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Optimizing the Location of Sensors Subject to Health Degradation

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Abstract— **One of the main hypotheses supporting the development of cooperative unmanned systems is that the deployment of mobile assets (sensors, weapons) in groups is expected to result in a more effective mission than if conducted with a single asset. Few researches have tackled the design of autonomous decision making for teaming UxVs (unmanned air and ground vehicles) operating under degraded conditions, even though it is common knowledge that real operations are more often than not conducted in less-than-ideal conditions. We consider a team of UxVs that have for mission to persistently monitor an area. We want to ensure they perform as best as possible assuming they are subject to a limited set of degraded conditions. We propose a model to account for variable sensor effectiveness as well as a method to optimize their placement based on a cost balancing heuristic. Numerical simulation suggests accounting for sensor effectiveness improves their placement.**

I. INTRODUCTION

A. Context

The development of unmanned vehicles (UxVs) has been mainly motivated by the desire to carry out missions that are too dull, dirty, or dangerous for humans. With advances in automation tools, electronics, communications, material, and propulsion, to name a few, unmanned vehicles offer a larger realm of possibilities.

One underlying principle supporting the development of new, innovative cooperative systems for teaming UxVs, is the fact that a group of assets is expected to carry out a mission more efficiently than a single individual. Before groups of UxVs are used in an operational context and their potential as a group is exploited, the vehicles and their onboard systems must offer a minimum level of effectiveness, safety and reliability under ideal and, importantly, non-ideal conditions.

A context of operation qualified as non-ideal refers to carrying out a mission under degraded conditions. Such a situation may refer to (1) the occurrence of onboard UxV system and component malfunctions, (2) unmanned assets being subject to hostile actions, and (3) the environment adversely affecting the performance of the UxVs. To ensure that a team of UxVs performs as well as possible under a limited set of degraded conditions, we propose a method to account for variable sensor effectiveness.

B. Persistent monitoring with a team of UxVs

We consider the task of persistently monitoring an area to detect any change in the surroundings. We focus on the placement of the unmanned assets and the control of their motion to achieve this placement. It is important to emphasize that (1) we are not concerned with the use of the information gathered by the team of UxVs, and (2) persistent monitoring is a subset of a more complex task known as persistent surveillance [1], [2].

We define the persistent monitoring (PM) mission as follows. A team of UxVs with limited sensing capabilities maintains information about a predetermined region for an extended period of time. There are many key elements to this definition. The first one is that the agents cooperate to achieve the task. Secondly, sensing capabilities are limited in the sense that sensor placement is not trivially achieved. The information must be maintained, in that the environment must be sensed constantly, for instance by deploying enough sensors. Finally, we want this monitoring to persist through time, despite possibly degrading health conditions of resource available to perform the task (i.e. sensors, vehicles, communications).

The problem of ensuring persistent monitoring is related to various monitoring problems that have been studied in the past. A particular case takes place when all the sensors can jointly cover the area. This coverage problem is often addressed through the locational optimization (LO) framework, also known as Voronoi coverage [3]. LO is interested in disseminating sensors throughout the environment to ensure coverage of an area. When the quality of the sensors is not high enough to ensure monitoring of the whole area at once, researchers proposed to develop strategies to move the robots (mobile sensors) in a way that optimizes a metric that indicates the level of awareness the robots have about their environment [4], [5]. While this approach allows for specifying regions of varying interest, a limitation is that the problem does not consider an awareness level that evolves over time. Some researchers propose to determine patrol routes [6], [7] that ensure monitoring of an area with a set of mobile sensors. These techniques enable some sort of persistence in the monitoring, with each vehicle following a cycle in the region of interest. A notable limitation with such technique is that it is not possible to specify regions of varying importance or to account for a sensor model that includes performance degradation with time, or performance that depends on range or angle. Information-theoretic approaches have also been proposed for monitoring tasks [8], [9]. There, the objective is to control the UxVs to optimize an information-theoretic criterion. While the approach is decentralized and explicitly accounts for the value of information, the health condition of the UxVs is still not part of the problem.

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In all the aforementioned techniques, the vehicles' health conditions are not explicitly incorporated in the formulation of the optimization problem. When sensors can cover the whole area at once, coverage is likely to be the correct way to decide on the placement of the mobile sensors. Yet, this is true under nominal (healthy) conditions, with the past research work not providing explicit support for enabling persistence when agents, or UxVs, do not operate nominally at all times.

