2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

A Derivative-Free Output Feedback Adaptive Control Architecture for Minimum Phase Dynamical Systems with Unmatched Uncertainties

Tansel Yucelen and Wassim M. Haddad

Abstract— **In this paper, we extend the model reference adaptive control architecture for minimum phase dynamical systems developed in [1] by constructing derivative-free adaptive update laws predicated on current information of the system states and system errors as well as delayed information of the update gains. The advantage of the proposed derivative-free adaptive control law architecture over adaptive update laws predicated on differentiation requiring an intermediate discretization step for implementation is that the former architecture can account for abrupt changes in system dynamics due to system faults or major variation in system parameters. In addition, the derivative-free architecture subverts high-gain feedback which can excite unmodeled system dynamics. Two illustrative numerical examples are given to demonstrate the efficacy of the proposed approach.**

I. Introduction

The purpose of feedback control is to achieve desirable system performance in the face of system uncertainty. Although system identification for estimating system parameters can reduce system uncertainty to some extent, residual modeling discrepancies of system parameters always remains. Controllers must therefore achieve desired disturbance rejection and/or tracking performance requirements in the face of such modeling uncertainty. For systems where the system model does not adequately capture the physical system due to idealized assumptions, model simplification, and model parameter uncertainty, adaptive control methods can be used to achieve system performance without excessive reliance on system models.

In a recent paper [1], we developed an output feedback adaptive control framework for continuous-time, minimum phase multivariable systems for output stabilization and command following. Specifically, a direct adaptive controller for a nonminimal state space model is constructed using the expanded states of the nonminimal realization and is shown to be effective for multi-input, multi-output minimum phase systems with unmatched uncertainties and unstable dynamics.

In this paper, we extend the model reference adaptive control architecture developed in [1] by constructing derivative-free adaptive update laws predicated on current information of the expanded system states and system errors as well as delayed information of the update gains. In particular, asymptotic output stabilization and command following is guaranteed by using a Lyapunov-Krasovskii functional. The advantage of the proposed derivative-free adaptive control law architecture over adaptive update laws predicated on differentiation requiring an intermediate discretization step for implementation is that the former architecture can account for abrupt changes in system dynamics due to system faults or major variation in system parameters. In addition, the derivative-free architecture subverts high-gain feedback which can excite unmodeled system dynamics [2], [3]. Finally, it is important to note that the derivative-free architecture for adaptive control was first proposed in [4]–[6]. Two illustrative numerical examples are provided to demonstrate the efficacy of the proposed approach.

The notation used in this paper is fairly standard. Specifically, $\mathbb{R}^n$ (resp., $\mathbb{C}^n$) denotes the set of $n \times 1$ real (resp., complex) column vectors, $\mathbb{R}^{n\times m}$ (resp., $\mathbb{C}^{n\times m}$) denotes the set of $n \times m$ real (resp., complex) matrices, $(\cdot)^{\mathrm{T}}$ denotes transpose, $(\cdot)^{-1}$ denotes inverse, and $\triangleq$ denotes equal by definition. Furthermore, we write $\lambda_{\min}(A)$ (resp., $\lambda_{\max}(A)$) for the minimum (resp., maximum) eigenvalue of the Hermitian matrix A, $\|\cdot\|_2$ for the Euclidian norm, $\|\cdot\|_{\mathrm{F}}$ for the Frobenius matrix norm, $\mathrm{tr}(\cdot)$ for the trace operator, $\mathrm{id}(A)$ for I_n (resp., $-I_n$) if $A \in \mathbb{R}^{n\times n}$ is positive-definite (resp., negative-definite), and $\mathrm{pd}(A)$ for A (resp., $-A$) if $A \in \mathbb{R}^{n\times n}$ is positive-definite (resp., negative-definite).

II. Nonminimal State Space Realization Formulation

In this section, we present a nonminimal state space realization architecture for continuous-time, linear multivariable uncertain dynamical systems. The nonminimal state space realization involves an expanded system state that consists entirely of the system filtered inputs and filtered outputs and their derivatives, which allows us to cast an output feedback control problem as a full-state feedback problem. Specifically, consider the controllable and observable linear uncertain dynamical system given by

$$\dot{x}_{\mathrm{p}}(t) = A_{\mathrm{p}}x_{\mathrm{p}}(t) + B_{\mathrm{p}}u(t), \quad x_{\mathrm{p}}(0) = x_{\mathrm{p}_0}, \quad t \geq 0, \tag{1}$$

$$y(t) = C_{\mathrm{p}}x_{\mathrm{p}}(t), \tag{2}$$

where $x_{\mathrm{p}}(t) \in \mathbb{R}^n$, $t \geq 0$, is the state vector, $u(t) \in \mathbb{R}^m$, $t \geq 0$, is the control input, $y(t) \in \mathbb{R}^l$, $t \geq 0$, is the system output, and $A_{\mathrm{p}} \in \mathbb{R}^{n\times n}$, $B_{\mathrm{p}} \in \mathbb{R}^{n\times m}$, and $C_{\mathrm{p}} \in \mathbb{R}^{l\times n}$ are *unknown* system matrices. An input-output equivalent nonminimal observer canonical state space model of (1) and (2) for $l > 1$ is given by ([7])

$$\dot{x}_{\mathrm{o}}(t) = A_{\mathrm{o}}x_{\mathrm{o}}(t) + B_{\mathrm{o}}u(t), \quad x_{\mathrm{o}}(0) = x_{\mathrm{o}_0}, \quad t \geq 0, \tag{3}$$

$$y(t) = C_{\mathrm{o}}x_{\mathrm{o}}(t), \tag{4}$$

where $x_{\mathrm{o}}(t) \in \mathbb{R}^{ln}$, $t \geq 0$, is the state vector,

This research was supported in part by the Air Force Office of Scientific Research under Grant FA9550-09-1-0429.

T. Yucelen and W. M. Haddad are with the School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA, 30332-0150, USA. E-mails: tansel@gatech.edu, wm.haddad@aerospace.gatech.edu.

$$A_{\mathrm{o}} = \begin{bmatrix} 0 & I_l & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & I_l \\ -a_0 I_l & -a_1 I_l & \cdots & -a_{n-1} I_l \end{bmatrix} \in \mathbb{R}^{ln\times ln}, \quad (5)$$

$$B_{\mathrm{o}} = \begin{bmatrix} C_{\mathrm{p}} B_{\mathrm{p}} \\ C_{\mathrm{p}} A_{\mathrm{p}} B_{\mathrm{p}} \\ \vdots \\ C_{\mathrm{p}} A_{\mathrm{p}}^{n-1} B_{\mathrm{p}} \end{bmatrix} \in \mathbb{R}^{ln\times m}, \quad (6)$$

and

$$C_{\mathrm{o}} = \begin{bmatrix} I_l & 0 & \cdots & 0 \end{bmatrix} \in \mathbb{R}^{l\times ln}. \quad (7)$$

Note that a_i, $i = 0, 1, \ldots, n-1$, in (5) are the coefficients of the characteristic polynomial of the matrix A_{p} in (1).

