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# Symmetric coverage of dynamic mapping error for mobile sensor networks

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***Abstract*— We present an approach to control design for a mobile sensor network tasked with sampling a scalar field and providing optimal space-time measurements. The coverage metric is derived from the mapping error in objective analysis (OA), an assimilation scheme that provides a linear statistical estimation of a sampled field. OA mapping error is an example of a consumable density field: the error decreases dynamically at locations where agents move and sample. OA mapping error is also a regenerating density field if the sampled field is time-varying: error increases over time as measurement value decays. The resulting optimal coverage problem presents a challenge to traditional coverage methods. We prove a symmetric dynamic coverage solution that exploits the symmetry of the domain and yields symmetry-preserving coordinated motion of mobile sensors. Our results apply to symmetric sampling regions that are non-convex and non-simply connected.**

## I. Introduction

A mobile sensor network used to observe a scalar field over a finite region can be made most efficient if it is designed for optimal measurement coverage. If the field varies spatially and temporally, sensing agents should be dynamically distributed in space and time to match the spatial and temporal scales of the field. This can be formulated as the problem of designing motion control laws for the agents that maximize information in the data collected.

Inspired by ocean sampling field experiments in Monterey Bay CA [1], we examine a coverage problem in which information derives from the classic objective analysis (OA) mapping error. OA is linear statistical estimation based on specified field statistics, and the mapping error provides a measure of the residual uncertainty in the model [2]. Since reduced uncertainty, equivalent to increased entropic information, implies better measurement coverage, the OA mapping error provides a useful density field for determining the coverage metric [2], [3].

In this context, optimal coverage is achieved by seeking maximum reduction of the residual OA mapping error; this problem is particularly challenging because mapping error changes with the spatial dynamics of the sensing agents. Indeed, error decreases near the locations where agents take measurements, reflecting the new information acquired, but as time passes, the relevance of past measurements decreases and the error increases.

Coverage for static density fields has been studied for convex and non-convex domains; see, e.g., [4]–[8]. In [8] convergence is proved in the case of slowly time-varying, nonuniform density fields. Related problems include control of a mobile sensor network in a noisy, possibly time-varying field for minimum-error spatial estimation [9], [10] and cooperative exploration of features [11]. In [12] agents are deployed in a distributed way in order to maximize the probability of event detection. The authors of [13] study the distributed implementation of maximizing joint entropy in measurements. However, none of these methods have been designed to specifically address a spatial-temporal density field that changes in response to the motion of the agents.

In this paper we propose a coverage strategy for a density field defined by OA mapping error in the case that the sampled field is time-varying. The approach, based on greedy search, exploits symmetry in the sampling region and yields symmetric coverage patterns of mobile agents. We show how the symmetry group that defines the spatial configuration of the mobile sensors relates to the symmetry of the region, and we prove that the search strategy preserves the symmetry of the spatial configuration. The method does not require a convex or simply connected sampling region; we illustrate with simulation examples.

In Section II, we review OA mapping error. We define the sensor network system and the coverage goals in Section III. In Section IV we show that a well-known coverage solution for static problems does not address the coverage problem for minimizing OA mapping error. We present our symmetry-based approach in Section V and prove invariance of symmetry in the spatial distribution of agents. We discuss performance and robustness issues in Section VI. In Section VII we discuss future directions.

## II. OA Mapping Error

In this section we review the method of *objective analysis* (OA); for further details see [2].

OA models a scalar sampling field observed at a point $\mathbf{x}$ and a time $t$ as a random process $T(\mathbf{x}, t)$. It is assumed that *a priori* information about this process is available, namely the mean value $\overline{T}$ and the covariance $B$ of fluctuations about the mean:

$$B(\mathbf{x}, t, \mathbf{x}', t') = \mathbb{E}\left[\left(T(\mathbf{x}, t) - \overline{T}(\mathbf{x}, t)\right) \times \left(T(\mathbf{x}', t') - \overline{T}(\mathbf{x}', t')\right)\right]. \quad (1)$$

OA is a linear estimator; it provides an estimate for the field as a linear combination of the discrete set of measurements obtained up to time $t$ as

$$\hat{T}(\mathbf{x}, t) = \overline{T}(\mathbf{x}, t) + \sum_{k=1}^{P} \eta_k(\mathbf{x}, t)\left(M_k - \overline{T}(\mathbf{x}_k, t_k)\right), \quad (2)$$

