2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

Hierarchical Distributed Model Predictive Control for Risk Mitigation: An Irrigation Canal Case Study

Ascensión Zafra-Cabeza, J. M. Maestre, Miguel A. Ridao, Eduardo F. Camacho and Laura Sánchez

***Abstract*— This paper presents a hierarchical distributed model predictive control approach applied to irrigation canals planning from the point of view of risk mitigation. In the lower control level, a distributed model predictive controller manipulates flows and gate openings in order to follow the water level set-points indicated by the upper control level, which in addition executes mitigation actions if risk occurrences are expected. This work shows how model predictive control can be used as a decision tool which takes into account different types of risks, affecting the operation of irrigation canals.**

I. Introduction

The operation of IC may be affected by many critical factors. These factors can be originated from different causes: political (changes in politics can change the water strategy), operation (water demands fails as forecast, water logging of adjacent land), financial, maintenance (failure in reach or devices due to wear and tear, seepage losses, thefts of sensors) or ecological. Most of these factors are sources of risks that may affect IC performance and should therefore be taken into account. Quantifying these risks and incorporating them into mathematical models of planning and operation may result in improved policies for water systems. In fact, the influence of drought in the performance of water systems have been studied intensively.Risk management (RM) is an area that is attracting a lot of interest from the scientific and industrial community [9]. The objective of RM in engineering systems is to establish risk-based policies to obtain better tradeoffs in safety and productivity.

From the point of view of IC control, many contributions can be found in the literature. There are applications ranging from classical approaches such as PI controllers to Model Predictive Control (MPC) applications [7]. MPC approaches have been widely applied in industry and also in water systems. Nevertheless, MPC is a technique with strong computational requirements which hinder its application to large-scale systems such as water or power networks. For this reason, most large scale control systems are based on a decentralized control architecture; that is, the system is divided into several subsystems, each controlled by a different control agent which may or may not share information with the rest. Each of the agents implements a MPC based on a reduced model of the system and on partial state information, which in general results in an optimization problem with a lower computational burden. In case that the agents communicate in order to obtain a cooperative solution we speak of distributed MPC (DMPC); otherwise the term decentralized MPC is used. Different DMPC schemes can be found in the literature. See [8] for an extensive review on the area. In this paper we will use an algorithm proposed in [5], which is the extension of the scheme [6] whose main feature is that agents can reach a cooperative solution with a low number of communications.

In this paper, a Hierarchical DMPC (HDMPC) approach is used to optimize the operation of IC and the benefits and the costs associated to the risk mitigation actions which can be carried out to reduce the exposure of the identified risks in the operation process. In the top level, a MPC sets the references for the water levels of reaches and determines what preventive actions are necessary. In the lower level, a DMPC distributes the water level regulation problem to control agents located in different geographical regions. The resulting optimization problem is a mixed integer quadratic problem (MIQP) which belongs to the class of NP-complete problems. The objective function is a multicriteria weighted function where the operating costs, demand satisfaction, mitigation actions and control efforts are involved.

The paper is organized as follows. First, a description of irrigation canals modeling is shown. Section III describes the risk model used. The optimization problem for planning is described in Section IV, where the DMPC controller and risk mitigation approach are joint in the objective function of the problem. In order to illustrate the benefits of the method, a simulation model of a IC and a risk structure are used in Section V with different configurations. Finally, some concluding remarks are provided in Section VI.

II. Irrigation Canal modelling for control

The considered system is an open-canal used for water distribution (for irrigation and supply of drinking water), composed of several reaches connected by gates with some reservoirs to store water and for regulation purposes. The dynamics of water flowing in irrigation open canals can be obtained by applying the Saint Venant equations [4], which are nonlinear partial differential equations. Because these equations are very complex to use directly for control, they are often linearized around a set point. First-order systems plus a delay are normally used to model the canal dynamic.

A typical irrigation canal may be divided into several sections separated by gates; the controlled variables are the

Ascensión Zafra-Cabeza, J. M. Maestre, Miguel A. Ridao and Eduardo F. Camacho are with the Department of Automatic and System Engineering of the University of Seville, Spain asun@esi.us.es, pepemaestre@cartuja.us.es, ridao@esi.us.es, eduardo@esi.us.es

Laura Sánchez is with INOCSA S.L.Ingeniería (SPAIN) laura.sanchez@inocsa.es

downstream water levels, $h_i(t) \in \mathbb{R}^+(m)$ and the manipulated variables are the check point to gates, $u_i(t) \in \mathbb{R}^+(m)$.

