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The Application of Domain of Danger in Autonomous Agent Team and Its Effect on Exploration Efficiency

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Abstract— **Prey animals in a group constantly face the trade-off between foraging gain and predation risk. The domain of danger concept was developed by ethologists to measure the predation risk taken by an individual within a group. In this work we make the analogy between information gathering for an autonomous agent team and food gathering for a group of animals to explore bio-inspired strategies for autonomous agent teams in an information gathering scenario. The sum of areas of domains of danger of the team is used as the measurement of risk taken by the whole team. Several movement rules that let an agent gather food nodes and/or shrink its domain of danger according to the current layout of the team and food nodes are proposed. The performance on foraging efficiency and team domain of danger of these movement rules are investigated by simulation. The results show that trying to shrink the domain of danger of each individual while no food can be gained in the near future doesn't degrade the foraging performance of the team and can effectively lower the risk taken by the whole team by prevention of over-spreading.**

I. INTRODUCTION

Autonomous agent teams have received a lot of attention in recent years. One of the most popular applications is to deploy autonomous teams composed of multiple agents to gather information or provide surveillance over a certain region. This scenario can be categorized into different sub-genres according to the nature of the goal and the resources at hand. We can employ the static coverage framework when we have enough sensory powers to provide satisfactory coverage to the region of interest with a static layout, or when our sensor platforms only have limited mobility after the initial deployment. A detailed overview of the basic approaches in static coverage is given in [1]. On the other hand, when the team does not provide enough sensory power to monitor the region with a stationary layout, dynamic coverage comes in. One approach in this area is cellular decomposition. The region to be covered is divided into cells and each agent is assigned to one or many cells. The cells are usually required to exhibit desirable geometric properties such as convexity so that agents can scan through the whole cell systematically. Various methods in this area are reviewed in [2].

Another approach to dynamic coverage that has drawn a lot of attention is inspired by the work from Hussein and Stipanovic [3]. They extended the locational optimization approaches used by the static coverage framework. The coverage of a location in the region is defined as the sum of the sensing power projected by all agents to that location over time. A distributive control law that guarantees the whole region will be satisfactorily covered is also proposed. When the mission only requires that each location be covered once, the problem fits more into the exploration framework. In the recent work of Haumann et al. [4], the dynamic coverage framework from Hussein and Stipanovic was combined with the frontier-based exploration in [5] using an optimization-based control scheme to control the orientation of each agent to efficiently explore the region of interest.

In this paper we make the analogy between information gathering for an autonomous agent team and food gathering for a group of animals. Consider a region with information as representative of food that our agents are trying to gather. A pack of animals foraging in a group is solving the same problem as the autonomous agent team. To forage efficiently as a group, individuals in a group have to avoid foraging interference when they get too close to each other. This is very similar to the way we want a team of autonomous agents to act on an exploration mission, avoiding the waste of sensory power. The tendency to avoid foraging interference serves as a imaginary repellent force that keep individuals from getting too close to one another. What keeps individuals within a group is the desire to avoid predation risk. An individual that is separated from the group is more likely to be targeted by a predator. The desire to forage without interference and to avoid predation risk guides individuals in the group foraging scenario. The purpose of this paper is to look into these two different aspects that have rich literature in ethology study and explore their potential in a collaborative autonomous agent team framework.

II. PRELIMINARIES

A. Domain of Danger

The domain of danger (DOD) idea was proposed by Hamilton in [6] as a measurement of predation risk taken by each prey individual within a group. It is assumed that a predator that is currently undetected by prey might show up anywhere within the field, even inside of the group. Also, when the predator shows up it will pursue the closest prey. Following this assumption, the domain of danger of a specific prey individual within the group is defined to be the region in which, if the predator shows up, the individual will be closer

