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# Time-Scale Features and Their Applications in Electric Power System Dynamic Modeling and Analysis

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*Abstract*—There was a major collection of publications between 1975 and 1985 that established fundamental theory and implementation of power system dynamic analysis. These results recognized the time-scale features of power systems and created strong modeling and analysis tools that are continuing to form the basis for simulation and analysis techniques today. This paper presents the fundamental time-scale properties of power system dynamic models that enabled many of these contributions. These properties have been utilized to create reduced-order dynamic models, efficient simulation algorithms, and to discover techniques for large-scale stability analysis.

## I. Introduction

Electric power systems can be broadly classified by mechanical equipment and electrical equipment as shown in Figure 1. While this figure separates the blocks by mechanical and electrical dominance, the mechanical equipment on the left requires a control system and communication infrastructure that depends on electrical

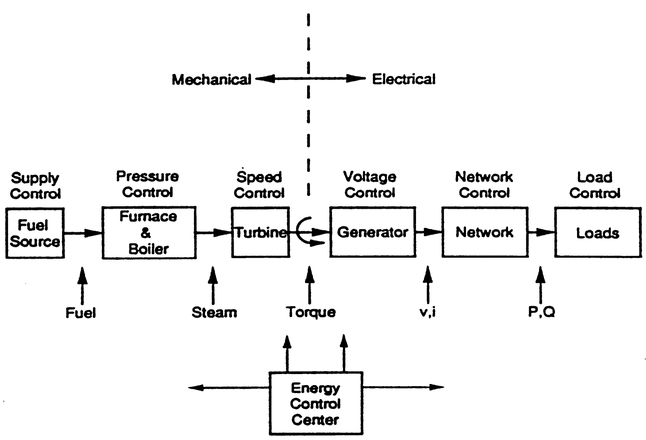


Figure 1. A typical Electromechanical system

components. Similarly the electrical equipment on the right includes mechanical components such as tap-changing-under-load (TCUL) transformers and massive numbers of rotating machines as loads.

The dynamics associated with the blocks in this figure range in time response from microseconds to hours. As such, the mathematical modeling of these dynamics presents both an interesting property and computational challenge. The phenomena that require understanding and analysis are shown in Figure 2.

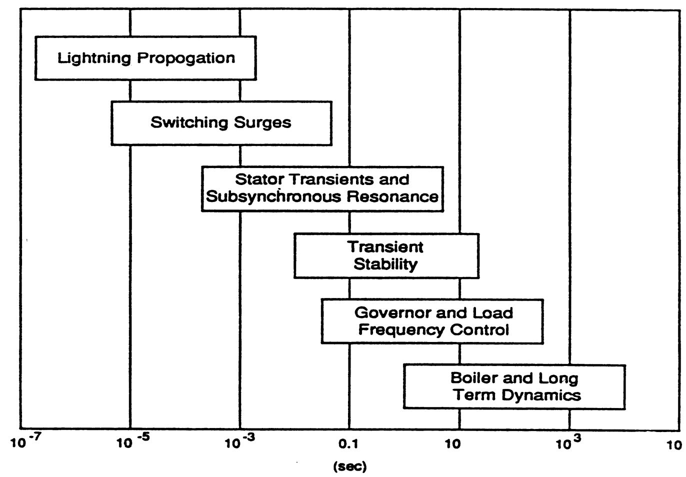


Figure 2. Phenomena time scales in electric power systems

The mathematical models for each of these phenomena often take drastically different forms because of the inherent time-scale features. Several of these are discussed in the following sections.

## II. Dynamic Modeling of Synchronous Machines

To see what some of the models for the equipment shown in Figure 1 look like, consider the middle of the time scale range. The heart of most electric power system dynamic analysis involves the dynamics of the generators that provide the majority of the power consumed by the loads. The models for these generators begin by considering the revolving field circuit in a synchronous machine being driven by a prime mover (typically a turbine of some type).

Figure 3 shows a typical schematic of that rotating field circuit and several other electrical coils. There are three coils on the stationary part of the machine (called the stator), and four additional coils on the rotating part of the machine (called the rotor). These 7 coils have labels – a, b, c for the stator, and fd, 1d, 1q, and 2q for the rotor. The magnetic axes of these 7 coils are shown on the diagram of Figure 3. The magnetic axis of the field winding (labeled fd) is aligned with the long dimension of the "dog bone" rotor. Perpendicular to this and 90 degrees ahead of this "d" axis is the quadrature axis – the "q" axis. The other 3 coils on the

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rotor (1d, 1q, 2q) are short circuited damper windings. The shaft position is measured from the stator "a" axis to the "q" axis, with assumed counterclockwise rotation.