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where $P$ is the number of measurements, $M_k$ is the $k$-th measurement, which is taken at location $\mathbf{x}_k$ at time instant $t_k$, and $\eta_k$ are coefficients that minimize the least square error of the estimate:

$$A(\mathbf{x},\, t,\, \mathbf{x},\, t) = \mathbb{E}\left[\left(T(\mathbf{x},\, t) - \hat{T}(\mathbf{x},\, t)\right) \times \left(T(\mathbf{x},\, t) - \hat{T}(\mathbf{x},\, t)\right)\right]. \quad (3)$$

An important aspect of OA is that the residual error $A(\mathbf{x},\, t,\, \mathbf{x},\, t)$ depends on the location and time of the measurements but not on the measured quantity (nor on $\bar{T}$).

The covariance between data points is given by

$$[C]_{jl} = \eta\delta_{jl} + B(\mathbf{x}_j,\, t_j,\, \mathbf{x}_l,\, t_l), \quad (4)$$

where $\eta$ is the measurement noise (given by the physical characteristic of the sensors) and $\delta_{ij}$ is Kronecker's delta.

As shown in [14], the error (3) can be written as

$$A(\mathbf{x},\, t,\, \mathbf{x}',\, t') = B(\mathbf{x},\, t,\, \mathbf{x}',\, t') - \sum_{k=1}^{P}\sum_{l=1}^{P}\Big[B(\mathbf{x},\, t,\, \mathbf{x}_j,\, t_j) \times \left[C^{-1}\right]_{jl} B(\mathbf{x}_l,\, t_l,\, \mathbf{x}',\, t')\Big]. \quad (5)$$

As is typical in ocean modeling [3], we assume that

$$B(\mathbf{x},\, t,\, \mathbf{x}',\, t') = \sigma_0 e^{-\left(\frac{\|\mathbf{x}-\mathbf{x}'\|}{\sigma}\right)^2 - \left(\frac{|t-t'|}{\tau}\right)^2}, \quad (6)$$

where $\sigma_0$ denotes the *space-time average* for the field covariance $B$, and $\sigma$ and $\tau$ represent its length and time scales, respectively.

We define the density field that describes the OA mapping error as $\rho(\mathbf{x},\, t) = A(\mathbf{x},\, t,\, \mathbf{x},\, t)$. For $\tau < \infty$, the mapping error is regenerating: if no new measurements are made, then $\rho(\mathbf{x},\, t) \to \sigma_0$ as $t \to \infty$. If $\tau = \infty$ the mapping error is not regenerating: if no new measurements are made in any time interval $I = [t_0,\, t_1]$, then $\rho(\mathbf{x},\, t)$ is constant for $t \in I$.

## III. Sampling System and Goals

Consider a set of $N$ autonomous agents (mobile sensors), indexed by $i = 1,\, 2,\, \ldots,\, N$, deployed in a bounded region $\mathcal{D} \subset \mathbb{R}^2$. As is traditional in the field, we model each agent $i$ as a first-order system [4], [5], [8], for which its location $\mathbf{p}_i \in \mathcal{D}$ evolves in time as

$$\dot{\mathbf{p}}_i = \mathbf{u}_i(\mathbf{p}_1,\, \ldots,\, \mathbf{p}_N), \quad (7)$$

where $\mathbf{u}_i$ is the control input for the $i$th agent. We assume that all the agents move at the same constant speed.

The coverage goal is to design control laws $\mathbf{u}_i$ to keep $\rho(\mathbf{x},\, t)$ small over space and time, i.e., to control the motion of the sensors so that the measurements they take minimize OA mapping error. For sufficiently small $\tau$, the sampled field changes so fast, it may be appropriate to use static sensors. However, more generally for finite $\tau$, the sensors will need to move continuously, because if they come to a static configuration, error will grow around them and performance will decline. For infinite $\tau$, the sensors will need to move continuously until $\rho(\mathbf{x},\, t)$ is zero over $\mathcal{D}$.

A coverage metric can be derived from $\rho(\mathbf{x},\, t)$. For example, entropic information $\mathcal{I}(t)$, computed as minus the log of the average over $\mathcal{D}$ of $\rho(\mathbf{x},\, t)$ at time $t$, provides one natural choice of coverage metric [3].

