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Synthesis of Robust PID Controllers Design with Complete Information On Pre-Specifications for the FOPTD Systems

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Abstract— **This paper provides a new design synthesis for the stable PID controllers to achieve the robustness to the system gain variations based on the first order plus time delay (FOPTD) systems. This designed PID controller is robust not only to the uncertainty of the plant steady-state gain, but also to the entire variations of the controller coefficients. The stability regions of the PID controller parameters are first determined according to a graphical stability criterion. According to two pre-specifications and the flat phase tuning constraint, a specific point in the three-dimension PID parameter-space can be determined. This designed PID controller can be, stable for sure as its parameter point is located in the stability region, and also robust to system gain variations according to the flat phase constraint. The important contribution of this proposed design synthesis is that, it provides the reliable procedures of designing the stable PID controller with robustness to the system gain variations, moreover, it can collect the complete information of the achievable design pre-specifications, which is the significant principle problem for the PID controllers synthesis in this paper.**

Index Terms— **Proportional-Integral-Derivative (PID) controllers, First Order Plus Time Delay (FOPTD) systems, stability region, robustness.**

I. I

The Proportional-Integral-Derivative (PID) controllers are so far overwhelmingly applied in industrial applications [1]. Because of its relatively simple structure and remarkable effectiveness of implementation, the PID controllers have been widely used in the process control industry [1]. It has been reported that, more than 95% of the control loops in process control industry are controlled by the PID controllers [2]. Hence, a small improvement in PID design and control can achieve a significant impact worldwide. In the past decides, many of the design techniques of the PID controllers are based on characterizing the dynamic response of the simple process plants to be controlled, e.g., the First Order Plus Time Delay (FOPTD) models [2].

For the PID controllers design, the stability issue is a fundamental problem. The solution of the stabilization problem is the minimal requirement to any PID controllers design and tuning. Recently, there has been a trend of calculating of stabilizing regions in the PID parameter-space [3][4][5][6][7]. In [3], a version of Hermite-Biehler Theorem derived by Pontryagin in [8] was investigated to determine the entire stabilizing region of PID parameters for the FOPTD models; and the same result of the stabilizing region of PID controllers as in [3] was achieved by a alternative way of the classical Nyquist stability criterion in [9]. In [10], the formulation, numerical scheme and numerical results for the computation of stabilizing fractional-order PID controllers for the fractional-order time delay systems are presented. This algorithm is simple and practical.

Meanwhile, the robust performance design is also a fundamental issue for the synthesis of PID controllers. The motivation of the robust performance design is to synthesis a controller with which the desired performance specifications are guaranteed in spite of the uncertainties of the plant and controller. In the process control industry, the steady-state gain of the plant dynamics which can be commonly characterized with the FOPTD systems, is easily to be affected under the complex industrial environment. At the same time, as the controller implementation is subject to the imprecision inherent in analog-digital and digital-analog conversion, resolution limitation of the measuring instruments and roundoff errors in numerical computations [11], the coefficients of the designed PID controller maybe change in the course of the control implementation. So, the robustness consideration for the steady-state gain of the plant and the coefficients of the PID controller is meaningful for the PID controller synthesis. Therefore, a flat phase tuning constraint is applied for our PID controller design. This tuning constraint makes the phase of open-loop system keeping flat around the pre-specified gain crossover frequency, which means the derivative of the phase w. r. t. the frequency at the gain crossover frequency point is zero, e.g., the system is robust to the system gain variations which include the uncertainty of the plant gain and the entire changes of the PID controller coefficients. Thus, the performance of the system with designed PID controller degrades gracefully as the system gain changes.

This paper provides a new design approach for the PID controllers to achieve the pre-specifications of phase margin and gain crossover frequency, and the flat phase requirement based on the FOPTD systems. This designed PID controller is not sensitive to the system gain variations, which means that it is robust not only to the uncertainty of the plant steady-state gain, but also to the entire variations of the controller coefficients. The stability regions of the PID controller parameters are first determined according to a graphical stability criterion. The regions are drawn graphically, instead of being calculated analytically. Whereafter, a three-dimension surface satisfying the pre-specified phase margin requirement, and a three-dimension curve on the surface above satisfying two pre-specifications, can be drawn in the PID parameter-space. Then, by defining a relative function according to the flat phase requirement, the specific point for determining the PID controller in the three-dimension-space can be fixed. This designed PID controller can be, stable as its parameters are located in the stability region for the first condition, and also robust to uncertainty of the plant steady-state gain and the entire variations of the controller coefficients. The important benefit of this proposed design synthesis is that it can give the complete information on the achievable pre-specified phase margin and gain crossover frequency, and provided the flexible design procedures of designing the PID controller with robustness to the system gain variations. Simulation example is presented to illustrate the advantages of the designed PID controller over the traditional Ziegler-Nichols PID controller [12], based on the FOPTD model.