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to the predator than any other individual in the group. In other words, the domain of danger of an individual is its Voronoi polygon. Consider a set of n agents $P = \{p_1, p_2, ..., p_n\}$ where $p_i \in \mathbb{R}^2$. The Voronoi polygon of an agent is denoted by:

$$V(p_i) = \{x | \|x - p_i\| \leq \|x - p_j\|; \forall j \neq i\}$$

However, under the standard definition of a Voronoi polygon, agents on the boundary will have a Voronoi polygon that extends to infinity. This will not only cause some inconvenience in the computation of the Voronoi polygon, but also implies that prey will be affected by predators that show up at a distance of infinity, which is unrealistic. To deal with this problem, James et al. proposed the limited domain of danger idea in [7]. The idea of the limited domain of danger is to limit the domain of danger within a certain radius from the individual so that it doesn't extend to infinity. It can be formally defined as:

$$DOD(p_i) = V(p_i) \cap C_{R_d}(p_i)$$
$$C_{R_d}(p_i) = \{x | \|x - p_i\| \leq R_d\}$$

It is the intersection of a circle centered at an agent with a radius R_d and its Voronoi polygon. In the seminal work of optimal escape theory from Ydenberg and Dill [8], it is stated that prey don't always react to the presence of a predator by fleeing. Prey will only start to flee from the predator when the predator is closer than a certain distance known as the optimal escape distance to preserve energy and valuable foraging time. By using the limited domain of danger concept with radius R_d, we assume that agents are not concerned about a predator further away than R_d.

The area of the domain of danger can be easily translated to the probability of being targeted by a predator in a scenario where we know that there is exactly one predator in a certain region. If we denote the probability of a predator being in a region of area A by ε, then the predation risk of a prey p_i is $\varepsilon \times DOD(p_i)/A$. Although the exact number of predators and the probability of their location in a certain region is usually unavailable, it's certain that the sum of the areas of domains of danger of the team is proportional to the predation risk taken by the whole team, which we denote by *PR*. We can write

$$PR \propto \sum_{i=1}^{n} DOD(p_i)$$

With the domain of danger concept, several movement rules for predator avoidance have been proposed. The simplest Nearest Neighbor rule where each prey moves toward its closest neighbor is proposed by Hamilton in [6]. Viscido et al. later on proposed the Local Crowded Horizon rule (LCH) where each prey moves toward the most crowded direction instead of just toward the closest neighbor in [9]. James et al. also proposed a Minimum Time movement rule (MT) in [7] which takes into account the orientation of the prey.

The original DOD model and movement rules are for the case where prey don't have explicit knowledge of the predator's location. In [10], Viscido et al. proposed a post-detection movement rule for the scenario where the predator has been detected. The resulting movement of this rule is a combination of moving away from the predator and shrinking one's DOD. The weighting on the away-from-predator direction is inversely proportional to the distance from the prey to the predator. Some empirical evidence for this model is provided in [11].

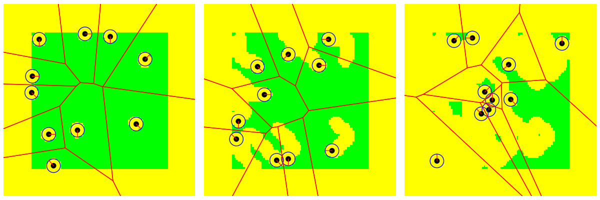
Almost all of the aforementioned movement rules are heuristic-based approaches to shrink one's domain of danger. In this paper, we propose a new approach that explicitly uses the size of DOD as the objective function to be minimized by each individual. The details of the new movement rule based on this approach will be described in section IV.

B. Producer-Scroungers/Information Sharing

Besides predation risk avoidance, another important aspect for group-living animals is foraging. The Producer-Scrounger (PS) model, as explained in [12], assumes some agents are producers that actually search for food and others are scroungers that just share the food found by producers. In most of the PS models, the finder of a food patch gets a finder's share from the patch and then shares the rest of the patch with scroungers that join the patch. The effect of finder's share on the dynamics of Producer-Scrounger interaction is investigated by Giraldeau et al. in [13]. Obviously a group with more producers will find food more efficiently. But individually, a scrounger might have higher food gain than a producer since it can join different food patches found by different producers without spending time searching for them. While most of the literature in this area of research uses game theoretical approaches to investigate the ratio between producers and scroungers, there is a set of research that focuses on the impact of the producer-scrounger role on the spatial properties of the group, such as the work of Flynn et al. in [14]. It is shown that the producers are more likely to be on the boundary of the group to avoid potential competitors, and the scroungers are more likely to stay near the center of the group to monitor multiple producers at once to maximize the chance of sharing a food patch.

