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A Control Allocation System for Automatic Detection and Compensation of Phase Shift Due To Actuator Rate Limiting

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Abstract— **This paper proposes a control allocation system that can detect and compensate the phase shift between the desired and the actual total control effort due to rate limiting of the actuators. Phase shifting is an important problem in control system applications since it effectively introduces a time delay which may destabilize the closed loop dynamics. A relevant example comes from flight control where aggressive pilot commands, high gain of the flight control system or some anomaly in the system may cause actuator rate limiting and effective time delay introduction. This time delay can instigate Pilot Induced Oscillations (PIO), which is an abnormal coupling between the pilot and the aircraft resulting in unintentional and undesired oscillations. The proposed control allocation system reduces the effective time delay by first detecting the phase shift and then minimizing it using constrained optimization techniques. Flight control simulation results for an unstable aircraft with inertial cross coupling are reported, which demonstrate phase shift minimization and recovery from a PIO event.**

I. Introduction

Actuator rate saturation is an important issue for closed loop control systems since it may create significant phase shift and amplitude reduction between the commanded and the actual system states (see Fig.1). Phase shift manifests itself as an effective time delay which, in general, decreases the phase margin of the system resulting in reduced robustness to uncertainties, oscillatory behavior and degraded performance. In flight control systems (FCS), for example, effective time delay introduction due to actuator rate saturation is encountered in almost all Pilot Induced Oscillation (PIO) events and consequently it is considered as an important instigator of PIOs. A PIO can be described as an inadvertent, sustained aircraft oscillation which is the consequence of an abnormal joint enterprise between the aircraft and the pilot [1]. An excellent overview of the effect of rate limiting in PIOs can be found in [2].

It is noted that even for the cases where phase shifting due to rate saturation does not cause a drastic event such as a PIO, it can and will degrade the performance and therefore must be addressed by the control system designer. In the literature, there are several successful approaches for phase compensation such as the "Differentiate-Limit-Integrate" (DLI) method [3], [4], [5], [6], [7], logic or feedback based based methods [8], [9], [10] and methods such as [11] that use describing function relationships developed in [12].

To the best of authors' knowledge, most, if not all, of the previously reported successful implementation results were for single-input single-output (SISO) applications without any redundant actuators. In such a SISO scenario, the phase shifted signals represented in Fig. 1 are the desired scalar control input and the actuator position. Therefore, the phase compensator should be placed between the control input and the actuator. However, in a multi-input multi-output (MIMO) system with redundant actuators, using a phase compensator for each and every individual actuator may not be necessary and may even create undesirable or unexpected results. An extension of the DLI method to multi-input multi-output (MIMO) applications was given in [4], however, the authors had to use "ganged" actuators for successful implementation. Ganging of the actuators may be undesirable as it prevents the use of redundant actuators for secondary objectives like drag minimization or reconfiguration after a failure. In addition, ganging becomes more cumbersome as the number of actuators increases [13].

In over-actuated systems, the designer may have an opportunity of treating secondary objectives using the redundancy of the actuators. For example, in a flight control system with redundant control surfaces, minimizing the drag can be a secondary objective together with the primary objective of trajectory following. This is achieved using a technique called control allocation. In a system with control allocation, first the total control input is calculated by the controller and then the control allocator distributes this total control effort among individual actuators. When there are redundant actuators, the allocator can pick actuator configurations so that a secondary objective can also be achieved. In addition to enabling achieving secondary objectives, control allocation also allows the separation of controller design and control effort distribution, which facilitates the reconfiguration of the actuators in case of a failure. For an introduction to control allocation see [14]. Also see [15] and [13] for different control allocation approaches.

The novel control allocation system proposed in this paper is an extension of authors' earlier work on CAPIO (a Control Allocation technique to recover from Pilot Induced Oscillations) [16], which was developed for MIMO applications in the presence of redundant actuators. In that work, the authors assumed that there exists a PIO detector (see [17] for an example) on board which turns on the synchronization mode of CAPIO in the case of a PIO event and once this mode is turned on, CAPIO helps the aircraft recover from the PIO