Each canal reach has an inflow from an upstream canal reach, $Q_{in,i} \in \mathbb{R}^+(m^3/s)$, and an outflow to a downstream canal reach, $Q_{o,i} \in \mathbb{R}^+(m^3/s)$. Also, other flows are considered as perturbation variables:

- $q_{in,i} \in \mathbb{R}^+(m^3/s)$, flows due to rainfall, failures in upstream gate...
- $q_{o,i} \in \mathbb{R}^+(m^3/s)$, known offtake outflows from farmers, considered as measurable perturbations.

The discrete model that has been considered using the previous variables is:

$$\begin{aligned}&A_i(h_i(k+1)-h_i(k)) = T_d(Q_{in,i}(k-t_d)+q_{in,i}(k)\\&-Q_{o,i}(k)-q_{o,i}(k))\end{aligned} \tag{1}$$

where $T_d(s)$ is the length of the sampling time, A_i the surface of the reach and t_d the delay of the input Q_{in} (the level is measured downstream).

The discharge through a submerged flow gate is usually determined [4]:

$$Q_o(t) = C_d L\sqrt{2g}u(t)\sqrt{h_{up}(t)-h_{dn}(t)}, \tag{2}$$

where C_d is the gate discharge coefficient, L is the gate width, $u(t)$ the gate opening and $h_{up}(t), h_{dn}(t)$ the upstream and downstream water levels, respectively.

III. Risk Modelling in IC

In this work, the term *risk* is defined as an event that could take place and cause impacts to some of the units $U = \{U_1 \dots n\}$ that make up the IC system. These units can be maintenance, operation or management departments. In order to define the risk strategy, several elements have to be previously identified:

- Manipulated and controlled variables.
- Process model.
- Operation policies, objectives and priorities.
- Risks that may cause impacts on the system, denoted by the set R.
- Strategic plan to reduce the exposures of the risks through mitigation actions, denoted by the set A.

Consider the set of parameters $Z = \{Z_1, \cdots, Z_{nc}\}$ of parameters that risks can change, with nc the number of parameters. Examples of these parameters can be time delays, demands or economic costs. We define $R = \{R_1, \cdots, R_m\}$ as the set of identified risks for the plant. Each risk R_r is characterized by a probability of occurrence in each time instant $P_r(t)$ and some initial impacts II_{rc}, with $c = 1, \dots, nc$ on the different parameters of the plant. Note that a unit can be influenced by any risk and each risk may have impacts on any parameter. Therefore, risk impacts can change the values of the parameter set Z when they occur and no mitigation action are carried out.

Once risk identification has been performed, the next step to undertake is the design of a strategic mitigation plan. In this way, each risk can be associated with a set of actions (A_i) that could mitigate these risks. We assume the mitigation action set as $A = \{A_1, \cdots, A_p\}$ with p representing the number of mitigation actions. Formally, each mitigation action is described by a set $A_a = \{u_{M_a}, F_a, G_a\}$, where the decision variable for the mitigation action $A : a$ is denoted by u_{M_a}. $F_a = \{f_{ca} : \Re \to \Re\}$ with $c = 1, \dots, nc$ is the set of functions that determine the risk impact reduction as a function of u_{M_a} in each time; thus, f_{ca} is the reduction of the initial impact affecting parameter Z_c when the action (A_a) is applied. Actions that are chosen to mitigate risks may have an associated cost of execution; this feature is modelled by defining functions $G_a = \{g_{ca} : \Re \to \Re\}$ that describes the extra values to be added if action A_a is carried out, also as a function of the corresponding decision variable u_{M_a}.

Figure 2 shows an example of a Risk-Based Structure (RBS) that illustrates the relationship between risks and actions in a possible strategic plan. It can be observed that a unit may be associated with some specific risks (i.e. $Market$ is susceptible to risks R_6, R_7 and R_8); a risk can be mitigated by different actions. In Figure 2 for example, R_1 is mitigated by A_1 and A_2. One action may mitigate different risks; note in Figure 2 how A_6 mitigates R_2 and R_3. Mitigating actions will reduce the initial impact of a risk, but usually, the system will incur additional costs as a result. Even if the impact is stochastic in nature (i.e., assessed only if the risk actually occurs), costs associated with mitigating actions will be incurred regardless. Mitigation action control variable u_{M_a} could either be a continuous ($u_{M_a} \in \Re$) or integer ($u_{M_a} \in \aleph$) variable.

