the case when the states are elements of $\mathbb{R}^n$ –*cf.* [14] while quaternions evolve on the manifold S^3 which is a subset of $\mathbb{R}^4$; see also [15] for precise definitions of stability and discussions.

B. Controller-Observer Design

We pose the following assumptions:

***Assumption** 3.1: The inertia matrix* $\mathbf{J}$ *is symmetric and positive definite, and satisfies the inequality*

$$\beta_j \le \|\mathbf{J}\| \le \beta_J, \tag{15}$$

with $\beta_j, \beta_J \in \mathbb{R}_+$.

***Assumption** 3.2: The disturbance moments* $\boldsymbol{\tau}_d^b$ *are bounded as*

$$\|\boldsymbol{\tau}_d^b(t)\| \le \beta_d, \tag{16}$$

with $\beta_d \in \mathbb{R}_+$.

***Assumption** 3.3: The desired angular velocity and the desired angular acceleration are bounded,* i.e.$\|\boldsymbol{\omega}_{i,d}^b(t)\| \le \beta_{\omega_{i,d}^b} \in \mathbb{R}_+$ *and* $\|\dot{\boldsymbol{\omega}}_{i,d}^b\| \le \beta_{\dot{\omega}_{i,d}^b} \in \mathbb{R}_+\ \forall\, t \ge t_0 \ge 0$.

***Assumption** 3.4: Let* $\tilde{\eta}(t_0) \ge 0$ *or* $\tilde{\eta}(t_0) < 0$, *and assume that* $\mathrm{sgn}(\tilde{\eta}(t_0)) = \mathrm{sgn}(\tilde{\eta}(t))\ \forall\ t \ge t_0 \ge 0$. *Moreover, we assume that there exists a constant* δ_η *such that either* $\eta_{e,b}(t) \ge \delta_\eta > 0$ *or* $\eta_{e,b}(t) \le -\delta_\eta < 0\ \forall\, t \ge t_0 \ge 0$.

The desired angular velocity of the spacecraft is usually given with reference to the inertial frame as $\boldsymbol{\omega}_{i,d}^i$. By rotating to the body frame we obtain

$$\boldsymbol{\omega}_{i,d}^b = \mathbf{R}_i^b\boldsymbol{\omega}_{i,d}^i \tag{17}$$

hence,

$$\dot{\boldsymbol{\omega}}_{i,d}^b = \dot{\mathbf{R}}_i^b\boldsymbol{\omega}_{i,d}^i + \mathbf{R}_i^b\dot{\boldsymbol{\omega}}_{i,d}^i \tag{18}$$

$$= -\mathbf{S}(\boldsymbol{\omega}_{i,b}^b)\boldsymbol{\omega}_{i,d}^b + \mathbf{R}_i^b\dot{\boldsymbol{\omega}}_{i,d}^i. \tag{19}$$

We see that to evaluate the derivative of the reference $\dot{\boldsymbol{\omega}}_{i,d}^b$ we need to know the actual velocity of the spacecraft $\boldsymbol{\omega}_{i,b}^b$. For control purposes we use the modified acceleration vector (*cf.* [16])

$$\mathbf{a}_d = -\mathbf{S}(\boldsymbol{\omega}_{i,d}^b)\boldsymbol{\omega}_{i,d}^b + \mathbf{R}_i^b\dot{\boldsymbol{\omega}}_{i,d}^i \tag{20}$$

$$= \mathbf{R}_i^b\dot{\boldsymbol{\omega}}_{i,d}^i. \tag{21}$$

***Proposition** 3.1: Let Assumptions 3.1–3.4 hold. Let* $\mathbf{e}_q$ *be defined either by* $\mathbf{e}_q = \mathbf{e}_{q+}$ *or* $\mathbf{e}_q = \mathbf{e}_{q-}$ *and* $\mathbf{e}_{eq} = \mathbf{e}_{eq+}$ *or* $\mathbf{e}_{eq} = \mathbf{e}_{eq-}$, *respectively. Either of the equilibrium points* $(\mathbf{e}_{q\pm}, \mathbf{e}_\omega, \mathbf{e}_{eq\pm}, \mathbf{e}_{e\omega}) = (\mathbf{0},\mathbf{0},\mathbf{0},\mathbf{0})$ *of the system* (5) *and* (7), *in closed loop with the control law*

$$\boldsymbol{\tau}_a^b = \mathbf{J}\mathbf{a}_d - \mathbf{S}(\mathbf{J}\boldsymbol{\omega}_{i,e}^b)\boldsymbol{\omega}_{i,d}^b \tag{22}$$

$$- k_p e^{k_1\mathbf{e}_q^\top\mathbf{e}_q}\mathbf{T}_e^\top\mathbf{e}_q - k_d e^{-k_2\mathbf{e}_q^\top\mathbf{e}_q}\boldsymbol{\omega}_{d,e}^b,$$