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by minimizing the phase shift between the commanded and actual total control effort. When the PIO ceases to exist, the PIO detector turns on the tracking mode of CAPIO which makes the actual total control effort follow the desired total control effort. In this paper, the authors propose an algorithm called "CAPIO System" which *automatically* detects and compensates the phase shift between the desired and the actual total control effort regardless of a PIO occurrence. As opposed to the earlier work [16], the switching between the synchronization and the tracking modes are based on the phase shift value, not PIO detection. In addition, it is not *assumed* that there exists an agent, like a PIO detector, for the switching decision, but this agent, a phase detector in this case, is actually *developed and integrated* with the CAPIO algorithm. This stand alone, integrated system is called "CAPIO System". This new system has two advantages over the previous CAPIO work. First, there is no need for a PIO detector, but instead, a phase detector is developed which is easier to implement. Second, this new system is a general phase compensator which eliminates the destabilizing effects of phase shifting and for which PIO recovery/prevention becomes a special case.

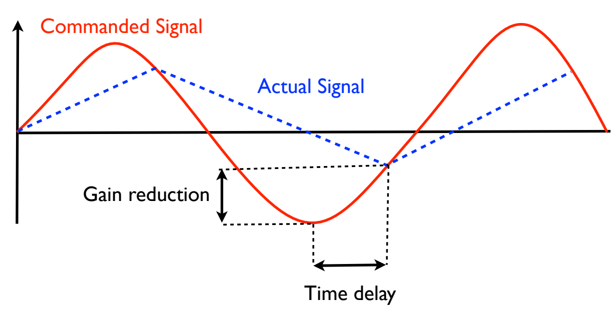

Fig. 1. Commanded and actual signals in the presence of rate saturation.

It is noted that in [18] Durham and Bordignon extended the direct control allocation scheme to make it easier to implement for the case of rate-limited actuators and consequently ended up with a "moment-rate allocation" scheme. Although in [18] there is no implementation result showing a PIO recovery/prevention example, the control allocation scheme in [18] has a potential to handle PIOs despite being more complicated than CAPIO System. Furthermore, the technique in [18] needs the calculation of a moment rate set which can introduce additional computational intensity.

The organization of this paper is as follows. In Section II, the phase detector is described. In Section III, the CAPIO System is explained using an unstable aircraft model with redundant actuators experiencing a PIO. First, an example of a PIO formation is presented when a conventional control allocation is used, then it is shown that with the CAPIO System phase shifting is minimized and PIO recovery is achieved. Finally, a summary of the paper is given.

II. Phase Detector

The main component in the phase detector developed in this paper is a signal peak detector which seeks the signal derivative sign changes. Once the peaks of the input and output signals are detected, individual signal frequencies and the time shift between the peaks of the input and the output signal can be calculated, which in turn is used to calculate the phase shift in real time.

There are certain assumptions that are used for this approach to online phase detection:

Assumption 1: All peaks belong to a piecewise-periodic signal.

Assumption 2: The point where the signal moves away from a rest (i.e. zero derivative) region is a peak.

Assumption 3: A local maximum (minimum) of the output signal always follows a local maximum (minimum) of the input signal.

Assumption 4: When the peak-to-peak amplitudes are smaller than a certain "meaningful signal value" (msv), the frequency of the signals are assumed to be unchanged.

Assumption 1 is necessary to eliminate the burden of determining whether or not the signals are periodic in the first place. This assumption is reasonable since in a rate saturation scenario which causes phase shift between the inputs and outputs, it appears to hold. *Assumption 2* is necessary to eliminate resetting the phase detector every time the system goes into an equilibrium state. *Assumption 3* is necessary to eliminate the burden of calculating the second derivative of the signals. In cases when this assumption does not hold, the phase shift is already larger than any threshold value to be used to switch CAPIO modes. The employment of this assumption will be clear after analyzing the following example. *Assumption 4* is necessary to eliminate very small oscillations that may disrupt phase detection.

In Fig. 2, an example of online phase detection is given. The first subfigure shows a desired (input) and achieved (output) system state in the presence of rate saturation. The other subfigures show the peak detection times, signal frequencies, peak to peak amplitudes and the phase shift between the signals, respectively. There are two important regions in this example that need special attention. The first region is the "moving away from equilibrium" region which corresponds to $t = 3s$. The peak detector, which is the second sub-figure, detects both the input and the output signal moving-away-from-equilibrium points. However, since the amplitude of the output signal is smaller than msv, which is set to 5 in this example, output signal frequency is assumed to be unchanged. The second region is the region where assumption 3 is violated, corresponding to $t \in [11 \;\; 12]$. The output signal maximum follows the input signal minimum and the phase detector gives a phase shift of 200 degrees (not shown in the figure). Note that this phase shift is the largest that the detector calculates and therefore will be larger than any threshold that one sets for the CAPIO System to switch modes, considering that at least one of the earlier calculated phase shifts is already unacceptable. A phase compensation system must prevent the system building such a large phase shift before it happens.

