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Linear Parameter Varying Controllers for the ADMIRE Aircraft Longitudinal Dynamics

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Abstract— **This paper details the design of various Linear Parametrically Varying (LPV) controllers for the ADMIRE fighter aircraft benchmark model. Attention is focused on the improvement of the ADMIRE longitudinal dynamics handling qualities. In this benchmark, performance and robustness must be guaranteed over an extensive portion of the flight envelope. To this end, three robust LPV gain-scheduling control design techniques are investigated. Each technique is assessed in terms of conservatism, off-line computation cost and ease of implementation. For each controller, the non-linear aircraft responses are given and discussed.**

I. INTRODUCTION

In this paper, LPV control techniques are used to design robust control augmentation systems for the ADMIRE fighter aircraft model [1]. Three robust LPV control design techniques are compared, namely full-block static multipliers [2], scaled bounded real lemma [3] and parameter gridding LPV control design techniques [4]. The multiplier approaches [3], [5], [2] assume a plant given in terms of Linear Fractional Transformations (LFTs). These LPV/LFT controller syntheses use static multipliers and thus assume unbounded parameter rate of variations. On the contrary, the parameter gridding approach does not require any particular modelling (mere linearized models suffice) and its formulation allows for parameter rate information [4].

In this paper, attention is restricted to the control of ADMIRE's longitudinal dynamics but it is clear that lateral-directional dynamics could be controlled using the same control techniques. Controllers are compared in terms of computational design cost, conservatism and implementation issues.

The paper is organized as follows: Sections II and III introduce the aircraft model, performance objectives, model augmentation structure and weighting function selection. Section IV briefly presents the different LPV controller synthesis techniques that will be used to control the aircraft. Section V shows the non-linear responses obtained with the LPV controllers. Conclusions are given in VI.

The notation is fairly standard:

M	Mach number
h	Altitude
α	Angle of attack
q	Pitch rate
N_z	Load factor
δ_c	Canard deflection
δ_e	Elevon deflection
LPV	Linear Parameter Varying
LFT	Linear Fractional Transformation
FBM	Full Block Multipliers
SBRL	Scaled Bounded Real Lemma
$\mathbf{R}^{m\times n}$	set of $m \times n$ real matrices
A^T	transpose of A
diag	Block Diagonal matrix
I_n	Identity matrix of size n
$A > 0$	A is a symmetric positive definite matrix
$\ker(A)$	Null-space of A

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The L_2 gain of a linear operator H is given by $\|H\| = \sup\{\|Hw\|/\|w\| : w \neq 0\}$ over square integrable signals w.

II. MODELLING AND CONTROL OBJECTIVES

A. ADMIRE

ADMIRE is a freely and publicly available advanced simulation model of a generic fighter aircraft. ADMIRE was developed and maintained by the Swedish Defense Research Agency [1]. The aircraft is featured with a delta wing, actuated canard configuration, inboard and outboard elevons and thrust vectoring. The model incorporates actuator and sensor models. In addition, the simulation package includes trim and linearisation routines. ADMIRE has been used to demonstrate various control techniques such as LPV control in [6], non-linear control using backstepping in [7].

B. Longitudinal dynamics

The linearised short-period longitudinal dynamic model is given by

$$\left\{\begin{array}{rcl} \dot{\alpha} & = & Z_\alpha \alpha + Z_q q + Z_{\delta_e}\delta_e + Z_{\delta_c}\delta_c \\ \dot{q} & = & M_\alpha \alpha + M_q q + M_{\delta_e}\delta_e + M_{\delta_c}\delta_c \\ N_z & = & n_{z_\alpha}\alpha + n_{z_\alpha} q + n_{z_{\delta_e}}\delta_e + n_{z_{\delta_c}}\delta_c \end{array}\right. \quad (1)$$

$$x = \left[\, \alpha \ q \,\right]^T,\ y = \left[\, \alpha \ q \ N_z \,\right]^T,\ u = \left[\, \delta_c \ \delta_e \,\right]^T$$

where x is the state vector, y the measured output and u the control input. Z_α is the partial derivative of normal force (Z) with respect to angle of attack (α) at equilibrium. Similarly, M_α denotes the partial derivative of the pitching moment with respect to α. We will assume that normal force and pitching moment derivatives depend only on altitude (h) and Mach number (M); variations with respect to incidence and control deflections will be neglected.