The rest of this paper is organized as follows. In Sec. II, the PID control design synthesis for achieving the robustness to the system gain variations is presented, the complete information of two pre-specifications is collected graphically and in Sec. III, the procedures of the proposed PID controller design are summarized with an example, in Sec. IV, the simulation example is illustrated to show the advantages of the proposed PID design synthesis over the

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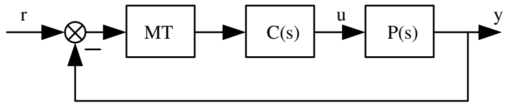


Fig. 1. Diagram of the feedback control system.

traditional Ziegler-Nichols PID optimization strategy. Conclusion is given in Sec. V.

II. R PID C D C I C A P -S

A. Preliminary

The design synthesis presented in this paper is based on the First Order Plus Time Delay (FOPTD) plants characterized by the following transfer function,

$$P(s) = \frac{K}{Ts+1}e^{-Ls}, \tag{1}$$

where, K represents the steady-state gain of the plant, T is the apparent time constant of the plant, and L represents the time delay.

Considering the feedback control system as shown in Fig. 1, where r is the command reference signal, y is the output signal of the plant, $P(s)$ given by (1) is the plant to be controlled, and $C(s)$ is the designed controller. In this paper, we focus on the Proportional-Integral-Derivative (PID) controller with a proportional item, an integral item and a derivative item, i.e.,

$$C(s) = K_p + \frac{K_i}{s} + K_d s, \tag{2}$$

where, K_p is the proportional gain, K_i is the integral gain, and K_d is the derivative gain.

In Fig. 1, MT is a Gain-Phase Margin Tester [13], which provides information for plotting the boundaries of constant gain margin and phase margin in a parameter plane [10]. The frequency transfer function of MT is given in the form below,

$$MT(A,\phi) = Ae^{-j\phi}. \tag{3}$$

Setting $\phi = 0$ in (3), the controller parameters can be obtained satisfying a given gain margin A of the control system as shown in Fig. 1; meanwhile, setting $A = 1$ in (3), one can get the controller parameters for a given phase margin ϕ.

B. Stability Region of the PID Parameters for FOPTD Plants

From Fig. 1 and expressions (1)(2)(3), the open-loop transfer function of the unity feedback control system in Fig. 1 is,

$$G(s) = MT(A,\phi)C(s)P(s). \tag{4}$$

The closed-loop transfer function can be expressed as,

$$\Phi(s) = \frac{MT(A,\phi)C(s)P(s)}{1+MT(A,\phi)C(s)P(s)}. \tag{5}$$

Substitute (1), (2) and (3) into (5), yield,

$$\Phi(s) = \frac{Ae^{-j\phi}Ke^{-Ls}(K_d s^2 + K_p s + K_i)}{s(Ts+1) + Ae^{-j\phi}Ke^{-Ls}(K_d s^2 + K_p s + K_i)}. \tag{6}$$

Hence, the characteristic equation of the closed-loop system is the denominator of (6) as,

$$\begin{aligned} & D(K_p, K_i, K_d, A, \phi; s) \\ = \; & s(Ts+1) + Ae^{-j\phi}Ke^{-Ls}(K_d s^2 + K_p s + K_i). \end{aligned} \tag{7}$$

With the given FOPTD model in (1), the problem is to compute a set of PID controllers stabilizing the FOPTD plant considered. The system stability is depending on the locations of the roots of the characteristic equation (7) with $A = 1$ and $\phi = 0°$. If all the roots of the polynomial (7) are located in the left-half of the s-plane, the closed-loop system (5) is bounded-input bounded-output stable. There are three parameters K_p, K_i and K_d for the PID controller. The stability region Q of these three controller parameters is defined as that, if $(K_p, K_i, K_d) \in Q$, all the roots of $D(K_p, K_i, K_d, A, \phi; s)$ are line in the left-half of the s-plane. The boundaries of the controller parameters stability region Q are determined by the real root boundary (RRB), complex root boundary (CRB) and infinity root boundary (IRB) [14][10][15][6].