Next, let

$$\begin{aligned} \bar{B}_0 &= C_{\mathrm{o}}(a_1 I_{ln} + \cdots + a_{n-1} A_{\mathrm{o}}^{n-2} + A_{\mathrm{o}}^{n-1}) B_{\mathrm{o}}, \quad (8) \\ \bar{B}_1 &= C_{\mathrm{o}}(a_2 I_{ln} + \cdots + a_{n-1} A_{\mathrm{o}}^{n-3} + A_{\mathrm{o}}^{n-2}) B_{\mathrm{o}}, \quad (9) \\ &\vdots \\ \bar{B}_{n-1} &= C_{\mathrm{o}} B_{\mathrm{o}}. \quad (10) \end{aligned}$$

Now, an alternative input-output equivalent nonminimal *controllable* state space realization of (1) and (2) is given by

$$\begin{aligned} \dot{x}_{\mathrm{f}}(t) &= A_{\mathrm{f}} x_{\mathrm{f}}(t) + B_{\mathrm{f}} u(t), \quad x_{\mathrm{f}}(0) = x_{\mathrm{f}_0}, \quad t \geq 0, \quad (11) \\ y(t) &= C_{\mathrm{f}} x_{\mathrm{f}}(t), \quad (12) \end{aligned}$$

where $x_{\mathrm{f}}(t) \in \mathbb{R}^{n_{\mathrm{f}}}$, $t \geq 0$, $n_{\mathrm{f}} \triangleq (m+l)n$, is the *known* filtered expanded state vector given by

$$x_{\mathrm{f}}(t) = \left[q_1^{\mathrm{T}}(t),\ \ldots,\ q_n^{\mathrm{T}}(t),\ v_1^{\mathrm{T}}(t),\ \ldots,\ v_n^{\mathrm{T}}(t)\right]^{\mathrm{T}}, \quad (13)$$

where $q_i(t) \triangleq y_{\mathrm{f}}^{(i-1)}(t)$, $v_i(t) \triangleq u_{\mathrm{f}}^{(i-1)}(t)$, $i = 1, 2, \ldots, n$, $z^{(n)}(t) \triangleq \mathrm{d}^n z(t)/\mathrm{d}t^n$, and where $x_{\mathrm{f}}(t)$ is obtained by filtering $u(t)$ and $y(t)$ through the filter $1/\Lambda(s)$, where

$$\Lambda(s) = (s+\lambda)^n = s^n + n\lambda s^{n-1} + \cdots + \lambda^n, \quad (14)$$

is a monic Hurwitz polynomial of degree n with $\lambda > 0$,

$$A_{\mathrm{f}} = \begin{bmatrix} 0 & I_l & 0 & \cdots & \cdots & & \cdots & 0 \\ \vdots & & \ddots & \cdots & \cdots & & & \vdots \\ 0 & \cdots & 0 & I_l & 0 & \cdots & \cdots & 0 \\ -a_0 I_l & \cdots & \cdots & -a_{n-1} I_l & \bar{B}_0 & \cdots & \cdots & \bar{B}_{n-1} \\ 0 & \cdots & & \cdots & 0 & I_m & 0 & 0 \\ \vdots & & & \cdots & \cdots & \ddots & \vdots & \\ \vdots & & & \cdots & \cdots & \cdots & 0 & I_m \\ 0 & \cdots & \cdots & 0 & -\lambda^n I_m & \cdots & \cdots & -n\lambda I_m \end{bmatrix} \in \mathbb{R}^{n_{\mathrm{f}}\times n_{\mathrm{f}}}, \quad (15)$$

$$B_{\mathrm{f}} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ I_m \end{bmatrix} \in \mathbb{R}^{n_{\mathrm{f}}\times m}, \quad (16)$$

and

$$C_{\mathrm{f}} = \begin{bmatrix} -a_0 I_l + \lambda^n I_l & \cdots & \cdots & -a_{n-1} I_l + n\lambda I_l \\ \bar{B}_0 & \cdots & \cdots & \bar{B}_{n-1} \end{bmatrix} \in \mathbb{R}^{l\times n_{\mathrm{f}}}. \quad (17)$$

Theorem 2.1 ([8]). System (1) and (2) is input-output equivalent to system (11) and (12).

Remark 2.1. The proof of Theorem 2.1 presents a construction of a nonminimal, albeit controllable, state space realization of (1) and (2) involving the expanded state $x_{\mathrm{f}}(t)$, $t \geq 0$, comprising of filtered versions of the inputs and outputs and their derivatives of the original system, without requiring differentiation of the actual input and output signals. It is important to note here that even though the original system is *unknown*, the expanded state vector $x_{\mathrm{f}}(t)$, $t \geq 0$, is *known*.

Remark 2.2. Since the *controllable* nonminimal state space realization of (11) and (12) is defined by a state that consists entirely of filtered inputs and outputs and their derivatives of the original system, an output feedback stabilization problem for (1) and (2) can be converted into a full-state feedback control design problem by equivalently considering (11) and (12). Furthermore, for an output feedback control design of the form (1) and (2) we typically require that $(A_{\mathrm{p}}, B_{\mathrm{p}})$ be controllable (or stabilizable) and $(A_{\mathrm{p}}, C_{\mathrm{p}})$ be observable (or detectable). In contrast, for a feedback control design using the input-output equivalent nonminimal state space model (11) and (12) we only require controllability of the pair $(A_{\mathrm{f}}, B_{\mathrm{f}})$, which is automatic. Finally, it is important to note that only the system matrix A_{f} in (11) is *partially unknown* for full-state feedback control design, whereas the triple $(A_{\mathrm{p}}, B_{\mathrm{p}}, C_{\mathrm{p}})$ is *unknown* in (1) and (2) for an output feedback control design.

Remark 2.3. Nonminimal state space realizations for discrete-time adaptive control have been extensively developed in the literature, see [9]–[11] and the references therein. The proposed nonminimal state space realization for continuous-time adaptive control developed in this section was first used in [12], [13] for active noise blocking and robust control and [8] for adaptive control.

III. Adaptive Control for the Nonminimal State Space Model

In this section, we introduce a direct adaptive state feedback control architecture for the nonminimal state space model (11) and (12) that guarantees adaptive output stabilization for the original system (1) and (2), as well as boundedness of the original system state $x_{\mathrm{p}}(t)$, $t \geq 0$.

Assumption 3.1. The system given by (1) and (2) is minimum phase and the smallest positive integer i such that the ith Markov parameter of (1) and (2) given by $C_{\mathrm{p}} A_{\mathrm{p}}^{i-1} B_{\mathrm{p}}$ is nonzero and known.

Letting d denote the smallest positive integer i in Assumption 3.1, it follows from (8)–(10) that

$$\begin{aligned} \bar{B}_{n-1} &= C_{\mathrm{o}} B_{\mathrm{o}} = C_{\mathrm{p}} B_{\mathrm{p}} = 0, \quad (18) \\ \bar{B}_{n-2} &= C_{\mathrm{o}}(a_1 I_{ln} + A_{\mathrm{o}}) B_{\mathrm{o}} \\ &= a_1 C_{\mathrm{p}} B_{\mathrm{p}} + C_{\mathrm{p}} A_{\mathrm{p}} B_{\mathrm{p}} = 0, \quad (19) \\ &\vdots \end{aligned}$$

$$\bar{B}_{n-d+1} = 0, \tag{20}$$
$$\bar{B}_{n-d} = C_{\rm p}A_{\rm p}^{d-1}B_{\rm p} \neq 0, \tag{21}$$

where (21) is the first nonzero Markov parameter of the original system (1) and (2).