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For ADMIRE the flight envelope is defined as $0.3 \leq M \leq 1.2$ and $100 \leq h \leq 6000$ [m].

C. Control Allocation

Elevons and canards can be used as primary control variables. However, because they both contribute to the pitching motion, they can advantageously be combined together in a, new, single control variable δ. Here, the simple control allocation method given in [8] is used. More precisely, if (A, B, C) are state-space matrices of a strictly proper LTI plant such that CB is full row rank, then, given a constant and invertible weight W_u, one can define the new control variable δ as

$$u = H\delta \tag{2}$$

where

$$H = W_u^{-1}(CB)^T((CB)W_u^{-1}(CB)^T)^{-1} \tag{3}$$

In this approach, W_u is used to weight the contribution of each actuator on the output specified by C. Note that C does not necessarily represent the output matrix of the plant to be controlled and, therefore, C can be chosen by the designer. In our case, we want to optimize the generation of pitching acceleration $\dot{q}$.

For ADMIRE, the flight envelope was discretized over a grid with a resolution of 0.04 in Mach number and 500 meter in altitude. At each point of the grid, a linearized state space model was computed. We selected $C = [0 \quad 1]$ and

$$W_u = \begin{bmatrix} 1/\delta_{c_m}^2 & 0 \\ 0 & 1/\delta_{e_m}^2 \end{bmatrix} \tag{4}$$

to improve the pitch rate to control ratio in the steady state. δ_{c_m} and δ_{e_m} weight, respectively, canard and elevon displacements. It was soon noticed that elevons have sufficient authority to meet pitch performance in the high speeds (i.e. over Mach 0.7), while the canard contribution was more significant at low velocity. This effect was taken into account by gradually increasing the value of δ_{e_m} with Mach number. δ_{c_m} equals 30 all over the flight envelope, δ_{e_m} is 30 at Mach 0.3 so that canards have the same contribution as elevons and increases linearly to 60 at Mach 0.7.

D. LFT modelling

Model (1) with $u = H\delta$ in not in a form that is suitable for LPV/LFT control techniques. But, from a family of linearised models, it is relatively easy to construct LFT models using polynomial interpolation. The basis (h, M, M^2, M^3) was used in a least-squares curve fit algorithm in [9]. In this case, we obtained an LFT model having 2 states, namely α and q, with a parameter block of size 12 ($\Delta = \mathsf{diag}(MI_9, hI_3)$). Figure 2 shows the LFT model against the original pitching moment derivative M_q. Similar plots were generated for the other longitudinal state-space coefficients. The general trend is that around Mach 1 (transonic regime) the LFT model did not capture well the aerodynamic variations. This is because low order polynomials were used. Better approximations could be obtained with higher order polynomials, however, at the expense of larger LFT models.