C. Degraded conditions and health

A wide array of degraded conditions can occur during the course of a mission, such as (1) loss of efficiency in sensing capabilities, (2) degraded communications, (3) reduced ability to maneuver/move, (4) the need to refuel and (5) the loss of one or more UxVs. In this paper, we focus on (1). Degraded communications can be modeled through limited range interactions; implications of those on Voronoi coverage control are studied in [10].

Health management at the mission planning level has been investigated by some researchers. For instance, [11] proposes a centralized mission planning system to plan the motion of a team of small unmanned vehicles, including a centralized health manager. This system controls the deployment of vehicles so that a predetermined number of vehicles remain deployed in a region of interest, despite refueling needs, expressed as a stochastic fuel consumption model. As opposed to a centralized health-aware system, we propose a decentralized decision-making system that takes into account degraded conditions. Centralized health management for a team of UxVs has been considered in the context of a task assignment problem in [12].

D. Proposed approach and contribution

In this paper we restrict ourselves to the problem of placing vehicles when their sensors may have variable effectiveness (health). For such situations, we propose: (1) a way to embed sensor health in the LO framework; (2) an optimization criterion; and (3) an algorithm method to optimize the cost using that criterion. We model health by introducing an additional parameter in the problem formulation. This parameter influences the quality of sensing performed by a sensor. The optimization procedure balances the loss in coverage quality resulting from health degradation with other nearby sensors. Simulation results suggest that the method proposed results in improved sensor placement.

II. PROBLEM SETTING

Consider a set of robots, indexed by $\mathcal{I}$, a set of cardinality $|\mathcal{I}|$. Assume the agents are located in $\mathcal{Q} \subset \mathbb{R}^2$, a convex subset of the Cartesian plane. The position of robot i is $\mathbf{p}_i \in \mathcal{Q}$. Let $P = \left(\mathbf{p}_1, \ldots, \mathbf{p}_{|\mathcal{I}|}\right)$ denote the position of all UxVs. We use $\partial\mathcal{Q} \subset \mathcal{Q}$ to denote the boundary of $\mathcal{Q}$. For simplicity, assume $(\mathbf{p}_i = \mathbf{p}_j) \equiv (i = j)$.

A. Space Partitioning

We consider a partition $\{\mathcal{Q}_1, \ldots, \mathcal{Q}_{|\mathcal{I}|}\}$ of $\mathcal{Q}$. By partition, we mean that $\cup_{i\in\mathcal{I}}\mathcal{Q}_i = \mathcal{Q}$ and that the intersection any two distinct elements $\mathcal{Q}_i$ and $\mathcal{Q}_j$ has zero area. Let $\Delta_{ij} = \partial\mathcal{Q}_i \cap \partial\mathcal{Q}_j$ be the frontier between $\mathcal{Q}_i$ and $\mathcal{Q}_j$. We say that i and j are neighbors when Δ_{ij} is neither empty nor a singleton set. Neighborhood is a symmetric relationship.

Similarly to [13], we consider generalized Voronoi diagrams, which are of the form:

$$\mathcal{Q}_i(P, \mathbf{w}) = \left\{\mathbf{q} \in \mathcal{Q} \,\middle|\, f(d(\mathbf{p}_i, \mathbf{q})) - w_i \leq f(d(\mathbf{p}_j, \mathbf{q})) - w_j\right\}, \quad (1)$$

where $d(\cdot,\cdot)$ is the Euclidean distance, $f(\cdot) : \mathbb{R} \to \mathbb{R}$ is a strictly increasing function and w_i is a weight associated to UxV i. The vector of all weights, $\mathbf{w} = \left(w_1, \ldots, w_{|\mathcal{I}|}\right)$ is taken from set U:

$$U = \left\{\mathbf{w} \in \mathbb{R}^{|\mathcal{I}|} \,\middle|\, |w_i - w_j| \leq f(d(\mathbf{p}_i, \mathbf{p}_j)) - f(0), \forall i, j \in \mathcal{I}\right\}.$$

The set U is compact [13] and describes the set of possible weight assignments resulting in $\mathcal{Q}_i \ni \mathbf{p}_i$. It is a restriction of a more general case where $\mathbf{w}$ is taken from $\mathbb{R}^{|\mathcal{I}|}$. We say that $\mathbf{p}_i$ is the generator of $\mathcal{Q}_i$. We now identify some specific choices for f and $\mathbf{w}$.

