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Adaptive Control of Piecewise Linear Systems with Applications to NASA GTM

Qian Sang and *Gang Tao*
Department of Electrical and Computer Engineering
University of Virginia
Charlottesville, VA 22904

***Abstract*—Nonlinear plants with their operating range covering multiple trim points are modeled as piecewise linear systems, where variations in operating points are modeled as switches between constituent linearized system dynamics. The adaptive state feedback for state tracking control problem for such systems is studied, for which piecewise linear reference model systems are used to generate desired state trajectories. Adaptive control schemes are developed, and it is proved that asymptotic tracking performance can be achieved if the reference input is sufficiently rich and the switches are sufficiently slow. Stability and tracking performance of the proposed adaptive control schemes are analyzed and evaluated on the high-fidelity Simulink model of the Generic Transport Model (GTM) developed at NASA Langley.**

I. INTRODUCTION

Many systems encountered in practice exhibit highly nonlinear behaviors, e.g., aircraft flight control systems. A practical way to control a nonlinear system is to design a linear controller based on a linearized model of the nonlinear system at some operating point [2], [4], [7]. Often this linearized model is taken to be time-invariant, which requires the nonlinear system to operate in a region around the operating point such that the linearized model is a valid approximation of the original nonlinear system. This requirement limits the applicability of a linearization-based design, and motivates the use of piecewise linear models of nonlinear systems for control design to expand system operating range. The piecewise linear system consists of a set of linear time-invariant (LTI) subsystems, each being a valid model of the nonlinear system within a neighborhood of an operating point. These neighborhoods are pieced together such that the operating range of interest is covered, and transitions of operating points are modeled as "switches" between the corresponding subsystems.

Not much effort has been made in the literature towards developing adaptive control strategies for such systems. An adaptive control scheme was presented in [1] for bimodal piecewise linear systems. However, the assumption of the system in canonical forms may limit its applicability. In this paper, we assume that the states of the controlled system are available for measurement, and study the state tracking control problem. A reference system is specified to generate the desired state trajectory. In conventional adaptive control [6], [13], the reference system is usually chosen to be LTI, for which such a control problem has been studied and closed-loop stability and asymptotic tracking was proved in [3]. However, an LTI reference system imposes stringent structural requirements on the constituent subsystems of the piecewise linear system; that is, each subsystem has to match the same LTI reference model through some (unknown) nominal controller parameters, which may not be feasible in practical applications. Thus it may be more realistic to specify an LTI reference model for each subsystem based on the knowledge of its desired behavior at that operating point. Stability of such a time-varying reference model system is studied based on the properties of switched linear systems [5], [8], [9]. A new adaptive state feedback controller structure is proposed for control of the piecewise linear systems. It can be proved that with sufficiently rich reference input signals and sufficiently slow switches, asymptotic state tracking can be achieved, in addition to stability.

The paper is organized as follows. In Section II, the state feedback for state tracking control problem for piecewise linear systems is formulated. Stability properties of a piecewise linear reference model system is studied in Section III. Adaptive state feedback control designs are proposed in Section IV, along with simulation results in Section V demonstrating their effectiveness on control of the NASA GTM [10]. Some concluding remarks are given in Section VI.

II. PROBLEM STATEMENT

The state feedback for state tracking control problem is formulated for piecewise linear systems. To motivate such a control problem, we first present the linearization and approximation of a nonlinear system at multiple operating points by a piecewise linear system.

A. *Linearization and Piecewise Linear System Model*

Consider a nonlinear system

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t)), \tag{1}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$ is the state vector, $\boldsymbol{u}(t) \in \mathbb{R}^m$ is the control input vector, and $\boldsymbol{f}(\cdot)$ is an n-dimensional nonlinear vector function. Let $\boldsymbol{\Omega} \subset \mathbb{R}^{n+m}$ be the region of interest for all possible system state and control vector $(\boldsymbol{x}, \boldsymbol{u})$, and denote $\boldsymbol{x}_{0i}$ and $\boldsymbol{u}_{0i}$, $i \in \mathcal{I} \triangleq \{1, 2, \ldots, l\}$, as a set of *equilibrium* operating points located at some representative (and properly separated) points inside $\boldsymbol{\Omega}$. Introduce a set of l regions $\boldsymbol{\Omega}_i$ centered at the chosen operating points $(\boldsymbol{x}_{0i}, \boldsymbol{u}_{0i})$, and denote their interiors as $\boldsymbol{\Omega}_{i0}$, $i \in \mathcal{I}$, such that $\boldsymbol{\Omega}_{j0} \cap \boldsymbol{\Omega}_{k0} = \{\emptyset\}$ for all $j \neq k$, and $\cup_{i=1}^{l} \boldsymbol{\Omega}_i = \boldsymbol{\Omega}$. With $\boldsymbol{x}_i(t) = \boldsymbol{x}(t) - \boldsymbol{x}_{0i}$

and $\boldsymbol{u}_i(t) = \boldsymbol{u}(t) - \boldsymbol{u}_{0i}$, a set of linear time-invariant system models can be obtained, i.e., for $i \in \mathcal{I}$, we have

$$\dot{\boldsymbol{x}}_i(t) = \boldsymbol{A}_i \boldsymbol{x}_i(t) + \boldsymbol{B}_i \boldsymbol{u}_i(t), \quad (\boldsymbol{x}(t), \boldsymbol{u}(t)) \in \boldsymbol{\Omega}_i \tag{2}$$

with $\boldsymbol{A}_i = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\Big|_{(\boldsymbol{x}_{0i}, \boldsymbol{u}_{0i})}$, $\boldsymbol{B}_i = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{u}}\Big|_{(\boldsymbol{x}_{0i}, \boldsymbol{u}_{0i})}$. Note that at each time instant t, $(\boldsymbol{x}(t), \boldsymbol{u}(t))$ belongs to only one $\boldsymbol{\Omega}_i$.

To formulate a piecewise linear system model for the nonlinear system (1), we rewrite (2) as

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_i \boldsymbol{x}(t) + \boldsymbol{B}_i \boldsymbol{u}(t) - \boldsymbol{A}_i \boldsymbol{x}_{0i} - \boldsymbol{B}_i \boldsymbol{u}_{0i}$$

for $(\boldsymbol{x}(t), \boldsymbol{u}(t)) \in \boldsymbol{\Omega}_i$, leading to

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}(t)\boldsymbol{x}(t) + \boldsymbol{B}(t)\boldsymbol{u}(t) - \boldsymbol{A}(t)\boldsymbol{x}_0(t) - \boldsymbol{B}(t)\boldsymbol{u}_0(t) \tag{3}$$

where $\boldsymbol{A}(t) = \boldsymbol{A}_i$, $\boldsymbol{B}(t) = \boldsymbol{B}_i$, $\boldsymbol{x}_0(t) = \boldsymbol{x}_{0i}$, $\boldsymbol{u}_0(t) = \boldsymbol{u}_{0i}$ for $(\boldsymbol{x}(t), \boldsymbol{u}(t)) \in \boldsymbol{\Omega}_i$. Note that $\boldsymbol{x}(t)$ is a "global" (instead of perturbed) state vector which is continuous and $\boldsymbol{u}(t)$ is a control input signal to be generated from a control law based on the piecewise linear system (3).