with $k_p, k_d, k_1, k_2 \in \mathbb{R}_+$ as tuning parameters with constraints, and the observer

$$\dot{\mathbf{z}} = \mathbf{a}_d + \mathbf{J}^{-1}\left[l_p e^{k_3 \mathbf{e}_{eq}^\top \mathbf{e}_{eq}} \mathbf{T}_{eq}^\top \mathbf{e}_{eq} - k_p e^{k_1 \mathbf{e}_q^\top \mathbf{e}_q} \mathbf{T}_e^\top \mathbf{e}_q\right], \quad (23)$$

$$\boldsymbol{\omega}_{i,e}^b = \mathbf{z} + 2\mathbf{J}^{-1} l_d \mathbf{T}_{eq}^\top \mathbf{e}_{eq}, \quad (24)$$

with $l_p, l_d, k_3, k_4 \in \mathbb{R}_+$ constants to be defined, is Uniformly Practically Asymptotically Stable (UPAS).

***Remark** 3.1: In [17] a PD+ based state feedback control law was presented using exponential gains with the derivative term as $-k_d e^{k_2 \mathbf{e}_\omega^\top \mathbf{e}_\omega} \boldsymbol{\omega}_{d,b}^b$. One difference is that in (22) the damping has a relatively small effect on the system while the solutions are located 'far' away from the equilibrium point, and increase when the attitude error is going towards zero. This helps in reducing overshoot.*

Proof: Without loss of generality, we show stability of the positive equilibrium points *i.e.*, let $\mathbf{e}_q = \mathbf{e}_{q+}$, $\mathbf{T}_e = \mathbf{T}_e(\mathbf{e}_{q+})$, $\mathbf{e}_{eq} = \mathbf{e}_{eq+}$ and $\mathbf{T}_{eq} = \mathbf{T}_{eq}(\mathbf{e}_{eq+})$. Let $\mathbf{x} := [\mathbf{e}_q^\top,\ \mathbf{e}_\omega^\top,\ \mathbf{e}_{eq}^\top,\ \mathbf{e}_{e\omega}^\top]^\top$, thus the error dynamics can be written on state space form $\dot{\mathbf{x}} = f(t, \mathbf{x})$ with

$$f(t,\mathbf{x}) = \begin{bmatrix} \mathbf{T}_e \mathbf{e}_\omega \\ \mathbf{J}^{-1}\xi_1 \\ \mathbf{T}_{eq}\mathbf{e}_{e\omega} \\ \mathbf{J}^{-1}\xi_2 \end{bmatrix}, \quad (25)$$

where

$$\xi_1 = \mathbf{S}(\mathbf{J}\boldsymbol{\omega}_{i,b}^b)\mathbf{e}_\omega + \mathbf{S}(\mathbf{J}\mathbf{e}_{e\omega})\boldsymbol{\omega}_{i,d}^b - k_p e^{k_1 \mathbf{e}_q^\top \mathbf{e}_q} \mathbf{T}_e^\top \mathbf{e}_q - k_d e^{-k_2 \mathbf{e}_q^\top \mathbf{e}_q}(\mathbf{e}_\omega - \mathbf{e}_{e\omega}) - \mathbf{J}\mathbf{S}(\boldsymbol{\omega}_{b,d}^b)\boldsymbol{\omega}_{i,d}^b + \boldsymbol{\tau}_d, \quad (26)$$

and

$$\xi_2 = \mathbf{S}(\mathbf{J}\boldsymbol{\omega}_{i,b}^b)\mathbf{e}_\omega - k_d e^{-k_2 \mathbf{e}_q^\top \mathbf{e}_q}\mathbf{e}_\omega + \mathbf{S}(\mathbf{J}\mathbf{e}_{e\omega})\boldsymbol{\omega}_{i,d}^b + k_d e^{-k_2 \mathbf{e}_q^\top \mathbf{e}_q}\mathbf{e}_{e\omega} - l_d[\eta_{e,b} + \mathbf{S}(\boldsymbol{\epsilon}_{e,b})]\mathbf{e}_{e\omega} - l_p e^{k_3 \mathbf{e}_{eq}^\top \mathbf{e}_{eq}} \mathbf{T}_{eq}^\top \mathbf{e}_{eq} + \boldsymbol{\tau}_d. \quad (27)$$