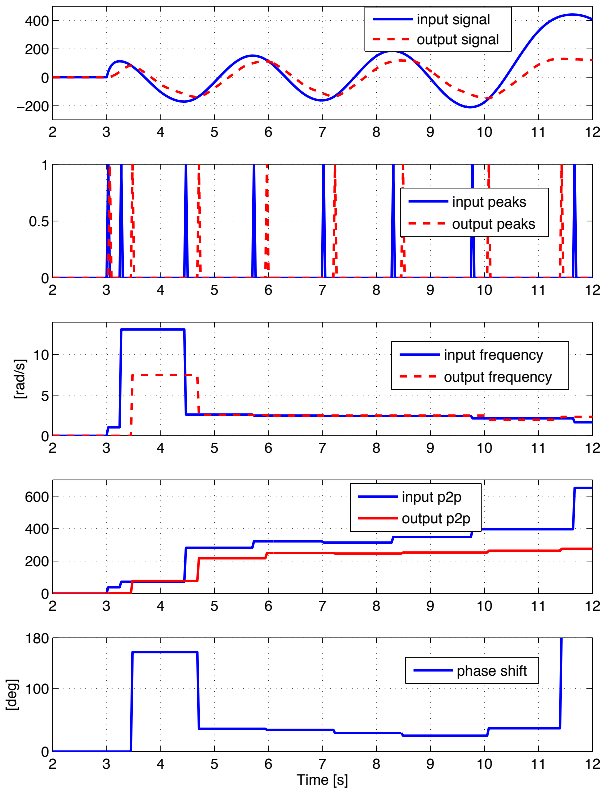

Fig. 2. An example of online phase shift detection.

III. CAPIO System

To demonstrate the capabilities of the CAPIO System, a flight control example using a simplified ADMIRE model from [19] is employed. Appropriate modifications were made to simulate inertial cross coupling. This model includes redundant actuators which makes the DLI method hard to apply directly.

The linearized aircraft model at Mach 0.22, altitude 3000 m is given by

$$
\begin{aligned}
x &= [\alpha \quad \beta \quad p \quad q \quad r]^T - x_{\text{lin}}, \\
y &= Cx = [p \quad q \quad r]^T - y_{\text{lin}}, \\
\delta &= [\delta_c \quad \delta_{re} \quad \delta_{le} \quad \delta_r]^T - \delta_{\text{lin}}, \\
u &= [u_c \quad u_{\text{re}} \quad u_{le} \quad u_r]^T - u_{\text{lin}} \\
\begin{bmatrix} \dot{x} \\ \dot{\delta} \end{bmatrix} &= \begin{bmatrix} A & B_x \\ 0 & -B_\delta \end{bmatrix} \begin{bmatrix} x \\ \delta \end{bmatrix} + \begin{bmatrix} 0 \\ B_\delta \end{bmatrix} u, \qquad (1)
\end{aligned}
$$

where α, β, p, q and r are the angle of attack, sideslip angle, roll rate, pitch rate and yaw rate, respectively. δ and u represent the actual and the commanded control surface deflections, respectively. Control surfaces are canard wings, right and left elevons and the rudder. $(.)_{\text{lin}}$ refers to values at the operating points where the linearization was performed. The actuators have the following position and rate limits

$$
\delta_c \in [-55, 25] \times \frac{\pi}{180}; \quad \delta_{re}, \delta_{le}, \delta_r \in [-30, 30] \times \frac{\pi}{180}
$$

$$
\dot{\delta}_c, \dot{\delta}_{re}, \dot{\delta}_{le}, \dot{\delta}_r \in [-70, 70] \times \frac{\pi}{180} \qquad (2)
$$

and have first order dynamics with a time constant of 0.05 seconds. It is noted that the position limits given are the same as the ones given in [19] but the rate limits are assumed to illustrate CAPIO System properties.