Besides the PS model, there is the Information Sharing (IS) model which is based on slightly different assumptions. The IS model assumes that an agent can search for food and monitor its groupmates for opportunities to join a food patch at the same time. The differences and similarities of the PS and IS model are throughly discussed in [13], [15], and [16]. Vickery et al., in [15], introduce an opportunistic foraging strategy as an intermediate strategy between producer and scrounger. An opportunistic agent can search for food and be aware of joining opportunities at the same time with a discounted efficiency. Various factors that affect the distribution of population using each strategy are also identified.

In this paper we will investigate movement rules that lead agents to switch between two strategies, gathering food by themselves or following other agents, according to various factors such as the distribution of food in the field and the layout of the team.



(a) 10th step (b) 30th step (c) 90th step

Fig. 1. (a),(b), and (c) are snapshots of a simulation run in different time steps. Green squares represent food nodes. Yellow grids are empty. Black dots are the agents. A circle in blue around each agent is the foraging region of that agent. The red line extended from the agent indicates the current heading of that agent. The red polygons around the agent represent the DOD of that agent.

III. PROBLEM STATEMENT AND SIMULATION

We define our field to be the R^2 space. The region of interest is a subset of the field. Agents are considered to live in the field, always moving with a constant speed and able to change their heading direction with an unlimited turning rate. We assume agents can see every food node as well as all other agents, and hence are capable of generating the DOD of current or nearby positions.

A food node is a point in the field that contains a unit amount of food. Food nodes are spread in the region of interest with a fixed density on a grid. This can be considered as discretizing the field and placing one food node on the center of each grid. An agent will collect every food node within its foraging radius. Once a food node is collected by an agent it is removed from the field. When foraging regions of multiple agents overlap, we randomly pick one of the agents to gather all the food nodes within the overlapped area.

The simulation is coded in Java using the MASON multi-agent simulation toolkit [1]. At the beginning of every simulation the region of interest is filled with food nodes and agents will only be initialized within the region of interest using a random seed. Note that although the placement of the food nodes involves discretization of the field, agents and food nodes still live in a continuous field. Once the agents are initialized, the simulation starts to advance in steps.

At the beginning of each step, agents are fed with the information of the position of all food nodes and other agents. Each agent then proposes a position to move to according to the movement rule it uses. After each agent has proposed a position, agents are moved from their current location to the proposed position in a random order. Food nodes that are within the foraging radius along the path are removed from the field and considered gathered by the agent. Fig.1 shows snapshots of different time steps of one simulation run.

IV. MOVEMENT RULES

In this section we describe different movement rule in detail in the form of pseudo-codes. Some basic notations and operations are explained as following:

$p \in R^2$: current position of the agent
$P = \{p_i, i = 1, \ldots, n\}$: set of n candidate positions
$F = \{f_i, i = 1, \ldots, m\}$: set of all available food positions
DOD_p: the domain of danger of agent at p
$FoodGain(p_a, p_b)$: food gain by moving from p_a to p_b
$RandomPick(P)$: randomly pick an element from set P
$NearestPosToFood(P, f)$: $p \in P$ that's closest to food f
$NearestFoodTo(p, F)$: nearest food to p in set F. If F is empty, then return $\emptyset$

The candidate position set P are filled with the possible positions the agent can be in the next time step. In the simulation we pick eight evenly spaced positions on a circle centered at the agent with a radius that is equal to the distance an agent can move in one step.

A. Greedy Foraging

Movement Rule A Greedy Foraging

```
1:  for all p_i ∈ P do
2:      FoodGainAt[p_i] ← FoodGain(p, p_i)
3:  end for
4:  if max(FoodGainAt[]) > 0 then
5:      return argmax(FoodGainAt[])
6:  else
7:      f_Near ← NearestFoodTo(p, F)
8:      if f_Near = ∅ then                    ▷ No food left
9:          return RandomPick(P)
10:     else
11:         return NearestPosToFood(P, f_Near)
12:     end if
13: end if
```

Agents using this movement rule compute the food to be gained by moving to each candidate position and picking the one with the most food gain. When there is no food to be gathered in one time step, the agent picks the position that will bring it closer to the closest food in the field. When there is no food left in the field, agents move randomly. Pseudo-code is shown in movement rule A.

B. Greedy Foraging with De-conflict

For this movement rule we need to define two more notations:

DOD_p: the limited Voronoi polygon for the agent at p. [2]
F_{DOD_p}: the set of available food that is within DOD_p.