The rest of the proof consists in showing that the conditions of [18, Theorem 10] hold[1]. Consider the Lyapunov function candidate

$$\mathcal{V}(\mathbf{x}) = V(\mathbf{x}) + \lambda W(\mathbf{x}) \quad (28a)$$

$$V(\mathbf{x}) = \frac{1}{2}\Big[\Big(\frac{k_p}{k_1} e^{k_1 \mathbf{e}_q^\top \mathbf{e}_q} - 1\Big) + \mathbf{e}_\omega^\top \mathbf{J}\mathbf{e}_\omega + \Big(\frac{l_p}{k_3} e^{k_3 \mathbf{e}_{eq}^\top \mathbf{e}_{eq}} - 1\Big) + \mathbf{e}_{e\omega}^\top \mathbf{J}\mathbf{e}_{e\omega}\Big], \quad (28b)$$

$$W(\mathbf{x}) = \mathbf{e}_q^\top \mathbf{T}_e \mathbf{J}\mathbf{e}_\omega + \mathbf{e}_{eq}^\top \mathbf{T}_{eq}\mathbf{J}\mathbf{e}_{e\omega} \quad (28c)$$

which is positive definite and proper, as we show next. We want to find functions $\underline{\alpha}(\mathbf{x}), \overline{\alpha}(\mathbf{x}) \in \mathcal{K}_\infty$ such that $\underline{\alpha}(\mathbf{x}) \leq \mathcal{V}(\mathbf{x}) \leq \overline{\alpha}(\mathbf{x})$. For the upper bound function we write

$$\mathcal{V} \leq \frac{1}{2}\left[\frac{k_p}{k_1}\left(e^{k_1\|\mathbf{x}\|^2} - 1\right) + \frac{l_p}{k_3}\left(e^{k_3\|\mathbf{x}\|^2} - 1\right) + 2\beta_J\|\mathbf{x}\|^2\right] + \lambda 2\beta_J \|\mathbf{x}\|^2 \quad (29)$$

$$\leq \max\left\{\frac{k_p}{k_1}, \frac{l_p}{k_3}, 2\beta_J, 4\lambda\beta_J\right\}\left(e^{\max\{k_1,k_3\}\|\mathbf{x}\|^2} - 1 + \|\mathbf{x}\|^2\right). \quad (30)$$

[1]That is, with the obvious modifications. Strictly speaking, we cannot show that the conditions on semi-globality of [18, Theorem 10] hold for arbitrarily large initial conditions in view of the topology of S^3.

We want to find a constant c such that $e^{c\|\mathbf{x}\|^2} - 1 \geq \|\mathbf{x}\|^2$ that is,

$$c \geq \sup_{\mathbf{x}\in\mathbb{R}^7} \frac{\ln(\|\mathbf{x}\|^2 + 1)}{\|\mathbf{x}\|^2} = 1$$

which in turn, leads us to define

$$\overline{\alpha}(\mathbf{x}) := c_1\left(e^{c_2\|\mathbf{x}\|^2} - 1\right), \quad (31)$$

where $c_1 := 2\max\{k_p/k_1, l_p/k_3, 2\beta_J, 4\lambda\beta_J\}$ and $c_2 := \max\{k_1, k_3, 1\}$. Now we find a quadratic lower bound on $\mathcal{V}$. For this we remark that

$$\left(e^{k_1 \mathbf{e}_q^\top \mathbf{e}_q} - 1\right) \geq k_1 \mathbf{e}_q^\top \mathbf{e}_q. \quad (32)$$

This can be seen recalling that

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \geq 1 + x\,. \quad (33)$$

Similarly for $e^{k_3 \mathbf{e}_{eq}^\top \mathbf{e}_{eq}} - 1$ hence we define $\underline{\alpha}(\mathbf{x}) := \mathbf{x}^\top q_m \mathbf{x}$ where $q_m > 0$ is the smallest eigenvalue of

$$\mathbf{Q} := \frac{1}{2}\begin{bmatrix} k_p\mathbf{I} & \lambda\mathbf{T}_e\mathbf{J} & \mathbf{0} & \mathbf{0} \\ \lambda\mathbf{J}\mathbf{T}_e^\top & \mathbf{J} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & l_p\mathbf{I} & \lambda\mathbf{T}_{eq}\mathbf{J} \\ \mathbf{0} & \mathbf{0} & \lambda\mathbf{J}\mathbf{T}_{eq}^\top & \mathbf{J} \end{bmatrix}. \quad (34)$$

Next, we evaluate the total time derivative of $\mathcal{V}$ along the closed-loop trajectories. To that end, we first compute the derivative of V. We have

$$\dot{V} = -k_d e^{-k_2 \mathbf{e}_q^\top \mathbf{e}_q}\mathbf{e}_\omega^\top \mathbf{e}_\omega + \mathbf{e}_\omega^\top \mathbf{S}(\mathbf{J}\mathbf{e}_{e\omega})\boldsymbol{\omega}_{i,d}^b - \mathbf{e}_\omega \mathbf{J}\mathbf{S}(\boldsymbol{\omega}_{b,d}^b)\boldsymbol{\omega}_{i,d}^b + \mathbf{e}_{e\omega}^\top \mathbf{S}(\mathbf{J}\boldsymbol{\omega}_{i,b}^b)\mathbf{e}_\omega + \mathbf{e}_{e\omega}\mathbf{S}(\mathbf{J}\mathbf{e}_{e\omega})\boldsymbol{\omega}_{i,d}^b - \left(l_d\eta_{e,b} - k_d e^{-k_2 \mathbf{e}_q^\top \mathbf{e}_q}\right)\mathbf{e}_{e\omega}^\top \mathbf{e}_{e\omega} + \left(\mathbf{e}_\omega^\top + \mathbf{e}_{e\omega}^\top\right)\boldsymbol{\tau}_d. \quad (35)$$

