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Gradient-based iterative parameter identification for multi-input multi-output OEMA-like models

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***Abstract*— This paper develops a hierarchical gradient-based iterative estimation algorithm for multi-input multi-output output error moving average (OEMA-like) models. In order to solve the difficulties that the noise-free outputs and the noise terms in the information vector/matrix of the corresponding identification model are unmeasurable, we replace the unknown variables in the information vector/matrix with their estimates. The simulation results show the effectiveness of the proposed algorithm.**

Index terms: Iterative estimation; Parameter identification; Hierarchical identification; Multivariable OEMA-like model; Multivariable CARMA-like model;

I. Introduction

The iterative methods are very important for solving matrix equations [1], [2], e.g., the famous Jacobi iteration and the Gauss-Seidel iteration for solving the equation $Ax=b$ [3]. In this literature, Ding, *et al.* extended the Jacobi iteration and the Gauss-Seidel iteration to general matrix equations and presented a large family of iterative methods for $Ax=b$ and $AXB=F$ [4], [5]. Furthermore, they presented a series of iterative algorithms, e.g., the least squares based iterative algorithms and the gradient based iterative algorithms [4]–[11] for (coupled) Sylvester matrix equations and general (coupled) matrix equations.

The iterative methods have important applications in solving matrix equations parameter identification, e.g., the least squares based parameter identification algorithms and gradient based parameter estimation algorithms [12]–[27]. Other identification methods can be found in [28]–[50]. The hierarchical identification principle is an effective method of dealing with identification of multivariable systems [51], [52]. Many hierarchical parameter estimation algorithms were reported for multivariable systems using the hierarchical identification principle [51]–[54]. This paper studies the hierarchical gradient-based iterative parameter estimation methods for multi-input multi-output OEMA-like systems using the hierarchical identification principle.

The paper is organized as follows. Section II describes the output error moving average system and derives its identification model. Section III derives an hierarchical gradient-based iterative parameter identification algorithm for an OEMA system. Section IV gives the version of the hierarchical gradient-based iterative algorithm with finite measurement data. Section V provides an illustrative example. Finally, concluding remarks are given in section VI.

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II. The identification model

Consider a multivariable output-error moving average (OEMA) system [44],

$$y(t)=\frac{Q(z)}{\alpha(z)}u(t)+D(z)v(t), \tag{1}$$

where $y(t)\in\mathbb{R}^m$ is the system output vector, $u(t)\in\mathbb{R}^r$ is the system input vector, $v(t)\in\mathbb{R}^m$ is a stochastic white noise vector with zero mean and variance σ^2, $\alpha(z)$ is a monic polynomial in the unit backward shift operator z^{-1} $[z^{-1}y(t)=y(t-1)]$, $Q(z)$ is a matrix polynomial in z^{-1}, $D(z)$ is a polynomial in z^{-1}, and defined by

$$\begin{aligned}
\alpha(z) &:= 1+\alpha_1z^{-1}+\alpha_2z^{-2}+\cdots+\alpha_nz^{-n}\in\mathbb{R}^1,\\
Q(z) &:= Q_1z^{-1}+Q_2z^{-2}+\cdots+Q_nz^{-n}\in\mathbb{R}^{m\times r},\\
D(z) &:= 1+d_1z^{-1}+d_2z^{-2}+\cdots+d_{n_d}z^{-n_d}\in\mathbb{R}^1.
\end{aligned}$$

Define the noise-free output,

$$x(t):=\frac{Q(z)}{\alpha(z)}u(t)\in\mathbb{R}^m. \tag{2}$$

Substitute (2) into (1) gives

$$y(t)=x(t)+D(z)v(t). \tag{3}$$

Define the parameter vectors $\vartheta_{\rm s}$, $\vartheta_{\rm n}$ and ϑ, the parameter matrix θ, the input information vector $\varphi(t)$ and the information matrices $\psi_{\rm s}(t)$, $\psi_{\rm n}(t)$ and $\psi(t)$ as