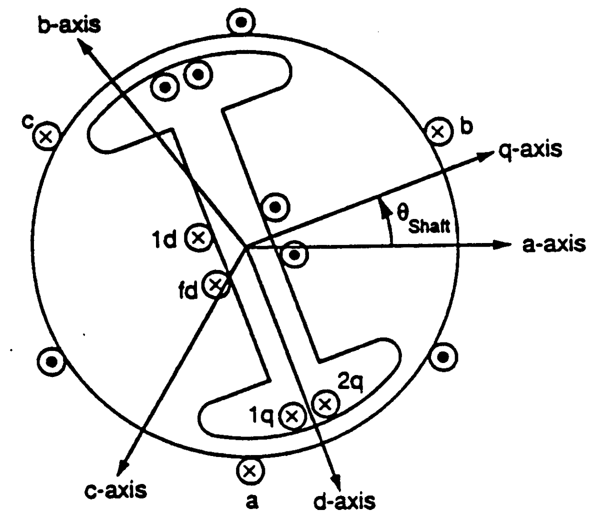


Figure 3. Synchronous machine coil locations

Virtually all of the electrical dynamics of interest in the synchronous machine arise from the dynamic equations associated with these 7 coils. Standard Kirchhoff (plus Ohm and Faraday) laws written in the time domain give the following differential equation for each coil:

$$v_i = i_i r_i + \frac{d\lambda_i}{dt} \tag{1}$$

The algebraic relationship between the flux linkages (λ) and currents (*i*) reflect the constraints of Ampere's law plus the magnetic property of the iron and air configuration. This relationship is normally assumed to be linear, although the flux linkages will include dependence on the rotor position $\theta_{shaft}$.

$$\lambda = L(\theta_{shaft})i \tag{2}$$

The fundamental Newton's Second Law for the rotating shaft of the machine has the following pair of differential equations:

$$\frac{d\theta_{\text{shaft}}}{dt} = \frac{2}{P}\omega \tag{3}$$

$$J\frac{2}{P}\frac{d\omega}{dt} = T_m - T_e - T_{f\omega} \tag{4}$$

where P is the number of magnetic poles per phase (Figure 3 shows a 2-pole machine). These "abc" equations are traditionally transformed into "dqo" variables through the following relationship [8]:

$$\begin{bmatrix} v_d \\ v_q \\ v_o \end{bmatrix} \triangleq T_{dqo}(\theta_{shaft}) \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = T_{dqo}^{-1}(\theta_{shaft}) \begin{bmatrix} v_d \\ v_q \\ v_o \end{bmatrix} \tag{6}$$

The same transformation is also performed on the flux linkages and currents. For time-scale modeling and analysis, the most valuable feature of power system dynamics is the use of "per-unit" scaling. The tradition of scaling voltages and currents to be relative to rated values makes the quantities of interest to be "near 1.0". For example, actual voltage divided by "rated voltage" will normally produce a value between 0.95 and 1.05 as that is the goal of most system controllers. This scaling feature allows the modeling to proceed directly into a well-defined multiple time-scale structure. In this scaled model the $\psi$ variables are per-unit scaled versions of the flux linkages and the capital I and V are the per-unit scaled versions of the currents and voltages. The stator dynamics are (with the terminal relationship between I and V yet to be specified):

$$\begin{aligned} \frac{1}{\omega_s}\frac{d\psi_d}{dt} &= R_s I_d + \left(1 + \frac{\varepsilon}{T_s}\omega_t\right)\psi_q + V_d \\ \frac{1}{\omega_s}\frac{d\psi_q}{dt} &= R_s I_q - \left(1 + \frac{\varepsilon}{T_s}\omega_t\right)\psi_d + V_q \\ \frac{1}{\omega_s}\frac{d\psi_o}{dt} &= R_s I_o + V_o \end{aligned} \tag{7}$$