- IRB: Although the theoretical method of calculating the IRB is difficult because of the time delay in the FOPTD system, which can generate infinite number of roots, the asymptotical location of roots far from the origin [16] can be used to figure out the IRB. The infinity root boundary is defined by the equation $D(K_p, K_i, K_d, A, \phi; s = \infty) = 0$, and the boundary can be obtained [10] as follows,

$$K_d = \pm\frac{T}{K}. \tag{8}$$

- RRB: The real root boundary is defined by the equation $D(K_p, K_i, K_d, A, \phi; s = 0) = 0$, so, one can get the boundary as,

$$K_i = 0.$$

- CRB: Substituting s with $j\omega$ in (7), the complex root boundary can be defined from $D(K_p, K_i, K_d, A, \phi; s = j\omega) = 0$ as follows,

$$\begin{aligned} & D(K_p, K_i, K_d, A, \phi; j\omega) \\ = \; & j\omega(j\omega T + 1) \\ & + Ae^{-j\phi}Ke^{-j\omega L}((j\omega)^2 K_d + j\omega K_p + K_i) \\ = \; & (-T\omega^2 + j\omega) + KAe^{-j(\phi+\omega L)}(-\omega^2 K_d + K_i + j\omega K_p) \\ = \; & (-T\omega^2 + j\omega) + KA(\cos(\phi+\omega L) \\ & - j\sin(\phi+\omega L))(-\omega^2 K_d + K_i + j\omega K_p) \\ = \; & -T\omega^2 + KA((-\omega^2 K_d + K_i)\cos(\phi+\omega L) \\ & + \omega K_p \sin(\phi+\omega L)) + j(\omega + KA(\omega K_p \cos(\phi+\omega L) \\ & - (-\omega^2 K_d + K_i)\sin(\phi+\omega L))) \\ = \; & 0. \end{aligned} \tag{9}$$

Considering the real part and the imaginary part of (9) respectively, one can obtain,

$$\begin{aligned} & -T\omega^2 + KA((-\omega^2 K_d + K_i)\cos(\phi+\omega L) \\ & + \omega K_p \sin(\phi+\omega L)) = 0; \end{aligned} \tag{10}$$

$$\begin{aligned} & \omega + KA(\omega K_p \cos(\phi+\omega L) \\ & - (-\omega^2 K_d + K_i)\sin(\phi+\omega L)) = 0. \end{aligned} \tag{11}$$

From (11),

$$\begin{aligned} & K_d \\ = \; & \frac{KA\sin(\phi+\omega L)K_i - KA\omega K_p \cos(\phi+\omega L) - \omega}{KA\omega^2 \sin(\phi+\omega L)}, \end{aligned} \tag{12}$$

substitute (12) into (10), yield,

$$K_p = \frac{T\omega\sin(\phi+\omega L) - \cos(\phi+\omega L)}{KA}, \tag{13}$$

so, one can get,

$$K_i = \frac{\omega\sin(\phi+\omega L) + T\omega^2\cos(\phi+\omega L)}{KA} + \omega^2 K_d. \tag{14}$$

Hence, given K_d, the curve of K_i w. r. t. K_p can be plotted with $\omega \rightarrow +\infty$.

So, with $A = 1$, $\phi = 0°$ and a fixed K_d, the parameter-space (K_p, K_i) is divided into stable and unstable regions by the IRB, RRB and CRB presented above. The stable region can be detected by testing one arbitrary point in every region. The system characteristic equation with PID controller chosen from the stable region has no root locating in the right-half of the s-plane. Inversely, if the PID is chosen from unstable region, the system characteristic equation must have some roots in the right-half of the s-plane. Thus, the stability region of the parameters K_i and K_p can be fixed by the RRB and CRB conditions with a given K_d in the interval determined by IRB condition (8). By sweeping over all the stabilizing K_d from IRB, the three-dimension stability region in the parameter-space for the three PID parameters can be determined, which is named as the *complete stability region*.

C. PID Parameters Design with Pre-Specifications of Phase Margin and Gain Crossover Frequency

Since the complete stability region is determined, the special surface in complete stability region of the parameter-space can be drawn to satisfy the designed phase margin ϕ_m with the set of $A = 1$ and $\phi = \phi_m$ in (9), or satisfy the designed gain margin A_m with the set of $\phi = 0^\circ$ and $A = A_m$ in (9).