Since the matrix $\mathbf{S}(\cdot)$ is linear in its arguments, we have [16]

$$\|\mathbf{S}(\mathbf{J}\mathbf{a})\mathbf{b}\| \leq \beta_J\|\mathbf{a}\|\|\mathbf{b}\|. \quad (36)$$

By applying (36), Young's inequality and Assumptions 3.1–3.4 we have

$$\mathbf{e}_\omega^\top \mathbf{S}(\mathbf{J}\mathbf{e}_{e\omega})\boldsymbol{\omega}_{i,d}^b \leq \frac{1}{2}\beta_J\beta_{\omega_{i,d}^b}(\|\mathbf{e}_\omega\|^2 + \|\mathbf{e}_{e\omega}\|^2) \quad (37)$$

$$\mathbf{e}_\omega \mathbf{J}\mathbf{S}(\boldsymbol{\omega}_{b,d}^b)\boldsymbol{\omega}_{i,d}^b \leq \beta_J\beta_{\omega_{i,d}^b}\|\mathbf{e}_\omega\|^2 \quad (38)$$

$$\mathbf{e}_{e\omega}^\top \mathbf{S}(\mathbf{J}\boldsymbol{\omega}_{i,b}^b)\mathbf{e}_\omega \leq \frac{1}{2}\beta_J(\|\mathbf{e}_\omega\|^2 + \|\mathbf{e}_{e\omega}\|^2)(\|\mathbf{e}_\omega\| + \beta_{\omega_{i,d}^b}) \quad (39)$$

$$\mathbf{e}_{e\omega}\mathbf{S}(\mathbf{J}\mathbf{e}_{e\omega})\boldsymbol{\omega}_{i,d}^b \leq \beta_J\beta_{\omega_{i,d}^b}\|\mathbf{e}_{e\omega}\|^2. \quad (40)$$

Inserting the bounds (37)–(40) into (35), and applying the fact that $\mathbf{e}_q^\top \mathbf{e}_q < 2$ for $\tilde{\eta} > 0$ we obtain

$$\dot{V} \leq -\phi(k_d, \|\mathbf{e}_\omega\|)\|\mathbf{e}_\omega\|^2 - \psi(k_d, l_d, \|\mathbf{e}_\omega\|)\|\mathbf{e}_{e\omega}\|^2 + \beta_d(\|\mathbf{e}_\omega\| + \|\mathbf{e}_{e\omega}\|), \quad (41)$$

$$\phi(k_d, \|\mathbf{e}_\omega\|) = k_d e^{-2k_2} - \beta_J\left(6\beta_{\omega_{i,d}^b} + \|\mathbf{e}_\omega\|\right)$$

$$\psi(k_d, l_d, \|\mathbf{e}_\omega\|) = l_d\delta_\eta - k_d - \frac{1}{2}\beta_J(2\beta_{\omega_{i,d}^b} + \|\mathbf{e}_\omega\|)$$

That is, $\dot{V}$ is negative semidefinite for bounded values of $\mathbf{e}_\omega$ and sufficiently large gains. Hence, the total time derivative of $\mathcal{V}$ along the closed-loop trajectories yields

$$\dot{\mathcal{V}}(\mathbf{x}) \leq = -\mathbf{x}^\top \mathbf{P}(\boldsymbol{\omega}_{i,b}^b)\mathbf{x} + 2\beta_d\|\mathbf{x}\| \tag{42}$$

where $\mathbf{P} = [\mathbf{p}_{ij}],\ \ i,j = 1,2,3,4$ with

$$\mathbf{p}_{11} = \lambda \mathbf{T}_e k_p e^{k_1 \mathbf{e}_q^\top \mathbf{e}_q} \mathbf{T}_e^\top \tag{43a}$$

$$\mathbf{p}_{12} = \mathbf{p}_{21}^\top = \frac{\lambda}{2}\mathbf{T}_e\Big[k_d e^{-k_2 \mathbf{e}_q^\top \mathbf{e}_q}\mathbf{I} - \mathbf{S}(\mathbf{J}\boldsymbol{\omega}_{i,b}^b) - \mathbf{J}\mathbf{S}(\boldsymbol{\omega}_{i,d}^b)\Big] \tag{43b}$$

$$\mathbf{p}_{13} = \mathbf{p}_{31}^\top = \mathbf{0} \tag{43c}$$

$$\mathbf{p}_{14} = \mathbf{p}_{41}^\top = \frac{\lambda}{2}\mathbf{T}_e\left[\mathbf{S}(\boldsymbol{\omega}_{i,d}^b)\mathbf{J} - k_d e^{-k_2 \mathbf{e}_q^\top \mathbf{e}_q}\mathbf{I}\right] \tag{43d}$$