In the next section, we show that an extension of a static coverage algorithm to the density field $\rho(\mathbf{x},\, t) = A(\mathbf{x},\, t,\, \mathbf{x},\, t)$ does not address the problem; indeed, the coverage algorithm converges to a static configuration. Our approach, presented in Section V, uses a kind of greedy search to prevent the sensors from becoming static and exploits symmetry to ensure that the domain is sufficiently well covered. We discuss performance with respect to the entropic information metric $\mathcal{I}(t)$ in Section VI.

## IV. Extension of Static Coverage

In this section we apply the static coverage law of Cortés et al. [4] to the consumable density field $\rho(\mathbf{x},\, t) = A(\mathbf{x},\, t,\, \mathbf{x},\, t)$. The approach in [4] has been successfully extended to include some time-varying fields, as in [15]. Substituting in the time-varying density $\rho(\mathbf{x}, t)$, the cost function of [4] to be minimized for optimal coverage becomes

$$J(t) = \sum_{i=1}^{N}\int_{V_i} \|\mathbf{x} - \mathbf{p}_i\|^2 \rho(\mathbf{x},\, t)\, d\mathbf{x}, \quad (8)$$

where $V_i$ is the Voronoi region associated to agent $i$. The coverage control law of [4] for agent $i$ is given by

$$\dot{\mathbf{p}}_i = -(C_{V_i} - \mathbf{p}_i), \quad (9)$$

where $C_{V_i}(t)$ is the centroid of $V_i$ at time $t$:

$$C_{V_i}(t) = \frac{1}{M_{V_i}}\int_{V_i} \mathbf{x}\,\rho(\mathbf{x}, t)\, d\mathbf{x}, \quad M_{V_i}(t) = \int_{V_i} \rho(\mathbf{x}, t)\, d\mathbf{x}.$$

The computation of Voronoi regions is independent of density $\rho$, but the control law (9) directs the $i$th agent to the density weighted centroid of its Voronoi region. This yields a static configuration of sensors that can initially reduce $\rho$ in some locations; however, it falls short of addressing the coverage problem at hand. As described above, continuous sensor dynamics are necessary for good coverage performance. In the next Theorem we prove that in the case of infinite $\tau$, the agents converge to a static configuration independent of how much the density field $\rho(\mathbf{x},\, t)$ has been reduced.

**Theorem 1** *Let $\tau$ be infinite. The control law* (9) *drives the system towards a stable, stationary equilibrium configuration.*

*Proof:* Observe that

$$\begin{aligned} \frac{dJ}{dt} &= \sum_i \frac{\partial J}{\partial \mathbf{p}_i}\dot{\mathbf{p}}_i + \frac{\partial J}{\partial t} \\ &= -2\sum_i M_{V_i}\|C_{V_i} - \mathbf{p}_i\|^2 \\ &\quad + \sum_j \int_{V_j} \|\mathbf{x} - \mathbf{p}_j\|^2 \frac{\partial \rho(\mathbf{x},\, \mathbf{t})}{\partial t} d\mathbf{x}. \end{aligned} \quad (10)$$

Since $\rho$ represents a non-regenerative resource, then $\partial\rho/\partial t \leq 0$ everywhere, and thus the integral on the right hand side of (10) is a non-positive function. The right hand side is negative unless all the agents are in the centers of their respective Voronoi regions (in which case, they are not moving, thus implying $\partial\rho/\partial t = 0$). This implies that

$$\frac{dJ}{dt} \leq \sum_i \|C_{V_i} - \mathbf{p}_i\|^2 \int_{V_i} \min_{\mathbf{x}\in V_i} \rho\left(\mathbf{x},\, t\right)\, dx \leq 0, \qquad (11)$$

with equality only when each of the agents is at the centroid of its respective Voronoi region. We can thus invoke LaSalle's principle for time varying systems (as in [16]), to conclude that the agents converge to the largest invariant set (under (9)) that is contained in the kernel of the right-hand side of (11). This is, the agents converge towards a centroidal Voronoi tessellation. ■

For infinite $\tau$, a good coverage solution is one in which the agents continue to move around and converge to a static configuration only when there is no more error in the field, i.e., $\rho$ is close to zero everywhere. Theorem 1 does not ensure that this will be the case. Indeed, in the simulation shown in panels (a)-(c) of Figures 1, where $\tau$ is infinite, the agents converge to a static configuration before the OA mapping error is significantly reduced; i.e., there remain subregions of very high density $\rho$. Panel (a) of Figure 1 shows the trajectories of the four agents. Panels (b) and (c) of Figure 1 show the initial and final density field $\rho(\mathbf{x},\, t) = A(\mathbf{x},\, t,\, \mathbf{x},\, t)$, which is plotted as a color map with white to black representing the range from high to low values. Panels (d)-(f) of Figure 1 show the same three plots for a simulation in the case of finite $\tau$. In this case too the agents converge to a static configuration despite the fact that OA mapping error is high around them.