We define $u = [u_o \;\; u_M]$ as the decision variable vector. $u_0(t)$ is the decision vector from the original problem (control variables of the plant) and $u_M = [u_{M_1}, ..., u_{M_p}]$ is the decision variable vector for the mitigation actions.

Taking into account all the previous information about risks, the term denoted by RE and named *Risk Exposure* is defined for each risk. Hence, $RE_{rc}(u_M, t)$ means the exposure of risk R_r affecting parameter Z_c. It takes the form:

$$\begin{aligned}RE_{rc}(u_M,t) = P_r(t)\big(II_{rc} - \textstyle\sum_{a=1}^{p} RA_{r,a} f_{ca}(u_{M_a})\big) + \\ + \textstyle\sum_{a=1}^{p} RA_{r,a} g_{ca}(u_{M_a}),\end{aligned} \tag{3}$$

where $P_r(t)$ is the probability of risk R_r at instant t and II_{rc} denotes the initial impact of risk R_r affecting the parameter Z_c; both of these can be time dependent. The sum of functions f means the reduction of the initial impact by taking actions; $RA_{r,a} = 1$ if risk R_r is mitigated by action A_a and otherwise $RA_{r,a} = 0$. $g_{ca}(u_a)$ is the extra value of mitigation action A_a on the parameter Z_c.

The next section shows how the planning of a IC plant can be carried out taking into account risk management.

IV. HDMPC and Planning of a IC

As it was before mentioned, two control levels are implemented in this approach. In the top level, risk mitigation is introduced to establish the set points of the reaches of the canal and to determine the mitigation actions to be be executed along the time. Hence, cost are optimized and the corrected set points are sent to the low level controller.

MPC has been selected to determine the decision variables. MPC is an optimal control strategy based on the explicit use of a dynamic model to predict the process output at future time instants [3] over a *prediction horizon* (N).

A. Top level: MPC

The objective function considered in this level is to minimize a multicriteria weighted function where the operating costs, mitigation actions and control efforts are involved. The manipulated variables are the mitigation actions to be executed to reduce costs. The index performance J takes into account the previous terms weighted by the variable $\beta = [\beta_1, ..., \beta_5]$:

$$\begin{aligned}&\min_u J = \beta_1 C_{oper}(u,t) \\ &+\beta_2 C_{elec}(u,t) - \beta_3 R_{ev}(u,t) + \beta_4 C_{effort}(u,t)\end{aligned} \tag{4}$$

where

- C_{oper} describes the operation and maintenance costs of the plant per sampling period. Therefore, the expression takes the form:
$$\begin{aligned}&C_{oper}(u,t) = \textstyle\sum_{j=1}^{N}[\hat{C}(t+j|t)+ \\ &\textstyle\sum_{r=1}^{m} RP_{oper,r}(t+j)RE_{oper,r}(u,t+j)]^2,\end{aligned} \tag{5}$$
with $\hat{C}(t+j|t)$, being the predicted cost at instant $(t+j)$. Note that risks can appear modifying the estimated cost. Therefore, the term $RE_{oper,r}(u,t+j)$ models the effect of risk R_r on the cost; it is the risk exposure affecting the operation/maintenance of the units of the plant as consequence of the lifetime. $RP_{oper,r}(t+j) = 1$ indicates that risk R_r can affect cost at time $t+j$; otherwise $RP_{oper,r}(t+j) = 0$. m denotes the total number of risks and N the prediction horizon.
- C_{elec} is the cost associated to energy consumption,
$$C_{elec}(u,t) = \textstyle\sum_{j=1}^{N} P_{KW}(t+j)E(t+j|t), \tag{6}$$
with P_{KW} the estimated price of the KW/h at $(t+j)$ and $E(t+j)$ the energy consumption at instant $(t+j)$.
- R_{ev} are the possible benefits from the sale of the water.
$$\begin{aligned}&R_{ev}(u,t) = \textstyle\sum_{j=1}^{N}[P_{fw}(t+j)W(t+j) \\ &+\textstyle\sum_{r=1}^{m} RP_{rev,r}(t+j)RE_{rev,r}(u,t+j)]\end{aligned} \tag{7}$$
with P_{fw} the contracted sale price of the water and W the water sold. $RE_{rev,r}(u,t+j)$ is the risk exposure due to revenues.
- C_{effort} represents the control effort for the controller.
$$C_{effort} = \textstyle\sum_{j=1}^{N} \triangle u(t+j-1)^2 \tag{8}$$

The output of the problem will depend strongly on the weights of the different terms. Additional terms can be added to the index function in order to incorporate other objectives.