It can be seen that the parameters in $\boldsymbol{A}(t)$ and $\boldsymbol{B}(t)$ vary in a piecewise constant pattern; that is, during different time periods, $(\boldsymbol{A}(t), \boldsymbol{B}(t))$ take on different values as specified by the parameter matrix sets $(\boldsymbol{A}_i, \boldsymbol{B}_i)$, where $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$ are unknown but constant parameter matrices representing the controlled plant dynamics at different operating points. To characterize such parameter discontinuities and for a simple notation, we introduce the *indicator functions*:

$$\chi_i(t) = \begin{cases} 1, & \text{if } (\boldsymbol{x}(t), \boldsymbol{u}(t)) \in \boldsymbol{\Omega}_i, \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

It follows that $\sum_{i=1}^{l} \chi_i(t) = 1$, $\chi_j(t)\chi_k(t) = 0$, $j \neq k$. Here we assume that the common boundary of two regions belongs to only one of them. The indicator functions contain knowledge of the durations of time of the parameter matrix set $(\boldsymbol{A}(t), \boldsymbol{B}(t))$ assumes and the time instants at which $(\boldsymbol{A}(t), \boldsymbol{B}(t))$ changes (switches) to another, which is useful for adaptive control design.

With the indicator functions $\chi_i(t)$, $i \in \mathcal{I}$, the piecewise linear system model becomes (3) with $\boldsymbol{A}(t) = \sum_{i=1}^{l} \boldsymbol{A}_i \chi_i(t)$ and $\boldsymbol{B}(t) = \sum_{i=1}^{l} \boldsymbol{B}_i \chi_i(t)$, where $\boldsymbol{A}_i$, $\boldsymbol{B}_i$ are unknown, while the indicator functions $\chi_i(t)$, defined in (4), are known because the information about $(\boldsymbol{x}(t), \boldsymbol{u}(t)) \in \boldsymbol{\Omega}_i$ is available.

B. Problem Statement

In this paper, the following multiple-input, multiple-state piecewise linear system is considered

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}(t)\boldsymbol{x}(t) + \boldsymbol{B}(t)\boldsymbol{u}(t) + \boldsymbol{c}(t), \tag{5}$$

where $\boldsymbol{A}(t) = \sum_{i=1}^{l} \boldsymbol{A}_i \chi_i(t)$, $\boldsymbol{B}(t) = \sum_{i=1}^{l} \boldsymbol{B}_i \chi_i(t)$, $\boldsymbol{c}(t) = \sum_{i=1}^{l} \boldsymbol{c}_i \chi_i(t)$ with unknown $\boldsymbol{A}_i$, $\boldsymbol{B}_i$, $\boldsymbol{c}_i = -\boldsymbol{A}_i \boldsymbol{x}_{0i} - \boldsymbol{B}_i \boldsymbol{u}_{0i}$, and $\boldsymbol{x}(t)$, $\boldsymbol{u}(t)$, $\boldsymbol{x}_{0i}$, $\boldsymbol{u}_{0i}$ defined as in Section II-A. The triple $(\boldsymbol{A}_i, \boldsymbol{B}_i, \boldsymbol{c}_i)$ is called a *mode* of (5), and when it is active (as indicated by $\chi_i(t) = 1$), (5) is said to be *operating in the ith mode*. For a basic study of the adaptive control problem for (5), we assume there are no internally forced switches, i.e., $\chi_i(t)$ in (4) does not depend on $(\boldsymbol{x}, \boldsymbol{u})$.

The control objective is to develop a state feedback control law for the plant (5) such that all the signals in the closed-loop system are bounded, and $\boldsymbol{x}(t)$ asymptotically track a reference trajectory $\boldsymbol{x}_m(t)$, i.e., $\lim_{t\to\infty}(\boldsymbol{x}(t) - \boldsymbol{x}_m(t)) = 0$, where $\boldsymbol{x}_m(t)$ is generated from a reference model system to be specified in the next section.

III. Time-Varying Reference Model Systems

A reference trajectory $\boldsymbol{x}_m(t)$ should be specified representing the desired system behaviors at each operating point and the transitions in between. Such a reference trajectory may be specified locally for each operating point, which are then pieced together to form $\boldsymbol{x}_m(t)$.

A. Piecewise Linear Reference Model System

It is natural and practical to specify a reference model for each operating point, resulting in a set of linear time-invariant reference systems

$$\dot{\boldsymbol{x}}_{mi}(t) = \boldsymbol{A}_{mi} \boldsymbol{x}_{mi}(t) + \boldsymbol{B}_{mi} \boldsymbol{r}(t), \tag{6}$$

where $\boldsymbol{r}(t) \in \mathbb{R}^m$ is a bounded piecewise continuous reference input signal, and the parameter matrices $\boldsymbol{A}_{mi} \in \mathbb{R}^{n\times n}$, $\boldsymbol{B}_{mi} \in \mathbb{R}^n$, $i \in \mathcal{I}$, are chosen with $\boldsymbol{A}_{mi}$ stable. When the plant is operating at the ith mode, the state trajectory $\boldsymbol{x}_{mi}(t)$ is desirable for the perturbed state $\boldsymbol{x}_i(t)$ to follow. To form a "global" reference trajectory for $\boldsymbol{x}(t)$ to track, with $\boldsymbol{x}_m(t) \triangleq \boldsymbol{x}_{mi}(t) + \boldsymbol{x}_{0i}$ for $(\boldsymbol{x}, \boldsymbol{u}) \in \boldsymbol{\Omega}_i$, the piecewise linear reference model system for (5) is

$$\dot{\boldsymbol{x}}_m(t) = \boldsymbol{A}_m(t)\boldsymbol{x}_m(t) + \boldsymbol{B}_m(t)\boldsymbol{r}(t) + \boldsymbol{c}_m(t), \tag{7}$$

where $\boldsymbol{A}_m(t) = \sum_{i=1}^{l} \boldsymbol{A}_{mi}\chi_i(t)$, $\boldsymbol{B}_m(t) = \sum_{i=1}^{l} \boldsymbol{B}_{mi}\chi_i(t)$, and $\boldsymbol{c}_m(t) = \sum_{i=1}^{l} \boldsymbol{c}_{mi}\chi_i(t)$ with $\boldsymbol{c}_{mi} = -\boldsymbol{A}_{mi}\boldsymbol{x}_{0i}$. Note here we require $\boldsymbol{x}_m(t)$ to be continuous, which is a meaningful reference trajectory for the continuous state vector $\boldsymbol{x}(t)$ to follow. This implies a (perturbed) reference state resetting whenever a mode switch from the ith to the jth mode occurs at a time instant t such that $\boldsymbol{x}_{mj}(t) + \boldsymbol{x}_{0j} = \boldsymbol{x}_{mi}(t^-) + \boldsymbol{x}_{0i}$.