Given one pre-specification – phase margin ϕ_m ($A = 1$), a *relative stability line* can be drawn in the (K_p, K_i)-space as $\omega \to \omega_0$ from zero with a certain fixed $K_{d1} \in [-T/K, T/K]$, by setting $A = 1$ and $\phi = \phi_m$ in (9). ω_0 is the maximum frequency guaranteing the pre-specified phase margin with the fixed K_{d1}. Sweeping all the K_d in $[-T/K, T/K]$, a surface in the three-dimension parameter-space can be generated satisfying the pre-specified ϕ_m, which is named as the *relative stability surface*. There exists a maximum frequency ω_0 satisfying the phase margin requirement with every fixed K_d in $[-T/K, T/K]$, so, one can figure out the boundary of the optional frequency.

Given the other pre-specification – gain crossover frequency ω_c, a point of the parameters K_p and K_i on the relative stability line can be determined with a fixed K_{d1}. Actually, from (5) and (7), one can get,

$$1 + M(A, \phi)C(s)P(s)|_{s=j\omega} = 0,$$

which means the open-loop transfer function G(s) is equal to -1 as below,

$$G(s) = M(A, \phi)C(s)P(s)|_{s=j\omega} = -1,$$

so, one can get,

$$|M(A, \phi)C(s)P(s)|_{s=j\omega}| = 1.$$

If $A = 1$ and $\phi = \phi_m$, all the $\omega \in (0, \omega_0]$ satisfying equation (7) can be treated as the gain crossover frequencies for the control plant (1) with the PID controller from that parameters point corresponding to ω.

So, with the pre-specified ω_c, ϕ_m and a certain K_{d1}, the other two PID parameters K_p and K_i can be determined on a point of the relative stability line. In the same way, sweeping all the K_d in $[-T/K, T/K]$, a curve in the three-dimension parameter-space can be determined, which is named as the *relative stability curve*. All the points on this curve can guarantee the two pre-specifications ω_c and ϕ_m.

D. PID Parameters Design with An Additional Constraint on Flat Phase

From (10) and (11), one can get,

$$\phi = \arctan \frac{T\omega^2 K_p + K_i - \omega^2 K_d}{-\omega K_p + \omega T K_i - \omega^3 T K_d} - \omega L + n\pi, \tag{15}$$

where, n is an integer which guarantees,

$$\phi + \omega L - n\pi = \arctan \frac{T\omega^2 K_p + K_i - \omega^2 K_d}{-\omega K_p + \omega T K_i - \omega^3 T K_d} \in (-\pi/2, \pi/2).$$

In order to enhance the system robustness to the system gain variations, which include the uncertainty of the plant steady-state gain and the entire change of the PID controller coefficients, the flat phase tuning constraint is proposed to design the PID controllers. The flat phase means the phase of the open-loop system is flat around the gain crossover frequency point in the Bode plot. With this constraint, the system phase can maintain almost the same value when the system gain changes in a certain interval, namely, the system with this designed PID is robust to the system gain variations, and even the overshoots of the step responses are almost the same with the variations of system gain in certain range. Hence, the control performance of the system with the designed PID controller degrades gracefully when the plants steady-state gain varies and the PID coefficients entirely change.

In order to satisfy the flat phase tuning constraint, the derivative of the open-loop system phase θ w. r. t. the frequency ω is force to be zero at the gain crossover frequency point, e.g.,

$$\frac{\mathrm{d}\theta}{\mathrm{d}\omega} = 0.$$

As mentioned in Sec. II-C, ϕ can be treated as the phase margin with $A = 1$ in (9). Thus, $\theta = \phi - \pi$, and,

$$\frac{\mathrm{d}\theta}{\mathrm{d}\omega} = \frac{\mathrm{d}\phi}{\mathrm{d}\omega} = 0.$$

From (15), one can get,

$$\frac{\mathrm{d}\phi}{\mathrm{d}\omega} = \frac{E}{B^2 + C^2} - L = 0, \tag{16}$$

where,

$$\begin{aligned} E &= -T\omega^2 K_p^2 + (T^2\omega^2 + 1)K_p K_i \\ &\quad + (T^2\omega^4 + \omega^2)K_p K_d + 2T\omega^2 K_d K_i - T\omega^4 K_d^2 - T K_i^2, \\ B &= -\omega K_p + \omega T K_i - \omega^3 T K_d, \\ C &= T\omega^2 K_p + K_i - \omega^2 K_d. \end{aligned}$$