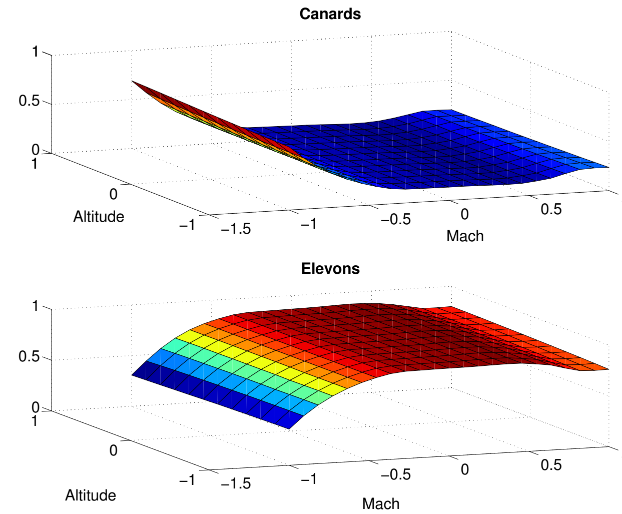

Fig. 1. Relative Control Allocation Function of Mach and Altitude

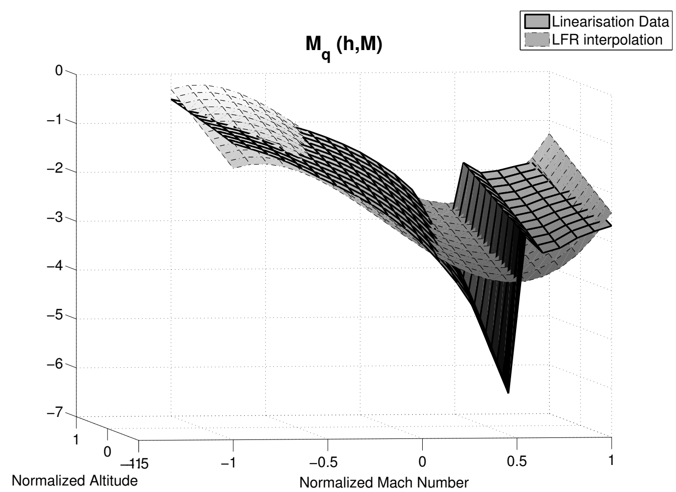

Fig. 2. LFT vs pitching moment derivative M_q

III. SYNTHESIS MODEL AND LPV CONTROLLER SYNTHESIS ALGORITHMS

The design of the LPV controllers is based on the synthesis model of Fig. 3. Load factor handling quality requirements are specified with a second order reference model of constant damping ratio $\xi = 0.8$ and natural frequency $3.5 \leq \omega_n \leq 6.5$ rad/s to translate the fact that the aircraft has better capabilities at high Mach numbers. Second order model characteristics is a standard way to depict aircraft handling qualities based on pilot ratings [10]. It should be noted that the reference model is also an LFT system to account for the natural frequency varying linearly with Mach number. Normal load factor tracking is enforced with the LTI low pass first order filter $W_{perf}(s) = 450 \cdot \frac{0.03 \cdot s+1}{30 \cdot s+1}$. Both elevons and canards were modelled with the same, first order, LTI system. Weights on the control signal (δ) and its time derivative ($\dot{\delta}$) are chosen constant ($W_{de} = 0.08$, $W_e = 0.1$).