Where the small parameter epsilon is the inverse of the electrical angular frequency:

$$\varepsilon = \frac{1}{\omega_s} \tag{8}$$

and a "transient speed" is defined as:

$$\omega_t = T_s(\omega - \omega_s) \text{ with } T_s = \sqrt{\frac{2H}{\omega_s}} \tag{9}$$

The "dqo" transformation removes the time variation of the scaled flux linkage vs current relationship to give (with stator current directions reversed):

$$\psi_d = X_d(-I_d) + X_{md} I_{fd} + X_{md} I_{1d}$$
$$\psi_{fd} = X_{md}(-I_d) + X_{fd} I_{fd} + X_{md} I_{1d} \quad (10)$$
$$\psi_{1d} = X_{md}(-I_d) + X_{md} I_{fd} + X_{1d} I_{1d}$$

$$\psi_q = X_q(-I_q) + X_{mq} I_{1q} + X_{mq} I_{2q}$$
$$\psi_{1q} = X_{mq}(-I_q) + X_{1q} I_{1q} + X_{mq} I_{2q} \quad (11)$$
$$\psi_{2q} = X_{mq}(-I_q) + X_{mq} I_{1q} + X_{2q} I_{2q}$$
$$\psi_0 = X_{ls}(-I_0) \quad (12)$$

This model also neglects leakage in the mutual inductances.

Three of the rotor quantities are typically scaled to be:

$$E'_q \triangleq \frac{X_{md}}{X_{fd}} \psi_{fd}, \quad E'_d \triangleq \frac{X_{mq}}{X_{1q}} \psi_{1q}, \quad E_{fd} \triangleq \frac{X_{md}}{R_{fd}} V_{fd} \quad (13)$$

A new angle variable is traditionally introduced as a "strobed" version of the actual shaft position as:

$$\delta \triangleq \frac{P}{2} \theta_{shaft} - \omega_s t \quad (14)$$

Where $\omega_s$ is rated speed in electrical radians per second. The rotor dynamics (4 electrical plus 2 mechanical) are then as follows.

$$T'_{do} \frac{dE'_q}{dt} = -E'_q - (X_d - X'_d) x$$

$$\left[ I_d - \frac{X'_d - X''_d}{(X'_d - X_{\ell s})^2} \left( \psi_{1d} + (X'_d - X_{\ell s}) I_d - E'_q \right) \right] + E_{fd} \quad (15)$$

$$T'_{qo} \frac{dE'_d}{dt} = -E'_d + \left( X_q - X'_q \right) x$$

$$\left[ I_q - \frac{X'_q - X''_q}{\left( X'_q - X_{\ell s} \right)^2} \left( \psi_{2q} + \left( X'_q - X_{\ell s} \right) I_q + E'_d \right) \right] \quad (16)$$

$$T''_{do} \frac{d\psi_{1d}}{dt} = -\psi_{1d} + E'_q - \left( X'_d - X_{\ell s} \right) I_d \quad (17)$$

$$T''_{qo} \frac{d\psi_{2q}}{dt} = -\psi_{2q} - E'_d - \left( X'_q - X_{\ell s} \right) I_q \quad (18)$$

$$\frac{d\delta}{dt} = \omega - \omega_s \quad (19)$$

$$\frac{2H}{\omega_s} \frac{d\omega}{dt} = T_M - \left( \psi_d I_q - \psi_q I_d \right) - T_{FW} \quad (20)$$

All parameters and variables are defined according to standard industry practice [8].

With the rotor coil currents eliminated, the stator flux linkages are related to the stator currents and rotor flux linkages by:

$$\psi_d = -X''_d I_d + \frac{\left( X''_d - X_{\ell s} \right)}{\left( X'_d - X_{\ell s} \right)} E'_q + \frac{\left( X'_d - X_{\ell s} \right)}{\left( X'_d - X_{\ell s} \right)} \psi_{1d} \quad (21)$$

$$\psi_q = -X''_q I_q - \frac{\left( X''_q - X_{\ell s} \right)}{\left( X'_q - X_{\ell s} \right)} E'_d + \frac{\left( X'_q - X''_q \right)}{\left( X'_q - X_{\ell s} \right)} \psi_{2q} \quad (22)$$

$$\psi_o = -X_{\ell s} I_o \quad (23)$$

There are of course dynamic models for the excitation systems and speed governors and turbines associated with each machine. These voltage and frequency control systems are discussed in [8], but do not play a strong role in the time-scale features that have been utilized for reduced-order modeling and stability analysis in the past 35 years.