To make this model suitable for control allocation implementation, the actuator dynamics are neglected and the control surfaces are viewed as pure moment generators; their influence on $\dot{\alpha}$ and $\dot{\beta}$ is neglected. It is noted that the actuator dynamics are present during the simulations, i.e. they are neglected only during the control allocation algorithm derivation. These assumptions lead to the following approximate model

$$
\begin{aligned}
\dot{x} &= Ax + B_u u = Ax + B_v v, \\
v &= Bu, \qquad (3)
\end{aligned}
$$

where

$$
B_u = B_v B, \quad B_v = \begin{bmatrix} 0_{2\times3} \\ I_{3\times3} \end{bmatrix},
$$

$$
A = \begin{bmatrix} -0.5432 & 0.0137 & 0 & 0.9778 & 0 \\ 0 & -0.1179 & 0.2215 & 0 & -0.9661 \\ 0 & -10.5128 & -0.9967 & 0 & 0.6176 \\ 2.6221 & -0.0030 & 0 & -0.5057 & 0 \\ 0 & 0.7075 & -0.0939 & 0 & -0.2127 \end{bmatrix},
$$

$$
B = \begin{bmatrix} 0 & -4.2423 & 4.2423 & 1.4871 \\ 1.6532 & -1.2735 & -1.2735 & 0.0024 \\ 0 & -0.2805 & 0.2805 & -0.8823 \end{bmatrix}.
$$

The virtual (total) control effort, v, consists of the angular accelerations in roll, pitch and yaw. To simulate the effects of inertial cross-coupling, we modify the A matrix so that a change in pitch angular velocity creates a moment in roll and yaw axes:

$$
A = \begin{bmatrix} -0.5432 & 0.0137 & 0 & 0.9778 & 0 \\ 0 & -0.1179 & 0.2215 & 0 & -0.9661 \\ 0 & -10.5128 & -0.9967 & 1 & 0.6176 \\ 2.6221 & -0.0030 & 0 & -0.5057 & 0 \\ 0 & 0.7075 & -0.0939 & 0.1 & -0.2127 \end{bmatrix} \qquad (4)
$$

In this flight control example the pilot task is to track a given pitch angle reference, θ_d, using a pitch rate, q_d, stick. In addition, roll rate, p, and the yaw rate, r, are to be controlled independently to track their references p_d and r_d. The overall system structure is given in figure 3.

The inner controller is a dynamic inversion controller which uses q_d, p_d and r_d as references and produces the necessary attitude accelerations, $v \in \Re^3$, to track these references. Dynamic inversion control laws, v, make the closed loop dynamics follow a desired reference model

$$
\dot{y}_m = A_m y_m + B_m r_m \qquad (5)
$$

where $y_m = [p_m \quad q_m \quad r_m]^T$ represents the desired output and $r_m = [p_d \quad q_d \quad r_d]^T$ is the reference input. In this example, $A_m = -2 \times I_{3\times3}$ and $B_m = 2 \times I_{3\times3}$. Reference

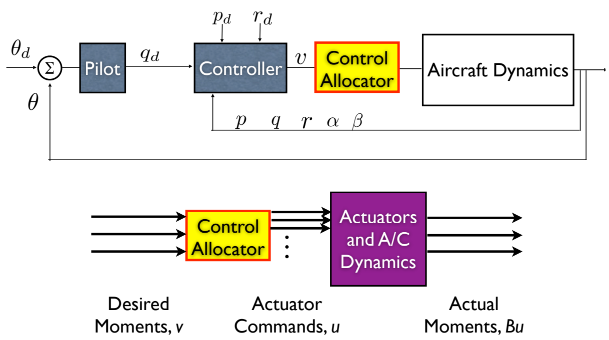

Fig. 3. Overall MIMO system structure

model tracking can be achieved by inverting the dynamics [13] as

$$v = (CB_v)^{-1}[A_m y + B_m r_m - CAx]. \tag{6}$$

The control allocator distributes this total control effort vector, v, to individual control surfaces via the actuator commands, $u \in \Re^4$. The control surfaces then produce actual attitude accelerations, Bu, where B is the control input matrix. The pilot is modeled as a pure gain for simplicity.