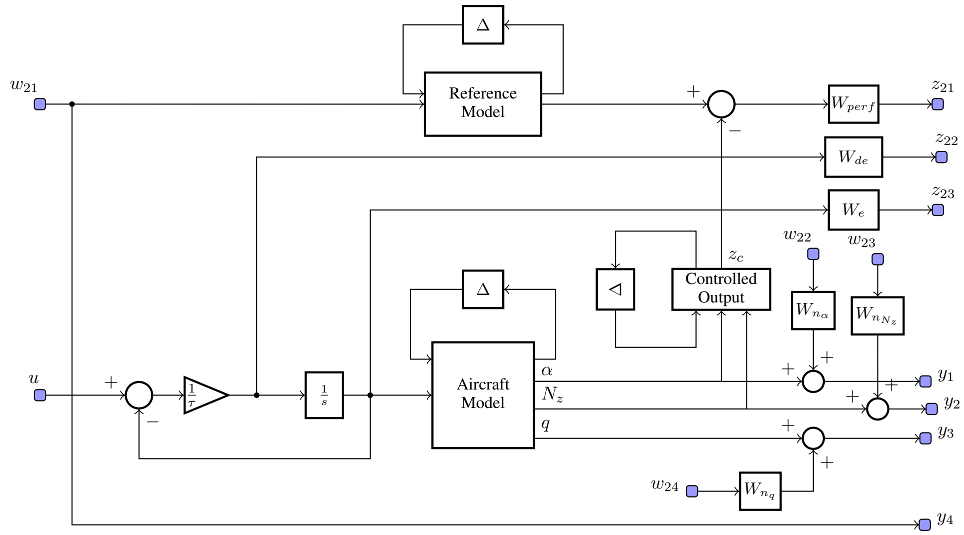

Fig. 3. Synthesis model

These values were chosen so that the inverse does not exceed the maximum performance of the actuators. The weighted command is mixed between canards and elevons according to a function of altitude and Mach (Fig. 1) so that a constant weight accounts for the variation of actuators need. Noise acting at sensors is taken into account with constant weights ($W_{n_\alpha} = 0.15$, $W_{n_z} = 0.05$, $W_{n_q} = 0.05$). Using standard LFT modelling manipulation tools [9], it is easy to construct an LPV/LFT representation of Fig. 3. Such a model takes the form:

$$\left(\begin{array}{c} \dot{x}(t) \\ \hline z_1(t) \\ z_2(t) \\ y(t) \end{array}\right) = \left(\begin{array}{c|ccc} A & B_1 & B_2 & B_3 \\ \hline C_1 & D_1 & D_{12} & D_{13} \\ C_2 & D_{21} & D_2 & D_{23} \\ C_3 & D_{31} & D_{32} & D_3 \end{array}\right) \left(\begin{array}{c} x(t) \\ \hline w_1(t) \\ w_2(t) \\ u(t) \end{array}\right) \tag{5}$$

with

$$w_1(t) = \Delta(t) z_1(t) \ , \ \Delta(t) \in \boldsymbol{\Delta}_e \in \mathbf{R}^{p_1 \times m_1} \tag{6}$$

where $\boldsymbol{\Delta}_e$ is the convex polytope with vertices $\{\Delta_1, \Delta_2, \ldots, \Delta_N\}$ and where closed-loop performance channel is from the error z_2 to the disturbance input w_2.

For ADMIRE, the LPV/LFT synthesis model obtained has 7 states (2 from the short-period longitudinal model, 4 from weights and reference model and 1 from the actuator) and a 14×14 parameter block (a block of dimension 3 for altitude and block of size 11 for Mach).

IV. LPV SYNTHESIS

In this section, the main LPV synthesis algorithms (full-block multipliers [2], scaled bounded real lemma [3] and parameter gridding LPV control design [4]) used to design the LPV controllers for ADMIRE are briefly described.

A. Full Block Multipliers LPV Control

The full block multipliers LPV/LFT controller existence conditions given in [2] are:

Theorem *Consider the LPV model (5)-(6). There exists an LPV/LFT controller such that the L_2 gain performance between z_2 and w_2 is less than or equal to γ if there exist symmetric positive definite matrices X, Y and multipliers P and $\tilde{P}$ satisfying:*

$$\begin{pmatrix} \Delta_i \\ I \end{pmatrix}^T P \begin{pmatrix} \Delta_i \\ I \end{pmatrix} > 0, \ \begin{pmatrix} I \\ -\Delta_i \end{pmatrix}^T \tilde{P} \begin{pmatrix} I \\ -\Delta_i \end{pmatrix} < 0 \ , \ i = 1, \ldots, N \tag{7}$$

where P and $\tilde{P}$ are partitioned according to Δ as

$$P = \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \ \textit{and} \ \tilde{P} = \begin{pmatrix} \tilde{Q} & \tilde{S} \\ \tilde{S}^T & \tilde{R} \end{pmatrix} \tag{8}$$

with

$$Q < 0 \ , \ \tilde{R} > 0 \tag{9}$$

and such that

$$\begin{pmatrix} X & I \\ I & Y \end{pmatrix} \geq 0 \tag{10}$$

$$\Psi^T \begin{pmatrix} * \end{pmatrix}^T \left(\begin{array}{cc|cc|cc} 0 & X & 0 & 0 & 0 & 0 \\ X & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & Q & S & 0 & 0 \\ 0 & 0 & S^T & R & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & -\gamma I & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\gamma} I \end{array}\right) \left(\begin{array}{ccc} I & 0 & 0 \\ A & B_1 & B_2 \\ \hline 0 & I & 0 \\ C_1 & D_1 & D_{12} \\ \hline 0 & 0 & I \\ C_2 & D_{21} & D_2 \end{array}\right) \Psi < 0 \tag{11}$$