$$\mathbf{p}_{22} = \phi(k_d, \|\mathbf{e}_\omega\|) - \frac{\lambda}{2}\left[\tilde{\eta}\mathbf{I} + \mathbf{S}(\tilde{\boldsymbol{\epsilon}})\right]\mathbf{J} \tag{43e}$$

$$\mathbf{p}_{23} = \mathbf{p}_{32}^\top = \frac{\lambda}{2}\left[k_d e^{-k_2 \mathbf{e}_q^\top \mathbf{e}_q}\mathbf{I} - \mathbf{S}(\mathbf{J}\boldsymbol{\omega}_{i,b}^b)\right]\mathbf{T}_{eq}^\top \tag{43f}$$

$$\mathbf{p}_{24} = \mathbf{p}_{42}^\top = \mathbf{0} \tag{43g}$$

$$\mathbf{p}_{33} = \lambda \mathbf{T}_{eq} l_p e^{k_3 \mathbf{e}_{eq}^\top \mathbf{e}_{eq}} \mathbf{T}_{eq}^\top \tag{43h}$$

$$\mathbf{p}_{34} = \mathbf{p}_{43}^\top = \frac{\lambda}{2}\mathbf{T}_{eq}\Big\{\mathbf{S}(\boldsymbol{\omega}_{i,d}^b)\mathbf{J} + l_d\left[\eta_{e,b}\mathbf{I} + \mathbf{S}(\boldsymbol{\epsilon}_{e,b})\right] - k_d e^{-k_2 \mathbf{e}_q^\top \mathbf{e}_q}\mathbf{I}\Big\} \tag{43i}$$

$$\mathbf{p}_{44} = \psi(k_d, l_d, \|\mathbf{e}_\omega\|) - \frac{\lambda}{2}[\eta_{e,b}\mathbf{I} + \mathbf{S}(\boldsymbol{\epsilon}_{e,b})]\mathbf{J}. \tag{43j}$$

Now, we claim that

$$\mathbf{e}_q^\top \mathbf{T}_e \mathbf{T}_e^\top \mathbf{e}_q \geq \frac{1}{8}\mathbf{e}_q^\top \mathbf{e}_q. \tag{44}$$

To see this we first notice that

$$\mathbf{e}_q^\top \mathbf{T}_e \mathbf{T}_e^\top \mathbf{e}_q = \frac{1}{4}\tilde{\boldsymbol{\epsilon}}^\top \tilde{\boldsymbol{\epsilon}}. \tag{45}$$

Also, in view of (12) we have

$$\frac{1}{8}\left((1-\tilde{\eta})^2 + \tilde{\boldsymbol{\epsilon}}^\top\tilde{\boldsymbol{\epsilon}}\right) = \frac{1}{8}\mathbf{e}_q^\top \mathbf{e}_q\,. \tag{46}$$

Assume that

$$\frac{1}{4}\tilde{\boldsymbol{\epsilon}}^\top\tilde{\boldsymbol{\epsilon}} < \frac{1}{8}\left((1-\tilde{\eta})^2 + \tilde{\boldsymbol{\epsilon}}^\top\tilde{\boldsymbol{\epsilon}}\right), \tag{47}$$

which is equivalent to

$$(1-\tilde{\eta})^2 > \tilde{\boldsymbol{\epsilon}}^\top\tilde{\boldsymbol{\epsilon}}\,. \tag{48}$$

In view of the quaternion constraint $\tilde{\boldsymbol{\epsilon}}^\top\tilde{\boldsymbol{\epsilon}} = 1 - \tilde{\eta}^2$ inequality (48) holds if and only if $2\tilde{\eta}(1-\tilde{\eta}) > 0$. In its turn, the latter holds only if $\tilde{\eta} < 0$ or $\tilde{\eta} > 1$. However, this does not hold by assumption *i.e.*, $\tilde{\eta} \in [0,1]$. We conclude that (47) does not hold. Therefore, from (45)–(48) we obtain that (44) holds. A similar reasoning is used for $\mathbf{e}_{eq}$. We conclude that there exist lower and upper bounds $p_{ij,m}$ and $p_{ij,M}$ on the norms of the sub-blocks $\mathbf{p}_{ij}$ of $\mathbf{P}$ respectively, such that, after applying the triangle inequality repeatedly, we obtain

$$\mathbf{x}^\top \mathbf{P}\mathbf{x} \geq \frac{1}{2}(p_{11,m}\|\mathbf{e}_q\|^2 + p_{22,m}\|\mathbf{e}_\omega\|^2 + p_{33m}\|\mathbf{e}_{eq}\|^2 + p_{44m}\|\mathbf{e}_{e\omega}\|^2). \tag{49}$$