A. Flight control with conventional control allocation

The conventional control allocation used in this example minimizes the following objective function

$$J = ||Bu - v||_2^2 + \epsilon||u||_2^2 \tag{7}$$

subject to constraints due to magnitude and rate limits $max(\dot{u}_{min}T + u^-, u_{min}) \leq u \leq \min(\dot{u}_{max}T + u^-, u_{max})$, where T is the sampling interval and u^- denotes the value of u at the previous time step. It is noted that norms, instead of square-norms, can be used in the objective function. Note that (7) is in the form of a typical objective function used in conventional control allocators [13], where the main objective is to minimize the error between the desired and the actual total control efforts. As $\epsilon \to 0$, minimizing (7) becomes equivalent to achieving the main objective explained above and picking the solution that gives the minimum control surface deflection, among different solutions. In our simulation example, $\epsilon = 10^{-5}$.

Figure 4 presents the simulation result with the conventional control allocation where the pilot receives a step pitch angle reference at $t = 3$ seconds and the inner loop controller receives a pulse yaw rate reference at $t = 0.5$ seconds and a zero roll rate reference at all times. The pilot is aggressive and has a gain of 4.11. Because of this high gain, the aircraft goes into a divergent PIO in the pitch axis. In addition, inertial cross coupling causes dangerous oscillations in the roll axis, which finally diverges. Yaw axis also becomes unstable. Canard wings and the ailerons saturate both in position and the rate. The results of saturation can best be observed as a phase shift between the desired pitch acceleration v_2 and the actual pitch acceleration created by the control surfaces Bu_2. This phase shift, or the effective time delay, is something that is almost always observed in PIO events due to actuator saturation.

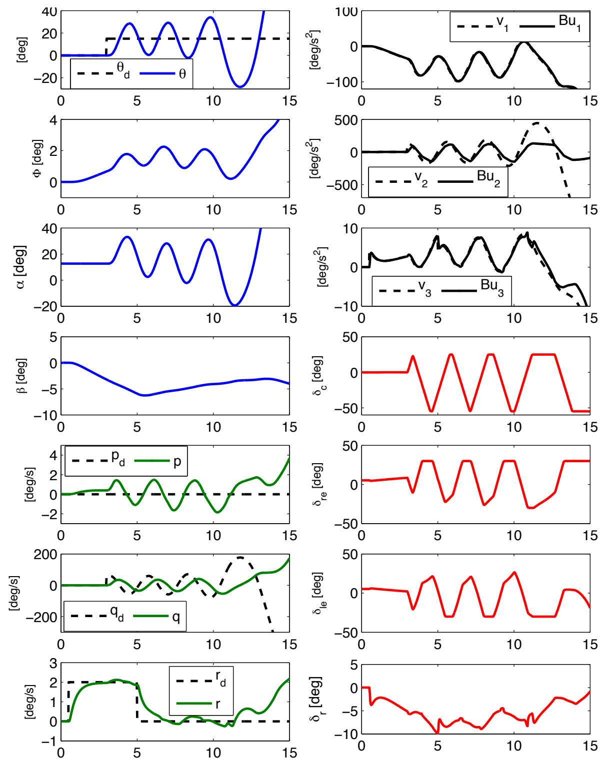

Fig. 4. Pitch and roll angles θ and ϕ, aircraft states x, on the left. Desired (commanded) and actual attitude accelerations v and Bu, and the control surface deflections δ, on the right, when a conventional control allocator is used.

B. Flight control with CAPIO System

To prevent the instability induced by the phase shifting (effective time delay) due to actuator rate saturation, CAPIO System forces the virtual (total) control effort v, to be in phase with the actual control effort Bu produced by the actuators. To achieve this, a derivative error term is added to objective function (7) to obtain the following CAPIO System objective function

$$J^{'} = ||Bu - v||_2^2 + ||W_d(B\dot{u} - \dot{v})||_2^2 + \epsilon||u||_2^2 \tag{8}$$

where $W_d \in R^{3\times3}$ represents a diagonal weighting matrix on the derivative term. The cost function $J^{'}$ is minimized with respect to u, with $\dot{u} = (u - u^-)/T$, where u^- denotes the value of u at the previous sampling instant. It is noted that with this modified objective function, the control allocator is trying to realize $\dot{v}$ as well as v. Very high values of W_d make the signals, v and Bu, have approximately the same derivative at all times, which eliminates the phase lag completely but causes a constant bias during tracking. On the other hand, very small values of W_d may not be sufficient for the control allocator to be any different than the conventional one and thus does not eliminate the destabilizing effects of phase shifting. Therefore, the designer needs to decide on suitable values of W_d that minimize the phase lag and at the same time prevent a bias. As an alternative, which is the proposed scheme in this paper, an automated system can "activate" W_d, i.e. set it to a constant matrix, only when one of the control axis phase shift becomes larger than a certain threshold value. In all other times the components of W_d are kept at zero. Phase shifts are calculated online by a phase detector as explained in Section II. W_d can also be utilized for axis prioritization.