B. Stability of the Reference Model System

The stability properties of the reference model system in (7) have been studied in [5], [8], [9] without considering the dynamics offset $\boldsymbol{c}_m(t)$. Following a similar line of arguments and derivations, it can be proved that the exponential stability of its homogeneous system implies stability of (7). Let the strictly increasing sequence $\{t_k\}_{k=1}^{\infty}$ denote the switching time instants, $T_0 \triangleq \min_k(t_k - t_{k-1})$, and $\boldsymbol{P}_i$, $\boldsymbol{Q}_i \in \mathbb{R}^{n\times n}$ be symmetric, positive definite satisfying

$$\boldsymbol{P}_i \boldsymbol{A}_{mi} + \boldsymbol{A}_{mi}^{\top} \boldsymbol{P}_i = -\boldsymbol{Q}_i, \quad i \in \mathcal{I}. \tag{8}$$

Due to the stability of $\boldsymbol{A}_{mi}$, there exist $a_{mi}, \lambda_{mi} > 0$ such that $\|e^{\boldsymbol{A}_{mi}t}\| \le a_{mi}e^{-\lambda_{mi}t}$. Define $a_m = \max_{i\in\mathcal{I}} a_{mi}$, $\lambda_m = \min_{i\in\mathcal{I}} \lambda_{mi}$, $\alpha = \max_{i\in\mathcal{I}} \lambda_{\max}[\boldsymbol{P}_{mi}]$, $\beta = \min_{i\in\mathcal{I}} \lambda_{\min}[\boldsymbol{P}_{mi}]$, with $\lambda_{\min}[\cdot]$ and $\lambda_{\max}[\cdot]$ denoting the minimum and maximum eigenvalues of a matrix. The following lemma gives a lower bound on T_0 that ensures exponential stability of its homogeneous system, thus stability of (7) [12]:

Lemma 1. The homogeneous system of (7) is exponentially stable with decay rate $\sigma \in (0, 1/2\alpha)$ if T_0 is such that

$$T_0 \geq \frac{\alpha}{1-2\sigma\alpha} \ln(1+\mu\Delta_{\boldsymbol{A}_m}),\ \mu = \frac{a_m^2}{\lambda_m \beta} \max_{i\in\mathcal{I}} \|\boldsymbol{P}_{mi}\|, \quad (9)$$

where $\Delta_{\boldsymbol{A}_m}$ stands for the largest difference between any two modes of $\boldsymbol{A}_m(t)$, i.e., $\Delta_{\boldsymbol{A}_m} = \max_{i,j\in\mathcal{I}} \|\boldsymbol{A}_{mi} - \boldsymbol{A}_{mj}\|$.

Remark 1: When all the constituent reference models $\boldsymbol{A}_{mi}$ are the same, or if there exists a common Lyapunov matrix $\boldsymbol{P}$ such that $\boldsymbol{P}\boldsymbol{A}_{mi} + \boldsymbol{A}_{mi}^{\mathsf{T}}\boldsymbol{P} < \boldsymbol{0}$, the stability of (7) follows for arbitrarily fast switches. □

IV. Adaptive Control Design

A new state feedback controller structure is proposed for the piecewise linear system (5) to achieve the control objective.

Assumptions. The following assumptions are made, $\forall i \in \mathcal{I}$:

(A1) There exist constant matrices $\boldsymbol{K}_{xi}^* \in \mathbb{R}^{n\times m}$ and $\boldsymbol{K}_{ri}^* \in \mathbb{R}^{m\times m}$ with $\boldsymbol{K}_{ri}^*$ nonsingular such that

$$\boldsymbol{A}_{mi} = \boldsymbol{A}_i + \boldsymbol{B}_i \boldsymbol{K}_{xi}^{*\mathsf{T}},\ \ \boldsymbol{B}_{mi} = \boldsymbol{B}_i \boldsymbol{K}_{ri}^*. \quad (10)$$

(A2.a) There is a known matrix $\boldsymbol{S}_i \in \mathbb{R}^{m\times m}$ such that $\boldsymbol{K}_{ri}^*\boldsymbol{S}_i$ is symmetric and positive definite.

A. *Controller Structure and Error Model*

If the plant parameter matrices $\boldsymbol{A}_i$, $\boldsymbol{B}_i$ were known, the nominal control law

$$\boldsymbol{u}(t) = \boldsymbol{u}_{0i}(t) + \boldsymbol{K}_x^{*\mathsf{T}}(t)\Delta\boldsymbol{x}(t) + \boldsymbol{K}_r^*(t)\boldsymbol{r}(t), \quad (11)$$

where $\boldsymbol{K}_x^*(t) = \sum_{i=1}^l \boldsymbol{K}_{xi}^*\chi_i(t)$, $\boldsymbol{K}_r^*(t) = \sum_{i=1}^l \boldsymbol{K}_{ri}^*\chi_i(t)$, $\Delta\boldsymbol{x}(t) = \boldsymbol{x}(t) - \boldsymbol{x}_{0i}(t)$ with $\boldsymbol{x}_{0i}(t) = \sum_{i=1}^l \boldsymbol{x}_{0i}\chi_i(t)$ and $\boldsymbol{u}_{0i}(t) = \sum_{i=1}^l \boldsymbol{u}_{0i}\chi_i(t)$, leads to the tracking error dynamics $\dot{\boldsymbol{e}}(t) = \boldsymbol{A}_m(t)\boldsymbol{e}(t)$ with $\boldsymbol{e}(t) = \boldsymbol{x}(t) - \boldsymbol{x}_m(t)$ converging to zero exponentially if T_0 satisfies (9).

When $\boldsymbol{A}_i$, $\boldsymbol{B}_i$ are unknown, (11) cannot be implemented, and we propose the following adaptive controller structure

$$\boldsymbol{u}(t) = \boldsymbol{u}_{0i}(t) + \boldsymbol{K}_x^{\mathsf{T}}(t)\Delta\boldsymbol{x}(t) + \boldsymbol{K}_r(t)\boldsymbol{r}(t), \quad (12)$$

where $\boldsymbol{K}_x(t) = \sum_{i=1}^l \boldsymbol{K}_{xi}(t)\chi_i(t)$, $\boldsymbol{K}_r(t) = \sum_{i=1}^l \boldsymbol{K}_{ri}(t)\chi_i(t)$ are the time-varying estimates of the nominal controller parameters $\boldsymbol{K}_x^*(t)$ and $\boldsymbol{K}_r^*(t)$, respectively. This control law leads to the error model

$$\dot{\boldsymbol{e}} = \sum_{i=1}^l \left(\boldsymbol{A}_{mi}\chi_i\boldsymbol{e} + \boldsymbol{B}_{mi}\boldsymbol{K}_{ri}^{*-1}\chi_i\left(\tilde{\boldsymbol{K}}_{xi}^{\mathsf{T}}\Delta\boldsymbol{x} + \tilde{\boldsymbol{K}}_{ri}\boldsymbol{r}\right)\right) \quad (13)$$

with $\tilde{\boldsymbol{K}}_{xi}(t) = \boldsymbol{K}_{xi}(t) - \boldsymbol{K}_{xi}^*$, $\tilde{\boldsymbol{K}}_{ri}(t) = \boldsymbol{K}_{ri}(t) - \boldsymbol{K}_{ri}^*$, $i \in \mathcal{I}$.