Assumption 3.2. The first nonzero Markov parameter given by (21) can be parameterized as

$$C_{\rm p}A_{\rm p}^{d-1}B_{\rm p} \quad = \quad \bar{B}\Lambda, \tag{22}$$

where $\bar{B} \in \mathbb{R}^{l\times m}$ is a *known* matrix and $\Lambda \in \mathbb{R}^{m\times m}$ is an *unknown* matrix given by

$$\Lambda \quad = \quad \text{block}-\text{diag}[\Lambda_{m_1}, \ldots, \Lambda_{m_s}], \tag{23}$$

where $\Lambda_{m_1} \in \mathbb{R}^{m_1\times m_1}$, ..., $\Lambda_{m_s} \in \mathbb{R}^{m_s\times m_s}$, and $m_1 + \cdots + m_s = m$. Furthermore, for each $i \in \{1, \ldots, s\}$, Λ_{m_i} is either positive definite or negative definite.

Note that it follows from Assumption 3.2 that Λ given by (23) can be written as $\Lambda = \mathrm{id}(\Lambda)\mathrm{pd}(\Lambda)$, where $\mathrm{id}(\Lambda) \triangleq \text{block}-\text{diag}[\mathrm{id}(\Lambda_{m_1}), \ldots, \mathrm{id}(\Lambda_{m_s})]$ is known and $\mathrm{pd}(\Lambda) \triangleq \text{block}-\text{diag}\ [\mathrm{pd}(\Lambda_{m_1}), \ldots, \mathrm{pd}(\Lambda_{m_s})]$ is unknown and positive-definite. For single-input, single-output dynamical systems without loss in generality letting $\bar{B} = 1$ in (22) gives $\Lambda = \mathrm{id}(C_{\rm p}A_{\rm p}^{d-1}B_{\rm p})\mathrm{pd}(C_{\rm p}A_{\rm p}^{d-1}B_{\rm p}) = \mathrm{sgn}(C_{\rm p}A_{\rm p}^{d-1}B_{\rm p})|C_{\rm p}A_{\rm p}^{d-1}B_{\rm p}|$, where $\mathrm{sgn}(y) \triangleq y/|y|$, $y \neq 0$, and $\mathrm{sgn}(0) \triangleq 0$. In this case, Assumption 3.2 implies that the sign of the first nonzero Markov parameter denoted by $\mathrm{id}(C_{\rm p}A_{\rm p}^{d-1}B_{\rm p})$ is known.

Next, consider the nonminimal state space model (11), where the *known* state vector $x_{\rm f}(t)$, $t \geq 0$, is given by (13), the *partially unknown* matrix $A_{\rm f}$ is given by (15), and the *known* input matrix $B_{\rm f}$ is given by (16), and note that (11) can be equivalently written as

$$\dot{q}(t) \;=\; A_0q(t) + B_0v_0(t) + B_1\Lambda\phi(t),\; q(0) = q_0,\; t \geq 0, \tag{24}$$
$$\dot{v}(t) \;=\; A_vv(t) + B_vu(t),\; v(0) = v_0, \tag{25}$$

where $q(t) \triangleq [q_1^{\rm T}(t),\ \ldots,\ q_n^{\rm T}(t)]^{\rm T} \in \mathbb{R}^{ln}$, $v_0(t) \triangleq [v_1^{\rm T}(t),\ \ldots,\ v_{n-d}^{\rm T}(t)]^{\rm T} \in \mathbb{R}^{m(n-d)}$, $\phi(t) \triangleq v_{n-d+1}(t) \in \mathbb{R}^m$, $v(t) \triangleq [v_1^{\rm T}(t),\ \ldots,\ v_n^{\rm T}(t)]^{\rm T} \in \mathbb{R}^{mn}$,

$$A_0 \triangleq \begin{bmatrix} 0 & I_l & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & I_l \\ -a_0I_l & -a_1I_l & \cdots & -a_{n-1}I_l \end{bmatrix} \in \mathbb{R}^{ln\times ln}, \tag{26}$$

$$B_0 \triangleq \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \\ \bar{B}_{\rm o} & \cdots & \bar{B}_{n-d-1} \end{bmatrix} \in \mathbb{R}^{ln\times m(n-d)}, \tag{27}$$

$$B_1 \triangleq \begin{bmatrix} 0 & \cdots & 0 & \bar{B}^{\rm T} \end{bmatrix}^{\rm T} \in \mathbb{R}^{ln\times m}, \tag{28}$$

$$A_v \triangleq \begin{bmatrix} 0 & I_m & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & I_m \\ -\zeta_1I_m & \cdots & \cdots & -\zeta_nI_m \end{bmatrix} \in \mathbb{R}^{mn\times mn}, \tag{29}$$

and

$$B_v \triangleq \begin{bmatrix} 0 & \cdots & 0 & I_m \end{bmatrix}^{\rm T} \in \mathbb{R}^{mn\times m}, \tag{30}$$

where $\zeta_1 \triangleq \lambda^n$, ..., $\zeta_n \triangleq n\lambda$. Note that A_0, B_0, and Λ in (24) are unknown, and hence, the dynamics in (24) are unknown, whereas the dynamics in (25) are completely known with A_v being Hurwitz. Hence, we use a two-stage design framework wherein we first design a *virtual* control signal $\phi(t)$, $t \geq 0$, that stabilizes the unknown dynamics in (24), and then design the *actual* control signal $u(t)$, $t \geq 0$, using the known dynamics in (25). The existence of such a virtual control signal $\phi(t)$, $t \geq 0$, is guaranteed under the following assumption.

Assumption 3.3. There exists $K_q \in \mathbb{R}^{ln\times m}$ and $K_v \in \mathbb{R}^{m(n-d)\times m}$ such that $A_{\rm m} \triangleq A_0 + B_1\Lambda K_q^{\rm T}$ is Hurwitz and $B_0 = B_1\Lambda K_v^{\rm T}$ holds.

Remark 3.1. It is important to note that if (1) and (2) is *square* (i.e., $m = l$) and $\bar{B}$ is nonsingular, then Assumption 3.3 is automatically satisfied.