Movement Rule B Greedy Foraging with De-conflict

Replace line 7 in movement rule A with:

$f_{Near} \leftarrow NearestFoodTo(p, F_{DOD_p})$

This movement rule is the same as movement rule A when there is still food to be gathered in one step. But when no food can be gathered in one step, the agent only tries to move

[1]MASON: Multi-Agent Simulator Of Neighborhoods/Network. http://cs.gmu.edu/~eclab/projects/mason/

[2]For the generation of Voronoi partition, we use the Mesh library which is kindly shared by Lee Byron on http://www.leebyron.com/

toward the closest food within its limited Voronoi polygon. Since every food node within the Voronoi polygon is closer to the agent than any other agents, the agent is guaranteed to get the food node it aims for before other agents. Adding this de-conflict mechanism eliminates the less efficient behavior where multiple agents aiming for the same food node will have to backtrack for other food nodes when the food is gathered by the closest agent.

C. Greedy Foraging then DOD

For this movement rule, one more operation is introduced: $Area(DOD(p_i))$: Area of DOD when agent is at p_i.

Movement Rule C Greedy Foraging then DOD

Replace line 7 in movement rule A with:

$f_{Near} \leftarrow NearestFoodTo(p, F_{DOD(p)})$

And replace line 9 in movement rule A with the following lines:

for all $p_i \in P$ **do**
    $DODAt[p_i] \leftarrow Area(DOD(p_i))$
end for
return $\arg\min(DODAt[])$

This movement rule takes into account the risk avoidance aspect which was left out in movement rule A and B. When there is no food left in one's DOD, instead of moving randomly, the agent tries to minimize the size of its DOD. It does this by calculating the size of the DOD at each candidate position assuming all other agents are stationary. In other words, when an agent can not contribute to the foraging efficiency of the team, it tries to shrink the DOD of itself and of the team. What will then often happen is after following a groupmate for a while, an agent's Voronoi polygon will again cover some available food nodes and the agent can gather them. This strategy of following group-mates and sharing the food found by them is similar to the scrounger strategy mentioned in section II-B.

D. Greedy DOD then Foraging

In this movement rule the risk avoidance aspect is given a higher priority than food gathering. A position in the candidate position set is safe if the size of DOD there is smaller than a safety threshold. When there are no safe positions, the agent moves to the position with the smallest DOD. When safe positions are present, the agent picks the one with the most food gain or brings it closer to the closest food within its DOD. If there is no food left in its DOD at this point, the agent will go to the safe position with the smallest DOD.

Movement Rule D Greedy DOD then Foraging

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for all $p_i \in P$ **do**
    $DODAt[p_i] \leftarrow Area(DOD(p_i))$
    if $Area(DOD(p_i)) \leq threshold$ **then**
        $SafePos[p_i] \leftarrow Area(DOD(p_i))$
    end if
end for
if $SafePos[] = \emptyset$ **then** ▷ No safe enough position
    return $\arg\min(DODAt[])$ ▷ Try to shrink DOD
else
    for all $p_i \in SafePos$ **do**
        $FoodGainAt[p_i] \leftarrow FoodGain(p, p_i)$
    end for
    if $\max(FoodGainAt[]) > 0$ **then**
        return $\arg\max(FoodGainAt[])$
    else
        $f_{Near} \leftarrow NearestFoodTo(p, F_{DOD_p})$
        if $f_{Near} = \emptyset$ **then** ▷ No food left in DOD
            return $\arg\min(DODAt[])$
        else
            return $NearestPosToFood(SafePos[], f_{Near})$
        end if
    end if
end if

V. RESULTS AND DISCUSSION

To investigate the performance of all these movement rules, we ran 200 simulations using the same set of randomly generated initial layouts of agents. The size of the food region is 100 by 100. Agents have a speed of 1, foraging radius of 5, and radius of DOD of 100. Each run will stop when the 500th step is reached or when the termination condition is met. We use different termination conditions for different movement rules. For movement rules A and B, once all food nodes are gathered, agents start to move randomly. The run is terminated when this happens. For movement rule C we terminate the simulation when there is no food in the region and when the team DOD stops changing for 20 steps. When the run is terminated before 500 steps we assume that the team DOD stays the same from termination until the 500th step. For movement rule D, we set the safety threshold for DOD as $\frac{1}{2} \times 100^2\pi$, which is half the size of the full DOD circle. No termination condition is enforced for movement rule D.

