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Cubature Information Filter and its Applications

Kumar Pakki. Bharani Chandra, Da-Wei Gu, and Ian Postlethwaite

Abstract— **This paper presents a new estimation algorithm called cubature information filtering for nonlinear systems. The proposed algorithm is developed from an extended information filter and a recently developed cubature Kalman filter. Unlike the extended Kalman filter, the proposed filter does not require the evaluation of Jacobians during state estimation. The efficacy of the proposed algorithm is demonstrated by simulation examples on frequency demodulation and localization problem and is compared with unscented information filtering.**

I. INTRODUCTION

Nonlinear state estimation has been an active research area for several decades. State estimation plays a major role in several practical applications like fault detection and isolation, guidance, navigation, control system design, communication, etc., where either the sensors are expensive or it is difficult to measure the states. The extended Kalman filter (EKF) has been an important approach for nonlinear state estimation over the last five decades. An algebraically equivalent form of EKF, the extended information filter (EIF), has been proposed in the literature to cope with some of the issues of EKF [13,5]. In EIFs, the parameters of interest are the information states and the inverse of covariance rather than states and covariance. Information filters are easy in initialization compared to conventional Kalman filters and the update stage is computationally economic. EIF has indeed several advantages over EKF; for more details see [5,13]. But, both EKFs and EIFs are only suitable for ‘mild’ nonlinearities where the first-order approximations of the nonlinear functions are suitable and require analytical Jacobians for state estimation. To overcome these limitations, an unscented Kalman filter (UKF) has been proposed [15], which has the ability to deal with some of the issues of EKF. UKF uses the deterministic sampling approach to capture the mean and covariances with sigma points and in general been shown to perform better than EKF in nonlinear estimation problems. There are a few other nonlinear estimation techniques found in the literature, namely, Rao-Blackwellised particle filters (RBPF) [7], which are improvised version of particle filters, Gaussian filters [8], state dependent Riccati equation (SDRE) filters [9], sliding mode observers [10], etc.

In the recent literature [1], the cubature Kalman filter (CKF) has been proposed for nonlinear state estimation. CKF is a Gaussian approximation of Bayesian filter, but provides a more accurate filtering estimates than existing Gaussian filters. CKF is shown to be more efficient than UKF [1]. The applicability and effectiveness of CKF for sensor data fusion for positioning is given in [12]. Our work is motivated from [1], where CKF is developed as the closest known direct approximation to the Bayesian filter and from [2], in which the unscented transformation in sigma point filters is fused into the EIF architecture for multiple sensor fusion.

In this work, we propose a cubature information filter by embedding CKF with EIF architecture for nonlinear systems and demonstrate its applicability for estimating the frequency modulation (FM) model states and localization problem. The rest of the paper is structured as follows. Section II includes the preliminaries of the EIF and CKF, and Section III describes the proposed cubature information filtering. Section IV is devoted to numerical simulations and Section V concludes the paper.

II. EXTENDED INFORMATION FILTER AND CUBATURE KALMAN FILTER

This section presents a brief introduction to EIF and CKF. For detailed formulation and derivation of these filtering algorithms, please see for example [5] for EIF and [1] for CKF.

A. Extended information filter

EIF is an algebraic equivalent of EKF, in which the parameters of interest are information states and the inverse of covariance matrix (information matrix) rather than states and covariance. EIF can be represented by a recursive process of time update and measurement update. The EIF equations are summarized below.

Consider the discrete nonlinear process and measurement models as

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) + \mathbf{w}_{k-1} \quad (1)$$

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{v}_k \quad (2)$$

where k is a current time index, $\mathbf{x}_k$ is a state vector, $\mathbf{u}_k$ is a control input, $\mathbf{z}_k$ is the measurement, $\mathbf{w}_{k-1}$ and $\mathbf{v}_k$ are the process and observation noises, respectively. The noises are assumed to be Gaussian-distributed random variables with zero mean and covariances of $\mathbf{Q}_{k-1}$ and $\mathbf{R}_k$. The predicted information state vector and the information matrix are given as

$$\widehat{\mathbf{y}}_{k|k-1} = \mathbf{Y}_{k|k-1}\widehat{\mathbf{x}}_{k|k-1} \quad (3)$$

$$\mathbf{Y}_{k|k-1} = \mathbf{P}^{-1}_{k|k-1} \quad (4)$$

$$= \left[\nabla\mathbf{f}_x \mathbf{Y}^{-1}_{k-1|k-1} \nabla\mathbf{f}^T_x + \mathbf{Q}_{k-1|k-1}\right]^{-1} \quad (5)$$

Kumar Pakki. Bharani Chandra is a PhD student with Control and Instrumentation Group, University of Leicester, UK, LE1 7RH, bcp3@leicester.ac.uk, bharanichandra@gmail.com.

Da-Wei Gu is the Head of the Control and Instrumentation Group, University of Leicester, UK, LE1 7RH, dag@leicester.ac.uk.

Ian Postlethwaite is Deputy Vice-Chancellor at Northumbria University, UK, NE1 8ST, ian.postlethwaite@northumbria.ac.uk.