The objective function (8) needs to be transformed into a form that can be minimized numerically. To achieve this goal, the derivatives in the objective function are approximated as $\dot{u} = (u - u^-)/T$. After some algebra, (8) can be rewritten as

$$\begin{aligned} J^{'} &= u^T \left(B^T T^2 B + B^T R B + \epsilon I_{4\times 4}\right) u \\ &\quad +2\left(-v^T T^2 B - {u^-}^T B^T R B - \dot{v}^T T R B\right) u \\ &\quad + v^T T^2 v + {u^-}^T B^T R B u^- + 2{u^-}^T B^T R T \dot{v} \\ &\quad + \dot{v}^T T^2 R \dot{v} \end{aligned} \tag{9}$$

subject to $max(\dot{u}_{min}T + u^-, u_{min}) \leq u \leq \min(\dot{u}_{max}T + u^-, u_{max})$, where $R = {W_d}^T W_d$.

Figure 5 presents the simulation result when CAPIO System is used as the control allocator. All the settings including the pilot gain are the same as in the previous example with the conventional control allocation. Since CAPIO System prevents the phase shift build-up, the aircraft now recovers from the PIO and no dangerous oscillation or divergence is observed in any axis after the recovery.

To show the difference that CAPIO System makes in control effort realization, the pitch axis accelerations are presented again in Fig. 6 for both cases together with the phase shift and CAPIO System modes for the case when phase compensation is active. CAPIO System switches between the synchronization (1) and tracking (0) modes depending on the phase shift value. The threshold value for the acceptable phase shift is set to 20 degrees. When the phase shift is larger than the threshold value synchronization mode becomes active, otherwise tracking mode is activated and the actual acceleration converges to the commanded acceleration without any bias formation.

The constrained optimization of the cost (9) is a low dimensional quadratic programming problem with linear inequality constraints. This problem depends on a vector of parameters, specifically, on $p = [u^- \quad v \quad \dot{v}]$. Note that the parameters enter linear in the cost and in the constraints and, hence, such a quadratic programming problem can be solved explicitly using off-line multi-parametric QP solvers. The solution is known to be a piecewise affine continuous function of the parameter vector and have the following form, $u = f_i p + g_i$, if $F_i + G_i \leq 0$, $i = 1, ..., N_r$ where N_r is the finite number of polyhedral regions and each region is associated with its set of linear inequality constraints and its affine map. Such an explicit solution is computed off-line and can be deployed on-line in the aircraft software using a set of simple if-then-else rules, additions, multiplications and

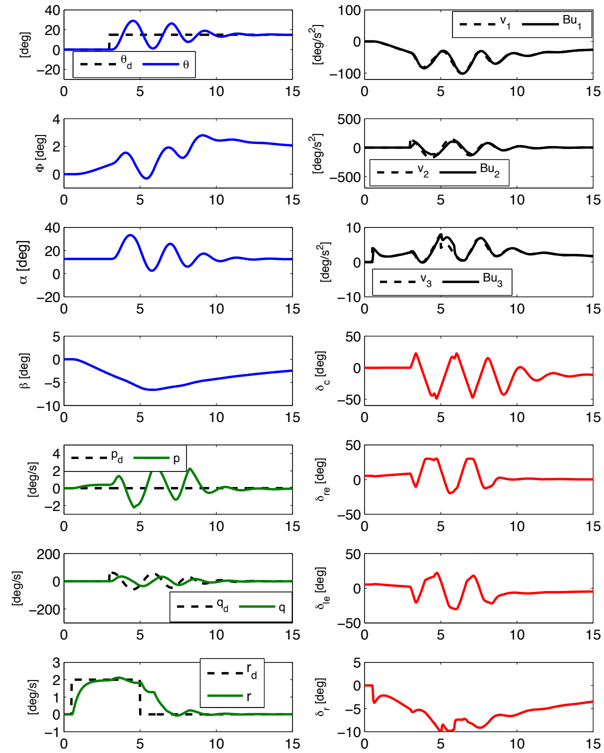

Fig. 5. Pitch and roll angles θ and ϕ, aircraft states x, on the left. Desired (commanded) and actual attitude accelerations v and Bu, and the control surface deflections δ, on the right, when CAPIO System is used.