Now, for any given Δ_ω let $\mathbf{e}_\omega \leq \Delta_\omega$. Hence $\boldsymbol{\omega}_{i,b}^b = \mathbf{e}_\omega + \boldsymbol{\omega}_{i,d}^b$ satisfies $\|\boldsymbol{\omega}_{i,b}^b\| \leq \Delta$ with $\Delta := \Delta_\omega + \beta_{\omega_{i,d}^b}$. It follows that (49) holds, that is, $\mathbf{P}$ is positive if defining

$$k_p^\star := 2 j_M \Delta, \tag{50}$$

$$l_p^\star := 2\left[j_m(\Delta + \beta_{\omega_{i,d}^b}) + l_d\right], \tag{51}$$

we set the gains $k_p > k_p^\star$, $l_p > l_p^\star$, and

$$\lambda \leq \min\left\{\frac{\phi(k_d, \Delta_\omega)}{k_d + \frac{1}{2}j_M(1 + 2\Delta + \beta_{\omega_{i,d}^b})}\,, \frac{\psi(k_d, l_d, \Delta_\omega)}{k_d + \frac{1}{2}j_M(1 + 2\beta_{\omega_{i,d}^b} + l_d)}, 1\right\}.$$

Thus,

$$\dot{\mathcal{V}} \leq -p_m\|\mathbf{x}\|^2 + 2\beta_d\|\mathbf{x}\|, \tag{52}$$

where $p_m(\Delta) > 0$ is a lower bound on the smallest eigenvalue of $\mathbf{P}(\Delta)$. The derivative $\dot{\mathcal{V}} < 0$ for all $\mathbf{x} \in \mathcal{H} := \{\mathbf{x} \in \mathcal{S}^3 \times \mathbb{R}^3 : \delta \leq \|\mathbf{x}\| \leq \Delta\}$, where $\delta := 2\beta_d/p_m$. Given any positive constants $\delta^\star$, $\Delta^\star$ such that $\delta^\star < \Delta^\star$, we have that there exists $\Delta > \delta > 0$ such that

$$\underline{\alpha}^{-1} \circ \overline{\alpha}(\delta) = \sqrt{\frac{c_1(\Delta)\left(e^{c_2\delta^2} - 1\right)}{q_m}} \leq \delta^\star \tag{53}$$

since δ decreases while c_1 and q_m increases monotonically with the gains, and c_2 is independent of δ, c_1 and q_m, and

$$\overline{\alpha}^{-1} \circ \underline{\alpha}(\Delta) = \sqrt{\frac{\ln\left(\frac{q_m\Delta^2 + 1}{c_1(\Delta)}\right)}{c_2}} \geq \Delta^\star \tag{54}$$

since while Δ increases, c_1 and q_m increases monotonically with the gains, and c_2 is independent of Δ, c_1 and q_m. In accordance with [18, Theorem 10], all the conditions are satisfied and we conclude that the equilibrium point $(\mathbf{e}_q, \mathbf{e}_\omega, \mathbf{e}_{eq}, \mathbf{e}_{e\omega}) = (\mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0})$ of the closed loop system is UPAS with in some sense a semi-global property for one part of the state.

The proof for the negative equilibrium points $\mathbf{e}_{q-}$, $\mathbf{T}_e(\mathbf{e}_{q-})$ and $\mathbf{e}_{eq-}$, $\mathbf{T}_{eq}(\mathbf{e}_{eq-})$ follows along similar lines hence, the equilibrium points $(\mathbf{e}_{q\pm}, \mathbf{e}_\omega, \mathbf{e}_{eq\pm}, \mathbf{e}_{e\omega}) = (\mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0})$ are UPAS.

□

For purpose of comparison we have the following proposition for a similar controller and estimator structure without variable gains.

***Proposition** 3.2:* *Assume that all assumptions made in Proposition 3.1 hold. Then the set of equilibrium points* $(\mathbf{e}_{q\pm}, \mathbf{e}_\omega, \mathbf{e}_{eq\pm}, \mathbf{e}_{e\omega}) = (\mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0})$ *of the system* (5) *and* (7), *in closed-loop with the control law*

$$\boldsymbol{\tau}_a = \mathbf{J}\mathbf{a}_d - \mathbf{S}(\mathbf{J}\boldsymbol{\omega}_{i,e}^b)\boldsymbol{\omega}_{i,d}^b - k_p\mathbf{T}_e^\top\mathbf{e}_q - k_d\boldsymbol{\omega}_{d,e}^b, \tag{55}$$

with $k_p, k_d, \in \mathbb{R}_+$ *considered as constant gains, and the observer*

$$\dot{\mathbf{z}} = \mathbf{a}_d + \mathbf{J}^{-1}\left[l_p\mathbf{T}_{eq}^\top\mathbf{e}_{eq} - k_p\mathbf{T}_e^\top\mathbf{e}_q\right], \tag{56}$$

$$\boldsymbol{\omega}_{i,e}^b = \mathbf{z} + 2\mathbf{J}^{-1}l_d\mathbf{T}_{eq}^\top\mathbf{e}_{eq}, \tag{57}$$

with $l_p, l_d \in \mathbb{R}_+$ considered as constant gains, are UPAS.