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and the updated information state vector and the information matrix are

$$\widehat{\mathbf{y}}_{k|k} = \widehat{\mathbf{y}}_{k|k-1} + \mathbf{i}_k \tag{6}$$

$$\mathbf{Y}_{k|k} = \mathbf{Y}_{k|k-1} + \mathbf{I}_k \tag{7}$$

The information state contribution and its associated information matrix are given

$$\mathbf{i}_k = \nabla\mathbf{h}_x^T\mathbf{R}_k^{-1}[\nu_k + \nabla\mathbf{h}_x\widehat{\mathbf{x}}_{k|k-1}] \tag{8}$$

$$\mathbf{I}_k = \nabla\mathbf{h}_x^T\mathbf{R}_k^{-1}\nabla\mathbf{h}_x \tag{9}$$

where

$$\nu_k = \mathbf{z}_k - \mathbf{h}(\widehat{\mathbf{x}}_{k|k-1})$$

and $\nabla\mathbf{f}_x$, and $\nabla\mathbf{h}_x$ are the Jacobians of **f** and **h** evaluated at **x**.

Initialization in the information space is easier than in EKF and the update stage of EIF is computationally simpler than EKF. Occasionally, EIF is shown to be more efficient than EKF. But some of the drawbacks inherent in the EKF still affect the EIF. These include the nontrivial nature of the Jacobian matrix derivation (and computation) and linearization instability [5].

B. Cubature Kalman filter

The use of EKF is not the best choice for many practical applications as it works well only in a 'mild' nonlinear environment, and hence can degrade the performance. The CKF is the closest known approximation to the Bayesian filter that could be designed in a nonlinear setting under the Gaussian assumption. Unlike the EKF, CKF filter does not require evaluation of Jacobians during the estimation process. It is an appealing option for nonlinear state estimation when compared with EKF or UKF [1]. The basic steps required for CKF are described in this section. One can see [1] for more details.

The key assumption of the cubature Kalman filter (CKF) is that the predictive density $p(\mathbf{x}_k|D_{k-1})$, where D_{k-1} denotes the history of input-measurement pairs up to $k-1$, and the filter likelihood density $p(\mathbf{z}_k|D_k)$ are both Gaussian, which eventually leads to a Gaussian posterior density $p(\mathbf{x}_k|D_k)$. Under this assumption, the cubature Kalman filter solution reduces to how to compute their means and covariances more accurately.

Consider the nonlinear system with additive noise defined by process and observation models in (1) and (2). The prediction and update stage for CKF is given below.

1) Prediction: In the prediction step, the CKF computes the mean $\widehat{\mathbf{x}}_{k|k-1}$ and the associated covariance $P_{k|k-1}$ of the Gaussian predictive density numerically using cubature rules. The predicted mean can be written as

$$\widehat{\mathbf{x}}_{k|k-1} = \mathrm{E}[\mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) + \mathbf{w}_{k-1}|D_{k-1}] \tag{10}$$

Since $\mathbf{w}_k$ is assumed to be zero-mean and uncorrelated with the measurement sequence, we get

$$\widehat{\mathbf{x}}_{k|k-1} = \mathrm{E}[\mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1})|D_{k-1}] \tag{11}$$

$$= \int_{R^n} \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1})p(\mathbf{x}_{k-1}|D_{k-1})d\mathbf{x}_{k-1} \tag{12}$$

$$= \int_{R^n} \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) \tag{13}$$

$$\mathcal{N}(\mathbf{x}_{k-1}; \widehat{\mathbf{x}}_{k-1|k-1}, P_{k-1|k-1})d\mathbf{x}_{k-1}$$

Similarly, the associated error covariance can be represented as

$$P_{k|k-1} = \mathrm{E}[(\mathbf{x}_k - \widehat{\mathbf{x}}_{k|k-1})(\mathbf{x}_k - \widehat{\mathbf{x}}_{k|k-1})^T|z_{k-1}] \tag{14}$$

$$= \int_{R^n} \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1})\mathbf{f}^T(\mathbf{x}_{k-1}, \mathbf{u}_{k-1})\mathcal{N}(\mathbf{x}_{k-1};$$

$$\widehat{\mathbf{x}}_{k-1|k-1}, P_{k-1|k-1})d\mathbf{x}_{k-1} - \widehat{\mathbf{x}}_{k|k-1}\widehat{\mathbf{x}}_{k|k-1}^T + Q_{k-1} \tag{15}$$