$$\begin{aligned}
\vartheta_{\rm s} &:= [\alpha_1,\alpha_2,\cdots,\alpha_n]^{\rm T}\in\mathbb{R}^n,\\
\vartheta_{\rm n} &:= [d_1,d_2,\cdots,d_{n_d}]^{\rm T}\in\mathbb{R}^{n_d},\\
\vartheta &:= \begin{bmatrix}\vartheta_{\rm s}\\ \vartheta_{\rm n}\end{bmatrix}\in\mathbb{R}^{n+n_d},\\
\theta^{\rm T} &:= [Q_1,Q_2,\cdots,Q_n]\in\mathbb{R}^{m\times(nr)},\\
\varphi(t) &:= [u^{\rm T}(t-1),u^{\rm T}(t-2),\cdots,u^{\rm T}(t-n)]^{\rm T}\in\mathbb{R}^{(nr)},\\
\psi_{\rm s}(t) &:= [x(t-1),x(t-2),\cdots,x(t-n)]\in\mathbb{R}^{m\times n},\\
\psi_{\rm n}(t) &:= [-v(t-1),-v(t-2),\cdots,-v(t-n_d)]\in\mathbb{R}^{m\times n_d},\\
\psi(t) &:= [\psi_{\rm s}(t),\psi_{\rm n}(t)]\in\mathbb{R}^{m\times(n+n_d)}.
\end{aligned}$$

Equation (2) can be rewritten as

$$x(t)=-\psi_{\rm s}(t)\vartheta_{\rm s}+\theta^{\rm T}\varphi(t). \tag{4}$$

From (3), we get the following identification model

$$y(t)+\psi(t)\vartheta=\theta^{\rm T}\varphi(t)+v(t). \tag{5}$$

III. THE GRADIENT ESTIMATION ALGORITHM

Because the identification model in (5) contains a parameter matrix in the left side and a parameter vector in the right side, general methods can not be applied directly. In this paper, we use the decomposition based hierarchical identification principle to derive the estimation algorithm of the parameter matrix θ and the parameter vector ϑ. That is, Equation (5) is decomposed into two virtual subsystems which contain the parameter vector ϑ and the parameter matrix θ, respectively, and then the parameters of these two subsystems are identified. The basic idea is to replace the unknown $v(t-i)$ with the estimated residual and the unknown $x(t-i)$ with the output of an auxiliary model.

Define two intermediate vectors,

$$b_1(t) := \theta^{\mathrm{T}}\varphi(t) \in \mathbb{R}^m, \quad b_2(t) := \psi(t)\vartheta \in \mathbb{R}^m.$$

Decompose (5) into the following two virtual subsystems

$$\begin{aligned} S_1: &\quad y(t) = -\psi(t)\vartheta + b_1(t) + v(t), \\ S_2: &\quad y(t) = \theta^{\mathrm{T}}\varphi(t) - b_2(t) + v(t). \end{aligned}$$

Consider the newest p data from $i = t-p+1$ to $i = t$ and define the stacked output vector $Y_1(t)$ and the stacked matrix $Y_2(t)$, the stacked information matrices $\psi(t)$ and $\Phi(t)$, the stacked white noise vector $V_1(t)$ and the stacked noise matrix $V_2(t)$, and the inner vector $B_1(t)$ and the inner matrix $B_2(t)$ as

$$Y_1(t) := \begin{bmatrix} y(t) \\ y(t-1) \\ \vdots \\ y(t-p+1) \end{bmatrix}, \quad \Psi(t) := \begin{bmatrix} \psi(t) \\ \psi(t-1) \\ \vdots \\ \psi(t-p+1) \end{bmatrix},$$

$$B_1(t) := \begin{bmatrix} b_1(t) \\ b_1(t-1) \\ \vdots \\ b_1(t-p+1) \end{bmatrix} = \begin{bmatrix} \theta^{\mathrm{T}}\varphi(t) \\ \theta^{\mathrm{T}}\varphi(t-1) \\ \vdots \\ \theta^{\mathrm{T}}\varphi(t-p+1) \end{bmatrix}, \tag{6}$$

$$\begin{aligned} V_1(t) &:= \begin{bmatrix} v(t) \\ v(t-1) \\ \vdots \\ v(t-p+1) \end{bmatrix}, \\ Y_2(t) &:= [y(t), y(t-1), \cdots, y(t-p+1)], \\ \Phi(t) &:= [\varphi(t), \varphi(t-1), \cdots, \varphi(t-p+1)], \\ B_2(t) &:= [b_2(t), b_2(t-1), \cdots, b_2(t-p+1)] \\ &= [\psi(t)\vartheta, \psi(t-1)\vartheta, \cdots, \psi(t-p+1)\vartheta], \\ V_2(t) &:= [v(t), v(t-1), \cdots, v(t-p+1)]. \end{aligned} \tag{7}$$

Then we have

$$\begin{aligned} S_1: &\quad Y_1(t) = -\Psi(t)\vartheta + B_1(t) + V_1(t), \\ S_2: &\quad Y_2(t) = \theta^{\mathrm{T}}\Phi(t) - B_2(t) + V_2(t). \end{aligned}$$