B. *Adaptive Laws*

Adaptive laws are developed based on (13). We first consider the case when a common Lyapunov matrix exists.

*1) **Adaptation when a common Lyapunov matrix $\boldsymbol{P}$ exists**:* If for the stable matrices $\boldsymbol{A}_{mi}$, $i \in \mathcal{I}$, there exists a common Lyapunov matrix $\boldsymbol{P} = \boldsymbol{P}^{\mathsf{T}} > 0$ such that

$$\boldsymbol{A}_{mi}^{\mathsf{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_{mi} < \boldsymbol{0}, \quad (14)$$

we propose the following adaptive laws:

$$\dot{\boldsymbol{K}}_{xi}^{\mathsf{T}}(t) = -\boldsymbol{S}_i^{\mathsf{T}}\boldsymbol{B}_{mi}^{\mathsf{T}}\chi_i(t)\boldsymbol{P}\boldsymbol{e}(t)\Delta\boldsymbol{x}^{\mathsf{T}}(t), \quad (15)$$

$$\dot{\boldsymbol{K}}_{ri}(t) = -\boldsymbol{S}_i^{\mathsf{T}}\boldsymbol{B}_{mi}^{\mathsf{T}}\chi_i(t)\boldsymbol{P}\boldsymbol{e}(t)\boldsymbol{r}(t). \quad (16)$$

We have the following stability and tracking properties:

Theorem 1. *If $\boldsymbol{A}_{mi}$, $i \in \mathcal{I}$, of the reference model system (7) satisfy (14) for some $\boldsymbol{P} = \boldsymbol{P}^{\mathsf{T}} > 0$, then all signals in the closed-loop system are bounded, and the state tracking error $\boldsymbol{e}(t) = \boldsymbol{x}(t) - \boldsymbol{x}_m(t)$ converges to zero asymptotically, for arbitrarily fast system mode switches.*

Proof. Let $\boldsymbol{A}_{mi}^{\mathsf{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_{mi} = -\boldsymbol{Q}_i$ for some $\boldsymbol{Q}_i = \boldsymbol{Q}_i^{\mathsf{T}} > \boldsymbol{0}$, $i \in \mathcal{I}$. Consider the Lyapunov function candidate

$$V = \boldsymbol{e}^{\mathsf{T}}\boldsymbol{P}\boldsymbol{e} + \sum_{i=1}^l \left(\mathrm{tr}[\tilde{\boldsymbol{K}}_{xi}\boldsymbol{M}_{si}^{-1}\tilde{\boldsymbol{K}}_{xi}^{\mathsf{T}}] + \mathrm{tr}[\tilde{\boldsymbol{K}}_{ri}^{\mathsf{T}}\boldsymbol{M}_{si}^{-1}\tilde{\boldsymbol{K}}_{ri}]\right)$$

where $\boldsymbol{M}_{si} = \boldsymbol{K}_{ri}^*\boldsymbol{S}_i$, $\tilde{\boldsymbol{K}}_{xi}^{\mathsf{T}}(t) = [\tilde{\boldsymbol{k}}_{xi1}, \tilde{\boldsymbol{k}}_{xi2}\ldots, \tilde{\boldsymbol{k}}_{xin}]$, $\tilde{\boldsymbol{K}}_{ri}(t) = [\tilde{\boldsymbol{k}}_{ri1}, \tilde{\boldsymbol{k}}_{ri2}, \ldots, \tilde{\boldsymbol{k}}_{rim}]$, and $\mathrm{tr}[\cdot]$ denotes the trace of a square matrix. Under Assumption (A2.a) and with facts $\mathrm{tr}[\boldsymbol{M}_1\boldsymbol{M}_2] = \mathrm{tr}[\boldsymbol{M}_2\boldsymbol{M}_1]$, $\mathrm{tr}[\boldsymbol{M}_3] = \mathrm{tr}[\boldsymbol{M}_3^{\mathsf{T}}]$ for any matrices $\boldsymbol{M}_i$, $i = 1, 2, 3$, of compatible dimensions, its time derivative along (15)–(16) is

$$\dot{V} \leq -\left(\min_{i\in\mathcal{I}} \lambda_{\min}[\boldsymbol{Q}_i]\right)\|\boldsymbol{e}\|^2. \quad (17)$$

It follows that $\boldsymbol{e}(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, $\boldsymbol{K}_{xi}(t), \boldsymbol{K}_{ri}(t) \in \mathcal{L}_\infty$, and with $\boldsymbol{x}_m(t) \in \mathcal{L}_\infty$ (see Remark 1), we have $\boldsymbol{u}(t), \dot{\boldsymbol{e}}(t) \in \mathcal{L}_\infty$. Therefore, all signals in the closed-loop system are bounded, and according to Barbălat Lemma, $\lim_{t\to\infty} \boldsymbol{e}(t) = \boldsymbol{0}$. ∇

Only a set of matrices with certain special structures is known to have a common $\boldsymbol{P}$ matrix, hence such a design cannot be extended to a general set of reference system matrices $\boldsymbol{A}_{mi}$. Next, we present and study the adaptive laws for the case when no such common $\boldsymbol{P}$ exists.

*2) **Adaptation when a common $\boldsymbol{P}$ does not exist**:* When no common Lyapunov matrix $\boldsymbol{P}$ satisfying (14) exists for the set of stable matrices $\boldsymbol{A}_{mi}$, $i \in \mathcal{I}$, we propose the parameter projection adaptive laws, with the assumption of certain knowledge of lower and upper bounds on the controller parameters, as follows:

$$\dot{\boldsymbol{K}}_{xi}^{\mathsf{T}}(t) = -\boldsymbol{S}_i^{\mathsf{T}}\boldsymbol{B}_{mi}^{\mathsf{T}}\chi_i(t)\boldsymbol{P}_i\boldsymbol{e}(t)\Delta\boldsymbol{x}^{\mathsf{T}}(t) + \boldsymbol{F}_{xi}(t), \quad (18)$$

$$\dot{\boldsymbol{K}}_{ri}(t) = -\boldsymbol{S}_i^{\mathsf{T}}\boldsymbol{B}_{mi}^{\mathsf{T}}\chi_i(t)\boldsymbol{P}_i\boldsymbol{e}(t)\boldsymbol{r}(t) + \boldsymbol{F}_{ri}(t), \quad (19)$$

to update the controller parameters in (12), where $\boldsymbol{P}_i = \boldsymbol{P}_i^{\mathsf{T}} > \boldsymbol{0}$, $i \in \mathcal{I}$, satisfy the Lyapunov equations $\boldsymbol{A}_{mi}^{\mathsf{T}}\boldsymbol{P}_i + \boldsymbol{P}_i\boldsymbol{A}_{mi} = -\boldsymbol{Q}_i$ for some $\boldsymbol{Q}_i = \boldsymbol{Q}_i^{\mathsf{T}} > 0$. For such an adaptive control design with parameter projection to be effective, we make a further assumption based on Assumption (A2.a):

(A2.b) The known matrix $\boldsymbol{S}_i$ in Assumption (A2.a) is such that $\boldsymbol{K}^*_{ri}\boldsymbol{S}_i$ is diagonal and positive definite.