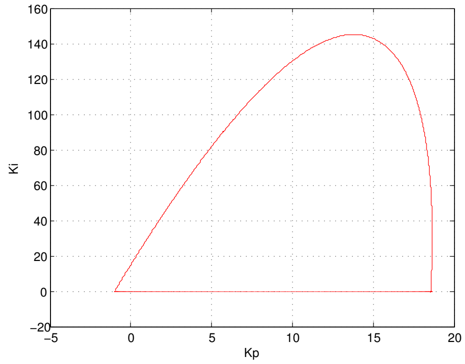
From Sec. II-C, all the PID parameters points on the relative stability curve in the three-dimension parameter-space satisfying both the pre-specified ϕ_m and ω_c, can be tested in the equation (16). If one point of the PID parameters (K_p, K_i, K_d) on the relative stability curve can be found to guarantee the relationship (16), this point is named as *plat phase stable point*. Thus, the designed PID controller from this plat phase stable point can achieve the robustness to the system gain changes. Pre-specifying different gain crossover frequency ω_c, the exitance of the flat phase stable point can be tested.

E. Information Collection for the Achievable Pre-Specifications for the PID Parameters Design

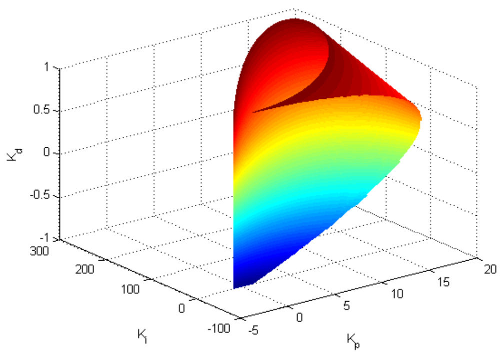
From Sec. II-C, there exists a maximum frequency ω_0 satisfying the phase margin ϕ ($A = 1$) requirement with every fixed K_{d1} in $[-T/K, T/K]$, so, sweeping all the K_d in $[-T/K, T/K]$, one can get a curve of ω_0 w. r. t. K_d. Then, the upper boundary ω_{0max} of the optional gain crossover frequency pre-specification can be figured out. Pre-specifying a phase margin ϕ_m, the existence of all the flat phase stable points can be detected with different $\omega_c \in (0, \omega_{0max}]$.

So, with a fixed phase margin, the region for choosing the gain crossover frequency to get the designed PID controller can be decided by searching the frequency in between $(0, \omega_{0max}]$. Whereafter, sweeping the the phase margin pre-specification from zero to infinity, the complete information for the achievable phase margin and gain crossover frequency pre-specifications guaranteing a PID controller satisfying the flat phase tuning constraint are collected.

According to this instructional information, choosing a phase margin ϕ_m and a gain crossover frequency ω_c properly, the designed stable and robust PID controller can be obtained following the proposed design synthesis in this paper.

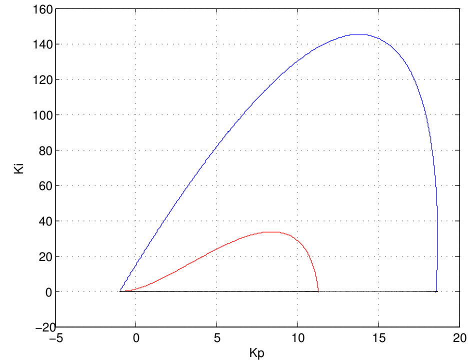


(a) Stability region of K_i w. r. t. K_p with K_d=0.5

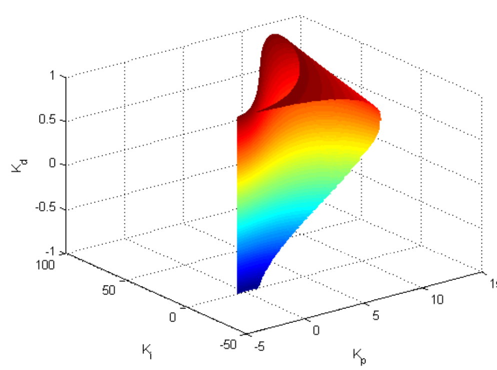


(b) Complete stability region of K_i, K_p and K_d

Fig. 2. Stability region of of K_i w. r. t. K_p with K_d=0.5 and complete stability region.



(a) Stability region comparison of K_i and K_p with $\phi_m = 0°$ and $\phi_m = 50°$ ($K_d = 0.5$)



(b) Three-dimension stability region of K_i, K_p and K_d with $\phi_m = 50°$.

Fig. 3. Stability region comparison of K_i and K_p and three-dimension stability region with $\phi_m = 50°$.