## V. Symmetric Greedy Coverage

We propose a coverage control law that directs the agents to move to those locations where $\rho(\mathbf{x},\, t) = A(\mathbf{x},\, t,\, \mathbf{x},\, t)$ is large. Given $\delta > 0$ we define $\mathbf{y}$ to be *$\delta$-close to* $\mathbf{p}_i$ if $\|\mathbf{y} - \mathbf{p}_i\| = \delta$. We define $\mathbf{p}_i^+(t)$ to be the point that is $\delta$-close to $\mathbf{p}_i(t)$ and maximizes the density field $\rho\left(\mathbf{x},\, t\right)$ over all points $\mathbf{x}$ that are $\delta$-close to $\mathbf{p}_i(t)$. In case there is more than one point $\mathbf{x}$ that satisfies these conditions, the agent chooses the first location found on a counterclockwise search, starting from its current direction of motion. The control law we propose directs each agent $i$ to move at time $t$ to $\mathbf{p}_i^+(t)$.

This approach is similar to a gradient control law, with the difference being that this gradient is not evaluated at the current location, but rather at positions that are $\delta$-close. This is necessary since the gradient of $\rho$ is zero when it is evaluated at the position $\mathbf{p}_i$ (i.e., $\mathbf{p}_i$ is a local minimum for $\rho$). The control law requires that each agent $i$ knows the values of $\rho$ in those locations that are $\delta$-close to it.

Like any local greedy algorithm, it is challenging to ensure global performance. Instead, to enforce coverage that is well distributed about the domain $\mathcal{D}$, we consider symmetric spatial distributions of agents. We let $\mathcal{D}$ be symmetric with $\mathcal{S}(\mathcal{D})$ its symmetry group[1], and we show what corresponding symmetry in the spatial distribution of agents will be invariant to the greedy control law.

We define a polygon to be *$\mathcal{D}$-friendly* if its center coincides with the center $O$ of $\mathcal{D}$. Assume that the agents are originally deployed in such a way that they describe a $\mathcal{D}$-friendly regular $N$-gon. We induce a set of headings on the agents in such a way that if each agent moves in the direction of its heading, together they still define a $\mathcal{D}$-friendly regular $N$-gon. We call such a set of headings a *symmetric* set of headings. The symmetry group of the regular $N$-gon is the dihedral group $D_N$.

Let $R_O : [0,\, 2\pi) \times \mathbb{R}^2 \to \mathbb{R}^2$ be a function such that $R_O\left(\theta,\, \mathbf{x}\right) \mapsto \mathbf{x}_\theta$ maps the point $\mathbf{x}$ to the point $\mathbf{x}_\theta$, after a rotation about $O$ by an angle $\theta \in [0,\, 2\pi)$. We define $\rho(\mathbf{x},\, t)$ to be a *symmetric density field at time $t$* if there exists a $\theta$ so that $\rho\left(\mathbf{x},\, t\right) = \rho\left(R_O\left(\theta,\, \mathbf{x}\right),\, t\right)$ for every $\mathbf{x}$ in $\mathcal{D}$. In the case where $\mathcal{S}\left(\mathcal{D}\right) = D_N$ and the positions of the agents define a $\mathcal{D}$-friendly regular $N$-gon at the initial time, then $\theta = 2\pi/N$. We now prove conditions such that for our greedy approach, the agents induce a $\mathcal{D}$-friendly regular $N$-gon at every time $t$.

**Claim 1** *Assume that at initial time $k = 0$, the agents are deployed inside $\mathcal{D}$ in such a way that they define a $\mathcal{D}$-friendly regular $N$-gon. Suppose that $\mathcal{S}(\mathcal{D}) = D_N$. Then $\rho$ is symmetric at time $k = 0$.*

This follows directly from the definition of the density field $\rho$ and the initial deployment in a $\mathcal{D}$-friendly regular $N$-gon.