1) Voronoi diagram: The partition is a Voronoi diagram [14] when $f : x \mapsto x^2$ and $\mathbf{w} = \alpha\mathbf{1}, \alpha \in \mathbb{R}$. In this case, $\mathcal{Q}_i$ is the set of points which are closer to $\mathbf{p}_i$ than to any other generator. An important property is that for any $\mathbf{q} \in \Delta_{ij}$, $d(\mathbf{q}, \mathbf{p}_i) = d(\mathbf{q}, \mathbf{p}_j)$. Furthermore Δ_{ij} is a line segment when i and j are neighbors. Finally, for each $P \in \mathcal{Q}^{|\mathcal{I}|}$, there is one and only one corresponding Voronoi diagram.

2) Power diagram: This special case of (1) corresponds to $f : x \mapsto x^2$ [15]. When compared to the Voronoi diagram, the power diagram adds $|\mathcal{I}|$ extra degrees of freedom: one can change $\mathbf{w}$ and obtain different space partitions, even for fixed P. We leverage this fact in Section IV to account for degraded sensor health. The power diagram has the useful property that Δ_{ij} is still a line segment, when i and j are neighbors. The frontier between two cells is not always at equal distance from the generators on either side of that frontier. In this paper, we always use power diagrams, hence we omit to note that $\mathcal{Q}_i$ depends on f.

B. Graph, Generalized Delaunay Graph, Graph Laplacian

Consider a finite undirected graph $G = (\mathcal{V}, \mathcal{E})$, where $\mathcal{E} \subseteq \mathcal{V}^2$. Let $\mathcal{N}_v(G) = \{w \in \mathcal{V} \mid (v, w) \in \mathcal{E}\}$ be the set of neighbors of v in G. The matrix of degrees is $D_G = \operatorname{diag}\left([|\mathcal{N}_v(G)|]_{v\in\mathcal{V}}\right)$. The adjacency matrix is defined as $A_G = [a_{uv}]$, where $a_{uv} = 1$ if $(u, v) \in \mathcal{E}$ and zero otherwise. When G, is undirected, A is symmetric. Finally, the graph Laplacian is defined as $L_G = D_G - A_G$.

For all space partitions, we introduce the generalized Delaunay graph. The Delaunay graph is a graph whose vertices are $\mathcal{I}$. There is an edge between i and j if and only if $i \neq j$ and $\Delta_{ij} \neq \emptyset$. Note that this neighborhood relationship is symmetric and that the Delaunay graph is undirected. The graph is connected if and only if $\mathcal{Q}$ is.

III. Locational Optimization With Sensor Health

A. Cost function

Locational optimization is a framework to model area coverage tasks. The quality of the coverage task is modeled through a cost function. Minimizing this cost results in increased surveillance quality. In this work, we introduce a sensor health parameter that modifies cost to represent varying health conditions. The quality of a sensor can vary for various reasons such as weather conditions, in the case of an outdoor vehicle.

The cost function has the following form:

$$J(P, H) = \sum_i \int_{\mathcal{Q}_i} \phi(\mathbf{q}) \psi(\mathbf{q}, \mathbf{p}_i, h_i)\, d\mathbf{q}. \quad (2)$$

In the previous definition, $\mathcal{Q}_i$ is an element of a space partition such as those described in Section II-A. Function $\phi : \mathcal{Q} \to \mathbb{R}_{\geq 0}$ gives the importance of $\mathbf{q}$ to the surveillance task being carried out. A more important location corresponds to larger value of ϕ. A time-varying ϕ can be used to implement target tracking tasks [16].

On the other hand, $\psi : \mathcal{Q} \times \mathcal{Q} \times \mathcal{H} \to \mathbb{R}_{\geq 0}$ measures the sensing quality of location $\mathbf{q}$ performed by an UxV located at $\mathbf{p}_i$, with health h_i. A smaller value of ψ corresponds to better surveillance. This is translated in ψ being a decreasing function of distance, i.e. $(\psi(\mathbf{q}, \mathbf{p}, h) < \psi(\mathbf{q}', \mathbf{p}, h)) \equiv (d(\mathbf{p}, \mathbf{q}) > d(\mathbf{p}, \mathbf{q}'))$, for all $\mathbf{q}, \mathbf{q}' \in \mathcal{Q}$ and $h \in \mathcal{H}$. An important difference between $\mathbf{p}_i$ and h_i is that $\mathbf{p}_i$ is a controlled variable while h_i is not.