The initial estimate of each element in $\boldsymbol{K}_{xi}(t)$, $\boldsymbol{K}_{ri}(t)$ is chosen to be within its known bounds. The projection terms $\boldsymbol{F}_{xi}(t)$, $\boldsymbol{F}_{ri}(t)$ are so defined as to confine the parameter estimates inside these bounds for all time: parameter adaptation is active (i.e., $\boldsymbol{F}_{xi}(t) = \boldsymbol{0}$, $\boldsymbol{F}_{ri}(t) = \boldsymbol{0}$) when the estimates are within those bounds, while it is deactivated otherwise, and the estimates are left unchanged (i.e., $\dot{\boldsymbol{K}}_{xi}(t) = \boldsymbol{0}$, $\dot{\boldsymbol{K}}_{ri}(t) = \boldsymbol{0}$).

With the definitions of a_m, λ_m, α, β, μ, $\Delta_{\boldsymbol{A}_m}$ in Lemma 1, we have the following stability and tracking properties:

Theorem 2. *Consider the closed-loop system with the plant (5), the reference model (7), and the controller (12) updated by the adaptive laws (18)–(19). If $T_0 \geq T_d = \alpha(1+\kappa)\ln(1+\mu\Delta_{\boldsymbol{A}_m})$, $\kappa > 0$, then all closed-loop signals are bounded, and the tracking error $\boldsymbol{e}(t)$ is small in the sense that*

$$\int_t^{t+T} \boldsymbol{e}^\mathsf{T}(\tau)\boldsymbol{e}(\tau)d\tau \leq \mu\Delta_{\boldsymbol{A}_m} c_0 \frac{T}{T_0} + c_1,\ t \geq t_0, T > 0 \quad (20)$$

with $c_1 = (1+\mu\Delta_{\boldsymbol{A}_m})c_0$, for some $c_0 > 0$.

Proof: Due to the fact $T_d > \alpha\ln(1+\mu\Delta_{\boldsymbol{A}_m})$, it follows from Lemma 1 that $T_0 \geq T_d$ ensures stability of (7) , i.e., $\boldsymbol{x}_m(t) \in \mathcal{L}_\infty$. Consider the piecewise continuous Lyapunov function

$$V = \boldsymbol{e}^\mathsf{T}\sum_{i=1}^{l}\boldsymbol{P}_{mi}\chi_i\boldsymbol{e} + \sum_{i=1}^{l}\left(\sum_{j=1}^{n}\tilde{\boldsymbol{k}}_{xij}^\mathsf{T}\boldsymbol{M}_{si}^{-1}\tilde{\boldsymbol{k}}_{xij} + \sum_{j=1}^{m}\tilde{\boldsymbol{k}}_{rij}^\mathsf{T}\boldsymbol{M}_{si}^{-1}\tilde{\boldsymbol{k}}_{rij}\right) \quad (21)$$

with $\boldsymbol{M}_{si} = \boldsymbol{K}^*_{ri}\boldsymbol{S}_i = \mathrm{diag}[m_{si1},\ldots,m_{sim}]^\mathsf{T} > \boldsymbol{0}$, $\tilde{\boldsymbol{K}}_{xi}^\mathsf{T} = [\tilde{\boldsymbol{k}}_{xi1},\ldots,\tilde{\boldsymbol{k}}_{xin}]$, $\tilde{\boldsymbol{K}}_{ri} = [\tilde{\boldsymbol{k}}_{ri1},\ldots,\tilde{\boldsymbol{k}}_{rim}]$. Here, without loss of generality, we consider the case $\boldsymbol{Q}_{mi} = \boldsymbol{I}_n$, $i \in \mathcal{I}$. Suppose that $\chi_i(t) = 1$ for $t \in [t_{k-1}, t_k)$, then over this time interval the derivative of V along (18)–(19) is

$$\dot{V} = -\boldsymbol{e}^\mathsf{T}\boldsymbol{e} + 2\left(\sum_{p=1}^{n}\sum_{q=1}^{m}\frac{1}{m_{siq}}\tilde{k}_{xipq}f_{xipq} + \sum_{p,q=1}^{m}\frac{1}{m_{sip}}\tilde{k}_{ripq}f_{ripq}\right)$$

with $\boldsymbol{K}_{xi}(t) = [k_{xipq}(t)]_{n\times m}$, $\boldsymbol{K}_{ri}(t) = [k_{ripq}(t)]_{m\times m}$, $\boldsymbol{K}^*_{xi} = [k^*_{xipq}]_{n\times m}$, $\boldsymbol{K}^*_{ri} = [k^*_{ripq}]_{m\times m}$, $\boldsymbol{F}_{xi}(t) = [f_{xipq}(t)]^\mathsf{T}_{n\times m}$, $\boldsymbol{F}_{ri}(t) = [f_{ripq}(t)]_{m\times m}$, $i \in \mathcal{I}$. It can be verified that $(k_{xipq}(t) - k^*_{xipq})f_{xipq}(t) \leq 0$ and $(k_{ripq}(t) - k^*_{ripq})f_{ripq}(t) \leq 0$, thus V is non-increasing whenever the system is operating at the ith mode. Furthermore, with the bounded parameter estimates $\boldsymbol{K}_{xi}$, $\boldsymbol{K}_{ri}$, there exists $c_p = \max_{i\in\mathcal{I}}[\sum_{j=1}^{n}\tilde{\boldsymbol{k}}_{xij}^\mathsf{T}\boldsymbol{M}_{si}^{-1}\tilde{\boldsymbol{k}}_{xij} + \sum_{j=1}^{m}\tilde{\boldsymbol{k}}_{rij}^\mathsf{T}\boldsymbol{M}_{si}^{-1}\tilde{\boldsymbol{k}}_{rij}] > 0$ such that

$$\dot{V} \leq -\frac{V}{(1+\kappa)\alpha} - \frac{\kappa V - (1+\kappa)lc_p}{(1+\kappa)\alpha},\ \kappa > 0; \quad (22)$$

that is, for $V > lc_p(1+\kappa)/\kappa$, V decays faster than exponentially at the rate $-1/(1+\kappa)\alpha$, and is non-increasing otherwise.

When a mode switch occurs at $t = t_k$, following the proof of Lemma 1 [12], we have $V(t_k) \leq (1+\mu\Delta_{\boldsymbol{A}_m})V(t_k^-)$, and the slow switching condition $T_0 \geq T_d$ ensures that

$$V(t_k) \leq \begin{cases} lc_p(1+\mu\Delta_{\boldsymbol{A}_m})\frac{1+\kappa}{\kappa}, & V(t_k^-) \leq lc_p\frac{1+\kappa}{\kappa}, \\ V(t_{k-1}), & V(t_k^-) > lc_p\frac{1+\kappa}{\kappa}. \end{cases}$$

Therefore, $V(t) \leq c_0 \triangleq \max\{lc_p(1+\mu\Delta_{\boldsymbol{A}_m})(1+\kappa)/\kappa, V(t_0)\}$, and closed-loop stability can be concluded.