The set points of the level (h_i) in reaches are set as follows:

$$h_i(t) = h_i(t) + \textstyle\sum_{r=1}^{m} RP_{dem,r}(t+j)RE_{dem,r}(u,t+j). \tag{9}$$

The final set points of reaches are sent to the low level.

Because decision variables of mitigation actions may be boolean, the resulting optimization problem is stated as a mixed integer quadratic problem (MIQP).

B. Low Level: Distributed MPC

In this paper we will use a simplified version of the DMPC algorithm presented in [5], which provides a reasonable trade-off between performance and communicational burden. In what follows it is assumed that, for each subsystem, there is an agent that has access to the model and the state of that subsystem. The agents do not have any knowledge of the dynamics of any of its neighbors, but can communicate freely among them in order to reach an agreement.

1) Problem Formulation: We consider the following class of distributed linear systems in which there are M_x subsystems coupled with their neighbors through M_u inputs.

$$x_i(t+1) = A_i x_i(t) + \textstyle\sum_{j\in n_i} B_{ij}u_j(t) \tag{10}$$

where $x_i \in \mathbb{R}^{q_i}$ with $i = 1, \ldots, M_x$ are the states of each subsystem, and $u_j \in \mathbb{R}^{r_j}$ with $j = 1, \ldots, M_u$ are the different inputs. The set of indices n_i indicates the set of inputs u_j which affect the state x_i and the set of indices m_j indicates the set of states x_i affected by the input u_j. Note that equation (10) has the same structure that (1). We consider the following linear constraints in the states and the inputs: $x_i \in \mathcal{X}_i, u_j \in \mathcal{U}_j$, where $\mathcal{X}_i$ and $\mathcal{U}_j$ are closed polyhedra that contain the origin in their interior.

2) DMPC Algorithm: The control objective of the proposed scheme is to minimize a global performance index defined as the sum of each of the local cost functions. The local cost function of agent i based on the predicted trajectories of its state and inputs is defined as

$$J_i(x_i, \{U_j\}_{j\in n_i}) = \textstyle\sum_{k=0}^{N-1} L_i(x_{i,k}, \{u_{j,k}\}_{j\in n_i})$$

where $U_j = \{u_{j,k}\}$ is the future trajectory of input j, N is the prediction horizon, $L_i(\cdot)$ with $i \in M_x$ is the stage cost function defined as

$$\begin{aligned}&L_i(x_i, \{u_j\}_{j\in n_i}) = (x_i - x_{r_i})^T Q_i (x_i - x_{r_i}) \\ &+\textstyle\sum_{j\in n_i} u_j^T S_{ij} u_j\end{aligned}$$

with $Q_i > 0, S_{ij} > 0$. Note that the term x_{r_i} stands for the agent i reference.

We use the notation $x_{i,k}$ to denote the state i, k-steps in the future obtained from the initial state x_i applying the input trajectories defined by $\{U_j\}_{j\in n_i}$.

At the end of the negotiation rounds, the agents decide a set of input trajectories denoted as U^d. The first input of these trajectories is applied and the rest of the values are used to generate the initial proposal U^s for the next sampling time. Note that the last value of these trajectories is repeated so that U^s has the proper size.

We define next the proposed distributed MPC scheme:

- Step 1: Each agent p measures its current state $x_p(t)$. The agents communicate in order to obtain $U^s(t)$ from $U^d(t-1)$. The initial value for the decision control vector $U^d(t)$ is set to the value of the shifted input trajectories, that is, $U^d(t) = U^s(t)$.

- Step 2: Randomly, each agent asks the neighbors affected if they are free to evaluate a proposal (each agent can only evaluate a proposal at the time). If all the neighbors acknowledge the petition, the algorithm continues. If not, the agent waits a random time before trying again. We will use the superscript p to refer to the agent which is granted permission to make a proposal.
- Step 3: In order to make its proposal, agent p solves:

$$
\begin{aligned}
&\{U_j^p(t)\}_{j\in n_p} = \arg\min_{\{U_j\}_{j\in n_p}} J_p(x_p, \{U_j\}_{j\in n_p})\\
&s.t.\\
&x_{p,k+1} = A_p x_{p,k} + \textstyle\sum_{j\in n_p} B_{pj} u_{j,k}\\
&x_{p,0} = x_i(t)\\
&x_{p,k} \in \mathcal{X}_p,\ k = 0, \dots N\\
&u_{j,k} \in \mathcal{U}_j,\ k = 0, \dots N-1,\ \forall j \in n_p\\
&U_j = U_j^d(t),\ \forall j \notin n_{prop}
\end{aligned}
\tag{11}
$$