2) Update: The predicted measurement density can be represented by

$$p(\mathbf{z}_k|D_{k-1}) = \mathcal{N}(\mathbf{z}_k; \widehat{\mathbf{z}}_{k|k-1}, P_{zz,k|k-1}) \tag{16}$$

where the predicted measurement and associated covariance are given by

$$\widehat{\mathbf{z}}_{k|k-1} = \int_{R^n} \mathbf{h}(\mathbf{x}_k)\mathcal{N}(\mathbf{x}_k; \widehat{\mathbf{x}}_{k|k-1}, P_{k|k-1})d\mathbf{x}_k \tag{17}$$

$$P_{zz,k|k-1} = \int_{R^n} \mathbf{h}(\mathbf{x}_k)\mathbf{h}^T(\mathbf{x}_k)\mathcal{N}(\mathbf{x}_k; \widehat{\mathbf{x}}_{k|k-1}, P_{k|k-1})d\mathbf{x}_k$$

$$-\widehat{\mathbf{z}}_{k|k-1}\widehat{\mathbf{z}}_{k|k-1}^T + R_k \tag{18}$$

and the cross-covariance is

$$P_{xz,k|k-1} = \int_{R^n} \mathbf{x}_k\mathbf{h}^T(\mathbf{x}_k)\mathcal{N}(\mathbf{x}_k; \widehat{\mathbf{x}}_{k|k-1}, P_{k|k-1})d\mathbf{x}_k$$

$$-\widehat{\mathbf{x}}_{k|k-1}\widehat{\mathbf{z}}_{k|k-1}^T \tag{19}$$

Once the new measurement $\mathbf{z}_k$ is received, the CKF computes the posterior density $p(\mathbf{x}_k|D_k)$ yielding

$$p(\mathbf{x}_k|D_k) = \mathcal{N}(\mathbf{x}_k; \widehat{\mathbf{x}}_{k|k}, P_{k|k}), \tag{20}$$

where

$$\widehat{\mathbf{x}}_{k|k} = \widehat{\mathbf{x}}_{k|k-1} + G_k(\mathbf{z}_k - \widehat{\mathbf{z}}_{k|k-1}) \tag{21}$$

$$P_{k|k} = P_{k|k-1} - G_kP_{zz,k|k-1}G_k^T \tag{22}$$

with the Kalman gain given as

$$G_k = P_{xz,k|k-1}P_{zz,k|k-1}^{-1} \tag{23}$$

It can be seen that in the above prediction and update equations, the Bayesian filter solution reduces to computing the multi-dimensional integrals, whose integrands are of the form $nonlinear function \times Gaussian$. The heart of the CKF is to find the multi-dimensional integrals using cubature rules.

3) Cubature Rules: The cubature rule to approximate an n-dimensional Gaussian weighted integral is as follows:

$$\int_{R^n} \mathbf{f}(\mathbf{x})\mathcal{N}(\mathbf{x};\mu,P)d\mathbf{x} \quad \approx \quad \frac{1}{2n}\sum_{i=1}^{2n}\mathbf{f}(\mu+\sqrt{P}\xi_i) \quad (24)$$

where $\sqrt{P}$ is a square-root factor of the covariance P, and satisfies the relation $P=\sqrt{P}\sqrt{P}^T$; the set of $2n$ cubature points are given by $\{\xi_i\}$ where ξ_i is the $i-th$ element of the following set:

$$\sqrt{n}\left\{\begin{pmatrix}1\\0\\\vdots\\0\end{pmatrix},\ldots,\begin{pmatrix}0\\\vdots\\0\\1\end{pmatrix},\begin{pmatrix}-1\\0\\\vdots\\0\end{pmatrix},\ldots,\begin{pmatrix}0\\\vdots\\0\\-1\end{pmatrix}\right\} \quad (25)$$

These cubature rules are required to numerically evaluate the multi-integrands in the prediction and update stage of the CKF. Finally, the evaluated mean and covariance matrices from (13), (15) and (24) are

$$\widehat{\mathbf{x}}_{k|k-1}=\frac{1}{2n}\sum_{i=1}^{2n}\boldsymbol{X}^*_{i,k|k-1} \quad (26)$$

$$P_{k|k-1}=\frac{1}{2n}\sum_{i=1}^{2n}\boldsymbol{X}^*_{i,k|k-1}\boldsymbol{X}^{*T}_{i,k|k-1}-\widehat{\mathbf{x}}_{k|k-1}\widehat{\mathbf{x}}^T_{k|k-1}+Q_{k-1} \quad (27)$$

and the predicted measurement and its associated covariances are given by

$$\widehat{\mathbf{z}}_{k|k-1}=\frac{1}{2n}\sum_{i=1}^{2n}\mathbf{Z}_{i,k|k-1} \quad (28)$$

$$P_{zz,k|k-1}=\frac{1}{2n}\sum_{i=1}^{2n}\mathbf{Z}_{i,k|k-1}\mathbf{Z}^T_{i,k|k-1}-\widehat{\mathbf{z}}_{k|k-1}\widehat{\mathbf{z}}^T_{k|k-1}+R_k \quad (29)$$

$$P_{xz,k|k-1}=\frac{1}{2n}\sum_{i=1}^{2n}\boldsymbol{X}_{i,k|k-1}\mathbf{Z}^T_{i,k|k-1}-\widehat{\mathbf{x}}_{k|k-1}\widehat{\mathbf{z}}^T_{k|k-1} \quad (30)$$

where ***X***, ***Z*** and ***X**** are cubature points of state and measurement equations and propagated cubature points, respectively.

See [1] for a detailed problem formulation and a solution for CKF.