$$\Phi^T \begin{pmatrix} * \end{pmatrix}^T \left(\begin{array}{cc|cc|cc} 0 & Y & 0 & 0 & 0 & 0 \\ Y & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & \tilde{Q} & \tilde{S} & 0 & 0 \\ 0 & 0 & \tilde{S}^T & \tilde{R} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & -\gamma I & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\gamma} I \end{array}\right) \left(\begin{array}{ccc} -A^T & -C_1^T & -C_2^T \\ I & 0 & 0 \\ \hline -B_1^T & -D_1^T & -D_{21}^T \\ 0 & I & 0 \\ \hline -B_2^T & -D_{12}^T & -D_2^T \\ 0 & 0 & I \end{array}\right) \Phi > 0 \tag{12}$$

where

$$\Psi = \mathsf{ker} \begin{pmatrix} C_3 & D_{31} & D_{32} \end{pmatrix} \ , \ \Phi = \mathsf{ker} \begin{pmatrix} B_3^T & D_{13}^T & D_{23}^T \end{pmatrix}$$

The LTI part of the LPV/LFT controller can be computed algebraically with the values of the Lyapunov matrices X

TABLE I

LPV SYNTHESIS COMPLEXITY

Method	Variables	LMIs
Scaled BRL [11]	$\sum_{i=1}^{N_p} d_i(d_i+1)+n(n+1)+1$	4
Full Block Multipliers [2]	$(m_1+p_1)(m_1+p_1+1)+n(n+1)+1$	$3+2^{N_p}+2^{N_p}$
Gridding (X(δ),Y(δ)) [4]	$(N_p+1)n(n+1)+1$	$N_{grid}(1+2^{N_p})$
Gridding (X(δ),Y_0) [4]	$(N_p+1)\frac{n(n+1)}{2}+1$	$N_{grid}(1+2^{N_p})$
Gridding (X_0,Y_0, fixed parameter) [4]	$(N_p+1)\frac{n(n+1)}{2}+1$	$2N_{grid}$

and Y and the multipliers P and $\tilde{P}$. The calculations are very similar to those involved in the construction of a central H_∞ controller. One peculiarity of Scherer's Full Block Multipliers LPV controller synthesis algorithms is the non-standard form of the scheduling controller matrix variable Δ_c. It is shown in [2] that the controller scheduling function is

$$\Delta_c(\Delta) = N_- V_-^T (S(\Delta)^T P S(\Delta) - V_- N_- V_-^T)^{-1} V_+(\Delta) \tag{13}$$

where

$$[V_-(\Delta) V_+(\Delta)] = S(\Delta)^T U \ , \ S(\Delta)^T = \left[\ \Delta^T \ \ I \ \right] \tag{14}$$

with matrix U columns form an orthogonal basis of $P-\tilde{P}^{-1}$ and N_- is a negative subspace of $P-\tilde{P}^{-1}$. The reader is referred to [2] for technical details. Note that the size of Δ_c can be different from the original Δ block of the LPV system. But, very often in practice, Δ_c and Δ have same dimensions.

B. Scaled Bounded Real Lemma LPV Synthesis (SBRL)

The SBRL can be viewed as a particular case of a full block multipliers synthesis if one considers $\boldsymbol{\Delta}_e$ a hypercube with $Q=-R$, $S=0$ and where Q is restricted to commute with the parameter structure. Thus, SBRL LPV synthesis is computationally simpler but always produces more conservative results than a full-block multiplier synthesis. Note that an SBRL LPV controller is scheduled with $\Delta_c(\Delta)=\Delta$, where Δ is the parameter block of the augmented plant.