Often, ψ is chosen to be the squared distance. We divert from this by choosing:

$$\psi(\mathbf{q}, \mathbf{p}_i, h_i) = h_i d(\mathbf{q}, \mathbf{p}_i)^2, \quad (3)$$

with $\mathcal{H} = \mathbb{R}_{>0}$. Therefore, h_i affects the performance of vehicle i's sensor. With all other variables constant, a smaller h_i results in a lower cost for vehicle i. The choice of ψ made above is of course arbitrary, as one could choose another parametrization. We present a principled way to improve the cost, for any ψ which is a decreasing function of distance.

The regular locational optimization setting corresponds to $h_i = 1$ for all $i \in \mathcal{I}$. When ϕ is constant over $\mathcal{Q}$, the Voronoi diagram is the space decomposition that achieves minimum cost for any P. This is because each cell is the set of points closest to its generator and ψ is a decreasing function of distance. In this case, the boundary of the regions corresponds to points where the sensing of the agents on either side is equally bad. This is no longer true when health is introduced, as the closest robot might not be the one providing best information about the frontier. Instead, it might be better to shift the frontier away from the most effective sensor.

Computing cost using a space decomposition such as Voronoi diagrams is a simplification because the sensors do not stop sensing exactly at the boundary of their cell. In a sense, the cost function is an upper bound on the actual cost that could be achieved without that simplification. For instance, if a vehicle's sensor provides meaningful information about points in another cell, some information fusion process could occur and provide better information. A study of various special cases of such interactions between neighbors is given in [17]. The rationale behind adjusting the frontier to minimize the cost can be interpreted as an attempt to minimize an upper bound on the cost; i.e. assuming a worst case scenario.

B. Centroidal configuration

To minimize (2), one simply has to take the derivative of the cost, with respect to robot positions:

$$\frac{\partial J}{\partial \mathbf{p}_i} = 2 M_{\mathcal{Q}_i} \left(C_{\mathcal{Q}_i} - \mathbf{p}_i \right) \quad (4)$$

where $M_{\mathcal{Q}_i}$ is the mass of $\mathcal{Q}_i$ and $C_{\mathcal{Q}_i}$ its centroid, which are defined as

$$M_{\mathcal{Q}} = \int_{\mathcal{Q}} \phi(\mathbf{q})\, d\mathbf{q}, \;\; \text{and} \;\; C_{\mathcal{Q}} = M_{\mathcal{Q}}^{-1} \int_{\mathcal{Q}} \mathbf{q}\phi(\mathbf{q})\, d\mathbf{q}. \quad (5)$$

The critical points of J correspond to the centroid of the regions. The configuration in which all generators are at the centroid of their region is called the centroidal configuration. This configuration corresponds to a local minimum of the cost function.

In this paper, we assume a vehicle can move to the centroid of its cell by using the following control law:

$$\mathbf{u} = k_i (C_{\mathcal{Q}_i} - \mathbf{p}_i), \quad (6)$$

with $k_i \in \mathbb{R}_{>0}$ a gain. A discussion about vehicle dynamics are amenable to Voronoi coverage control is provided in [3].

A useful property of (6) is that it depends only on self and neighbors' positions to compute the Voronoi cell. This is desirable in practice because the number of neighbors is typically much less than the total number of vehicles.

IV. COST BALANCING

In Section III we extended the LO framework so that it would allow for modeling UxVs whose sensors have variable health level. We have also outlined that the Voronoi diagram is not satisfactory in this setting.

We further motivate cost balancing with an example. Suppose that two vehicles are performing a coverage task and that under nominal conditions, they have identical sensors. For some reason, one of these UxVs suffers from degraded sensor performance, so that its region is covered less efficiently. In agreement with the sensor model and the cost function described in Section III, this is modeled as an increased cost for that UxV. This increased cost is actually a cost increase for the whole team, because the cost function is the sum of all individual cost functions. By Equation (6), we also know that this UxV would remain at the centroid of its cell. The minimum of its cost function is larger, but its location remains unchanged. Meanwhile, the cost incurred by the other (still nominally operating) UxV on its own cell has not changed. There is thus a difference between the cost of two neighbors. Cost balancing provides a way to update the boundary, by moving it towards the neighboring region of higher cost, when there is a cost difference. The motivation for doing this is to further minimize the cost. We use a simple optimization criterion, namely that the cost for covering each cell is the same.

Before we describe how we account for variable sensor health, we present recent related work upon which we build.