III. The Cubature Information Filter

This section describes the CIF algorithm, which uses CKF in an EIF framework. The CIF algorithm is summarized in Algorithm 1. The factorization of the error covariance matrix, evaluation of cubature points and propagated cubature points for the process model is required for CIF, and is shown in (31-33).

Let the information state vector and information matrix be given by $\widehat{\mathbf{y}}_{k|k-1}$ and $\mathbf{Y}_{k|k-1}$. The factorization of the inverse information matrix is required to evaluate S, which is required for cubature points propagation.

$$[Y_{k-1|k-1}]^{-1} \quad = \quad S_{k-1|k-1}S^T_{k-1,k-1} \quad (31)$$

The evaluation of cubature points and propagated cubature points can then be given as

$$\boldsymbol{X}_{i,k-1|k-1} \quad = \quad S_{k-1|k-1}\xi_i+\widehat{\mathbf{x}}_{k-1|k-1} \quad (32)$$

$$\boldsymbol{X}^*_{i,k-1|k-1} \quad = \quad \mathbf{f}(\boldsymbol{X}_{i,k-1|k-1},\mathbf{u}_{k-1}) \quad (33)$$

where, $i=1,2,...,2n$ and n is the size of the state vector.

From (4) and (27), and (3) and (26)

$$\begin{aligned}\mathbf{Y}_{k|k-1} &= \mathbf{P}^{-1}_{k|k-1}\\ &=\left[\frac{1}{2n}\sum_{i=1}^{2n}\boldsymbol{X}^*_{i,k-1|k-1}\boldsymbol{X}^{*T}_{i,k-1|k-1}-\widehat{\mathbf{x}}_{k|k-1}\widehat{\mathbf{x}}^T_{k|k-1}+Q_{k-1}\right]^{-1}\\ \widehat{\mathbf{y}}_{k|k-1} &= \mathbf{P}^{-1}_{k|k-1}\widehat{\mathbf{x}}_{k|k-1}\\ &= \mathbf{Y}_{k|k-1}\widehat{\mathbf{x}}_{k|k-1}\\ &= \frac{1}{2n}\left[\mathbf{Y}_{k|k-1}\sum_{i=1}^{2n}\boldsymbol{X}^*_{i,k-1|k-1}\right]\end{aligned}$$

In the measurement update of CIF, the first two steps involve the evaluation of propagated cubature points and the predicted measurement and are given below.

The propagated cubature points for measurement model can be evaluated as

$$\mathbf{Z}_{i,k|k-1} \quad = \quad \mathbf{h}(\boldsymbol{X}_{i,k|k-1},\mathbf{u}_k) \quad (34)$$

and the predicted measurement can be given as

$$\widehat{\mathbf{z}}_{k|k-1} \quad = \quad \frac{1}{2n}\sum_{i=1}^{2n}\mathbf{Z}_{i,k|k-1} \quad (35)$$

The information state contribution and its associated information matrix in (8) and (9) are explicit functions of the linearized Jacobian of the measurement model. But the CKF algorithm does not require the Jacobians for measurement update and hence it cannot be directly used in the EIF framework. However, by using the following linear error propagation property [11, 2], it is possible to embed the CKF update in the EIF framework. The linear error propagation property for the error covariance and cross covariance can be approximated as

$$P_{zz,k|k-1} \quad \simeq \quad \nabla\mathbf{h}_x\mathbf{P}_{k|k-1}\nabla\mathbf{h}^T_x \quad (36)$$

$$P_{xz,k|k-1} \quad \simeq \quad \mathbf{P}_{k|k-1}\nabla\mathbf{h}^T_x \quad (37)$$

By multiplying $\mathbf{P}^{-1}_{k|k-1}$ and $\mathbf{P}_{k|k-1}$ on the RHS of (8) and (9) we get

$$\mathbf{i}_k=\mathbf{P}^{-1}_{k|k-1}\mathbf{P}_{k|k-1}\nabla\mathbf{h}^T_x\mathbf{R}^{-1}_k[\nu_k+\nabla\mathbf{h}_x\mathbf{P}^T_{k|k-1}\mathbf{P}^{-T}_{k|k-1}\widehat{\mathbf{x}}_{k|k-1}] \quad (38)$$

$$\mathbf{I}_k=\mathbf{P}^{-1}_{k|k-1}\mathbf{P}_{k|k-1}\nabla\mathbf{h}^T_x\mathbf{R}^{-1}_k\nabla\mathbf{h}_x\mathbf{P}^T_{k|k-1}\mathbf{P}^{-T}_{k|k-1} \quad (39)$$

By using (36) and (37) in (38) and (39) we get

$$\mathbf{I}_k=\mathbf{P}^{-1}_{k|k-1}P_{xz,k|k-1}\mathbf{R}^{-1}_kP^T_{xz,k|k-1}\mathbf{P}^{-T}_{k|k-1} \quad (40)$$

$$\mathbf{i}_k=\mathbf{P}^{-1}_{k|k-1}P_{xz,k|k-1}\mathbf{R}^{-1}_k[\upsilon_k+P^T_{xz,k|k-1}\mathbf{P}^{-T}_{k|k-1}\widehat{\mathbf{x}}_{k|k-1}] \quad (41)$$

where

$$P_{xz,k|k-1} \quad = \quad \frac{1}{2n}\sum_{i=1}^{2n}\boldsymbol{X}_{i,k|k-1}\mathbf{Z}^T_{i,k|k-1}-\widehat{\mathbf{x}}_{k|k-1}\widehat{\mathbf{z}}^T_{k|k-1}$$

The updated information state vector and information matri for the CIF can be obtained by using $\mathbf{I}_k$ and $\mathbf{i}_k$ from (4(and (41) in (6) and (7).