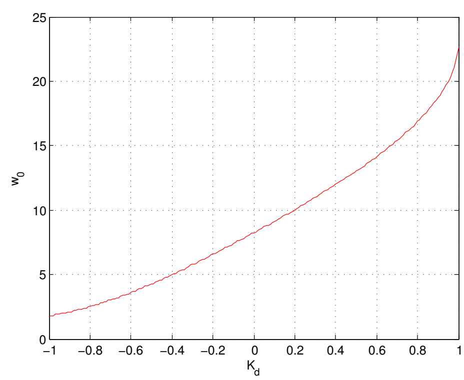


Fig. 4. Relation curve of ω_0 v.s. K_d, $\omega_{0max} = 22.73\ rad/s$.

III. D P S A E

In this section, we present an examples to summarize the procedures of the proposed PID controller design and information collection for the achievable pre-specifications, as shown below.

Step 1: Give the FOPTD plant with $K = 1$, $T = 1s$ and $L = 0.1s$, pre-specify the phase margin $\phi_m = 50°$ and gain crossover frequency $\omega_c = 10\ rad/s$.

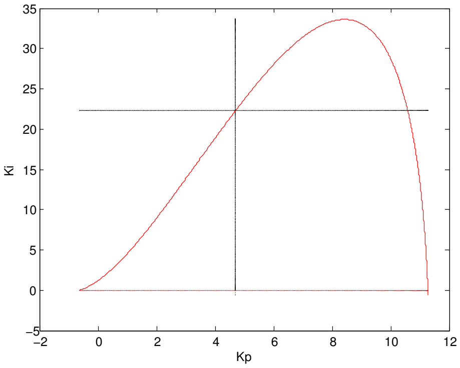
Step 2: Get the stability interval of K_d, $K_d \in [-1, 1]$, from (8) according to IRC; choose $K_d = 0.5$ and draw the line of K_p w. r. t. K_i in the (K_p, K_i)-space, detect the stable region with random point test as shown the red line surrounded section in Fig. 2(a); obtain the complete stability region as shown in Fig. 2(b), without constraints following the scheme introduced in Sec. II-B.

Step 3: With the pre-specified $\phi_m = 50°$ and fixed $K_d = 0.5$, the relative stability line in the (K_p, K_i)-space can be drawn as shown the red line and $K_i = 0$ surrounded section with $\omega \in (0, \omega_0]$ in Fig. 3(a), which can be compared with the stability region as shown the blue line and $K_i = 0$ surrounded section for $\phi_m = 0°$; get the relative stability surface by sweeping $K_d \in [-1, 1]$ in Fig. 3(b) in the three-dimension parameter-space, satisfying the pre-specified phase margin $\phi_m = 50°$.

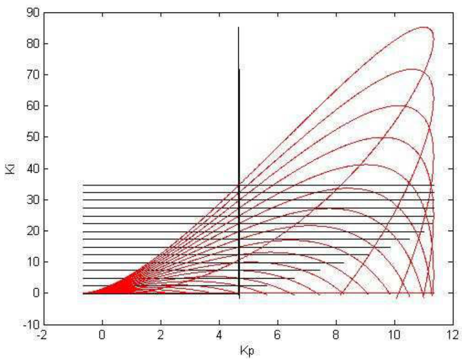
Step 4: According to the relative stability surface in Fig. 3(b), the relation curve of ω_0 w. r. t. K_d can be drawn in Fig. 4, it can be seen that, the maximum value of ω_0 is $\omega_{0max} = 22.73\ rad/s$; given gain crossover frequency $\omega_c = 5\ rad/s$, phase margin $\phi_m = 50°$ and $K_d = 0.5$, the other two parameters K_p and K_i can be determined from the intersection in Fig. 5(a) with $\omega = \omega_c = 5\ rad/s$; sweeping all the $K_d \in [-1, 1]$ as shown in Fig. 5(b), one can get the relative stability curve presented by the blue dashed line in Fig. 6(a) on the relative stability surface with $\phi_m = 50°$.

Step 5: Test all the points on the the relative stability curve to find a solution of equation (16), which is illustrated as a red star point on the relative stability curve in the three-dimension parameter-space of Fig. 6(b). Get the point in the three-dimension parameter-space to fix the PID controller satisfying the pre-specified phase margin, gain crossover frequency and the flat phase constraint. This point is the *flat phase stable point*. As shown in Fig. 6(b), this parameter point which satisfies the flat phase requirement is presented on the three-dimension curve of the K_i, K_p and K_d satisfying the phase margin $\phi = 50°$ and the gain crossover frequency $\omega_c = 5\ rad/s$.