A. Spatial Load Balancing

Recently, a method was proposed to balance the mass of the cells so that the *masses* (cf. Eq. (5)) match some given objective [13]. We present this method before we modify it to perform cost balancing.

Let $\mathbf{m}^* \in \mathbb{R}^n_{>0}$ be a vector representing a feasible mass assignment. An assignment is feasible if and only if:

$$||\mathbf{m}^*||_1 = \int_{\mathcal{Q}} \phi(\mathbf{q})\, d\mathbf{q}.$$

For convenience, let

$$M(P, \mathbf{w}) = \left(M_{\mathcal{Q}_1(P,\mathbf{w})}, \ldots, M_{\mathcal{Q}_n(P,\mathbf{w})}\right)$$

be the vector of masses corresponding to weight assignment $\mathbf{w}$ and UxV positions P. Also, let

$$\mathbf{F}(P, \mathbf{w}) = M(P, \mathbf{w}) - \mathbf{m}^*. \tag{7}$$

An important result from [13] is that for fixed P, there *exists* a $\mathbf{w}^* \in \mathbb{R}^n$ such that

$$\mathbf{F}(P, \mathbf{w}^*) = \mathbf{0}. \tag{8}$$

Furthermore, the sequence

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \gamma \text{diag}\left(\frac{\partial F_1(P, \mathbf{w}_k)}{\partial w_1}, \ldots, \frac{\partial F_n(P, \mathbf{w}_k)}{\partial w_n}\right)^{-1} \mathbf{F}(P, \mathbf{w}_k) \tag{9}$$

converges to $\mathbf{w}^*$, for fixed P. In the previous equation,

$$\frac{\partial F_i}{\partial w_j} = \int_{\Delta_{ij}(P,\mathbf{w})} \phi(\mathbf{q}) \mathbf{n}_i^T(\mathbf{q}) \frac{\partial \mathbf{q}}{\partial w_j} d\mathbf{q}, \tag{10}$$

where $\mathbf{n}_i(\mathbf{q})$ is the outer normal to $\mathcal{Q}_i$ at point $\mathbf{q}$. A corollary is that $\partial F_i / \partial w_j$ is identically zero unless i and j are neighbors.

B. Cost Balancing

The cost balancing mechanism is based on the existing result presented in Section IV-A. Rather than being subject to mass constraints, we pose the problem through cost constraints and we propose a method to adjust weights to satisfy these cost constraints. The focus is on online adjustment of the parameters ($\mathbf{w}$), based on the current costs. More specifically, we are interested in achieving equal cost for all robots. Ultimately, we also want to have the UxVs in a centroidal configuration with respect to this uniform cost partitioning.

For the sake of brevity, let $\psi_i(\mathbf{q}) = \psi(\mathbf{q}, \mathbf{p}_i, h_i)$. Then,

$$J_i(P, \mathbf{w}) = \int_{\mathcal{Q}_i(P,\mathbf{w})} \phi(\mathbf{q}) \psi_i(\mathbf{q})\, d\mathbf{q},$$

where ψ_i was introduced assuming fixed h_i and P. Note that while we derive the equations for fixed P and h, these variables may change, although not at the same time as $\mathbf{w}$.

We are interested in finding a weight assignment $\mathbf{w}^*$ that achieves equal cost for all robots. The constraint we pose on the costs (the analogue of $\mathbf{m}^*$) is

$$J_i^*(P, \mathbf{w}^*) = J_j^*(P, \mathbf{w}^*), \forall i \in \mathcal{I}, j \in \mathcal{N}_i(P, \mathbf{w}^*), \tag{11}$$

where $\mathcal{N}_i(P, \mathbf{w}^*)$ is the set of neighbors of i in the partition, given P and $\mathbf{w}$. However the total cost is not known *a priori* and it can change with health and position. Because of this, we cannot explicitly state feasible cost assignments. We thus state the desired cost assignment as a function of neighbors' costs. An equivalent statement to (11) is

$$J_i^*(P, \mathbf{w}^*) = |\mathcal{N}_i(P, \mathbf{w}^*)|^{-1} \sum_{j \in \mathcal{N}_i(P,\mathbf{w}^*)} J_j^*(P, \mathbf{w}^*), \forall i \in \mathcal{I}. \tag{12}$$

In words, we want the cost incurred by each UxV to match its neighbors' average cost. When the Delaunay graph is connected, it is easy to show that (11) and (12) are equivalent.