C. LPV synthesis by gridding

This LPV technique is based on an extension of the Bounded Real Lemma to LPV systems [4]. This framework accounts for incorporating parameter time derivative information with parameter dependent Lyapunov functions. If Lyapunov functions are both chosen parameter dependent, then the parameter time derivative is needed in real-time for updating the controller. If only one of the Lyapunov functions is parameter dependent, then one gets more conservative results, but with the advantage that parameter rate is no longer required in real time implementation. The method relies on checking a finite number of LMIs at some points, N_{grid}, in the parameter space [4]. Good L_2 performance is usually obtained on coarse grid. However, a controller designed on a coarse grid may have poor non-linear performance. In such situation, it is necessary to redesign the controller on a denser grid. Thus, LPV synthesis by gridding can be extremely time demanding. Also, real-time implementation of the controller is complicated since, at each sampling instant, one matrix inversion and several matrices multiplications are required.

Table I shows the computational cost associated with the different methods. In this table, n is the dimension of the augmented plant state vector, N_p is the number of parameters, N_{grid} represents the number of points chosen in parameter space and m_1 (resp. p_1) is the number of columns (resp. rows) of Δ. d_i is the size of repeated block i associated to parameter i. For the ADMIRE augmented plant of this paper we have $n=7$, $\Delta=\mathsf{diag}(MI_{11}, hI_3)$ and thus, $N_p=2$, $d_1=11$, $d_2=3$, $p_1=m_1=14$. Typically, $N_{grid} \gg 2^{N_p}$ and thus N_{grid} grows exponentially with the number of parameters. SBRL is by far the less computationally expensive but also it is the most conservative method.

V. DESIGN AND SIMULATIONS RESULTS

A. Scaled BRL controller design

This method is by far the most conservative and failed to give a solution on the full-flight flight envelope. SBRL requires to describe the parameter space by an hypercube and as Fig. 4 suggests only part of the flight envelope suits this description. Thereby a controller was obtained for the flight envelope over Mach 0.7. It should be noted that a robust controller was also found through DK iteration for the same partial flight envelope. A refined description of the parameter space should be employed as can be with the following methods.

B. Full block multipliers controller design

A first synthesis was carried out with the smallest hypercube covering the flight envelope. The L_2 performance level obtained, γ, was about 400, an unacceptable performance level for satisfactory tracking performance. It is well-known that control surface efficiency decreases quickly with speed and air density. In order to achieve good controller design, we had to remove the low speed/high altitude region of the flight envelope in which good tracking cannot be achieved. Removing the problematic flight envelope corner can easily be done in a FBM controller synthesis because the polytope is not restricted to be a hypercube. We considered the 5 vertices polytope of Fig. 4 which excludes the low aircraft performance region. In this case γ dropped from 400 to 4.3.

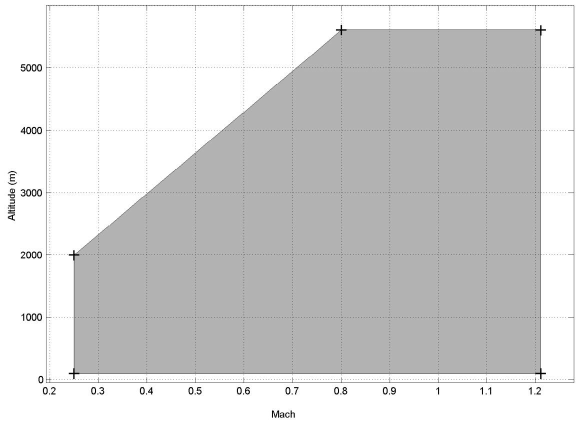

Fig. 4. Polytopic cover for FBM LPV/LFT controller design

In this application, FBM synthesis involved 759 optimization variables and required about 20 seconds of computation time on a 2GHz PC.

C. LPV controller design by gridding

For the parameter gridding approach, we decided that X will vary linearly with altitude and with the square of Mach number whereas Y will remain constant. The weights, reference model and polytope were the same as those used in the previous section. To get a satisfactory controller, providing good non-linear responses, it was necessary to use a grid of more than 206 points. As expected, this led to a huge computational effort; the controller synthesis took about 6200 seconds of CPU time.