Algorithm 1 Cubature Information Filter

```
Time Update
1: Evaluate the information matrix and the information state vector
       $\mathbf{Y}_{k|k-1} = \mathbf{P}^{-1}_{k|k-1}$
       $\hat{\mathbf{y}}_{k|k-1} = \mathbf{Y}_{k|k-1}\frac{1}{2n}\sum_{i=1}^{2n}\boldsymbol{X}^{*}_{i,k-1|k-1}$
   where,
       $P_{k|k-1} = \frac{1}{2n}\sum_{i=1}^{2n}\boldsymbol{X}^{*}_{i,k|k-1}\boldsymbol{X}^{*T}_{i,k|k-1} - \hat{\mathbf{x}}_{k|k-1}\hat{\mathbf{x}}^{T}_{k|k-1} + Q_{k-1}$
Measurement Update
1: Evaluate the information state contribution and its associated information matrix
       $\mathbf{I}_k = \mathbf{Y}_{k|k-1}P_{xz,k|k-1}\mathbf{R}^{-1}_k P^{T}_{xz,k|k-1}\mathbf{Y}^{T}_{k|k-1}$
       $\mathbf{i}_k = \mathbf{Y}_{k|k-1}P_{xz,k|k-1}\mathbf{R}^{-1}_k[\upsilon_k + P^{T}_{xz,k|k-1}\mathbf{Y}^{T}_{k|k-1}\hat{\mathbf{x}}_{k|k-1}]$
   where
       $P_{xz,k|k-1} = \frac{1}{2n}\sum_{i=1}^{2n}\boldsymbol{X}_{i,k|k-1}\mathbf{Z}^{T}_{i,k|k-1} - \hat{\mathbf{x}}_{k|k-1}\hat{\mathbf{z}}^{T}_{k|k-1}$
2: The estimated information vector and information matrix of CIF are given as:
       $\mathbf{Y}_{k|k} = \mathbf{Y}_{k|k-1} + \mathbf{I}_k$
       $\hat{\mathbf{y}}_{k|k} = \hat{\mathbf{y}}_{k|k-1} + \mathbf{i}_k$
Recovery of Estimated State
       $\hat{\mathbf{x}}_{k|k} = \mathbf{Y}^{-1}_{k|k}\hat{\mathbf{y}}_{k|k}$
```

The derived CIF algorithm has some of the common properties of both CKF and EIF. For example, it has a derivative free filter, state propagation is in the information space, easy initialization of the filter, etc.

IV. Applications of CIF

In order to show the effectiveness of the proposed algorithm, two examples are included. First example deals with estimation of frequency and phase of the frequency demodulation model, and the second example deals with simultaneous localization and mapping (SLAM).

A. Frequency Demodulation

Consider the case of demodulating the frequency modulation (FM) signal. The discrete nonlinear FM model [16] can be given by

$$\begin{bmatrix} \omega_k \\ \varphi_k \end{bmatrix} = \begin{bmatrix} \mu\omega_{k-1} \\ \arctan(\lambda\varphi_{k-1} + \omega_{k-1}) \end{bmatrix} + [\mathbf{w}] \qquad (42)$$

and the output model can be given by

$$\begin{bmatrix} y_k \end{bmatrix} = \begin{bmatrix} \cos(\varphi_k) \\ \sin(\varphi_k) \end{bmatrix} + [\mathbf{v}] \qquad (43)$$

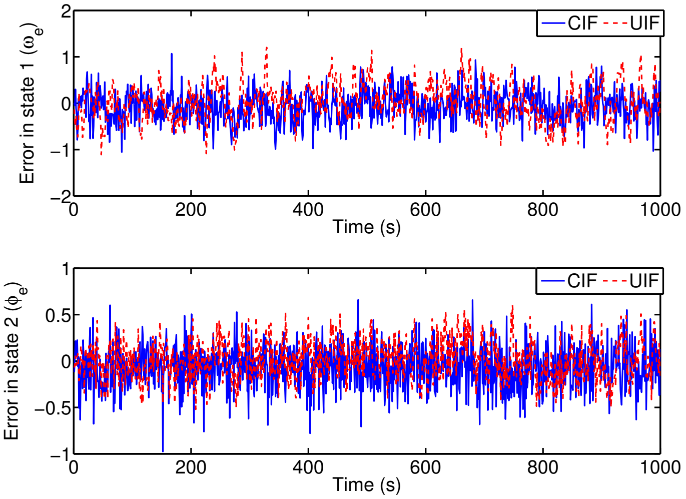

Fig. 1. Error of the FM model states using UIF and CIF. Solid line and dotted lines represents error in states using CIF and UIF algorithms, respectively.