To state the analogue of (7), we would need to know J_i^*. We cannot know this quantity unless we know $\mathbf{w}^*$, which is what we are looking for. Instead, we approximate of J_i^* by using the current $\mathbf{w}$ rather than $\mathbf{w}^*$. This yields the following alternate criterion:

$$\begin{aligned} &G_i(P, \mathbf{w}) \\ &= |\mathcal{N}_i(P, \mathbf{w})|^{-1} \sum_{j \in \mathcal{N}_i(P,\mathbf{w})} \left[J_i(P, \mathbf{w}) - J_j(P, \mathbf{w})\right]. \qquad (13) \\ &\approx J_i(P, \mathbf{w}) - J_i^*(P, \mathbf{w}^*). \end{aligned}$$

If it exists, a $\mathbf{w}$ such that $G_i(P, \mathbf{w}) = 0$ entails $\mathbf{w}^* = \mathbf{w}$. The existence of a such $\mathbf{w}^*$ is discussed later in this section.

Similarly to (10), we have that:

$$\frac{\partial J_i}{\partial w_j} = \int_{\Delta_{ij}(P,\mathbf{w})} \phi(\mathbf{q}) \psi_i(\mathbf{q}) \mathbf{n}_i(\mathbf{q}) \frac{\partial \mathbf{q}}{\partial w_j} d\mathbf{q}. \tag{14}$$

This is obtained by substituting $\phi(\mathbf{q})$ by $\phi(\mathbf{q})\psi_i(\mathbf{q})$ in (10). We can now obtain

$$\begin{aligned} &|\mathcal{N}_i(P, \mathbf{w})| \frac{\partial G_i}{\partial w_i} \\ &= \sum_{j \in \mathcal{N}_i(P,\mathbf{w})} \left[\frac{\partial J_i}{\partial w_i}(P, \mathbf{w}) - \frac{\partial J_j}{\partial w_i}(P, \mathbf{w})\right] \qquad (15) \\ &= \sum_{j \in \mathcal{N}_i(P,\mathbf{w})} \int_{\Delta_{ij}} \phi(\mathbf{q}) \Big[\psi_i(\mathbf{q}) + \psi_j(\mathbf{q})\Big] \mathbf{n}_i^T(\mathbf{q}) \frac{\partial \mathbf{q}}{\partial w_i}\, d\mathbf{q}. \qquad (16) \end{aligned}$$

The last line is obtained by substituting (14) in (15) and by then using the fact that $\mathbf{n}_i(\mathbf{q})$ exists and is equal to $-\mathbf{n}_j(\mathbf{q})$ for almost all $\mathbf{q} \in \Delta_{ij}$. For of power diagrams, $\mathbf{n}_i^T(\mathbf{q}) \partial \mathbf{q} / \partial w_i = 1$ for all $i \in \mathcal{I}$.

We can now state an algorithm similar to (9) to balance the cost across neighboring cells as

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \gamma\, \text{diag}\left(\frac{\partial G_1(P, \mathbf{w}_k)}{\partial w_1}, \ldots, \frac{\partial G_n(P, \mathbf{w}_k)}{\partial w_n}\right)^{-1} \mathbf{G}(P, \mathbf{w}_k)$$

Since $|\mathcal{N}_i(P, w)|$ appears in both the inverted diagonal matrix and $\mathbf{G}$, they cancel out, to yield:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \gamma \left(I_{|\mathcal{I}|} \circ (L\mathbb{J}[J]\,(P, \mathbf{w}_k)) \right)^{-1} LJ(P, \mathbf{w}_k), \quad (17)$$

where L is the graph Laplacian of the Delaunay graph corresponding to the partition induced by $(P, \mathbf{w}_k)$, J is the current cost vector, $\mathbb{J}[J]$ is the Jacobian matrix of J and $\circ$ is the Schur product. Factoring out the graph Laplacian matrix outlines that the dependency between the components of $\mathbf{w}$ has the same structure as the Delaunay graph: it only requires communication with neighbors in the partition.

Existence of a $\mathbf{w}^*$ that achieves cost constraints (12) can be proved using the same scheme ([13, Proposition IV.4]) as the existence proof for for mass constraints (8).

We must ensure that $\mathbf{w} \in U$. This can be achieved by forcing w_i to the interval

$$\left[\max_{j \in \mathcal{N}_i} \{ w_j - f(d(\mathbf{p_i}, \mathbf{p}_j)) \}, \min_{j \in \mathcal{N}_i} \{ w_j + f(d(\mathbf{p_i}, \mathbf{p}_j)) \} \right].$$

The choice of γ in eq. (17) must be made so that the increments to $\mathbf{w}$ are small enough that it remains in U.