Next, we write (24) as

$$\begin{aligned} \dot{q}(t) \;&=\; A_{\rm m}q(t) - B_1\Lambda K_q^{\rm T}q(t) + B_0v_0(t) + B_1\Lambda\phi(t) \\ &=\; A_{\rm m}q(t) - B_1\Lambda\tilde{K}_q^{\rm T}(t)q(t) + B_1\Lambda\tilde{K}_v^{\rm T}(t)v_0(t) \\ &\quad +B_1\Lambda\big[\phi(t) - \hat{K}_q^{\rm T}(t)q(t) + \hat{K}_v^{\rm T}(t)v_0(t)\big] \\ &=\; A_{\rm m}q(t) + B_1\Lambda\tilde{W}^{\rm T}(t)q_w(t) + B_1\Lambda\big[\phi(t) \\ &\quad +\hat{W}^{\rm T}(t)q_w(t)\big], \quad q(0) = q_0, \quad t \geq 0, \end{aligned} \tag{31}$$

where $\tilde{K}_q(t) \triangleq K_q - \hat{K}_q(t) \in \mathbb{R}^{ln\times m}$, $t \geq 0$, $\hat{K}_q(t) \in \mathbb{R}^{ln\times m}$, $t \geq 0$, $\tilde{K}_v(t) \triangleq K_v - \hat{K}_v(t) \in \mathbb{R}^{m(n-d)\times m}$, $t \geq 0$, $\hat{K}_v(t) \in \mathbb{R}^{m(n-d)\times m}$, $t \geq 0$, $q_w(t) \triangleq \big[-q^{\rm T}(t),\ v_0^{\rm T}(t)\big]^{\rm T} \in \mathbb{R}^{\tilde{n}}$, $t \geq 0$, $\tilde{n} \triangleq m(n-d) + ln$, $\tilde{W}(t) \triangleq W - \hat{W}(t) \in \mathbb{R}^{\tilde{n}\times m}$, $t \geq 0$, $W \triangleq \big[K_q^{\rm T},\ K_v^{\rm T}\big]^{\rm T} \in \mathbb{R}^{\tilde{n}\times m}$, and $\hat{W}(t) \triangleq \big[\hat{K}_q^{\rm T}(t),\ \hat{K}_v^{\rm T}(t)\big]^{\rm T} \in \mathbb{R}^{\tilde{n}\times n}$, $t \geq 0$, satisfies the derivative-free update law

$$\begin{aligned} \hat{W}(t) \;&=\; \hat{W}(t-\tau) + \kappa q_w(t)q^{\rm T}(t)P_{\rm m}B_1\mathrm{id}(\Lambda), \\ &\qquad \hat{W}(0) = \hat{W}_0, \quad t \geq 0, \end{aligned} \tag{32}$$

where $\tau > 0$, $\kappa > 0$, and $P_{\rm m}$ is a positive-definite solution of the Lyapunov equation

$$0 \quad = \quad A_{\rm m}^{\rm T}P_{\rm m} + P_{\rm m}A_{\rm m} + R_{\rm m}, \tag{33}$$

where $R_{\rm m} \in \mathbb{R}^{ln\times ln}$ is a symmetric positive-definite matrix. Note that since $A_{\rm m}$ is Hurwitz, it follows from converse Lyapunov theory [14] that there exists a unique symmetric positive-definite matrix $P_{\rm m}$ satisfying (33) for a given symmetric positive definite matrix $R_{\rm m}$.

Proposition 3.1. Consider the uncertain dynamical system given by (24) and the virtual control signal

$$\phi(t) = -\hat{W}^{\rm T}(t)q_w(t), \quad t \geq 0, \tag{34}$$

with derivative-free update law (32), and assume that Assumptions 3.1, 3.2, and 3.3 hold. Then, the solution $(q(t),\ \hat{W}(t))$ of the system (31) and (32) is Lyapunov stable for all $(q_0,\ \hat{W}_0,) \in \mathbb{R}^{ln} \times \mathbb{R}^{\tilde{n}\times m}$ and $t \geq 0$, and $q(t) \to 0$ as $t \to \infty$.

Proof. Using (32), $\tilde{W}(t)$, $t \geq 0$, is given by

$$\tilde{W}(t) \quad = \quad \tilde{W}(t-\tau) - \kappa q_w(t)q^{\rm T}(t)P_{\rm m}B_1\mathrm{id}(\Lambda). \tag{35}$$

Now, using (34) and (35), (31) can be rewritten as

$$\begin{aligned} \dot{q}(t) \;&=\; A_{\rm m}q(t) + B_1\Lambda\big[\tilde{W}(t-\tau) - \kappa q_w(t)q^{\rm T}(t)P_{\rm m}B_1 \\ &\quad \times\mathrm{id}(\Lambda)\big]^{\rm T}q_w(t), \quad q(0) = q_0, \quad t \geq 0. \end{aligned} \tag{36}$$

Next, consider the Lyapunov-Krasovskii functional candidate

$$V(q,\tilde{W}) = q^{\mathrm{T}}P_{\mathrm{m}}q+\rho\mathrm{tr}\int_{-\tau}^{0}\tilde{W}^{\mathrm{T}}(\sigma)\tilde{W}(\sigma)\mathrm{d}\sigma\mathrm{pd}(\Lambda), \tag{37}$$

where $P_{\mathrm{m}} > 0$ satisfies (33). Note that (37) satisfies $\hat{\alpha}(\|\varphi\|)\leq V(\varphi)\leq\hat{\beta}(\|\varphi\|)$, where $\varphi\triangleq[q^{\mathrm{T}},\ \bar{w}^{\mathrm{T}}]^{\mathrm{T}}$, with $\bar{w}^{\mathrm{T}}(t)\bar{w}(t)\triangleq\mathrm{tr}\int_{-\tau}^{0}\tilde{W}^{\mathrm{T}}(\sigma)\tilde{W}(\sigma)\mathrm{d}\sigma\mathrm{pd}(\Lambda)$, and $\hat{\alpha}(\|\varphi\|)=\hat{\beta}(\|\varphi\|)=\|\varphi\|^2$, with $\|\varphi\|^2\triangleq e^{\mathrm{T}}P_{\mathrm{m}}e+\rho\bar{w}^{\mathrm{T}}(t)\bar{w}(t)$. Furthermore, note that $\hat{\alpha}(\cdot)$ is a class $\mathcal{K}_\infty$ function.