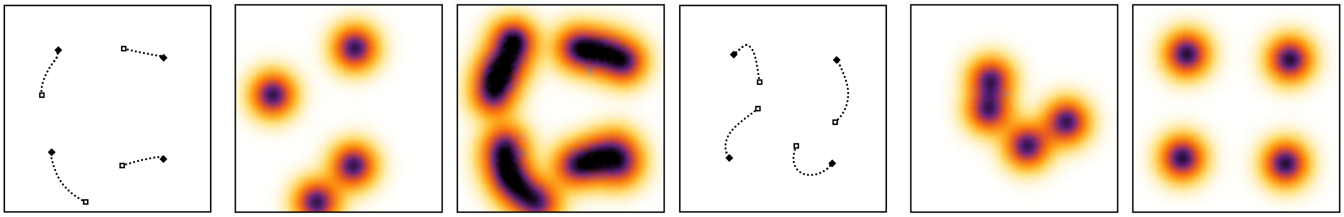
**Claim 2** *Suppose that $\mathcal{S}(\mathcal{D}) = D_N$. Consider the time instant $k > 0$. Suppose that at each time instant $j \leq k$, the formation induces both a $\mathcal{D}$-friendly regular $N$-gon and a symmetric $\rho$ on $\mathcal{D}$. Then, at time $k+1$, the formation induces both a $\mathcal{D}$-friendly regular $N$-gon and a symmetric $\rho$ on $\mathcal{D}$.*

*Proof:* By hypothesis, at some time $k$, the formation induces both a $\mathcal{D}$-friendly regular $N$-gon and a symmetric $\rho$ with respect to $\mathcal{D}$. For each agent $i$, let $\mathbf{p}_i^+$ be the $\delta$-close point to $\mathbf{p}_i$ such that $\rho\left(\mathbf{p}_i^+,\, k\right)$ attains its maximum. If $R_O\left(2\pi/N, \mathbf{p}_i^+\right) = \mathbf{p}_{i+1}^+$, with $\mathbf{p}_{N+1}^+ \equiv \mathbf{p}_1^+$, then the formation induces a $\mathcal{D}$-friendly regular $N$-gon at time $k+1$, and by construction $\rho$ is symmetric at time $k+1$ on $\mathcal{D}$.

Suppose then, that there is an index $i$ such that $R_O\left(2\pi/N, \mathbf{p}_i^+\right) \neq \mathbf{p}_{i+1}^+$. This means that $\rho\left(\mathbf{p}_{i+1}^+,\, k\right) > \rho\left(R_O\left(2\pi/N, \mathbf{p}_i^+\right),\, k\right) = \rho\left(\mathbf{p}_i^+,\, k\right)$ because of the assumed symmetry at time $k$. But this implies $\rho\left(\mathbf{p}_{i+1}^+,\, k\right) = \rho\left(R_O\left(-2\pi/N,\, \mathbf{p}_{i+1}^+\right),\, k\right) > \rho\left(\mathbf{p}_i^+,\, k\right)$. Since $R_O$ is an isometry, then $R_O\left(-2\pi/N,\, \mathbf{p}_{i+1}^+\right)$ is a $\delta$-close point to $\mathbf{p}_i$. Thus, the last inequality contradicts the definition of $\mathbf{p}_i^+$. This concludes the proof. ■

Observe that in Claims 1 and 2, it is essential that $\mathcal{S}\left(\mathcal{D}\right) \equiv D_N$, since this leads to the symmetric behavior of the density

[1]The group of all isometries under which $\mathcal{D}$ is invariant, having composition as the operation.



(a) Trajectories of the agents, $\tau = \infty$. (b) $\rho(\mathbf{x}, t)$ at deployment, $\tau = \infty$. (c) $\rho(\mathbf{x}, t)$ after the agents have reached equilibrium, $\tau = \infty$. (d) Trajectories of the agents, finite $\tau$. (e) $\rho(\mathbf{x}, t)$ at deployment, finite $\tau$. (f) $\rho(\mathbf{x}, t)$ after the agents have reached equilibrium, finite $\tau$.

Fig. 1. Evolution of density $\rho(\mathbf{x}, t)$ with four agents in a square domain using the coverage control law (9). Panels (a)-(c) illustrate the case for infinite $\tau$, and panels (d)-(f) illustrate the case for finite $\tau$. In panels (a) and (d), the squares and filled rhombuses represent the initial and final positions of the agent trajectories, respectively. In both cases (infinite and finite $\tau$), the agents converge to a stationary equilibrium configuration. The density is plotted according to a color map with white to black representing the range from high to low values; this convention is used for all subsequent figures.