D. Non-linear time responses

A discrete version of the FBM controller was implemented as a Simulink C++ mex function. The LTI part of the controller was discretized at 100Hz using Tustin approximation. The scheduling function was computed using (13). A LU factorisation was needed for the inversion of the central term and realized with standard linear algebra C++ routines available in [12]. Finally, a unit time delay was added to the feedback loop to prevent an algebraic loop or a further matrix inversion.

A similar discrete implementation is used for the gridding controller. At each time step, the controller state is computed using basic matrix operations (addition, multiplication and inversion). Discrete dynamics are then obtained via Tustin approximation using a few additional simple matrix operations and another matrix inversion. [13] shows how to reduce implementation complexity. Note that this implementation requires the state-space representation of the synthesis model and of the controller. The controller is implemented with the use of lookup tables and a linear interpolation scheme. As expected, such a control architecture may require a considerable memory space as the controller complexity increases with the size of the synthesis model, the structure of Lyapunov functions and the number of grid points.

Figures 5 and 6 show the non-linear longitudinal time responses obtained with the largest admissible load factor demand, a step of 9g, with full-block multipliers and gridding LPV controllers respectively. In both cases, simulation results were obtained at a starting altitude of 1000m. The 9g manoeuvres were restricted to Mach numbers higher than 0.7. Actually, at speeds below Mach number 0.7 there were not enough engine power to maintain a sufficient airspeed to avoid stall. At Mach 0.7, the aircraft stalls shortly after the 5 second window of the figures. Similar responses were obtained for a 2.4g step demand as shown in Fig. 7 and 8. This time, the Mach envelope was entirely covered. As for the 9g pull-up at Mach 0.7, the manoeuvre performed at Mach 0.4 or at Mach 0.5 makes the aircraft approaching stall.

In all cases, it is observed that both controllers deliver very good performance across the operational flight envelope.

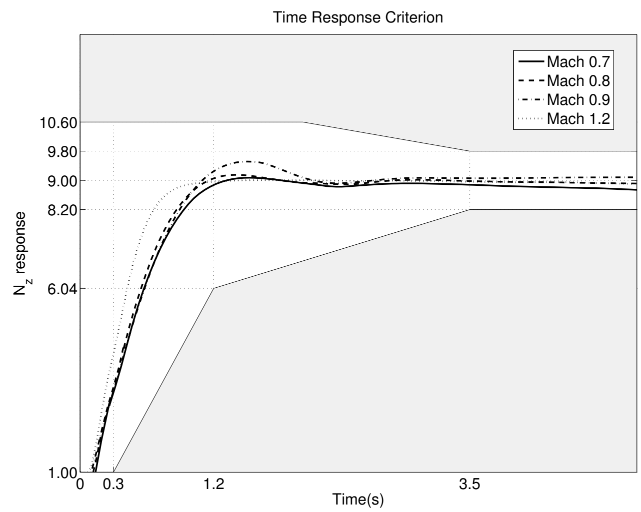

Fig. 5. Nonlinear simulation with FBM LPV/LFT controller: 9g pull-up

VI. Conclusions

Three LPV synthesis algorithms have been used to design a longitudinal controller longitudinal dynamics of the ADMIRE aircraft model. Two proved very successful, namely the full-block multipliers and the LPV gridding controller synthesis techniques. In addition, the latter allow for a general polytopic description of the parameter space which was very useful to achieve good design by avoiding ill-suited parameter combinations.

The Scaled Bounded Real Lemma LPV/LFT synthesis (SBRL), which looks potentially attractive from a computational point of view, did not lead to any satisfactory solution. This is because the multipliers class is too small and assumption on the parameter space (a hyper-rectangle) make SBRL too conservative. In contrast, the gridding method (which is perhaps the most flexible method as it does not require an LFT representation of the plant and allows for parameter dependent Lyapunov functions) may require