Step 6: With the maximum frequency ω_{0max} from *Step 4*, obtain the optional values ω_c of the pre-specification gain crossover frequency in $(0, \omega_{0max}]$ under the pre-specified phase margin $\phi_m = 50°$,



(a) The designed K_i and K_p satisfying $\omega_c = 5\ rad/s$, $\phi_m = 50°$ with $K_d = 0.5$



(b) The designed K_i and K_p satisfying $\omega_c = 5\ rad/s$, $\phi_m = 50°$ sweeping all the optional K_d

Fig. 5. The designed K_i and K_p with $K_d = 0.5$ and with sweeping all the optional K_d.

for guaranteing the existence of the flat phase stable point. So, the information of the pre-specification gain crossover frequency ω_c is collect from Fig. 7 with $\phi_m = 50°$, the achievable ω_c interval is corresponding to the nonzero solutions of the flat phase stable point $K_{pfp}/K_{ifp}/K_{dfp}$. Test different phase margin ϕ_m, the complete information of the pre-specifications phase margin and gain crossover frequency can be collected as plotted the red region in Fig. 8(a).

Remark 3.1: The FOPTD system (1) can be normalized as shown below,

$$P_n(s) = \frac{1}{Ts+1}e^{-(L/T)Ts} = \frac{1}{s'+1}e^{-L's'}, \tag{17}$$

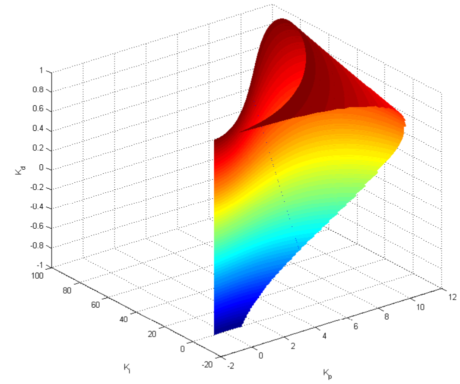
where, $s' = Ts$ and $L' = L/T$. The parameter K in (1) can be normalized as 1, since the steady-state gain of the plant can always be treated as part of the gain of the PID controller.

So, if the complete information of the pre-specifications phase margin ϕ_m and gain crossover frequency ω_m are collected for the standard form of the control system plants below,

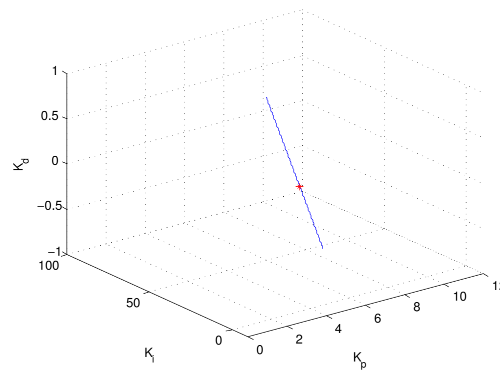
$$P_0(s) = \frac{1}{s+1}e^{-Ls}, \tag{18}$$

where, L is equal to L' in (17), then, the complete information of the pre-specifications phase margin and gain crossover frequency for the normalized FOPTD system (17) can be easily figured out with the proportional change of the ω_c axis, $\omega_c = \omega'_c/T$ as $s' = Ts$.

From the example in Sec. III, the complete information of ϕ_m and ω_m are collected for the standard form of the control system (18) with $L = 0.1s$. In order to validate Remark 3.1, by repeating the procedures $Step$ 1 to $Step$ 6 in Sec. III, the complete information of ϕ_m and ω_m are also collected for the normalized control plant



(a) The relative stability curve on the relative stability surface



(b) The flat phase stable point on the relative stability curve

Fig. 6. The relative stability curve and the flat phase stable point in the three-dimension parameter-space.

(17) with $T = 10s$ and $L = 1s$, where $L' = L/T = 0.1s$ which is equal to the time delay L in (18) above. As shown in Fig. 8(b), the figure is the same with Fig. 8(a) except the $1/T = 0.1$ times proportional relationship of the ω_c axis.

For the standard form (18) of the control systems, different complete information of the ϕ_m and ω_m can be collected with different time delay L. Following the procedures in Sec. III, the pre-specification information is collected with $L = 1s$ and $L = 10s$ in Fig. 9(a) and Fig. 9(b), respectively.

IV. S I

Omitted due to space limit.