At each time step, the percentage of the food left with respect to the initial amount of food nodes at the beginning of each run is logged. Also, the ratio of the size of the team DOD to the maximum possible size of the team DOD is logged. The team DOD is at its maximum when the DOD of each agent is a full circle with a radius of 100. We also compute an ideal optimal foraging rate as a reference point for the foraging performance of the team in different movement rules. The rate is calculated by assuming that at every step each agent gathers the maximum amount of food possible according to the size of its foraging region. For example, an agent with a foraging radius of 5 and a speed of 1 can cover an area of $(5 \times 2) \times 1 = 10$. Assuming this area is full of food nodes, we can calculate the ideal optimal foraging rate with a given food density. This ideal optimal rate can only be achieved in some very special cases but nevertheless serves as a good upper bound of the foraging performance.

A. Foraging Performance of Different Movement Rules

From Fig. 2 we can see that all four movement rules give similar performance in foraging efficiency. The offset of the initial food left from 100% is due to the fact that at step zero, agents gather all the food without even moving. In the first 50 steps, all four movement rules have almost the same efficiency as the ideal optimal foraging rate. For movement rules A, B, and C, this is because when the field is still densely populated with food nodes, agents are likely to still have some candidate position that will give the ideal optimal food gain. It is the same for movement rule D but agents might not be able to move to these positions due to constraints on DOD size. At the beginning of each run agents are more likely to have a DOD bigger than the threshold, so agents will try to gather together to shrink their DOD. However, food nodes in the field are still abundant at this stage, so agents can still have a very high foraging rate even if they are not trying to gather the most food they can.

After the first 50 steps, we can see that the foraging efficiency of all four movement rules start to degrade gradually. The difference from the ideal optimal rate starts to get more noticeable around step 100. At this stage there are only about 60% of the food nodes left in the field. For most of the agents, there are now no candidate positions that can provide the ideal optimal foraging rate. The four different movement rules still perform fairly similarly.

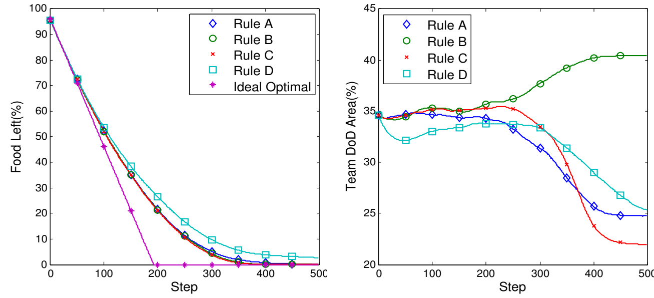
When there is less food left in the field, movement rule D performs worse than other rules. This is due to the fact that once agents gather into a packed group, the threshold on DOD will prevent them from moving too far away from the group. This hampers the ability of the team to gather food nodes near the boundary of the region of interest.

From step 300 to the end, we can see that movement rules B and D perform very similarly and are slightly better than movement rule A. At this stage, there are usually few left-over food nodes distributed sparsely in the region. Movement rule A doesn't have a de-conflict mechanism, so multiple agents will often move toward the same food node and hence converge into a small pack; when that food node is collected by one of the agents, this small pack of agents will again aim for the same food node that is closest to all of them. The lack of de-conflict mechanism greatly impacts the foraging efficiency when food nodes are sparse.

B. Domain of Danger Performance

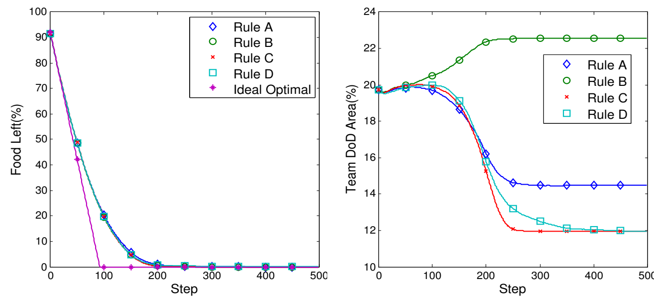
Fig.2(b) shows how each movement rule performs in the domain of danger aspect. Since we use the same set of 200 layouts for all movement rules, they have exactly the same team DOD in the beginning.