$\rho$. The same result can be expected if $D_N$ is a subgroup of $\mathcal{S}(\mathcal{D})$. We state this result without proof because of space limitations.

**Theorem 2** *Assume that at initial time $k = 0$, the agents are deployed inside $\mathcal{D}$ in such a way that they define a $\mathcal{D}$-friendly regular $N$-gon. Suppose that $D_N$ is a subgroup of $\mathcal{S}(\mathcal{D})$. Then, for any time step $k \geq 0$, the formation induces a $\mathcal{D}$-friendly regular $N$-gon and $\rho$ is symmetric.*

We illustrate Theorem 2 by example in the two simulations shown in Figure 2. The domain $\mathcal{D}$ is a circle in the first example, shown in panels (a)-(c) of Figure 2 and an annulus in the second example, shown in panels (d)-(f) of Figure 2. The symmetry group for both the circle and the annulus is infinite dimensional. In each example, we have initially deployed seven agents such that they define a $\mathcal{D}$-friendly regular heptagon (which has symmetry group $D_7$). Since $D_7$ is a subgroup of the symmetry group of the circle and the annulus, Theorem 2 implies that for every time $k \geq 0$ the positions of the agents define a $\mathcal{D}$-friendly regular heptagon. The simulations of Figure 2 illustrate how the results of Theorem 2 apply in both simply connected and non-simply connected domains.

Observe that Theorem 2 assumes that the $N$ agents define a single $\mathcal{D}$-friendly regular polygon (the regular $N$-gon). The same result can be expected if the $N$ agents were to define $r$ $\mathcal{D}$-friendly regular $M$-gons with $rM = N$. Suppose the agents are deployed initially in this way. We induce a set of headings on the agents such that if each agent moves in the direction of its heading, together they still define $r$ $\mathcal{D}$-friendly regular $M$-gons. We call such a set of headings an *$r$-symmetric* set of headings. We now state the result when the agents define multiple copies of the regular $M$-gon. Again, we state the result without proof because of space limitations.

**Theorem 3** *Assume that at initial time $k = 0$, the agents are deployed inside $\mathcal{D}$ in such a way that they define $r$ $\mathcal{D}$-friendly regular $M$-gons, with $rM = N$, and their headings correspond to an $r$-symmetric set. Suppose that $D_M$ is a subgroup of $\mathcal{S}(\mathcal{D})$. Then, for any time step $k \geq 0$, the formation induces $r$ $\mathcal{D}$-friendly regular $M$-gons and $\rho$ is symmetric.*

We illustrate Theorem 3 by example in the simulation shown in Figure 3. The domain $\mathcal{D}$ is a six-pointed star with symmetry group $D_6$. We have initially deployed nine agents defining a $\mathcal{D}$-friendly regular nonagon, which has symmetry group $D_9$. $D_9$ is not a subgroup of $D_6$, and hence Theorem 2 does not apply. However, the regular nonagon can be seen as three $\mathcal{D}$-friendly equilateral triangles, each of which has symmetry group $D_3$. In the language of Theorem 3, we have that $r = 3$ and $M = 3$ and thus the condition $rM = N = 9$ holds. We observe that, although the three equilateral triangles are preserved for every time $k \geq 0$ as dictated by Theorem 3, they can dilate relative to one another with time. The simulation of Figure 3 illustrates how the results of Theorem 3 apply even in non-convex regions.

## VI. Performance and robustness

### A. Evolution of the performance metric

Entropic information $\mathcal{I}(t)$, as defined in Section III, can be used as a coverage performance metric [3], as it quantifies the information richness of the data gathered as the agents move inside $\mathcal{D}$. If we want our coverage approach based on symmetry groups to be useful, we have to be able to guarantee a quantitative advantage with respect to other methods that could also address this task. Numerical studies in [17] on performance of OA mapping error coverage with respect to $\mathcal{I}$ for prescribed coordinated motion patterns showed that the best performance was achieved by formations maintaining $\mathcal{D}$-friendly regular configurations; although in these studies there was no discussion of the relation of the particular motion with the symmetry groups of the sampling domain. Further, we have made numerical comparisons between our approach and others. In Figure 4 we show $\mathcal{I}(t)$ as a function of time for our symmetry-based approach and for a greedy search when the agents define the same $\mathcal{D}$-friendly regular polygon, but with a random first step, after which they use our greedy coverage approach. The results in Figure 4 show that the symmetric approach outperforms the greedy search with random first step. We are currently working on proving conditions under which our approach has a performance advantage.