comparisons. The need to embed a quadratic programming solver to perform constrained optimization of the cost (9) within aircraft software can thus be avoided altogether. A cross-section of the explicit solution polyhedral regions is illustrated in figure 7; the explicit solution has $N_r = 223$ regions.

IV. SUMMARY

In this paper, a "CAPIO System" was proposed which is a control allocation system that automatically detects and compensates the phase shift between the commanded and the actual total control input vectors. This new system was formed by developing an online phase detector and integrating it with previously developed control allocator CAPIO. CAPIO System switches between its synchronization mode and tracking mode depending on the phase shift between the commanded and the actual total control inputs. When the phase shift is larger than a certain threshold value, CAPIO System goes into synchronization mode where it minimizes the error between the derivatives of the signals which minimizes the phase shift. When the phase shift is smaller than the threshold value, CAPIO System switches to the tracking mode where it behaves like a conventional control allocator and minimizes the error between the commanded and the actual total control inputs. This dual behavior results in prevention of destabilizing effects of phase shifting, for example PIO formation, without any side effects like bias formation.

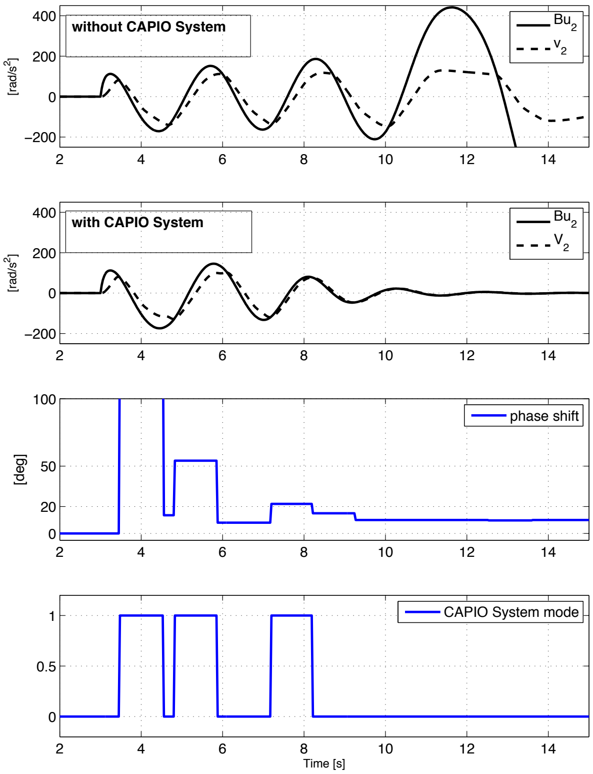

Fig. 6. Desired (commanded) and actual attitude accelerations v_2 and Bu_2 with and without the CAPIO System, phase shift between the desired and actual accelerations when CAPIO System is working and CAPIO System modes during operation. 0 and 1 corresponds to tracking and synchronization CAPIO System modes.

It was shown that CAPIO System works effectively in an inertially cross-coupled unstable MIMO system unlike the previously presented control allocators in the literature.

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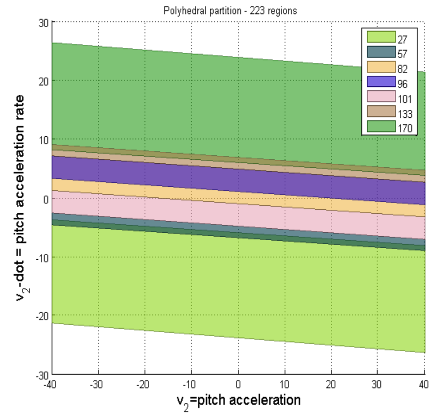

Fig. 7. Cross-section of regions of explicit solution by pitch acceleration - pitch acceleration rate plane.

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