The proof is omitted, but follows along the same lines as Proposition 3.1.

***Remark** 3.2: Note that instead of using $\boldsymbol{\omega}_{i,d}^b$ in the modified acceleration vector* (20)*, the estimated angular velocity $\boldsymbol{\omega}_{i,e}^b$ might be utilized. This will lead to added restrictions for the l_d gain, but might result in better performance when the estimation error has converged.*

IV. SIMULATION RESULTS

We present simulation results for a spacecraft in an elliptic LEO. The simulations were performed in Simulink using a fixed sample-time Runge-Kutta ODE4 solver with 10^{-2} s step size. The moments of inertia were chosen as $\mathbf{J} = \mathrm{diag}\{4.35,\ 4.33,\ 3.664\}\ \mathrm{kgm}^2$, and the spacecraft orbit was chosen with perigee at $600\ \mathrm{km}$, apogee at $750\ \mathrm{km}$, inclination at $71°$, and the argument of perigee and the right ascension of the ascending node at $0°$.

Simulations were performed using (22)–(24) and (55)–(57) for sake of comparison, to show the improved performance using variable gains. To evaluate and compare the performance of the controllers we use the functionals

$$J_q = \int_{t_0}^{t_f} \tilde{\boldsymbol{\epsilon}}^\top \tilde{\boldsymbol{\epsilon}} dt,\ \ J_{eq} = \int_{t_0}^{t_f} \boldsymbol{\epsilon}_{e,b}^\top \boldsymbol{\epsilon}_{e,b} dt,\ \ J_p = \int_{t_0}^{t_f} \boldsymbol{\tau}_a^{b,\top} \boldsymbol{\tau}_a^b dt,$$

where t_0 and t_f defines the start and end of the simulation window, respectively. The functionals J_q and J_{eq} describes the integral functional error of the attitude between body and desired frame, and body and estimated frame, respectively, while J_p describes the integral of the applied control torque.

We introduce measurement noise as $\sigma\mathbb{B}^n = \{x \in \mathbb{R}^n : \|x\| \leq \sigma\}$ and add a suitable amount to the error functions according to $\tilde{\mathbf{e}}_q = (\mathbf{e}_q + 0.01\mathbb{B}^4)/\|\mathbf{e}_q + 0.01\mathbb{B}^4\|$. Since we are applying a slightly elliptic LEO, we only consider the disturbance torques which are the major contributors to these kind of orbits; namely, gravity gradient torque [12], and torques caused by atmospheric drag [19] and J_2 effect [20]. The J_2 effect is caused by non-homogeneous mass distribution of a planet, and the torques generated by atmospheric drag and J_2 are induced because of $\mathbf{r}_c^b = [0.1,\ 0,\ 0]^\top$ m displacement of the center of mass. All disturbances are considered continuous and bounded. For our simulations we have chosen the initial conditions as $\mathbf{q}_{i,b} = \mathbf{q}_{i,e} = [0.3772,\ -0.4329,\ 0.6645,\ 0.4783]^\top$, $\boldsymbol{\omega}_{i,b}^b = [0.1,\ 0.2,\ -0.3]^\top$ rad/s, $\mathbf{z} = [0\ 0\ 0]^\top$, $t_0 = 0$ s and $t_f = 15$ s. The control laws were tuned to achieve similar performance for sake of comparison thus using parameters $k_p = 10$, $k_d = 7$, $l_p = 100$, $l_d = 75$, and $k_1 = k_2 = k_3 = 1$ for (22)–(24), and $k_p = 49$, $k_d = 11$, $l_p = 240$ and $l_d = 150$ for (55)–(57). The spacecraft were commanded to follow smooth sinusoidal trajectories around the origin with velocity profile

$$\boldsymbol{\omega}_{i,d}^i = [3.2\cos(2\times10^{-3}t),\ 0.12\sin(1\times10^{-3}t),\ -3.2\sin(4\times10^{-3}t)]^\top \times 10^{-6}\ \mathrm{rad/s}. \quad (58)$$

In the following, we summarize our simulation results for the attitude maneuver described above, comparing results between the control laws presented in Proposition 3.1 and 3.2, where the performance functionals are presented in Table I, attitude error, angular velocity error of the dynamics and estimation error and integral of the control torque are depicted in Figure 1 and the control torque are depicted in Figure 2. The performance functionals show that both controllers have similar performance as should be expected based on the tuning of the controller gains. Two differences are that the controller with variable gains utilizes higher angular velocity throughout the maneuver because of the $e^{-k_2\mathbf{e}_q^\top\mathbf{e}_q}$ term which means that the damping is reduced while the attitude error has not yet converged *–cf.* Remark 3.1, while the instantaneous maximum control torque is smaller since controller gains are smaller compared to using static gains.