Differentiating (37) along the trajectories of (35) and (36) yields

$$\begin{aligned}
&\dot{V}(q(t),\tilde{W}_t)\\
&\quad= -q^{\mathrm{T}}(t)P_{\mathrm{m}}q(t)+2q^{\mathrm{T}}(t)P_{\mathrm{m}}B_1\Lambda\big[\tilde{W}(t-\tau)\\
&\qquad -\kappa q_w(t)q^{\mathrm{T}}(t)P_{\mathrm{m}}B_1\mathrm{id}(\Lambda)\big]^{\mathrm{T}}q_w(t)+\rho\mathrm{tr}\big[\tilde{W}^{\mathrm{T}}(t)\\
&\qquad \times\tilde{W}(t)-\tilde{W}^{\mathrm{T}}(t-\tau)\tilde{W}(t-\tau)\big]\mathrm{pd}(\Lambda)\\
&\quad= -q^{\mathrm{T}}(t)P_{\mathrm{m}}q(t)+2q^{\mathrm{T}}(t)P_{\mathrm{m}}B_1\Lambda\big[\tilde{W}(t-\tau)\\
&\qquad -\kappa q_w(t)q^{\mathrm{T}}(t)P_{\mathrm{m}}B_1\mathrm{id}(\Lambda)\big]^{\mathrm{T}}q_w(t)\\
&\qquad +\rho\mathrm{tr}\big[\tilde{W}^{\mathrm{T}}(t-\tau)\tilde{W}(t-\tau)-2\kappa\tilde{W}^{\mathrm{T}}(t-\tau)\\
&\qquad \times q_w(t)q^{\mathrm{T}}(t)P_{\mathrm{m}}B_1\mathrm{id}(\Lambda)+\kappa^2\mathrm{id}(\Lambda)B_1^{\mathrm{T}}P_{\mathrm{m}}q(t)\\
&\qquad \times q_w^{\mathrm{T}}(t)q_w(t)q^{\mathrm{T}}(t)P_{\mathrm{m}}B_1\mathrm{id}(\Lambda)-\tilde{W}^{\mathrm{T}}(t-\tau)\\
&\qquad \times\tilde{W}(t-\tau)\big]\mathrm{pd}(\Lambda)\\
&\quad= -q^{\mathrm{T}}(t)P_{\mathrm{m}}q(t)+2q^{\mathrm{T}}(t)P_{\mathrm{m}}B_1\Lambda\tilde{W}^{\mathrm{T}}(t-\tau)\\
&\qquad \times q_w(t)-2\kappa q^{\mathrm{T}}(t)P_{\mathrm{m}}B_1\Lambda\mathrm{id}(\Lambda)B_1^{\mathrm{T}}P_{\mathrm{m}}q(t)\\
&\qquad \times q_w^{\mathrm{T}}(t)q_w(t)+\rho\mathrm{tr}\big[-2\kappa\tilde{W}^{\mathrm{T}}(t-\tau)q_w(t)q^{\mathrm{T}}(t)\\
&\qquad \times P_{\mathrm{m}}B_1\mathrm{id}(\Lambda)+\kappa^2\mathrm{id}(\Lambda)B_1^{\mathrm{T}}P_{\mathrm{m}}\\
&\qquad \times q(t)q_w^{\mathrm{T}}(t)q_w(t)q^{\mathrm{T}}(t)P_{\mathrm{m}}B_1\mathrm{id}(\Lambda)\big]\mathrm{pd}(\Lambda).
\end{aligned} \tag{38}$$

Letting $\kappa=1/\rho>0$ and noting that if $\Lambda>0$ (resp., $\Lambda<0$), then $\Lambda\mathrm{id}(\Lambda)=\Lambda>0$ (resp., $\Lambda\mathrm{id}(\Lambda)=-\Lambda>0$), it follows from (38) that

$$\begin{aligned}
\dot{V}(q(t),\tilde{W}_t) &= -q^{\mathrm{T}}(t)R_{\mathrm{m}}q(t)-\kappa\big[q^{\mathrm{T}}(t)P_{\mathrm{m}}B_1\\
&\quad \times\Lambda\mathrm{id}(\Lambda)B_1^{\mathrm{T}}P_{\mathrm{m}}q(t)\big]q_w^{\mathrm{T}}(t)q_w(t)\\
&\leq -q^{\mathrm{T}}(t)R_{\mathrm{m}}q(t)\\
&\leq 0,\quad t\geq 0.
\end{aligned} \tag{39}$$

Hence, the solution $(q(t),\ \hat{W}(t))$ of the system (31) and (32) is Lyapunov stable for all $(q_0,\ \hat{W}_{0},)\in\mathbb{R}^{ln}\times\mathbb{R}^{\tilde{n}\times m}$ and $t\geq 0$. Now, by the LaSalla-Yoshizawa theorem [14], $\lim_{t\to\infty}q^{\mathrm{T}}(t)R_{\mathrm{m}}q(t)=0$ and, hence, $q(t)\to 0$ as $t\to\infty$. □

Proposition 3.1 shows that the virtual control signal $\phi(t)$, $t\geq 0$, given by (34) ensures that $q(t)\to 0$ as $t\to\infty$. Next, we construct the actual control signal $u(t)$, $t\geq 0$, using the known dynamics in (25). For this case, it follows from (25) that

$$\begin{aligned}
u(t) &= \dot{v}_n(t)+\zeta_{n-1}v_{n-1}(t)+\zeta_{n-2}v_{n-2}(t)+\cdots\\
&\quad +\zeta_{n-d+2}v_{n-d+2}(t)+\zeta_{n-d+1}v_{n-d+1}(t)\\
&\quad +\zeta_{n-d}v_{n-d}(t)+\cdots+\zeta_2v_2(t)+\zeta_1v_1(t),\ t\geq 0.
\end{aligned} \tag{40}$$

Using $\phi(t)$, $t\geq 0$, (40) can be equivalently rewritten as

$$\begin{aligned}
u(t) &= \phi^{(d)}(t)+\zeta_{n-1}\phi^{(d-1)}(t)+\zeta_{n-2}\phi^{(d-2)}(t)+\cdots\\
&\quad +\zeta_{n-d+2}\dot{\phi}(t)+\zeta_{n-d+1}\phi(t)+\zeta_{n-d}\left[\int_0^t\phi(\sigma_1)\mathrm{d}\sigma_1\right]\\
&\quad +\cdots+\zeta_2\left[\int_0^t\cdots\int_0^t\left[\int_0^t\phi(\sigma_1)\mathrm{d}\sigma_1\right]\mathrm{d}\sigma_2\cdots\right.\\
&\quad \left.\times\mathrm{d}\sigma_{n-d-1}\right]+\zeta_1\left[\int_0^t\cdots\int_0^t\left[\int_0^t\phi(\sigma_1)\mathrm{d}\sigma_1\right.\right.\\
&\quad \left.\left.\times\mathrm{d}\sigma_2\cdots\mathrm{d}\sigma_{n-d}\right],\quad t\geq 0.\right.
\end{aligned} \tag{41}$$

The following theorem presents the main result of this section.

Theorem 3.1. Consider the uncertain dynamical system given by (11) and the control signal (41) with (32) and (34), and assume that Assumptions 3.1, 3.2, and 3.3 hold. Then, $x_{\mathrm{p}}(t)$, $t\geq 0$, satisfying (1) is bounded for all $x_{\mathrm{p}}(0)\in\mathbb{R}^n$ and $y(t)\to 0$ as $t\to\infty$.

Proof. It follows from Proposition 3.1 that the solution $(q(t),\ \hat{W}(t))$ of the system (31) and (32) is Lyapunov stable for all $(q_0,\ \hat{W}_{0},)\in\mathbb{R}^{ln}\times\mathbb{R}^{\tilde{n}\times m}$ and $t\geq 0$, and $q(t)\to 0$ as $t\to\infty$. Since the first l components of $q(t)$, $t\geq 0$, correspond to the filtered output of the original system, it follows that $y_{\mathrm{f}}(t)\to 0$ as $t\to\infty$. Now, since the filter given by (14) is asymptotically stable, it follows that $y(t)\to 0$ as $t\to\infty$.