The two states ω and φ, represents the frequency and the phase of the signal, y denotes the observations, and $\mathbf{w}$ and $\mathbf{v}$ are the process and measurement noises with covariances $\mathbf{Q}$ and $\mathbf{R}$. The parameters used in the simulations are $\mu = 0.9$, $\lambda = 0.99$, and standard deviations of process and measurement noises are 1 and 0.002, respectively. By choosing the standard deviation of process noise as 1, the states behaviour is substantially nonlinear. The objective is to estimate the frequency message ω_k from the noisy in-phase and quadrature observations y_k [16]. The estimation of the FM model states is done by using two estimation algorithms, unscented information filtering (UIF) algorithm [2] and the proposed CIF. The UIF is the closest counterpart of CIF, as both are Bayesian filters built in Gaussian domain and both uses the information domain for the state propagation. To compare these two state estimation algorithms for FM model, 100 independent Monte-Carlo simulations were performed, and the average of these 100 simulations have considered for the analysis. All the filters were initialized randomly. The tuning parameters used for UIF filter are $\alpha = 0.001$, $\kappa = 0$, and $\beta = 3$. One can note that, CKF or CIF algorithms does not require these tuning parameters for state estimation. The Fig.1 shows the error between the actual states and the estimated states. One can see that the average error in both the states are slightly higher in case of UIF algorithm as compared to the proposed CIF algorithm. It is hard to see the effectiveness of the proposed algorithm from Fig.1. Hence, we have used the root-mean square error (RMSE) of the frequency and phase (of the FM model) as performance metric to compare the proposed method with UIF. Fig.2 shows the RMSE of both the algorithms, once can see that the overall RMSE of the UIF is higher than the proposed CIF algorithm. The average values of RMSE (over simulated time interval) are 4.0541 and 4.4337 using CIF and UIF algorithms, respectively.

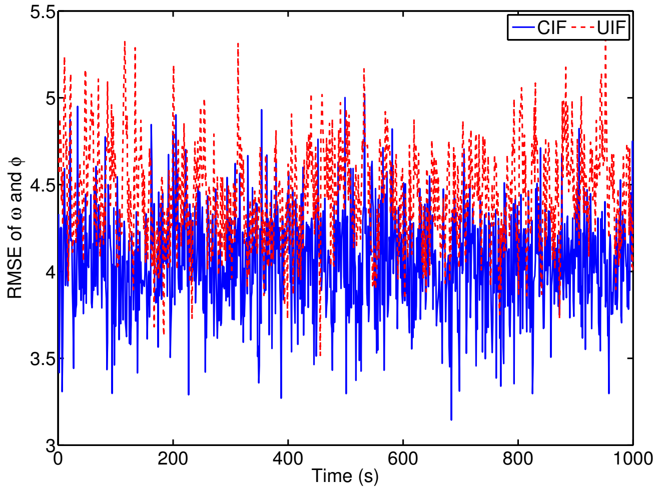

Fig. 2. RMSE of ω and φ using UIF and CIF. Solid line represents RMSE using CIF and dotted line represents RMSE using UIF.

B. SLAM

SLAM is the process of simultaneously building a map and locating a vehicle in it, and can be used for autonomous navigation [3]. SLAM can be performed by storing the vehicle poses and landmarks in a single state vector, and estimating it by a recursive process of prediction and update. In SLAM, the vehicle typically starts at an unknown location without *a priori* knowledge of landmark locations. The vehicle is mounted with a sensor which is capable of identifying the landmarks. The most common sensor used for SLAM is a laser, which takes the observation of the landmarks and outputs the range and bearing of the landmarks. While continuing in motion, the vehicle builds a complete map of landmarks and uses these to provide estimates of the vehicle location. By using the relative position between the vehicle and landmarks in the environment, both the positions of the vehicle and the positions of the features or landmarks can be estimated simultaneously. The basic EKF SLAM package is available in [17], and is modified for this work. The different models used for the SLAM application are given below.

1) Vehicle Model: The vehicle model used in this paper is the common bicycle model, assuming that the control inputs are given by the wheel velocity, V_{k-1}, and steering angle, γ_{k-1} and L is the distance between the front or rear set of wheels and the time interval ΔT denotes the time from $k-1$ to k [6], [4]. The vehicle's state vector represents its location and orientation.

$$\begin{bmatrix} \mathbf{x}_{v_k} \end{bmatrix} = \begin{bmatrix} x_{v_{k-1}} + \Delta T V_{k-1} \cos(\phi_{v_{k-1}} + \gamma_{k-1}) \\ y_{v_{k-1}} + \Delta T V_{k-1} \sin(\phi_{v_{k-1}} + \gamma_{k-1}) \\ \phi_{v_{k-1}} + \Delta T V_{k-1} \frac{\sin(\gamma_{k-1})}{L} \end{bmatrix}$$

In the above equations, the process noise $\mathbf{w}_{k-1}$ is eliminated. One popular way to include the process noise in the process model is to insert the noise terms into the control signal $\mathbf{u}$ such that

$$\mathbf{u}_{k-1} = \mathbf{u}_{n_{k-1}} + \mathbf{w}_{k-1} \tag{44}$$

where $\mathbf{u}_{n_{k-1}}$ is a nominal control signal and $\mathbf{w}_{k-1}$ is a zero mean Gaussian distribution noise vector with covariance matrix, $\mathbf{Q}$.