Simulation results for one orbital period $(5896\ \mathrm{s})$ is presented in Table II and as can be seen by looking at the performance functionals, the control law using variable gains is less effected by sensor noise from a energy consumption point of view compared to the control law using static gains, and it follows that the states are less affected as can be seen in Figure 3. This is because as $\mathbf{e}_q \approx \mathbf{0}$, the controller gains are $k_p e^{k_1\mathbf{e}_q^\top\mathbf{e}_q} \approx k_p$ and similar for k_d and l_p, and since the gains in general are smaller for (22)–(24) compared to (55)–(57) for similar performance during a maneuver, the noise has less effect on the performance functionals. From Figure 4 we see similar behavior for the observer; the noise has less effect on both attitude and angular velocity between estimated and body frame.

TABLE I

VALUES OF PERFORMANCE FUNCTIONALS FOR ATTITUDE MANEUVER

	J_q	J_{eq}	J_p
Static gains	0.778	0.013	96.3
Variable gains	0.800	0.013	96.1

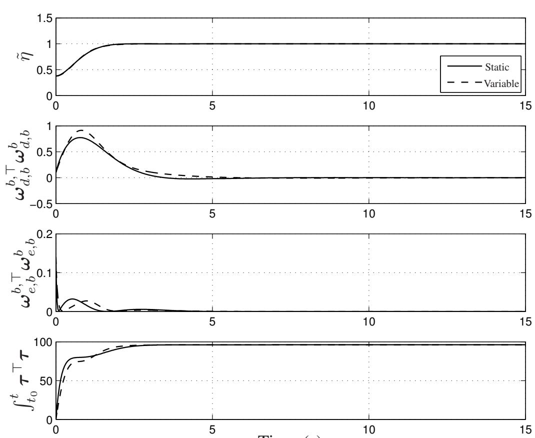

Fig. 1. Attitude, angular velocity and angular velocity estimation error, and power consumption using PD+ based output feedback with static and variable gains during spacecraft attitude maneuver.

V. CONCLUSION

We solved the attitude tracking control problem for a rigid body via PD+ based output feedback control. The controller stabilizes the system in a practical sense as it is illustrated through numerical simulation.

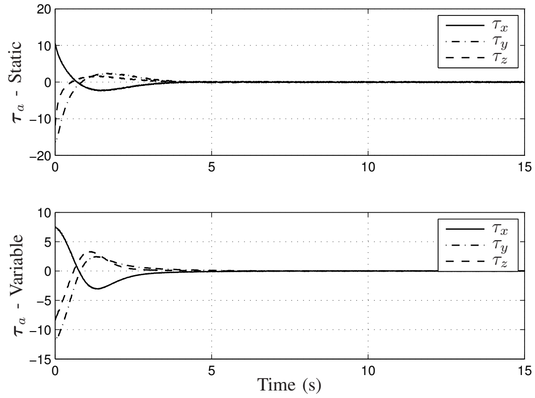

Fig. 2. Control torque for PD+ based output feedback with static and variable gains during spacecraft attitude maneuver.

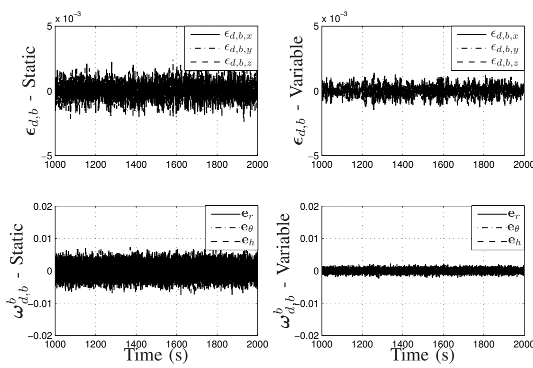

Fig. 3. Attitude and angular velocity error for static gains (left) and variable gains (right).

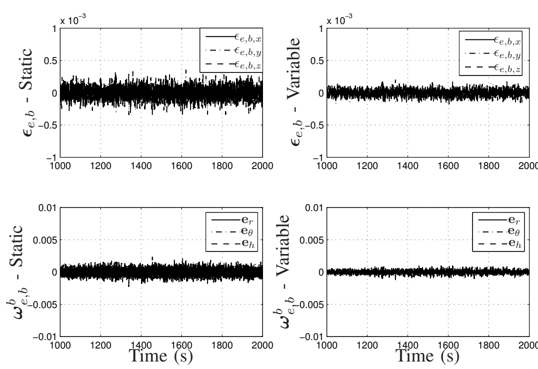

Fig. 4. Attitude and angular velocity estimation error for static gains (left) and variable gains (right).

TABLE II

VALUES OF PERFORMANCE FUNCTIONALS FOR ATTITUDE MANEUVER OVER ONE ORBITAL PERIOD (5896 s)

	J_q	J_{eq}	J_p
Static gains	0.785	0.014	236.9
Variable gains	0.803	0.013	156.7

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