To show that $x_{\mathrm{p}}(t)$, $t\geq 0$, satisfying (1) is bounded, note that since the solution $(q(t),\ \hat{W}(t))$ of the system (31) and (32) is Lyapunov stable for all $(q_0,\ \hat{W}_{0},)\in\mathbb{R}^{ln}\times\mathbb{R}^{\tilde{n}\times m}$ and $t\geq 0$, and $q(t)\to 0$ as $t\to\infty$, it follows from the dynamics in (31) with the virtual control signal defined in (34) that $v_0(t)$, $t\geq 0$, is bounded. In addition, since the first m components of $v_0(t)$, $t\geq 0$, correspond to the filtered input of the original system, it follows that $u_{\mathrm{f}}(t)$, $t\geq 0$, is bounded. Now, since the filter given by (14) is asymptotically stable, it follows that $u(t)$, $t\geq 0$, is bounded. Furthermore, since $u(t)$, $t\geq 0$, is bounded and A_v is Hurwitz, it follows from (25) that $v(t)$, $t\geq 0$, is bounded. Similarly, $\dot{y}(t),\ldots,y^{(n-1)}(t)$, and $\dot{u}(t),\ldots,u^{(n-1)}(t)$, $t\geq 0$, are bounded, and hence, uniformly continuous. Hence, it follows from the minimality of $(A_{\mathrm{p}},B_{\mathrm{p}},C_{\mathrm{p}})$ that $x_{\mathrm{p}}(t)$, $t\geq 0$, is bounded. □

To elucidate the structure of the control architecture (41), consider a second-order, single-input, single-output system with $d=1$. In this case, the actual control signal given by (41) becomes

$$\begin{aligned}
u(t) &= \dot{\phi}(t)+\zeta_2\phi(t)+\zeta_1\int_0^t\phi(\sigma)\mathrm{d}\sigma\\
&= \dot{\phi}(t)+2\lambda\phi(t)+\lambda^2\int_0^t\phi(\sigma)\mathrm{d}\sigma,
\end{aligned} \tag{42}$$

which involves a proportional-integral-derivative control architecture. To further elucidate the controller structure (42), assume that the adaptive gains $\hat{K}_q(t)$, $t\geq 0$, and $\hat{K}_v(t)$, $t\geq 0$, converge to $\hat{K}_{q\infty}=\big[\hat{k}_{q1},\ \hat{k}_{q2}\big]^{\mathrm{T}}$ and $\hat{K}_{v\infty}=\hat{k}_v$, respectively. In this case, using (34) with $q(t)=\big[q_1(t),\ q_2(t)\big]^{\mathrm{T}}=$

$\left[y_{\rm f}(t),\ \dot{y}_{\rm f}(t)\right]^{\rm T}$ and $v_0(t) = v_1(t) = u_{\rm f}(t)$, it follows that

$$u(s) \quad = \quad \frac{\hat{k}_{q2}s + \hat{k}_{q1}}{s + \hat{k}_v} y(s), \tag{43}$$

which involves a lead/lag-type compensator. Note that *unstable pole-zero cancelation* in (43) is precluded by Assumption 3.1 since (1) and (2) is assumed to be minimum phase.

IV. Adaptive Command Following for the Nonminimal State Space Model

In this section, we extend the adaptive control architecture developed in Section 3 to the case of command following. To address system tracking, consider the additional integrator state satisfying

$$\dot{q}_{\rm int}(t) = -y_{\rm f}(t) + r_{\rm f}(t) = -q_1(t) + r_{\rm f}(t),\ t \geq 0, \tag{44}$$

where $r_{\rm f}(t) \in \mathbb{R}^l$, $t \geq 0$, is a filtered (through the filter $\Lambda(s)$ defined by (14)) command of a given bounded piecewise continuous reference command $r(t) \in \mathbb{R}^l$, $t \geq 0$. Now, (24) can be augmented with the integrator state (44) to give

$$\begin{aligned}\dot{q}_{\rm a}(t) \quad &= \quad A_{\rm a0}q_{\rm a}(t) + B_{\rm a0}v_0(t) + B_{\rm a1}\Lambda\phi(t) + B_{\rm am}r_{\rm f}(t), \\ & \qquad q_{\rm a}(0) = q_{\rm a0}, \quad t \geq 0, \end{aligned}\tag{45}$$

where $q_{\rm a}(t) \triangleq [q^{\rm T}(t),\ q_{\rm int}^{\rm T}(t)]^{\rm T} \in \mathbb{R}^{l(n+1)}$,

$$A_{\rm a0} \triangleq \begin{bmatrix} 0 & I_l & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & I_l & \vdots \\ -a_0 I_l & -a_1 I_l & \cdots & -a_{n-1}I_l & 0 \\ -I_l & 0 & \cdots & 0 & 0 \end{bmatrix} \in \mathbb{R}^{l(n+1)\times l(n+1)}, \tag{46}$$

$$B_{\rm a0} \triangleq \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \\ \bar{B}_{\rm o} & \cdots & \bar{B}_{\rm n-d-1} \\ 0 & \cdots & 0 \end{bmatrix} \in \mathbb{R}^{l(n+1)\times m(n-d)}, \tag{47}$$

$$B_{\rm a1} \triangleq \begin{bmatrix} 0 & \cdots & 0 & \bar{B}^{\rm T} & 0 \end{bmatrix}^{\rm T} \in \mathbb{R}^{l(n+1)\times m}, \tag{48}$$

and

$$B_{\rm am} \triangleq \begin{bmatrix} 0 & \cdots & 0 & 0 & I_l \end{bmatrix}^{\rm T} \in \mathbb{R}^{l(n+1)\times m}. \tag{49}$$

Note that in this case (25) remains unchanged. Analogous to Assumption 3.3, we have the following assumption.

Assumption 4.1. There exists $K_{\rm a\mathit{q}} \in \mathbb{R}^{l(n+1)\times m}$ and $K_{\rm av} \in \mathbb{R}^{m(n-d)\times m}$ such that $A_{\rm am} \triangleq A_{\rm a0} + B_{\rm a1}\Lambda K_{\rm a\mathit{q}}^{\rm T}$ is Hurwitz and $B_{\rm a0} = B_{\rm a1}\Lambda K_{\rm av}^{\rm T}$ holds.

Remark 4.1. Once again, note that if (1) and (2) is square (i.e., $m = l$) and $\bar{B}$ is nonsingular, then Assumption 4.1 is automatically satisfied.

Next, consider the reference system given by

$$\dot{q}_{\rm am}(t) \ = \ A_{\rm am}q_{\rm am}(t) + B_{\rm am}r_{\rm f}(t),\ q_{\rm am}(0) = q_{\rm am_0},\ t \geq 0, \tag{50}$$

where $q_{\rm am}(t) \in \mathbb{R}^{l(n+1)}$, $t \geq 0$, is the reference system state vector. Since $A_{\rm am}$ is Hurwitz, it follows from converse Lyapunov theory that there exist a positive-definite matrix $R_{\rm am} \in \mathbb{R}^{l(n+1)\times l(n+1)}$ and a positive-definite matrix $P_{\rm m} \in \mathbb{R}^{l(n+1)\times l(n+1)}$ such that

$$0 \quad = \quad A_{\rm am}^{\rm T}P_{\rm am} + P_{\rm am}A_{\rm am} + R_{\rm am}. \tag{51}$$

Finally, note that since $r(t)$ is bounded for all $t \geq 0$ and the filter given by (14) is asymptotically stable, it follows that $r_{\rm f}(t)$ is bounded for all $t \geq 0$. Furthermore, $q_{\rm am}(t)$ is uniformly bounded for all $q_{\rm am_0} \in \mathbb{R}^{l(n+1)}$ and $t \geq 0$.