From the centralized point of view, the proposal at time step t of agent p is defined as

$$U^p(t) = \{U_j^p(t)\}_{j\in n_p} \biguplus U^d(t)$$

where the operation $\biguplus$ stands for the update of the components relatives to $\{U_j^p(t)\}_{j\in n_p}$ in $U^d(t)$.
- Step 4: Each agent i affected by the proposal evaluates the difference between the cost of the new proposal $U^p(t)$ and the cost of the current accepted proposal $U^d(t)$ as

$$
\begin{aligned}
\Delta J_i^p(t) = J_i(x_i(t), \{U_j^p(t)\}_{j\in n_i})\\
-J_i(x_i(t), \{U_j^d(t)\}_{j\in n_i})
\end{aligned}
$$

This difference $\Delta J_i^p(t)$ is sent back to the agent p. If the proposal does not satisfy the constraints of the corresponding local optimization problem, an infinite cost increment is assigned. This implies that unfeasible proposals will never be chosen.
- Step 5: Once agent p receives the local cost increments from each neighbor, it can evaluate the impact of its proposal $\Delta J^p(t)$, which is given by the following expression

$$\Delta J^p(t) = \textstyle\sum_{i\in\bigcup_{j\in n_{prop}} m_j} \Delta J_i^p(t) \tag{12}$$

This global cost increment is used to make a cooperative decision on the future inputs trajectories. If $\Delta J^p(t)$ is negative, the agent will broadcast the update on the control actions involved in the proposal and the joint decision vector $U^d(t)$ will be updated to the value of $U^p(t)$, that is $U^d(t) = U^p(t)$. Else, is discarded.
- Step 6: The algorithm goes back to step 1 until the maximum number of proposals have been made or the sampling time ends. We denote the optimal cost corresponding to the decided inputs as

$$J(t) = \textstyle\sum_{i=1}^{M_x} J_i(x_i(t), \{U_j^d(t)\}_{j\in n_i}) \tag{13}$$

- Step 7: The first input of each optimal sequence in $U^d(t)$ is applied and the procedure is repeated the next sampling time.

V. CASE STUDY

The proposed algorithm will be tested with data of a real system, a section of the 'postrasvase Tajo-Segura' in the South-East of Spain. The 'postrasvase Tajo-Segura' is a set of canals which distribute water coming from the Tajo River in the basin of the Segura River. This water is mainly used for irrigation (78%), although a 22% of it is drinking water. The selected section is a Y-shape canal, a main canal that splits into two canals with a gate placed at the input of each one of them.

- Canal de la Pedrera, the total length of this canal is 6,680 kilometres.
- Canal de Cartagena; in our case-study only a part of this canal is used (17,444 kilometres).

The total length of the canals is approximately of 24 kilometres. At the end of the whole 'Canal de Cartagena' there is a reservoir with limited capacity.

The main elements in the canals are the main gates, which regulate the level of water along the canals, and also the off-take gates, where the farmers take water from the canals for irrigation. There are 7 main gates and 17 off-take gates in the section selected.

Figure 1 shows a description of the gates, the off-take gates, and the milestones where they are located.