Movement rule A and B do not take DOD into account, yet they display fairly constant trends in team DOD. The team DOD for movement rule A stays around the same level for the first 200 steps and then drops gradually after that. As mentioned in section V-A, when food nodes are sparse agents using movement rule A tend to gather into a small pack, which results in a small team DOD. On the other hand, agents using movement rule B tend to spread out due to the



(a) Average foraging performance (b) Average team DOD performance

Fig. 2. Average performance of different movement rules over 200 runs with 5 agents.



(a) Average foraging performance (b) Average team DOD performance

Fig. 3. Average performance of different movement rules over 200 runs with 10 agents.

de-conflict mechanism when the food distribution becomes sparser. So the team DOD rises significantly toward the end of each run.

Movement rule C has a similar performance in team DOD as movement rule B until the 200th step. This is predictable in that agents using these 2 movement rules act exactly the same when there are still food nodes within the agents' DOD. When the food nodes become sparse, agents using rule B will spread and the team DOD will grow. There is no mechanism to shrink the team DOD. Agents using movement rule C will also spread due to the de-conflict mechanism, but when an agent has no food within its DOD, it tries to shrink it. This counteracts the spread and result in the shrinkage of team DOD. The fact that these 2 movement rules have almost identical foraging efficiency indicates that trying to shrink the DOD when there is no food to be gained does not degrade the foraging efficiency and helps the team tremendously in maintaining a smaller DOD.

For movement rule D, the team DOD drops in the first 50 steps because most agents will have a DOD above the safety threshold initially and are actively trying to shrink it. As mentioned in section V-A, this makes food nodes on the boundary hard to gather. With a lowered team foraging efficiency, fewer agents will have empty DODs and be in DOD shrinking mode. As a result the performance in team DOD of this rule is lower than that of rule C.

C. Scaling of performance

Another 200 runs of simulation with 10 agents are performed with the same set of parameters to investigate the scaling of performance. The result is shown in Fig. 3. With 10 agents, the ideal foraging rate is doubled and the finishing time is cut in half. Fig. 3 (a) and Fig. 2 (a) show the same characteristic. All movement rules perform closely to the ideal case in the beginning, and then the foraging performance degrades gradually. What is slightly different from the 5-agent case is that the foraging performance of movement rule D is no longer significantly different from other rules. It can also be observed from Fig. 3 (b) that the dip of team DOD in the beginning of the simulation is not that noticeable anymore. The drop toward the end is faster than that of rule A. This indicates that with double the amount of agents but still the same region, the initial layout of the team no longer placed that many agents above the safety threshold. Agents using movement rule D can start gathering food greedily right away. Also even toward the end of each run when the food nodes are sparse, agents can still gather food without suffering too much from the safety constraint.

From Fig. 3 (b) we can also see that the performance data on different movement rules are smoother, which indicates the performances are more consistent throughout the 200 runs. Teams using movement rule B still spread out when the food is sparse, while the performance of other rules are very similar, especially before 200th step. When the agent-to-area ratio is high, agents will have food nodes in their DOD most of the time, where the behavior from rules A, C, and D are similar. Also, in this situation rule A although lacking a de-conflict mechanism, doesn't suffer much in foraging efficiency.

VI. CONCLUSIONS

In this paper we proposed and investigated several movement rules inspired by the DOD concept and the producer-scrounger game in animal behavior studies. DOD is used by ethologists to measure and model the predation risk taken by an individual within a group. The producer-scrounger game focuses on two different tactics used by group-living animals in group foraging situations. Producers search for food by themselves, and scroungers follow producers to share their findings.

This paper formulates a problem where a team of agents with perfect information have to gather all the food within the region of interest and also keep the team DOD small. Four movement rules are proposed and their performances in foraging and team DOD are investigated. All four movement rules have similar foraging efficiency when the food is still abundant in the region and the performances gradually degrade as the food becomes sparser. Most of the time the team takes twice as long as the ideal optimal case to finish collecting all the food nodes, except rule D. The constraint on size of DOD degrades the foraging performance significantly when food nodes are sparse. In most runs with 5 agents, movement rule D is not capable of collecting all the food nodes in time. When the agent-to-food ratio is higher, the difference in foraging performance and team DOD of different movement rules becomes smaller. A Voronoi-based de-conflict mechanism can indeed enhance the foraging performance of the team, but without a counteract mechanism the team DOD performance will suffer greatly when food is sparse. Trying to shrink the DOD while there is no food to be gained in the near future doesn't degrade the foraging performance and also results in good performance on team domain of danger.