Next, define $e(t) \triangleq q_{\rm a}(t) - q_{\rm am}(t)$ and note that it follows from the augmented dynamics (45) and the reference system (50) that

$$\begin{aligned}\dot{e}(t) \ &= \ A_{\rm am}e(t) - B_{\rm a1}\Lambda\tilde{K}_{\rm a\mathit{q}}^{\rm T}(t)q_{\rm a}(t) + B_{\rm a1}\Lambda\tilde{K}_{\rm av}^{\rm T}(t)v_0(t) \\ &\quad + B_{\rm a1}\Lambda\left[\phi(t) - \hat{K}_{\rm a\mathit{q}}^{\rm T}(t)q_{\rm a}(t) + \hat{K}_{\rm av}^{\rm T}(t)v_0(t)\right] \\ &= \ A_{\rm am}e(t) + B_{\rm a1}\Lambda\tilde{W}_{\rm a}^{\rm T}(t)q_{\rm a\mathit{w}}(t) + B_{\rm a1}\Lambda\big[\phi(t) \\ &\quad + \hat{W}_{\rm a}^{\rm T}(t)q_{\rm a\mathit{w}}(t)\big], \quad e(0) = e_0, \quad t \geq 0, \end{aligned}\tag{52}$$

where $\tilde{K}_{\rm a\mathit{q}}(t) \triangleq K_{\rm a\mathit{q}} - \hat{K}_{\rm a\mathit{q}}(t) \in \mathbb{R}^{l(n+1)\times m}$, $t \geq 0$, $\hat{K}_{\rm a\mathit{q}}(t) \in \mathbb{R}^{l(n+1)\times m}$, $t \geq 0$, $\tilde{K}_{\rm av}(t) \triangleq K_{\rm av} - \hat{K}_{\rm av}(t) \in \mathbb{R}^{m(n-d)\times m}$, $t \geq 0$, $\hat{K}_{\rm av}(t) \in \mathbb{R}^{m(n-d)\times m}$, $t \geq 0$, $q_{\rm a\mathit{w}}(t) \triangleq \left[-q_{\rm a}^{\rm T}(t),\ v_0^{\rm T}(t)\right]^{\rm T} \in \mathbb{R}^{\tilde{n}_{\rm a}}$, $t \geq 0$, $\tilde{n}_{\rm a} \triangleq l(m+1) + m(n-d)$, $\tilde{W}_{\rm a}(t) \triangleq W_{\rm a} - \hat{W}_{\rm a}(t) \in \mathbb{R}^{\tilde{n}_{\rm a}\times m}$, $t \geq 0$, $W_{\rm a} \triangleq \left[K_{\rm a\mathit{q}}^{\rm T},\ K_{\rm av}^{\rm T}\right]^{\rm T} \in \mathbb{R}^{\tilde{n}_{\rm a}\times m}$, and $\hat{W}_{\rm a}(t) \triangleq \left[\hat{K}_{\rm a\mathit{q}}^{\rm T}(t),\ \hat{K}_{\rm av}^{\rm T}(t)\right]^{\rm T} \in \mathbb{R}^{\tilde{n}_{\rm a}\times n}$, $t \geq 0$, satisfies the derivative-free update law

$$\begin{aligned}\hat{W}_{\rm a}(t) \quad &= \quad \hat{W}_{\rm a}(t-\tau_{\rm a}) + \kappa_{\rm a}q_{\rm a\mathit{w}}(t)e^{\rm T}(t)P_{\rm am}B_{\rm a1}{\rm id}(\Lambda), \\ & \qquad \hat{W}_{\rm a}(0) = \hat{W}_{\rm a0}, \quad t \geq 0, \end{aligned}\tag{53}$$

where $\tau_{\rm a} > 0$ and $\kappa_{\rm a} > 0$.

Theorem 4.1. Consider the uncertain dynamical system given by (11) and the control signal (41) with

$$\phi(t) = -\hat{W}_{\rm a}^{\rm T}(t)q_{\rm a\mathit{w}}(t), \quad t \geq 0, \tag{54}$$

and with derivative-free update law (53), and assume that Assumptions 3.1, 3.2, and 4.1 hold. Then, the solution $(e(t),\ \hat{W}_{\rm a}(t))$ to (52) and (53) is Lyapunov stable for all $(e_0,\ \hat{W}_{\rm a0}) \in \mathbb{R}^{l(n+1)} \times \mathbb{R}^{\tilde{n}_{\rm a}\times m}$ and $t \geq 0$, and $e(t) \to 0$ as $t \to \infty$. Furthermore, $x_{\rm p}(t)$, $t \geq 0$, satisfying (1) is bounded for all $x_{\rm p}(0) \in \mathbb{R}^n$.

Proof. The proof is similar to the proofs of Proposition 3.1 and Theorem 3.1 with the Lyapunov-Krasovskii functional given by

$$V(q, \tilde{W}) \quad = \quad e^{\rm T}P_{\rm am}e + \rho{\rm tr}\int_{-\tau_{\rm a}}^{0}\tilde{W}_{\rm a}^{\rm T}(\sigma)\tilde{W}_{\rm a}(\sigma){\rm d}\sigma{\rm pd}(\Lambda), \tag{55}$$

where $P_{\rm am} > 0$ satisfies (51). □

Remark 4.2. Theorem 4.1 shows that $x_{\rm p}(t)$, $t \geq 0$, is bounded and $q_{\rm a}(t) \to q_{\rm am}(t)$ as $t \to \infty$. Since the first l components of $q_{\rm a}(t)$, $t \geq 0$, correspond to the filtered output of the original system $y_{\rm f}(t)$, $t \geq 0$, we can always choose an appropriate reference system for (50) that captures a desired tracking behavior for $y_{\rm f}(t)$, $t \geq 0$. Hence, Theorem 4.1 guarantees adaptive command following for the original uncertain dynamical system (1) and (2), as well as boundedness of the original system state $x_{\rm p}(t)$, $t \geq 0$.

V. Illustrative Numerical Examples

In this section, we present two numerical examples to illustrate the efficacy of the proposed adaptive control architectures for adaptive output stabilization and command following.

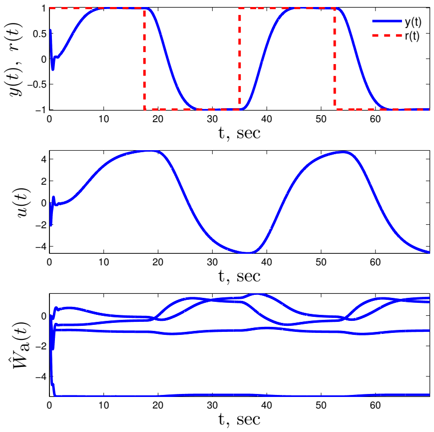

Fig. 1. Closed-loop response of the unstable plant in Example 5.1. The adaptive controller (41) with (54) and (53) with $\kappa_{\mathrm{a}} = 1$ and $\tau_{\mathrm{a}} = 0.01$ tracks the reference $r(t)$.