Id	Code	Type	P/G	Description	Kilometer
15 Canal del Campo de Cartagena					
				Starting of the canal Campo de Cartagena	0,000
1501	CCMICAR01	Gate	Gravity	Initial gate	0,200
1504	MICAR01	Offtake	Gravity	Offtake 5 -Fuensanta y Estafeta	1,170
1505	MICAR02	Offtake	Gravity	Offtake 5' -Palacete	2,540
1506	MICAR03	Offtake	Pump	Offtake 6 -Santo Domingo	2,840
1507	CCMICAR04	Gate		Gate Canal Pedrera	4,485
1508	MICAR04	Offtake	Pump	Offtake 7 -Campo Salinas	5,970
1509	MICAR05	Offtake	Gravity	Offtake 8 -San Miguel	6,550
1510	MICAR06	Offtake	Gravity	Offtake 9 -Las Cañadas	8,050
1511	MICAR07	Offtake	Gravity	Offtake 10 -San Miguel	9,390
1512	MICAR08	Offtake	Pump	Offtake 11 -Campo Salinas	9,590
1513	CCMICAR05	Gate		Gate Tunel San Miguel	10,480
1514	MICAR09	Offtake	Gravity	Offtake 12 -San Miguel	12,630
1515	MICAR10	Offtake	Pump	Offtake 13 -Campo Salinas	12,780
1516	CCMICAR06	Gate		Gate Rambla La Fayona (start)	14,433
1517	CCMICAR07	Gate		Gate Rambla La Fayona (end)	14,579
1518	MICAR11	Offtake	Pump	Offtake14 -Villamartín	16,540
1519	CCMICAR08	Gate		Gate Cañada La Estacada	17,444
16 Canal de La Pedrera					
1601	CCMIPED01	Gate		Starting of the canal La Pedrera	0,000
1602	MIPED01	Offtake	Gravity	Offtake 1P -Santo Domingo	0,770
1603	MIPED02	Offtake	Gravity	Offtake 2P -Santo Domingo y Mengoloma	3,740
1604	MIPED03	Offtake	Pump	Offtake 3P -Santo Domingo	4,260
1605	MIPED04	Offtake	Gravity	Offtake Riegos Levante 1	5,260
1606	MIPED05	Offtake	Gravity	Offtake 4P -Santo Domingo	6,440
1607	MIPED06	Offtake	Gravity	Offtake Riegos Levante 2 y 3	6,680

Fig. 1. Data of the canals.

The main target is to manage the water in the canals in order to guarantee flows requested by users. To this end, it is necessary to maintain the level of the canal over the off-take gate when flow is requested. The controlled variables are the upstream levels beside the gates. There are maximum and minimum level constraints regarding these variables. The manipulated variables are the flow at the head of the canal and the position of the gates. There is a constraint on the flow at the head: the total amount of water over a determined time period is limited. There are also physical constraints regarding the gates: maximum and minimum openings. In addition, the level of the reservoir at the end of the Canal of Cartagena must be maintained between minimum and maximum operating limits. Another objective to be considered is the minimization of the leaks and evaporation (function of the levels) and also to minimize maintenance costs (the maintenance of concrete blocks and junctions is better if they are submerged, so high levels are preferred for that purpose). The goal of the optimization process is to minimize the energy consumption and operation costs by satisfying an estimated water demand. The model of the plant and the index performance are taken from equation (1) and (4), respectively.

A. Risks and actions identification

A number of potential risks can be encountered during the operation of the IC system. Table I shows the risks that have been considered in this example by considering the above described system. Some risks have been taken from [1] and [2]. Initial impacts (II) are expressed on the parameters $Z = \{Z_1, Z_2\}$, with Z_1 being the cost (euros) and Z_2 the water demand.

Risks R_1, R_4, R_5, R_6 and R_7 have an impact on the cost per quarter; on the other hand, R_2 and R_3 may change the initial estimated water level reference. The last column of the table corresponds to the probability of the risk. The probability of R_2, R_3 and R_6 changes with time. Hence, the function probability of R_2 is higher during the summer season and the probability of R_3 depends on the forecast for the city of Murcia. The risk R_6 has been established as the possibility that changes in government modify the water strategy for the plant.

The description of the actions used to mitigate risks is shown in Table II. There are five mitigation actions and therefore, five additional control variables ($\{u_{M_1}, \cdots, u_{M_5}\}$), where u_{M_5} is real and the rest are boolean. Note that functions f and g are described in this table and some of them depend proportionally on the impacts. The execution of the mitigation actions are carried out every three months. In this period, the mean values of the risks are considered for mitigation. The risk-based structure with the links between risks and actions is shown in Fig. 2.

B. Results and discussion

The results that are presented aims to a hypothetic prices and costs. The results of the MIQP have been obtained using the commercial solver Cplex.

First, the outcomes of the top controller are shown. The study period has been set to 365 days, the sampling time 1 day and the horizon $N = 5$. The weight vector have been set to $\beta = [1\ 0\ 0\ 1]$. The rainfall forecast of the city of Murcia and the discharge of farmers have been considered along the 2009 year. According that, the initial set points of the level of reaches are modified. Figure 3 shows the initial level

TABLE II

MITIGATION ACTIONS DESCRIPTION.