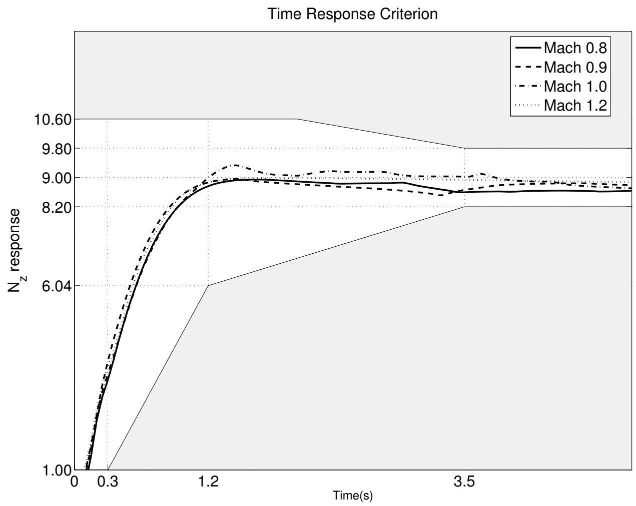

Fig. 6. Nonlinear simulation with LPV gridding controller: 9g pull-up

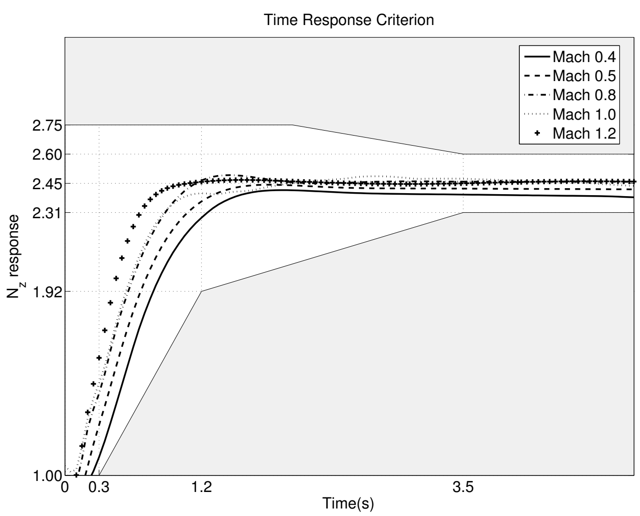

Fig. 7. Nonlinear simulation with FBM LPV/LFT controller: 2.45g pull-up

a considerable amount of points for a successful design. For ADMIRE, we had to use the weighting functions of a previous design (i.e. the full block multiplier controller design) to get a satisfactory controller. Complexity both in terms of implementation (large memory space requirements for look-up tables, matrix operations in real time) and off line computation are perhaps the most serious drawbacks of the gridding approach. On the contrary, the FBM controller synthesis provides a good trade-off between performance, complexity and implementation. Its main drawback is that it needs an LFT model. The derivation of LFT models from a non linear model always introduces parametric uncertainties which are due to fitting errors. The effect of these errors on design and analysis can be difficult to characterize.

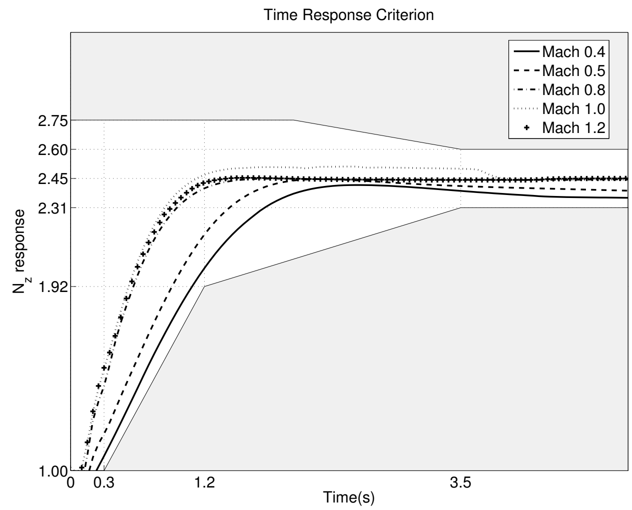

Fig. 8. Nonlinear simulation with LPV gridding controller: 2.45g pull-up

VII. ACKNOWLEDGEMENTS

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