For evaluating the tracking performance, there are four possible cases depending on the integration interval $[t, t+T]$:

(i) $T \leq T_0$, $t_{k-1} \leq t \leq t+T < t_k$. There is no mode switch over $[t, t+T]$, and we have $\int_t^{t+T}\boldsymbol{e}^\mathsf{T}(\tau)\boldsymbol{e}(\tau)d\tau \leq c_0$.

(ii) $T \leq T_0$, $t < t_k \leq t+T$. There is one and only one switch at $t = t_k$. We have $\dot{V} \leq -\boldsymbol{e}^\mathsf{T}(t)\boldsymbol{e}(t) + \boldsymbol{e}^\mathsf{T}(t)\Delta\boldsymbol{P}_{m(k)}\delta(t-t_k)\boldsymbol{e}(t)$ with $\delta(t)$ being the unit impulse function, and $\int_t^{t+T}\boldsymbol{e}^\mathsf{T}(\tau)\boldsymbol{e}(\tau)d\tau \leq V(t) - V(t+T) + \boldsymbol{e}^\mathsf{T}(t_k)\Delta\boldsymbol{P}_{m(k)}\boldsymbol{e}(t_k) \leq (1+\mu\Delta_{\boldsymbol{A}_m})c_0$.

(iii) $T > T_0$, $t < t_k$, $t+T < t_{k+N}$, where N is the largest integer less than or equal to T/T_0. There are at most N mode switches at $t = t_k, t_{k+1}, \ldots, t_{k+N-1}$, respectively, and $\int_t^{t+T}\boldsymbol{e}^\mathsf{T}(\tau)\boldsymbol{e}(\tau)d\tau \leq V(t) - V(t+T) + \sum_{j=0}^{N-1}\boldsymbol{e}^\mathsf{T}(t_{k+j})\Delta\boldsymbol{P}_{m(k+j)}\boldsymbol{e}(t_{k+j}) \leq \mu\Delta_{\boldsymbol{A}_m}c_0\frac{T}{T_0} + c_0$.

(iv) $T > T_0$, $t < t_k$, $t+T \geq t_{k+N}$. There are at most $N+1$ mode switches at $t = t_k, t_{k+1}, \ldots, t_{k+N}$, respectively, so that $\int_t^{t+T}\boldsymbol{e}^\mathsf{T}(\tau)\boldsymbol{e}(\tau)d\tau \leq \mu\Delta_{\boldsymbol{A}_m}c_0\frac{T}{T_0} + (1+\mu\Delta_{\boldsymbol{A}_m})c_0$.

It can be concluded from (i)–(iv) that (20) is satisfied. ∇

3) ***Adaptation with sufficiently rich reference input*** $\boldsymbol{r}(t)$*:* If some of the system modes with indices $i \in \mathcal{I}^* \subset \mathcal{I}$ are no longer active after a finite time $T_i \geq t_0$, while other modes are active intermittently over infinitely many intervals, then under the persistency of excitation condition, we have the following stability and tracking properties for $t \geq T^* = \max_{i\in\mathcal{I}^*}\{T_i\}$:

Theorem 3. *Consider the closed-loop system with the controller (12) updated by the adaptive laws*

$$\dot{\boldsymbol{K}}_{xi}^\mathsf{T}(t) = -\boldsymbol{S}_i^\mathsf{T}\boldsymbol{B}_{mi}^\mathsf{T}\chi_i(t)\boldsymbol{P}_i\boldsymbol{e}(t)\Delta\boldsymbol{x}^\mathsf{T}(t), \quad (23)$$

$$\dot{\boldsymbol{K}}_{ri}(t) = -\boldsymbol{S}_i^\mathsf{T}\boldsymbol{B}_{mi}^\mathsf{T}\chi_i(t)\boldsymbol{P}_i\boldsymbol{e}(t)\boldsymbol{r}(t). \quad (24)$$

Suppose each element $r_i(t)$, $i = 1, 2, \ldots, m$, of the reference input $\boldsymbol{r}(t)$ is sufficiently rich of order $n+1$ and uncorrelated, and $(\boldsymbol{A}_{mi}, \boldsymbol{B}_{mi})$, $i \in \mathcal{I}$, are controllable. If the switching time intervals are sufficiently large, then all closed-loop signals are bounded for $t \geq t_0$; $\boldsymbol{e}(t)$, $\tilde{\boldsymbol{K}}_{xi}(t)$, $\tilde{\boldsymbol{K}}_{ri}(t)$ converge to zero exponentially, $i \in \mathcal{I} - \mathcal{I}^*$, $t \geq T^*$; and $\boldsymbol{K}_{xi}(t) = \boldsymbol{K}_{xi}(T_i)$, $\boldsymbol{K}_{ri}(t) = \boldsymbol{K}_{ri}(T_i)$, $i \in \mathcal{I}^*$, $t \geq T_i$.

The proof of this corollary follows the same line as that in [11] for the single-input case, and is omitted here.

V. Simulations on NASA GTM

Simulations are performed to demonstrate the system stability and tracking performance with the proposed adaptive control schemes applied to the piecewise linear system model of the longitudinal dynamics of the NASA GTM at multiple operating points and the nonlinear GTM [10].

A. *Linearized Aircraft Longitudinal Model and Reference Model System*

An operating point for a nonlinear aircraft system is specified by (V, h), with V and h the vehicle speed and altitude, respectively. For one specific operating point (V_i, h_i), a trim point (equilibrium) $(\boldsymbol{x}_{0i}, \boldsymbol{u}_{0i})$ may be found, where $\boldsymbol{x}_{0i}$ is the nominal state vector, and $\boldsymbol{u}_{0i}$ is the nominal input vector to the system. In steady-state, straight, level flight, the longitudinal and lateral-directional dynamics of an aircraft can be decoupled from each other, and the linearized longitudinal model of an aircraft around $(\boldsymbol{x}_{0i}, \boldsymbol{u}_{0i})$ can be represented by (2), i.e.,

$$\dot{\boldsymbol{x}}_i = \boldsymbol{A}_i \boldsymbol{x}_i + \boldsymbol{B}_i \boldsymbol{u}_i, \boldsymbol{x}_i = [u, w, q, \theta]^{\mathsf{T}}, \boldsymbol{u}_i = [\delta_e, \delta_T]^{\mathsf{T}}, \quad (25)$$

where u, w, q, θ are the perturbed aircraft velocity components along the x- and z-body-axis (fps), angular velocity along the y-body-axis (crad/s), and pitch angle (crad), respectively; that is, $\boldsymbol{x}_i(t) = \boldsymbol{x}(t) - \boldsymbol{x}_{0i}$ with $\boldsymbol{x}(t)$ being the aircraft longitudinal state vector. The control input vector $\boldsymbol{u}_i(t)$ consists of the perturbed elevator deflection δ_e and throttle input δ_T, i.e., $\boldsymbol{u}_i(t) = \boldsymbol{u}(t) - \boldsymbol{u}_{0i}$ with $\boldsymbol{u}(t)$ being the total control applied to the aircraft. In terms of the original state and control vector $\boldsymbol{x}(t)$ and $\boldsymbol{u}(t)$, the linearized longitudinal model is $\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_i \boldsymbol{x}(t) + \boldsymbol{B}_i \boldsymbol{u}(t) + \boldsymbol{c}_i$ with $\boldsymbol{c}_i = -\boldsymbol{A}_i \boldsymbol{x}_{0i} - \boldsymbol{B}_i \boldsymbol{u}_{0i}$.