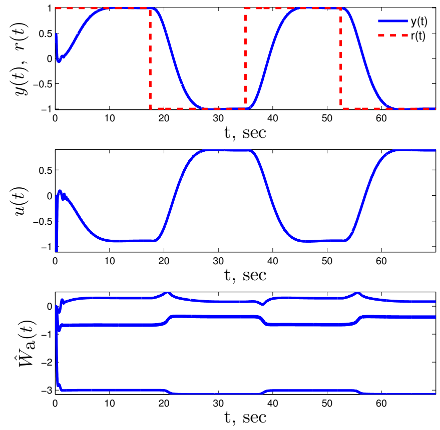

Fig. 2. Closed-loop response of the unstable plant in Example 5.2. The adaptive controller (41) with (54) and (53) with $\kappa_{\mathrm{a}} = 1$ and $\tau_{\mathrm{a}} = 0.01$ tracks the reference $r(t)$.

Example 5.1 (Adaptive command following of an unstable plant). Consider the plant given by

$$\dot{x}(t) = \begin{bmatrix} 0.5 & 1 \\ -2 & 1 \end{bmatrix} x(t) + \begin{bmatrix} -0.1 \\ 1 \end{bmatrix} u(t), \ t \geq 0, \quad (56)$$

$$y(t) = \begin{bmatrix} 1 & 2 \end{bmatrix} x(t), \quad (57)$$

with $x^{\mathrm{T}}(0) = [0.5, -0.5]$, and poles $\{0.75 \pm 1.39\jmath\}$ and zero $\{-0.26\}$. Let $\lambda = 2.5$ and

$$A_{\mathrm{am}} = \begin{bmatrix} 0 & 1 & 0 \\ -0.69 & -1.22 & 0.15 \\ -1 & 0 & 0 \end{bmatrix}. \quad (58)$$

Furthermore, let $R_{\mathrm{am}} = 5I_3$, $\kappa_{\mathrm{a}} = 1$, $\tau_{\mathrm{a}} = 0.01$ seconds, and $\bar{B} = 1$. Finally, assume $\mathrm{id}(\Lambda) = \mathrm{id}(C_{\mathrm{p}}B_{\mathrm{p}}) = 1$. Here, our aim is to track a given square-wave reference command $r(t)$, $t \geq 0$. The closed-loop response along with the control signal and adaptive gains is shown in Figure 1. △

Example 5.2 (Adaptive command following of an unstable plant). Consider the plant given by

$$\dot{x}(t) = \begin{bmatrix} 0.5 & 5 \\ 2 & 0.5 \end{bmatrix} x(t) + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(t), \ t \geq 0, \quad (59)$$

$$y(t) = \begin{bmatrix} 1 & 2 \end{bmatrix} x(t), \quad (60)$$

with $x^{\mathrm{T}}(0) = [0.5, -0.5]$, and poles $\{3.66, \ -2.66\}$ and zero $\{-2.75\}$. Here, we use the same control design as in Example 5.1 and assume $\mathrm{id}(\Lambda) = \mathrm{id}(C_{\mathrm{p}}B_{\mathrm{p}}) = 1$. Once again, our aim is to track a given square-wave reference command $r(t)$, $t \geq 0$. The closed-loop response along with the control signal and adaptive gains is shown in Figure 2. △

VI. Conclusion

We presented an adaptive control architecture for minimum phase uncertain dynamical systems with unmatched uncertainties and unstable dynamics predicated on derivative-free update laws. The framework is particularly effective for systems undergoing a failure mode and/or large variations in system parameters, which would require high adaptation gains using conventional adaptive update laws predicated on differentiation. Future work will include extensions to nonminimum phase systems, systems with unmatched disturbances, and nonlinear uncertain dynamical systems.

References

[1] T. Yucelen and W. M. Haddad, "Output Feedback Adaptive Stabilization and Command Following for Minimum Phase Dynamical Systems with Unmatched Uncertainties," *Proc. Amer. Contr. Conf.*, San Francisco, CA, June 2011.

[2] E. W. Bai and S. S. Sastry, "Persistency of Excitation, Sufficient Richness and Parameter Convergence in Discrete-Time Adaptive Control," *Sys. Contr. Lett.*, vol. 6, pp. 153–163, 1985.

[3] C. Rohrs, L. Valavani, M. Athans, and G. Stein, "Robustness of Continuous-Time Adaptive Control Algorithms in the Presence of Unmodeled Dynamics," *IEEE Trans. Autom. Contr.*, vol. 30, pp. 881–889, 1985.

[4] T. Yucelen and A. J. Calise, "Derivative-Free Model Reference Adaptive Control," *Proc. AIAA Guid., Nav., and Contr. Conf.*, Toronto, ON, 2010.

[5] T. Yucelen and A. J. Calise, "Derivative-Free Model Reference Adaptive Control of a Generic Transport Model," *Proc. AIAA Guid., Nav., and Contr. Conf.*, Toronto, ON, 2010.

[6] T. Yucelen and A. J. Calise, "Adaptive Control for the Generic Transport Model: A Derivative-Free Approach," *Proc. AIAA Infotech Conf.*, Atlanta, GA, 2010.

[7] P. J. Antsaklis and A. N. Michel, *Linear Systems*. Boston, MA: Birkhäuser, 2005.

[8] T. Yucelen, W. M. Haddad, and A. J. Calise, "Output Feedback Adaptive Command Following for Nonminimum Phase Uncertain Dynamical Systems," *Proc. Amer. Contr. Conf.*, Baltimore, MD, pp. 123–128, 2010.

[9] J. B. Hoagg, M. A. Santillo, and D. S. Bernstein, "Discrete-time Adaptive Command Following and Disturbance Rejection with Unknown Exogenous Dynamics," *IEEE Trans. Autom. Control*, vol. 53, pp. 912-928, 2008.

[10] G. C. Goodwin and K. S. Sin, *Adaptive Filtering, Prediction, and Control*. Englewood Cliffs, NJ: Prentice-Hall, 1984.

[11] C. J. Taylor, A. Chotai, and P. C. Young, "State Space Control System Design Based on Nonminimal State-Variable Feedback: Further Generalization and Unification Results," *Int. J. Control*, vol. 73, pp. 1329-1345, 2000.

[12] T. Yucelen, "Real-time $\mathcal{H}_\infty$ Approach for Robust Optimal Control." M.Sc. Thesis, Southern Illinois University, Carbondale, IL, 2008.

[13] T. Yucelen and F. Pourboghrat, "Active Noise Blocking: Nonminimal Modeling, Robust Control, and Implementation," *Proc. Amer. Contr. Conf.*, St. Louis, MO, pp. 5492-5497, 2009.

[14] W. M. Haddad and V. Chellaboina, *Nonlinear Dynamical Systems and Control: A Lyapunov-Based Approach*. Princeton, NJ: Princeton University Press, 2008.