Let $\|X\|^2 := \mathrm{tr}[XX^{\mathrm{T}}]$, define two criterion functions:

$$\begin{aligned} J_1(\vartheta) &:= \|Y_1(t) + \Psi(t)\vartheta - B_1(t)\|^2, \\ J_2(\theta) &:= \|Y_2(t) - \theta^{\mathrm{T}}\Phi(t) + B_2(t)\|^2. \end{aligned}$$

Let $k = 1, 2, \cdots$ be an iteration variable, $\hat{\vartheta}_k(t)$ and $\hat{\theta}_k(t)$ represent the estimates of ϑ and θ at iteration k, $\mu_k(t) \geqslant 0$ is the time-varying iterative step-size (time-varying convergence factor). Minimizing $J_1(\vartheta)$ and $J_2(\theta)$ using the negative gradient search leads to the iterative algorithm of estimating ϑ and θ as follows:

$$\begin{aligned} \hat{\vartheta}_k(t) &= \hat{\vartheta}_{k-1}(t) - \frac{\mu_k(t)}{2}\mathrm{grad}[J_1(\hat{\vartheta}_{k-1}(t))] \\ &= \hat{\vartheta}_{k-1}(t) - \mu_k(t)\Psi^{\mathrm{T}}(t)[Y_1(t) - B_1(t) + \Psi(t)\hat{\vartheta}_{k-1}(t)], \\ \hat{\theta}_k(t) &= \hat{\theta}_{k-1}(t) - \frac{\mu_k(t)}{2}\mathrm{grad}[J_2(\hat{\theta}_{k-1}(t))] \\ &= \hat{\theta}_{k-1}(t) + \mu_k(t)\Phi(t)[Y_2(t) - \hat{\theta}_{k-1}^{\mathrm{T}}(t)\Phi(t) + B_2(t)]^{\mathrm{T}}. \end{aligned}$$

Substituting $B_1(t)$ in (6) and $B_2(t)$ in (7) into the above equations, respectively, gives

$$\begin{aligned} \hat{\vartheta}_k(t) &= \hat{\vartheta}_{k-1}(t) - \mu_k(t)\Psi^{\mathrm{T}}(t) \\ &\quad \times \left(Y_1(t) - \begin{bmatrix} \theta^{\mathrm{T}}\varphi(t) \\ \theta^{\mathrm{T}}\varphi(t-1) \\ \vdots \\ \theta^{\mathrm{T}}\varphi(t-p+1) \end{bmatrix} + \Psi(t)\hat{\vartheta}_{k-1}(t) \right), \end{aligned} \tag{8}$$

$$\begin{aligned} \hat{\theta}_k(t) &= \hat{\theta}_{k-1}(t) \\ &\quad + \mu_k(t)\Phi(t)\{Y_2(t) - \hat{\theta}_{k-1}^{\mathrm{T}}(t)\Phi(t) \\ &\quad + [\psi(t)\vartheta, \psi(t-1)\vartheta, \cdots, \psi(t-p+1)\vartheta]\}^{\mathrm{T}}. \end{aligned} \tag{9}$$

The difficulty is that the above two equations contain the unknown parameter matrix θ and parameter vector ϑ, so the algorithm in (8) and (9) is impossible to realize. Here in order to solve such a difficulty, we use the hierarchical identification principle [51], [52] and replacing θ in (8) and ϑ in (9) with their iterative estimates $\hat{\theta}_{k-1}(t)$ and $\hat{\vartheta}_{k-1}(t)$ at the preceding iteration $k-1$ to get

$$\begin{aligned} \hat{\vartheta}_k(t) &= \hat{\vartheta}_{k-1}(t) - \mu_k(t)\Psi^{\mathrm{T}}(t) \\ &\quad \times \left(Y_1(t) - \begin{bmatrix} \hat{\theta}_{k-1}^{\mathrm{T}}(t)\varphi(t) \\ \hat{\theta}_{k-1}^{\mathrm{T}}(t)\varphi(t-1) \\ \vdots \\ \hat{\theta}_{k-1}^{\mathrm{T}}(t)\varphi(t-p+1) \end{bmatrix} + \Psi(t)\hat{\vartheta}_{k-1}(t) \right), \end{aligned} \tag{10}$$

$$\begin{aligned} \hat{\theta}_k(t) &= \hat{\theta}_{k-1}(t) + \mu_k(t)\Phi(t)\{Y_2(t) - \hat{\theta}_{k-1}^{\mathrm{T}}(t)\Phi(t) \\ &\quad + [\psi(t)\hat{\vartheta}_{k-1}(t), \psi(t-1)\hat{\vartheta}_{k-1}(t), \cdots, \\ &\quad \psi(t-p+1)\hat{\vartheta}_{k-1}(t)]\}^{\mathrm{T}}. \end{aligned} \tag{11}$$