C. A Distributed Algorithm

In [13], the Move-to-center-and-compute-weight (MCCW) method is proposed to optimize both the weights and the robots position. The idea is to alternate moving to the centroid (i.e. optimizing P using a control law such as (6)) and optimizing the regions (i.e. optimizing $\mathbf{w}$ using (9)). It is also proved that doing this results in converges to a generalized centroidal configuration. In such a configuration, the analogue of (2) (without health) is at a local minimum and the mass constraints are satisfied.

In order to adjust the position online, we adjust the weights when the robots are at the centroid, similarly to the MCCW method described above. Although using this method ensures convergence to a centroidal configuration with equal cell costs, we do not know if balancing cost while in a near-centroidal configuration has an effect on the total cost of the configuration the vehicles will converge to. To balance the costs, UxV i needs only share the cost on its cell, $J_i = \int_{Q_i} \psi_i(\mathbf{q})\phi(\mathbf{q})\, d\mathbf{q}$, and the cost on the frontier $\int_{\Delta_{ij}} \psi_i(\mathbf{q})\phi(\mathbf{q})\, d\mathbf{q}$ with each of its neighbors ($j \in \mathcal{N}_i$).

The computations required to implement cost balancing are not significantly more difficult to carry out than those required to perform standard Voronoi coverage. In the latter case, every vehicle must be able to compute the mass and centroid of its cell (eq. (5)). To implement cost balancing, each vehicle must also compute the cost of each frontier with a neighbor. The latter is a one dimensional integral, and is therefore not significantly harder to compute than the centroid and mass, which are two dimensional integrals defined on the same quantities, namely ϕ and ψ. A useful special case is that where ϕ is a strictly positive constant function. In this case, the centroid and mass of each region are easily computed because each region is a polygon. The frontier costs are also conveniently expressed in terms of the length of the sides of the region.

V. SIMULATION

A. Simulation Setup

We present simulation results of the method proposed in Section IV. Three vehicles with different sensor health conditions ($[h_1\ h_2\ h_3] = [1\ 3\ 9]$) were randomly placed on a unit square $\phi(\mathbf{q}) = 1$. The health states were held fixed throughout the simulation. Vehicles are initially in a centroidal configuration. The cost balancing gain was set to $\gamma = 1 \cdot 10^{-3}$ and the weights were initialized to $\mathbf{w}(0) = \mathbf{0}$, which corresponds to a Voronoi diagram. In these simulations, time is discrete and the time unit is a time step.

B. Simulation Results and Discussion

Figures 1 and 2 show the behavior of the vehicles for healthy and degraded conditions respectively. Cost balancing achieves equal cost as shown in Figures 1(c) and 2(c). This results in some unequal weight assignment among Vehicles, which is shown in Figures 1(b) and 2(b).

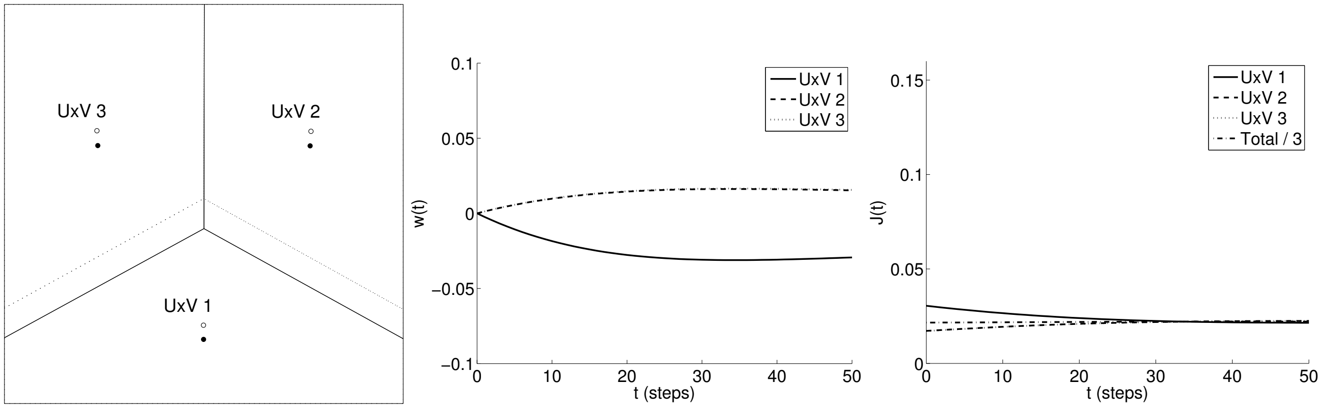
With equal health variables, the Voronoi diagram does not correspond to a configuration where all cells have equal weight. This fact is highlighted by Figure 1(a). In fact, when all vehicles have equal sensor health, cost balancing results in a higher total cost than the use of a plain Voronoi diagram, as shown in Figure 2(c). However, having each cost matching the average cost also lowers the cost incurred by the less effective sensor. Balancing cost can therefore be interpreted as a trade-off between less effective and more effective sensors. When some sensors are much less effective than others, this tradeoff can result in a significant improvement over the use of a Voronoi diagram as shown in Figure 2(c).