VII. FUTURE WORK

The authors are currently focusing on formalizing the DOD-based risk avoidance behavior in an optimization framework as a gradient descent algorithm. They are also developing a modified domain of danger definition to take into account additional factors such as vigilance, cues from predators, and obstacles, to further enhance the bio-inspired risk avoidance algorithm.

REFERENCES

[1] F. Bullo, J. Cortés, and S. Martínez, *Distributed Control of Robotic Networks*, ser. Applied Mathematics Series. Princeton University Press, 2009, electronically available at http://coordinationbook.info.

[2] H. Choset, "Coverage for robotics a survey of recent results," *Annals of Mathematics and Artificial Intelligence*, vol. 31, no. 1, pp. 113–126, Oct. 2001.

[3] I. Hussein and D. Stipanovic, "Effective coverage control for mobile sensor networks with guaranteed collision avoidance," *Control Systems Technology, IEEE Transactions on*, vol. 15, no. 4, pp. 642–657, 2007.

[4] A. Haumann, K. Listmann, and V. Willert, "DisCoverage: a new paradigm for multi-robot exploration," in *2010 IEEE International Conference on Robotics and Automation (ICRA)*, 2010, pp. 929–934.

[5] B. Yamauchi, "A frontier-based approach for autonomous exploration," in *Proceedings of the 1997 IEEE International Symposium on Computational Intelligence in Robotics and Automation*, 1997, p. 146.

[6] W. D. Hamilton, "Geometry for the selfish herd," *Journal of Theoretical Biology*, vol. 31, no. 2, pp. 295–311, 1971.

[7] R. James, P. G. Bennett, and J. Krause, "Geometry for mutualistic and selfish herds: the limited domain of danger," *Journal of Theoretical Biology*, vol. 228, no. 1, pp. 107–113, 2004.

[8] R. Ydenberg, L. Dill, and C. B. J. S. Rosenblatt, "The economics of fleeing from predators." Academic Press, 1986, vol. Volume 16, pp. 229–249.

[9] S. V. Viscido, M. Miller, and D. S. Wethey, "The dilemma of the selfish herd: The search for a realistic movement rule," *Journal of Theoretical Biology*, vol. 217, no. 2, pp. 183–194, 2002.

[10] S. V. Viscido, M. Miller, and D. Wethey, "The response of a selfish herd to an attack from outside the group perimeter," *Journal of Theoretical Biology*, vol. 208, no. 3, pp. 315–328, Feb. 2001.

[11] S. V. Viscido and D. S. Wethey, "Quantitative analysis of fiddler crab flock movement: evidence for 'selfish herd' behaviour," *Animal behaviour*, vol. 63, no. 4, pp. 735–741, 2002.

[12] C. Barnard and R. Sibly, "Producers and scroungers: A general model and its application to captive flocks of house sparrows," *Animal Behaviour*, vol. 29, no. 2, pp. 543–550, May 1981.

[13] L. A. Giraldeau and G. Beauchamp, "Food exploitation: searching for the optimal joining policy," *Trends in Ecology & Evolution*, vol. 14, no. 3, pp. 102–106, 1999.

[14] R. E. Flynn and L. Giraldeau, "Producer-Scrounger games in a spatially explicit world: Tactic use influences flock geometry of spice finches," *Ethology*, vol. 107, no. 3, pp. 249–257, 2001.

[15] W. L. Vickery, L. Giraldeau, J. J. Templeton, D. L. Kramer, and C. A. Chapman, "Producers, scroungers, and group foraging," *The American Naturalist*, vol. 137, no. 6, pp. 847–863, Jun. 1991.

[16] G. Beauchamp and L. Giraldeau, "Group foraging revisited: Information sharing or Producer-Scrounger game?" *The American Naturalist*, vol. 148, no. 4, pp. 738–743, Oct. 1996.