Ac	Description	f_{1i}, g_{1i} on Z_1(cost)	u_{M_i}
A_1	Periodic water analysis .	$f_{11} = 0.7II_1 u_{M_1}$, $g_{11} = 1400u_{M_1}$	B
A_2	Control weed growth	$f_{12} = 0.3II_1 u_{M_2}$, $g_{12} = 1500u_{M_2}$	B
A_3	Appropriate monitoring over devices	$f_{13} = II_{13} u_{M_3}$, $g_{13} = 2800u_{M_3}$	B
A_4	Lining Irrigation Canal	$f_{14} = 0.95II_{15} u_{M_4}$, $g_{14} = 50000u_{M_4}$	B
A_5	Insurance policy	$f_{15} = 175u_{M_5}$, $g_{15} = u_{M_5}$	R
A_6	Modify set-points of water levels	$f_{16} = 0$,$g_{16} = 0$	R

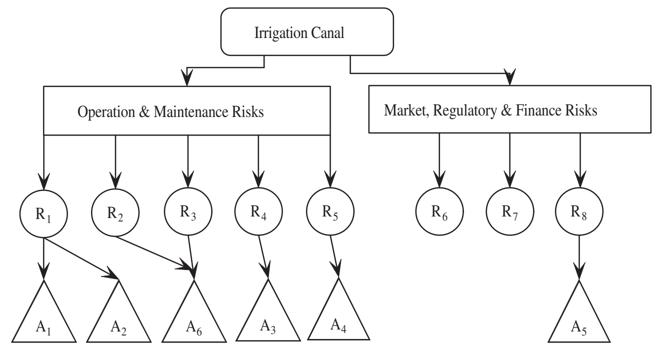

Fig. 2. Risk-based structure for the case study.

(dotted green line) that is modified by R_2 and R_3, giving rise to an actual level reference depicted by the solid red line. Note that in summer reason the level is increased due to farmers may demand more water as consequence of the drought.

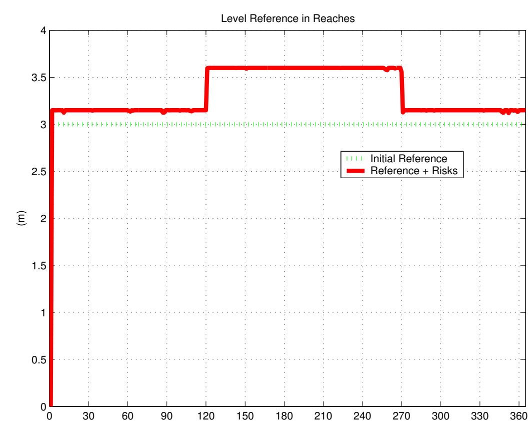

Fig. 3. Level references in reaches by considering risks.

Besides getting the desired setpoints, the method reduces the effects of the impacts in the cost of the plant. Figure 4 shows the associated costs to operation and maintenance, C_{oper} (see equation 5). Note how the no risks option always presents the lowest cost and the no mitigation option reflects the highest cost. As expected, the proposed cost is between the no risks and no mitigation lines. To reach these costs,

TABLE I

RISK DESCRIPTION (CASE STUDY)

Risk	Description	Impacts Cost(euros/quarter)/ Demand	$P_r(t)$
	Operation & Maintenance Risks		
R_1	Non adequate fresh water quality.	$II_{11} = 8{\cdot}10^4$/ $II_{12} = 0$	0.1
R_2	Farmers water demand fails to keep as forecast	$II_{22} = 0$/ $II_{22} = +0.15W_{FD}$	$P_2(t)$
R_3	Rainfalls changes water level of canal, producing water logging of adjacent lands	$II_{31} = 12000$/ $II_{32} = 0$	$P_3(t)$
R_4	Failure in devices due to wear and tear	$II_{41} = 10000$/ $II_{42} = 0$	0.5
R_5	Seepage losses	$II_{51} = 3000$/ $II_{52} = 0$	0.1
	Market, Regulatory & Finance Risks		
R_6	Changes in politics modify the strategy	$II_{61} = 10000$ $II_{62} = 0$	$P_6(t)$
R_7	State policies provide incentives for IC systems	$II_{71} = -200000$/ $II_{72} = 0$	0.01
R_8	Uninsured events of force majeure	$II_{81} = 6{\cdot}10^5$/ $II_{82} = 0$	0.01

mitigation actions to be executed and instants to be launched are shown in Figure 5. Notice that mitigation actions are undertaken every 3 months. The mitigation actions are chosen by considering the probabilities of risk with time. Action A_4 is never executed due to the fact the impact of R_5 and its probability is lower than the cost of the action. All the actions are boolean, except A_5 (insurance).