The proposed approach may fail to optimize the costs if $\phi(\mathbf{q}) = 0$ for all $\mathbf{q}$ at some frontier Δ_{ij}. In this case, a difference between the cost incurred by i and j, might not result in a change of either w_i or w_j. In such situations, it is possible that updates to w_i or w_j occur through interaction with other neighbors and that at some point, Δ_{ij} will have moved so that the integral on Δ_{ij} becomes non-zero. A pathological case of this is when *all* the frontiers of a robot have a zero cost. The proposed cost balancing strategy will not adjust the frontiers for that agent. A way to counter this is to let $\phi(\mathbf{q}) > 0$ for all $\mathbf{q}$.

VI. CONCLUSIONS AND FUTURE WORKS

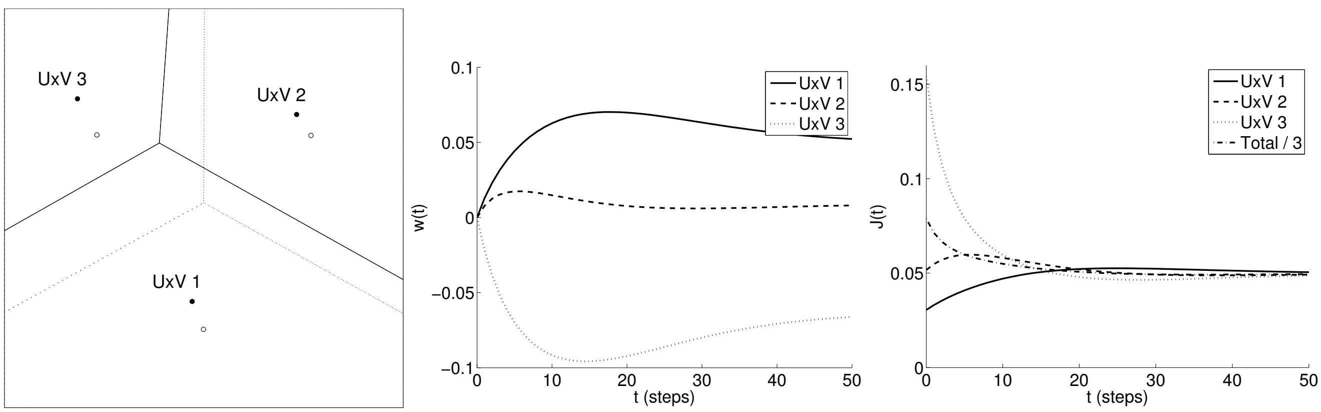
We presented a way to model variable sensor effectiveness through health and to optimize the placement of such sensors. The method has the benefits of begin computationally efficient and to depend only on local information. When sensor effectiveness is unequal, simulations showed that cost balancing achieves lower total cost than Voronoi coverage, while providing only little degradation under nominal conditions.

Future works includes integrating the method described here in a system that accounts for other ways in which the performance of one or many vehicles can be degraded, such as a variable number of agents, variable communication capabilities and movement capabilities. We are also interested in adding a mission planning component to the system.



(a) Voronoi (dotted) and cost-balanced (full).
(b) Weights
(c) Cost

Fig. 1. Cost balancing under nominal conditions ($h_1 = 1, h_2 = 1, h_3 = 1$).



(a) Voronoi (dotted) and cost-balanced (full).
(b) Weights
(c) Cost

Fig. 2. Cost balancing under degraded conditions ($h_1 = 1, h_2 = 3, h_3 = 9$).

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