The desired longitudinal behavior of the aircraft within a neighborhood of $(\boldsymbol{x}_{0i}, \boldsymbol{u}_{0i})$ is specified by a reference model in the form $\dot{\boldsymbol{x}}_{mi} = \boldsymbol{A}_{mi} \boldsymbol{x}_{mi} + \boldsymbol{B}_{mi} \boldsymbol{r}$, where $\boldsymbol{r}(t)$ is the reference input vector that can generate the desired (perturbed) state trajectory $\boldsymbol{x}_{mi}(t)$. Here $\boldsymbol{x}_{mi}(t)$ is defined with regard to $\boldsymbol{x}_{0i}$. In terms of a "global" reference trajectory $\boldsymbol{x}_m(t) = \boldsymbol{x}_{mi}(t) + \boldsymbol{x}_{0i}$, with $\boldsymbol{c}_{mi} = -\boldsymbol{A}_{mi} \boldsymbol{x}_{0i}$, we have the reference model system

$$\dot{\boldsymbol{x}}_m(t) = \boldsymbol{A}_m(t) \boldsymbol{x}_m(t) + \boldsymbol{B}_m(t) \boldsymbol{r}(t) + \boldsymbol{c}_m(t).$$

In this simulation study, we first design controllers in the form $\boldsymbol{u}_{\text{nom}}(t) = \boldsymbol{u}_{0i}(t) + \boldsymbol{K}_{xi}^{*\mathsf{T}} \Delta \boldsymbol{x}(t) + \boldsymbol{r}(t)$ using LQ techniques, based on $(\boldsymbol{A}_i, \boldsymbol{B}_i)$ (not used in adaptive control design), and the reference model systems are chosen such that Assumption (A1) is satisfied with $\boldsymbol{K}_{ri}^* = \boldsymbol{I}_n$. In other words, the nominal linearized closed-loop system dynamics are chosen as the reference model system. Note that with such a choice, the Assumptions (A2.a) and (A2.b) are satisfied as well. In particular, $\boldsymbol{S}_i$ can be any positive definite diagonal matrix.

B. *Switches of Operating Points*

Extensive simulations are performed to determine the valid linearization regions $\boldsymbol{\Omega}_1$, $\boldsymbol{\Omega}_2$, around the trim points $(\boldsymbol{x}_{01}, \boldsymbol{u}_{01})$, $(\boldsymbol{x}_{02}, \boldsymbol{u}_{02})$, respectively, and a decent switching surface in between. In particular, for each trim point $(\boldsymbol{x}_{0i}, \boldsymbol{u}_{0i})$, $i = 1, 2$, a reference input vector signal $\boldsymbol{r}(t)$ relatively small in magnitude is chosen such that the GTM longitudinal states $\boldsymbol{x}(t)$ stay within $\boldsymbol{\Omega}_i$. A $\boldsymbol{r}(t)$ with relatively large magnitude is also determined which can drive $\boldsymbol{x}(t)$ to cross the switching surface, corresponding a desired change of operating point.

C. *Design and Simulation Parameters*

For simplicity of presentation, we choose $l = 2$, and trim the GTM at steady-state, straight, wings-level flight condition at 75 knots and 85 knots at 800 ft., respectively, to obtain a piecewise linear longitudinal system model. The reference model system is specified by LQ designs with $\boldsymbol{Q} = \boldsymbol{I}_4$, $\boldsymbol{R} = 10\boldsymbol{I}_2$. It is found that a common Lyapunov matrix exists such that (14) is satisfied, thus the adaptive design in Section IV-B1 may be applied. The matrices $\boldsymbol{S}_i$ are chosen based on the observed closed-loop GTM system performance. Here, we choose $\boldsymbol{S}_1 = 0.05\boldsymbol{I}_4$ and $\boldsymbol{S}_2 = 0.05\boldsymbol{I}_4$.

The reference input signal $\boldsymbol{r}(t)$ is selected as $\boldsymbol{r}(t) = [2\sin(0.02\pi t),\ 0]^{\mathsf{T}}$ to specify a longitudinal reference state trajectory for the GTM at each operating point; in the nonlinear simulations, it is set to be $\boldsymbol{r}(t) = [5,\ 0]^{\mathsf{T}}$ whenever there is a desired transition from the first operating point (75 knots, 800 ft.) to the second (85 knots, 800 ft.), and $\boldsymbol{r}(t) = [-5,\ 0]^{\mathsf{T}}$, otherwise. Since the parameter matrices of the linearized longitudinal model of the GTM are not sensitive to altitude variations within a relatively small range (± 100 ft.), as can be verified by linearizing the GTM at the same airspeed but different altitudes, the switching plane is specified by V only. In this simulation study, V is chosen to be $V = 80$ knots.

For all the simulations, the GTM is initially trimmed at 75 knots, 800 ft., steady-state, straight, wings-level flight. A switch of operating point is commanded (through the setting of $\boldsymbol{r}(t)$) every 100s. The initial tracking error is $\boldsymbol{e}(0) = [7\ -4\ 0\ 50]^{\mathsf{T}}$, and the initial parameter estimates are set as 60% of their nominal values.

D. *Simulation Results*

*1) **Simulations for linear system:*** Figure 1 shows the state tracking error $\boldsymbol{e}(t)$ for the adaptive control scheme in Section IV-B1 applied to the piecewise linear model of the GTM. A convergence of the state tracking error $\boldsymbol{e}(t)$ to zero is observed.

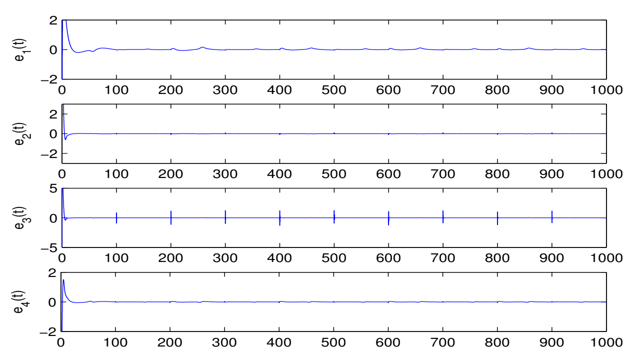

Fig. 1. Simulations for linear system: state tracking error $\boldsymbol{e}(t)$.