V. C

This paper provides a new design synthesis for PID controllers to achieve two pre-specifications (phase margin and gain crossover frequency) and flat phase tuning constraint for the first order plus time systems. This designed PID controller is robust not only to the uncertainty of the plant steady-state gain, but also to the entire variation of the controller coefficients. The complete stability region of the PID controller parameters is first determined according to a graphical stability criterion. Whereafter, the relative stability surface satisfying the pre-specified phase margin, and the relative stability curve on relative stability surface above guaranteing two pre-specifications, can be determined in the parameter-space. Then, by defining concerned functions according to the flat phase tuning constraint, the flat phase stable point can be fixed in the parameter-space. This designed PID controller will be, stable for sure first as its parameters are located in the complete stability region, and

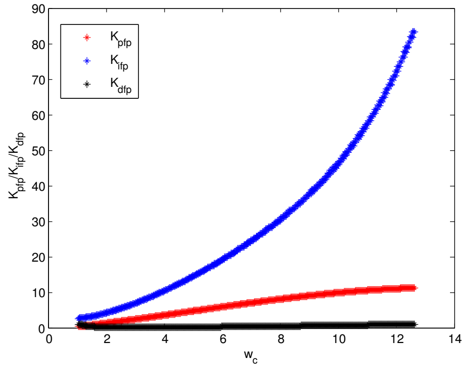
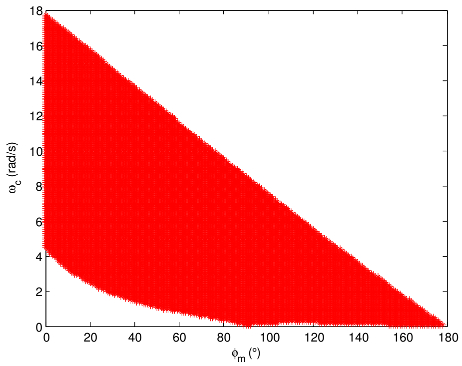
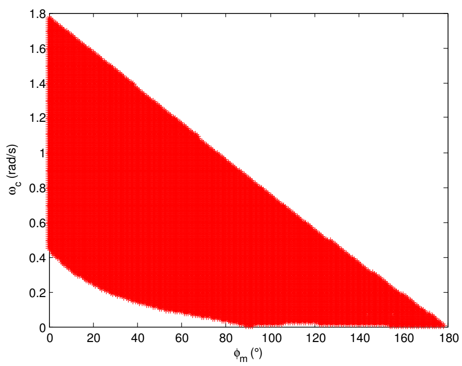


Fig. 7. K_{pfp}, K_{ifp} and K_{dfp} v.s. ω_c with $\phi_m = 50°$.

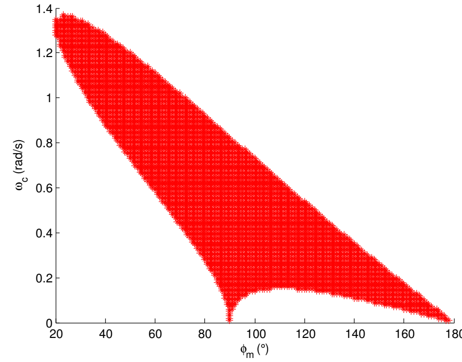


(a) $T = 1s$ and $L = 0.1s$

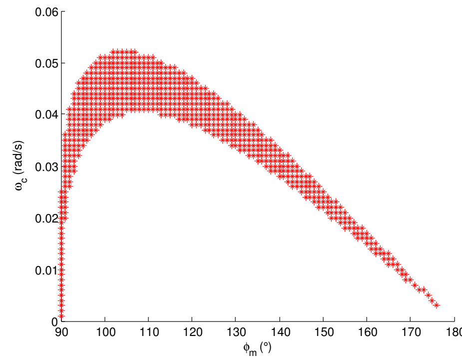


(b) $T = 10s$ and $L = 1s$

Fig. 8. The achievable regions of choosing ω_c v.s. ϕ_m with $T = 1s$ and $L = 0.1s$, $T = 10s$ and $L = 1s$.



(a) $T = 1s$ and $L = 1s$



(b) $T = 1s$ and $L = 10s$

Fig. 9. The achievable regions of choosing ω_c v.s. ϕ_m with $T = 1s$ and $L = 1s$, $T = 1s$ and $L = 10s$.

also robust to the system gain changes. The benefit of this proposed design synthesis is that it can give the complete information on the achievable pre-specifications (phase margin and gain crossover frequency), and provided the flexible design procedures of designing the PID controller with robustness to the system gain variations. Our immediate effort is to extend the results of this paper to fractional order PI controller design [17] with complete information for FOPTD models.

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