## III. Dynamic Modeling of the Network Interconnection

The interconnection of all the synchronous machines in the power network is not included in the above formulation. The dynamics of that interconnection include the effects of the magnetic and electric fields associated with the transmission lines. The proper technique to analyze the electrical dynamics associated with these fields is the set of partial differential equations describing these fields. However, there is a traditional approximation that utilizes a lumped-parameter model which can be used to together with the above dynamic models to create powerful reduced-order models that sufficiently capture the relatively slow phenomena of rotor synchronism and coherent behavior. The use of this lumped parameter approximation results in series inductance and shunt capacitance with their associated resistances. This step of going from partial differential equations for the electric and magnetic fields to the lumped-parameter approximation should actually be thought of as the first time-scale feature that is traditionally done to avoid that complex dynamic analysis associated with the exact Maxwell Equations.

If shunt capacitors are neglected, a graph methodology involving independent branches and nodes of interconnected 3-phase lumped-parameter elements results in the following network model (including passive loads) for "branch" flux linkages, currents, and voltages relative to a synchronously rotating reference frame:

$$\varepsilon \frac{d\psi_{Dbranch}}{dt} = R_{branch} I_{Dbranch} + \psi_{Qbranch} + V_{Dbranch}$$
$$\varepsilon \frac{d\psi_{Qbranch}}{dt} = R_{branch} I_{Qbranch} - \psi_{Dbranch} + V_{Qbranch} \quad (24)$$
$$\varepsilon \frac{d\psi_{Obranch}}{dt} = R_{branch} I_{Obranch} + V_{Obranch}$$

The epsilon small parameter is the same as that for the stator transients given in (8).

It is straightforward to extend this to independent "loop" flux linkages, currents, and voltages. With the help of per-unit scaling, the complete dynamic model of a power system with interconnected synchronous machines is of the standard two-time scale form. Formally setting the small parameter epsilon to zero gives the dynamic model in the slow time scale where the fast stator and network currents and flux linkages are infinitely fast (change instantaneously). The resulting algebraic equations that replace the differential equations have the following general form for the synchronous machine:

$$\begin{aligned} 0 &= R_s I_d + \psi_q + V_d \\ 0 &= R_s I_q - \psi_d + V_q \\ 0 &= R_s I_o + V_o \end{aligned} \tag{25}$$

together with (10)-(12) above and the following form for transmission lines and impedance loads:

$$\begin{aligned} 0 &= R_i I_{Di} + \psi_{Qi} + V_{Di} \\ 0 &= R_i I_{Qi} - \psi_{Di} + V_{Qi} \\ 0 &= R_i I_{Oi} + V_{Oi} \end{aligned} \tag{26}$$

with the linear flux linkage/current relationship:

$$\begin{aligned} \psi_{Di} &= -X_{epi} I_{Di} \\ \psi_{Qi} &= -X_{epi} I_{Qi} \\ \psi_{Oi} &= -X_{eoi} I_{Oi} \end{aligned} \tag{27}$$

This huge reduction in model complexity gives a very useful "circuit" view of the algebraic equations created by the singular perturbation. For balanced systems, the "o" quantities will be zero and the remaining pairs of equations can be written as complex equations by adding the "d" equation plus j times the "q" equation as follows:

$$0 = R_s (I_d + jI_q) + (\psi_q - j\psi_d) + (V_d + jV_q) \tag{28}$$

The machine flux linkages are related to the machine currents through equations (10) and (11). Doing the same for the network transmission lines, loads, and transformers gives:

$$0 = R_i (I_{Di} + jI_{Qi}) + jX_{epi} (I_{Di} + jI_{Qi}) + (V_{Di} + jV_{Qi}) \tag{29}$$

To connect the machine "circuit" equations to the network "circuit" equations, it is necessary to transform the machine "dq" variables into the synchronously rotating "DQ" reference frame. This is done by multiplying by an exponential term that includes the strobed rotor shaft position angle delta, i.e.:

$$\left(I_{di} + jI_{qi}\right) e^{j\left(\delta_i - \pi/2\right)} = \left(I_{Di} + jI_{Qi}\right) \tag{30}$$

With this complex variable notation, the algebraic equations look very similar to traditional "steady state" phasor equations. This is not surprising since the stator/network singular perturbation leaves the slow subsystem modeled in what is often called the "quasi steady state" form. It is also often called "neglecting stator transients".