*2) **Simulations on the GTM:*** To show the effectiveness of the proposed adaptive control scheme on system performance improvement, the nonlinear GTM system response is shown next, along with those obtained by applying two other control schemes to the GTM. The first is a fixed control scheme in the form of (11) with controller parameters to be 60% of their nominal values, and the second is a comparison adaptive controller (12) with parameter adaptive laws:

$$\dot{\boldsymbol{K}}_{xi}^{\top}(t) = -\boldsymbol{S}_i^{\top}\boldsymbol{B}_{mi}^{\top}\boldsymbol{P}\boldsymbol{e}(t)\Delta\boldsymbol{x}^{\top}(t), \qquad (26)$$

$$\dot{\boldsymbol{K}}_{ri}(t) = -\boldsymbol{S}_i^{\top}\boldsymbol{B}_{mi}^{\top}\boldsymbol{P}\boldsymbol{e}(t)\boldsymbol{r}(t), \qquad (27)$$

with the aforementioned common $\boldsymbol{P}$ matrix. Note that the difference of the adaptive laws above from those in (15)–(16) is the absence of the indicator functions $\chi_i(t)$, $i = 1, 2$.

Figure 2 shows the state tracking error $\boldsymbol{e}(t)$ when the adaptive control designs in Section IV-B1, the 60% fixed control, and the comparison adaptive control scheme (26)–(27), respectively, are applied to the nonlinear GTM. All other system signals are bounded (not shown due to space limit).

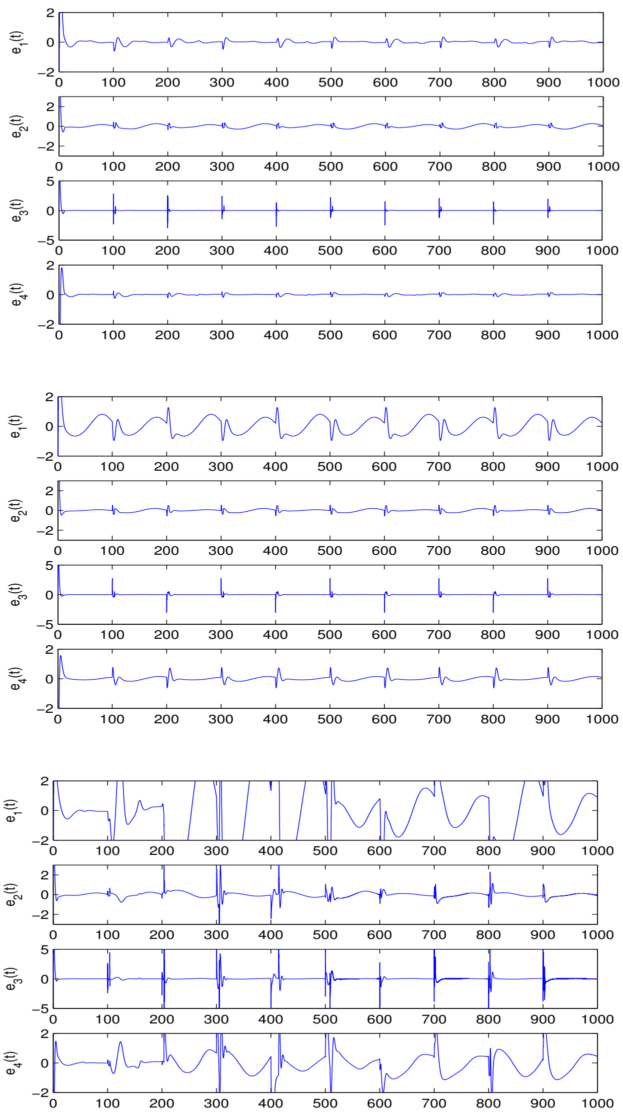

Fig. 2. GTM simulation: $\boldsymbol{e}(t)$ for the control scheme in Section IV-B1 (above), for the 60% fixed control law (middle), and for the comparison control scheme (26)–(27) (below).

From the simulation results, we can see that the closed-loop stability is achieved for all the simulations. As for the state tracking, the proposed adaptive control schemes provide substantially improved performance over the fixed control law under the same flight conditions. The comparison adaptive control scheme in (26)–(27) leads to unacceptable tracking performance. The simulation results demonstrate the effectiveness of the proposed linearization-based adaptive control designs applied to the nonlinear GTM system.

VI. Conclusions

Adaptive state feedback for state tracking problem for piecewise linear systems was considered in this paper. Unlike conventional MRAC designs, the reference model was chosen to be piecewise linear. Adaptive control schemes were proposed, and their stability and tracking performance was evaluated. The presented adaptive control scheme with parameter projection ensures signal boundedness. One way to achieve asymptotic tracking is to impose persistent excitation conditions such that each closed-loop subsystem is exponentially stable and the tracking errors decay to zero, when the switch is sufficiently slow. Effectiveness of the adaptive control scheme is demonstrated on control of the longitudinal dynamics of the nonlinear GTM at multiple operating points. Substantial performance improvement over a fixed control law and a comparison adaptive control design was observed in the simulation results.

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References

[1] M. di Bernardo, U. Montanaro, and S. Santiti, “Minimal control synthesis adaptive control of continuous bimodal piecewise affine systems,” *SIAM J. Control Optim.*, vol. 48, no. 7, pp. 4242–4261, 2010.

[2] L. Crespo, J. Jang, M. Matsutani, T. Gibson, and A. Annaswamy, “Design and verification of an adaptive controller for the Generic Transport Model,” *Proc. AIAA Guid. Navi. Contr. Conf.*, AIAA-2009-5618, Chicago, IL, August 2009.

[3] W. O. Dennis and G. Tao, “Adaptive control of plants with jumping parameters,” *the 3rd SIAM Conf. Contr. Appl.*, St. Louis, MO, 1995.

[4] E. Lavretsky, “Combined/composite model reference adaptive control,” *Proc. AIAA Guid. Navi. Contr. Conf.*, AIAA-2009-6065.

[5] J. P. Hespanha and A. S. Morse, “Stability of switched systems with average dwell-time,” tech. rep., EE-Systems, Univ. of Southern California/Lab. for Control Science & Eng., Yale Univ., Jan. 1999.

[6] P. Ioannou and J. Sun, *Robust Adaptive Control*, Prentice Hall, 1996.

[7] E. Lavretsky, R. Gadient, and I. Gregory, “Predictor-based model reference adaptive control,” *Proc. AIAA Guid. Navi. Contr. Conf.*, AIAA-2009-5853, Chicago, IL, August 2009.

[8] D. Liberzon, *Switching in Systems and Control*. Boston, MA: Birkhäuser, 2003.

[9] D. Liberzon and A. S. Morse, “Basic problems in stability and design of switched systems,” *IEEE Contr. Sys. Mag.*, vol. 19, no. 5, 1999.

[10] A. M. Murch, “A flight control system architecture for the NASA AirSTAR flight test infrastructure,” *Proc. AIAA Guid. Navi. Contr. Conf.*, Honolulu, Hawaii, Aug. 18-21, 2008.

[11] Q. Sang and G. Tao, “Adaptive control of piecewise linear systems: the state tracking case,” accepted to *IEEE Trans. Automat. Contr.*.

[12] Q. Sang and G. Tao, “Adaptive control of piecewise linear systems: the output tracking case,” *Proc. American Contr. Conf.*, San Francisco, CA, 2011.

[13] G. Tao, *Adaptive Control Design and Analysis*, New York: Wiley, 2003.