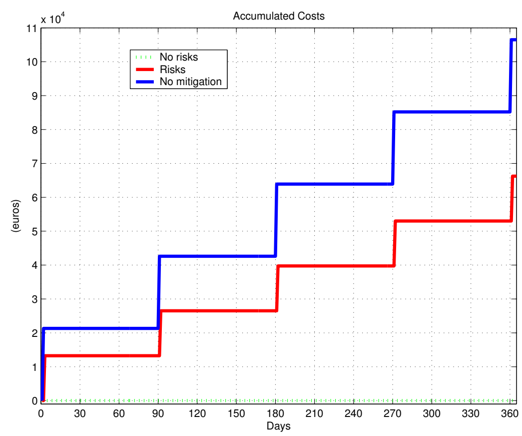

Fig. 4. Optimization of the cost by considering risks.

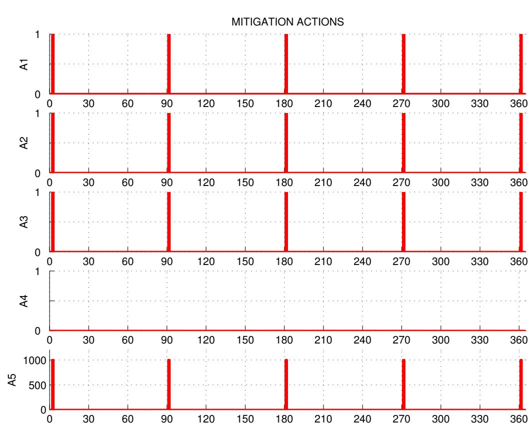

Fig. 5. Mitigation actions to be undertaken to reduce risks impacts.

If reference changes, this controller sends the modifications to the DMPC in the lower level. For this controller, the sample time has been considered 1 minute and the prediction horizon, N_p, has been set to 5 plus the delay time. The weights of the local costs in the canals grow with 2^i, that is, the farther a node is from the beginning , the more important is. This way of weighting the error facilitates a faster flow of water towards the last canals. Finally, the matrix that weights the control effort S_i has been set to zero for simplicity.

VI. SUMMARY AND CONCLUSIONS

This paper describes a control-based methodology for decision-making in irrigation canals to address prevention and control problems in the plant. The objective is to optimize the operation of the system, taking into account explicitly modelled risks that can be identified prior to the planning. Finally, the distributed approach at low level simplifies the implementation of the scheme in real world applications.

VII. ACKNOWLEDGEMENTS

This research has been supported by the European 7th framework STREP project ‘Hierarchical and distributed model predictive control (HD-MPC)’, contract number INFSO-ICT-223854.

REFERENCES

[1] S. Baggett, P. Jeffrey, and B. Jefferson. Risk perception in participatory planning for water reuse. *Desalination*, 187:149–158, 2006.

[2] Susan Baines, Kumar Sinha, and Kathi Kirschchenheiter. Risk mitigation by managing water treatment system interfaces in the power industry. In *International Water Conference*, 2004.

[3] Eduardo F. Camacho and Carlos Bordons. *Model Predictive Control*. 2nd Edition. Springer-Verlag, London, 2004.

[4] V. T. Chow. *Open-Channel Hydraulics*. Mc-Graw-Hill Book, 1859.

[5] J. M. Maestre, D. Muñoz de la Peña, E. F. Camacho, and T. Alamo. Distributed model predictive control based on agent negotiation. *Journal of Process Control*, 2011.

[6] J. M. Maestre, David Muñoz de la Peña, and E. F. Camacho. Distributed model predictive control based on a cooperative game. *Optimal Control Applications and Methods*, 32:153–176, 2011.

[7] Rudy R. Negenborn, Peter-Jules van Overloop, Tamas Keviczky, and Bart de Schutter. Distributed model predictive control of irrigation canals. *Networks and Heterogeneous Media*, 4(2):359–380, 2009.

[8] R. Scattolini. Architectures for distributed and hierarchical model predictive control - a review. *Journal of Process Control*, 19:723–731, 2009.

[9] A. Zafra-Cabeza, M.A. Ridao, and E.F. Camacho. Applying risk management to combined heat and power plants. *IEEE Transactions on Power Systems*, 23:938–945, 2008.