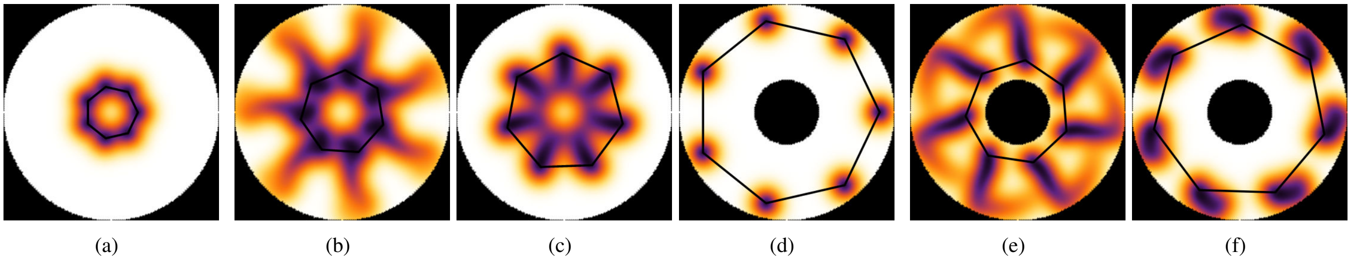


Fig. 2. Density field $\rho(\mathbf{x}, t)$ and the heptagon induced by seven agents at three different instants of time inside a circular domain (panels (a)-(c)) and inside an annulus (panels (d)-(f)). Panels (a) and (d) show the initial deployment and density field in each example. We observe that the coverage algorithm preserves the $\mathcal{D}$-friendly heptagon and the symmetry of the density field even for the annular domain, which is not simply connected.

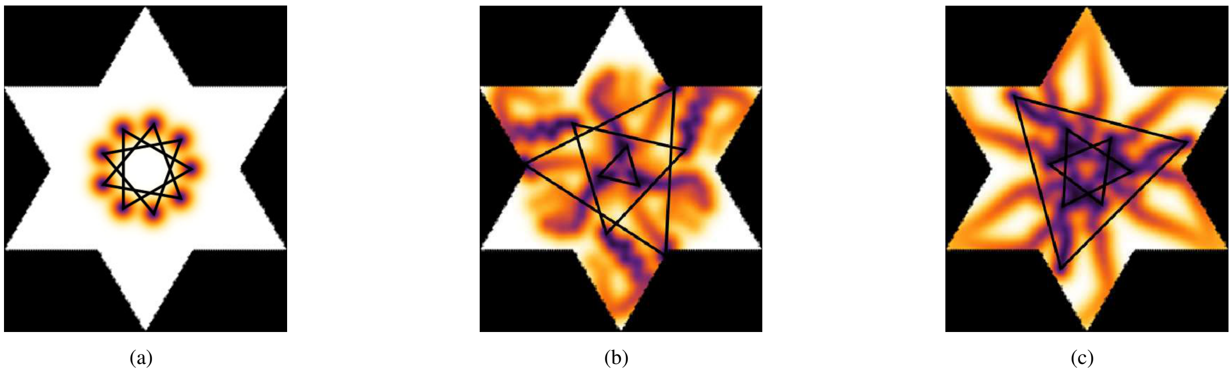


Fig. 3. Density field $\rho(\mathbf{x}, t)$ and the three equilateral triangles induced by nine agents at three different instants of time in the six-pointed star domain. Panel (a) shows the initial deployment and density field, and panels (b) and (c) show subsequent time instants. We observe that the coverage algorithm preserves the three $\mathcal{D}$-friendly triangles and the symmetry of the density field even for the six-pointed star domain, which is not convex.

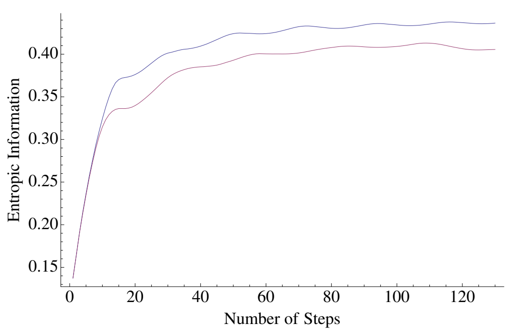



Fig. 4. Entropic information as a function of time for our symmetric greedy coverage (blue line) and the greedy approach with random first step (red line) in the case of four agents in a square domain. In each case the agents are deployed in the same $\mathcal{D}$-friendly square. We observe that the performance is the same in each case for the first few time steps, but then the symmetric approach outperforms the approach where the first step is random for each of the agents.