2) Landmark Model: In the context of SLAM, a landmark is a feature of the environment that can be observed using vehicle's sensor. Different kinds of landmark are used in SLAM like point landmarks, corners, lines, etc. In our case, we assumed the landmarks as point features. For the SLAM algorithm, the feature states are assumed to be stationary. Landmarks can be represented by the following expression

$$\mathbf{x}_{m_k} = \mathbf{x}_{m_{k-1}} \tag{45}$$

The SLAM map is defined by an augmented state vector formed by the concatenation of the vehicle and feature map state.

$$\mathbf{x}_a(k+1) = \begin{bmatrix} \mathbf{x}_{v_k}^T & \mathbf{x}_{m(k+1)}^T \end{bmatrix}^T \tag{46}$$

3) Sensor Model: In this paper, it is assumed that the sensor is equipped with a range-bearing sensor that takes observations of the features of the environment. Laser and sonar sensors are two examples of range-bearing sensors that can be used on a vehicle. Given the current vehicle position $\mathbf{x}_{v_k}$ and the position of an observed feature $\mathbf{x}_{m_k}$, the range and bearing can be modelled as

$$\mathbf{z}_{i_k} = \begin{bmatrix} \sqrt{(x_{v_k} - x_{i_k})^2 + (y_{v_k} - y_{i_k})^2} \\ \arctan \frac{y_{v_k} - y_{i_k}}{x_{v_k} - x_{i_k}} \end{bmatrix} + \begin{bmatrix} v_{r_k} \\ v_{\theta_k} \end{bmatrix} \tag{47}$$

where 'i' denotes the feature number.

In the following numerical experiments, the velocity of the vehicle is $3m/s$, the maximum steering angle is $-30^\circ < \gamma < 30^\circ$ and the maximum rate of change in steer angle is 20 deg/sec. The controls are updated every 0.025 seconds and observations occur every 0.2 seconds. The range-bearing sensor has a forward-facing 180° field-of-view and maximum range of 30 metres. The process and observation noises for both the filters are $\sigma_v = 0.3m/s, \sigma_\gamma = 3^\circ$ and $\sigma_r = 0.1m, \sigma_\theta = 1^\circ$, respectively. The trajectory of the vehicle is known and sixty two landmarks were randomly spread, the simulation scenario is shown in Fig. 3.

The CIF SLAM is compared with its closest counterpart, the unscented information filter (UIF) [2] SLAM. The SLAM results with CIF and UIF are shown in Fig. 4. The few peaks in the figure are because of the steering angle constraints. The average norm error of the CIF and UIF SLAM are 0.1294 and 0.3553, respectively. Hence, the CIF SLAM performance is better than the UIF SLAM. One can note that purpose of this paper is to propose the new estimation algorithm, and to verify its effectiveness. The estimation of the SLAM states using the proposed approach in a very large-scale environments is beyond the scope of the paper. However, one of our future works is to verify the CIF SLAM using real robots or by using the real data-sets like Victoria Park data-set, etc.

The obvious structure of the CIF, makes it suitable for multiple sensor fusion and the square root version of CIF

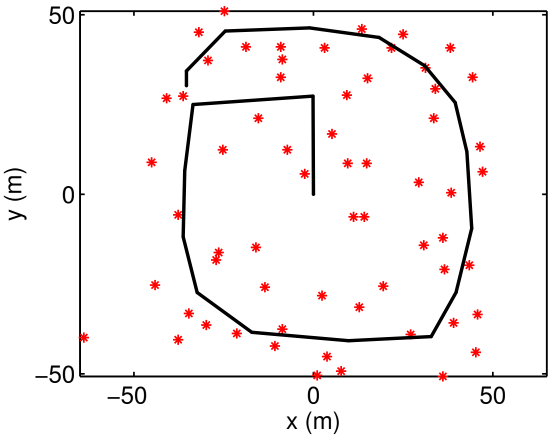

Fig. 3. Simulation scenario showing the trajectory of the vehicle (solid line) and landmarks (‘*’).