This completes the description of the reduced-order modeling benefits of the time-scale features of the synchronous machine and transmission grid, and leads directly to another benefit which builds on these features. The beauty of this result is that it provides a systematic network algorithm for dealing with interconnection dynamics and power flows. The algebraic equations in complex form fit a circuit representation that allows standard topology processing to create a fully interconnected dynamic model. In this simplest form, all of the dynamics have been confined to the generator locations. In this formulation, the interconnected network does not contain elements with additional dynamics. Now, in real models, there are devices in the network that do contain dynamic phenomena on the same time scale. Examples are Tap-Changing-Under-Load (TCUL) transformers, Flexible AC Transmission (FACTS) devices with their controls, and of course dynamic loads.

But, to illustrate a second major benefit of the time-scale features, it is useful to consider the concepts of "weak connections" and coherent machines within weakly coupled areas. These coherent machines are groups of synchronous machines that respond in similar ways to disturbances. For example, if a fault in the network causes two machines to both accelerate together, they could be considered "coherent" and therefore perhaps be combined into one. When taken on a system-wide basis, this coherent grouping of machines makes a powerful tool for equivalencing. Common sense indicates that machines that are tightly connected electrically should respond in a similar way to disturbances. And, machines that are weakly connected electrically should not respond in a similar way. Therefore, one approach to finding coherent machines in a system would be to quantify the strength of interconnections and identify extremes. This would lead to "areas" which behave as single large machines rather than many smaller machines. This was extensively investigated in the earlier years with the notion that within a tightly connected area, the dynamics are predominantly fast while between weakly connected areas, the dynamics are predominantly slow.

## IV. Coherency, Time Scales, Aggregation, and Weak Coupling

In addition to the reduced-order dynamic modeling for synchronous machines, there were transformative contributions to large-scale system dynamic modeling and analysis between 1975 and 1985. While the seeds for this work were found even in earlier times, one major program created the environment for "systems engineering for power" [1]. This conference and the subsequent Department of Energy program created this environment which eventually stimulated research programs around the world that exist in various forms today. The time-scale features of electric power systems include phenomena such as multi-mass oscillations found in the shaft train of generating stations involving multiple stage turbines, generator rotors, and exciters. The notion of weak and strong coupling applies to these mechanical dynamics to model and analyze shaft oscillations. When couplings are made infinitely fast, the mass train becomes one solid mass with all sections moving together. If the purpose of the analysis is to study how this single mass behaves relative to other single masses at other locations, the coupling between individual component masses is made stiff. This is equivalent to a singular perturbation that results in a "slow" model of the mass dynamics that are interconnected by the transmission grid. Alternatively, considering multiple masses connected together and their relative swings on the tandem shafts results in a "fast" model. Taking this same concept one level higher, masses that are strongly coupled within an "area" can be aggregated even further, creating reduced order models on a system-wide basis. This concept of coherency is still being used today for creating islanding schemes.

## V. Conclusions

The time-scale features of electric power systems are rich in mathematical properties that enable powerful modeling, simulation, and analysis tools. The work between 1975 and 1985 was a unique collection of fundamental contributions that established benchmark methods for dynamic modeling and analysis. The references below are but the tip of a very big iceberg that has emerged and formed the foundation for modern power system dynamic analysis of electric power systems.

## VI. Acknowledgements

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## Biography

Peter W. Sauer received the BSEE degree from the University of Missouri at Rolla, in 1969, and the MSEE and PhD degrees from Purdue University, West Lafayette, IN, in 1974 and 1977, respectively. He served in the U.S. Air Force on active duty from 1969 to 1973 and in the reserves from 1982 to 1998. He served as the program director for power systems in the ECS division of the National Science Foundation from August 1991 to August 1992. He is a cofounder of PowerWorld Corporation and the Power Systems Engineering Research Center (PSERC). He is currently the Grainger Chair Professor of Electrical Engineering at the University of Illinois at Urbana-Champaign. He is a registered Professional Engineer in Virginia and Illinois a Fellow of the IEEE, and a member of the U.S. National Academy of Engineering.