## *B. Asymmetric regions*

Our approach restricts to domains for which $\mathcal{S}(\mathcal{D})$ is non-trivial; this includes most of the domains on which coverage algorithms are typically tested (square or rectangular domains). However, it is of interest to consider extending to asymmetric domains; this requires asking if it is possible to modify the approach to deal with domains $\mathcal{D}$ with $\mathcal{S}(\mathcal{D}) = I$, the identity transformation. Although there exists a diffeomorphism $\mathcal{T} : \mathcal{D} \to \mathbb{D}^1$, that sends an asymmetric domain $\mathcal{D}$ to the unit disk $\mathbb{D}^1$, in order for our approach to be applicable under this diffeomorphism, we also need for the induced density $\tilde{\rho}$ in $\mathbb{D}^1$ to be symmetric. Otherwise, solving the problem in $\mathbb{D}^1$ and then mapping back to the original $\mathcal{D}$ would not be sufficient since, in general, this diffeomorphism is not an isometry and thus would change the isotropic properties of $B$. One of the alternatives we are currently studying includes the use of some *special* agents that are dedicated to taking measurements near the boundary of an asymmetric $\mathcal{D}$ in such a way that the rest of the agents need only address coverage in the remaining interior symmetric region.

It is also possible that the domain will be only slightly perturbed from an idealized symmetric region. It is a topic of future work to investigate how robust our approach is to small perturbations from a symmetric domain.

## *C. Asymmetric initial conditions*

We have assumed symmetric initial positions for the agents that induce $\mathcal{D}$-friendly regular polygons. To make this practical we consider that a feedback control law be used to move the agents from an arbitrary initial deployment to the desired initial symmetric location. Then the density field $\rho(\mathbf{x}, t)$ can be initialized, and the coverage algorithm can be initiated. In this way the symmetry of $\rho$ is not affected by previous measurements and the spatial distribution of the agents would thus evolve as described by our theory.

With this approach we cannot expect that the agents will attain the desired initial configuration exactly. It is thus reasonable to ask if there exists a neighborhood of the family of desirable initial positions (and headings) such that the greedy search starting from any configuration in this neighborhood yields a collective behavior that is close to the one that evolves from a desired set of initial conditions. This is related to the question in the previous section on the robustness of our approach to a domain that is only slightly asymmetric.

Simulation results have shown that the entropic information is very similar when the perturbation to the initial condition is *small* (for instance, a difference in the relative headings of the agents bounded by $\pm\pi/20$). This suggest that the family of desired initial conditions has a neighborhood around it to which the solution of our symmetric coverage algorithm is robust. It is a topic of future work to fully characterize the restrictions of such variations.

## VII. Conclusions and Future Work

We have presented a symmetric approach for the challenging problem of designing motion control laws for a group of mobile agents covering a scalar density field defined by OA mapping error. This density field varies in both time and space as the agents move inside the sampling domain; error decreases at locations recently sampled and error grows back as the value of the measurements decay. We first show that the application of the static coverage approach of [4] does not address this type of problem: the agents converge to a static equilibrium configuration and do not continue to visit high density regions. This motivates our derivation of an alternative approach.

The approach we propose uses greedy search to respond to the changing density in the domain $\mathcal{D}$ and exploits symmetry in the domain to enable a symmetric coverage pattern that provides well distributed coverage. We prove the nature of the symmetric coverage and show through simulation that the agents keep moving in response to the changing density. Our approach applies to domains that are non-convex as well as to domains that are non-simply connected. Simulations suggest that the performance, as measured by entropic information in measurements collected, is relatively high for our symmetric approach.

In future work, we will investigate conditions that guarantee optimal coverage. We will also prove robustness of the method to asymmetries in the domain and in the initial configuration of the agents. The greedy search is a local decision by each agent – each agent only needs to know density locally. However, currently the computation of the density field, i.e., the OA mapping error, is centralized. We are exploring alternatives to compute a distributed approximation of the OA mapping error using the fact that the correlation between the value of the field at different points in the domain decreases with distance, i.e., the mapping error local to an agent will depend most on information on measurements taken local to that agent. Preliminary simulation results are supportive of the correctness of this approach.

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