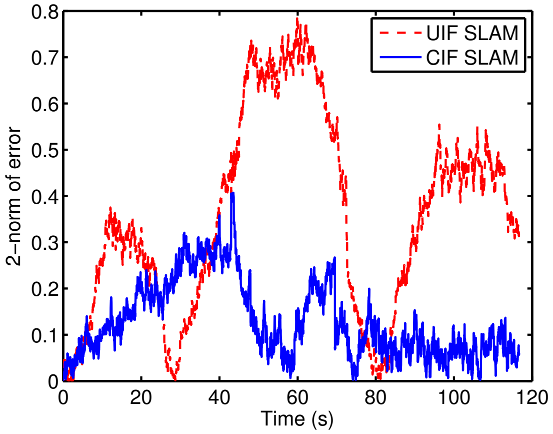

Fig. 4. 2-norm error of the vehicle positions $[x_v, y_v]^T$ of UIF and CIF SLAM. Solid line represents CIF SLAM and the dashed line represents UIF SLAM.

can be used for numerical stability. One may note that, the objective of this paper is to embed the EIF and CKF frameworks to form an efficient filter which has the desired properties of both the filters and the sparsity analysis of the CIF is beyond the scope of this work. One of our future work is to perform convergence-, stability-, and computational complexity analysis of the proposed CIF.

V. Conclusions

In this paper, we have proposed a cubature information filter for nonlinear systems. The proposed filter is derived from an extended information filter and the recently developed cubature Kalman filter. The CIF have the following desirable properties:

1) It does not require the evaluation of Jacobians during the prediction and update stage.
2) CIF is expressed in information space and hence can be easily initialized compared to the CKF or EKF.
3) The update step of CIF is computationally simpler than CKF or EKF and the update structure makes it a promising approach for decentralized data fusion.

The efficacy of the proposed algorithm is verified by simulations. The CIF algorithm is applied for state estimation of FM model and for SLAM application. The proposed algorithm is also compared with the UIF for both applications. It is also shown that the CIF, when applied to FM model estimation and SLAM, outperforms UIF.

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References

[1] I. Arasaratnam and S. Haykin, “Cubature Kalman Filters,” IEEE Transactions on Automatic Control, vol. 54, no. 6, 2009, pp. 1254-1269.

[2] D.-J Lee, “Nonlinear Estimation and Multiple sensor fusion using unscented information filtering,” IEEE Signal Processing Letters, vol. 15, 2008, pp. 861-864.

[3] M.W.M.G. Dissanayake, P. Newman, S. Clark, H. F. Durrant-Whyte, and M. Csorba, “ A Solution to the Simultaneous Localization and Map Building (SLAM) Problem,” IEEE Tramsactions on Robotics and Automation, vol. 17, no.3, 2001, pp. 229-241.

[4] T. Bailey, J. Nieto, J. Guivant, M. Stevens and E. Nobot, “Consistency of EKF-SLAM Algorithm”, in Proceedings of the Intelligent Robots and Systems, 2006, pp. 3562-3568.

[5] A.G.O. Mutambara, “Decentralized Estimation and Control for Multisensor Systems,” CRC Press, 1998.

[6] S. J. Julier, and J. K. Uhlmann “ Unscented filtering and nonlinear estimation,” Proceedings of the IEEE, vol. 92, no.3, 2004, pp. 401-422.

[7] S. Arulampalam, S. Maskell, N. Gordon, and T Clapp “ A tutorial on particle filters for on-line nonlinear/non-Gaussian Bayesian tracking,” IEEE Transactions on Signal Processing, vol. 50, no. 2, 2002, pp. 174-188.

[8] K. Ito and K. Xiong, “Gaussian Filters for Nonlinear Filtering Problems,” IEEE Transactions on Automatic Control, vol. 45, no. 5, 2000, pp. 910-927.

[9] C. P. Mracek, J. R. Cloutier, and C. A. DSouza, “A New Technique for Nonlinear Estimation,” Proceedings of IEEE Conference on Control Applications, 1996, pp.338-343.

[10] C. Edwards, S. K. Spurgeon and R. J. Patton, “Sliding mode observers for fault detection and isolation,” Automatica, vol. 36, no. 4, 2000, pp. 541-553.

[11] G. Sibley, G. Sukhatme, and L. Matthies, “The iterated sigma point filter with applications to long range stereo,” in Proceddings of Robotics: Science and Systems, Philadelphia, PA, 2006.

[12] C. Fernandez-Prades, and J. Vila-Valls, “Bayesian Nonlinear Filtering Using Quadrature and Cubature Rules Applied to Sensor Data Fusion for Positioning,” in Proceedings of IEEE International Conference on Communications, Cape Town, 2010.

[13] B.D.O. Anderson and J.B. Moore, “Optimal Filtering,” Prentice-Hall, 1979.

[14] P.S. Maybeck, “Stochastic Models, Estimation and Control,” Academic Press, vol. 1, 1979.

[15] S. Julier, J.K. Uhlmann “A New Method of the Nonlinear Transformation of Means and Covariances in Filters and Estimators,” IEEE Transactions on Automatic Control, vol. 45, no. 3, pp. 477-482, 2000.

[16] G.A. Einicke and L.B. White, “Robust Extended Kalman Filtering,” IEEE Transactions on Signal Processing, vol. 47, no. 9, 1999, pp. 2596-2600.

[17] [Online]. Available: http://www-personal.acfr.usyd.edu.au/tbailey/software/slam_simulations.htm. Jan. 2010.