Another difficulty is that $\Psi(t)$ (that is $\psi(t)$) contains unknown vectors $v(t-i)$ and $x(t-i)$. Define

$$\begin{aligned} \hat{\psi}_k(t) &:= [\hat{\psi}_{\mathrm{s},k}(t), \hat{\psi}_{\mathrm{n},k}(t)] \in \mathbb{R}^{m\times(n+n_d)}, \\ \hat{\psi}_{\mathrm{s},k}(t) &:= [\hat{x}_{k-1}(t-1), \hat{x}_{k-1}(t-2), \cdots, \hat{x}_{k-1}(t-n)] \in \mathbb{R}^{m\times n}. \\ \hat{\psi}_{\mathrm{n},k}(t) &:= [-\hat{v}_{k-1}(t-1), -\hat{v}_{k-1}(t-2), \cdots, \\ &\qquad -\hat{v}_{k-1}(t-n_d)] \in \mathbb{R}^{m\times n_d}. \end{aligned}$$

From (4) and (5), we have

$$\begin{aligned} x(t-i) &= -\psi_{\mathrm{s}}(t-i)\vartheta_{\mathrm{s}} + \theta^{\mathrm{T}}\varphi(t-i), \\ v(t-i) &= y(t-i) + \psi(t-i)\vartheta - \theta^{\mathrm{T}}\varphi(t-i). \end{aligned}$$

Replacing $\psi_{\mathrm{s}}(t-i)$, $\psi(t-i)$, ϑ_{s}, ϑ and θ with $\hat{\psi}_{\mathrm{s},k}(t-i)$, $\hat{\psi}_k(t-i)$, $\hat{\vartheta}_{\mathrm{s},k}(t)$, $\hat{\vartheta}_k(t)$ and $\hat{\theta}_k(t)$, the iterative estimates

$\hat{v}_k(t-i)$ and $\hat{x}_k(t-i)$ of $v(t-i)$ and $x(t-i)$ at iteration k can be computed by

$$\begin{aligned}\hat{v}_k(t-i) &= y(t-i)+\hat{\psi}_k(t-i)\hat{\vartheta}_k(t)-\hat{\theta}_k^{\mathrm{T}}(t)\varphi(t-i),\\ \hat{x}_k(t-i) &= -\hat{\psi}_{\mathrm{s},k}(t-i)\hat{\vartheta}_{\mathrm{s},k}(t)+\hat{\theta}_k^{\mathrm{T}}(t)\varphi(t-i).\end{aligned} \tag{12}$$

Define

$$\hat{\Psi}_k(t) := \begin{bmatrix}\hat{\psi}_k(t)\\ \hat{\psi}_k(t-1)\\ \vdots\\ \hat{\psi}_k(t-p+1)\end{bmatrix} \in \mathbb{R}^{(mp)\times(n+n_d)}.$$

Let I be an identity matrix of appropriate sizes and $\mathbf{1}_{m\times n}$ be an $m\times n$ matrix whose entries are all 1. Replacing $\Psi(t)$ and $\psi(t)$ in (10) and (11) with $\hat{\Psi}_k(t)$ and $\hat{\psi}_k(t)$ gives

$$\begin{aligned}&\hat{\vartheta}_k(t) = \hat{\vartheta}_{k-1}(t)-\mu_k(t)\hat{\Psi}_k^{\mathrm{T}}(t)\\ &\times\left(Y_1(t)-\begin{bmatrix}\hat{\theta}_{k-1}^{\mathrm{T}}(t)\varphi(t)\\ \hat{\theta}_{k-1}^{\mathrm{T}}(t)\varphi(t-1)\\ \vdots\\ \hat{\theta}_{k-1}^{\mathrm{T}}(t)\varphi(t-p+1)\end{bmatrix}+\hat{\Psi}_k(t)\hat{\vartheta}_{k-1}(t)\right),\end{aligned} \tag{13}$$

$$\begin{aligned}\hat{\theta}_k(t) = \hat{\theta}_{k-1}(t)+\mu_k(t)\Phi(t)\{&Y_2(t)-\hat{\theta}_{k-1}^{\mathrm{T}}(t)\Phi(t)\\ &+[\hat{\psi}_k(t)\hat{\vartheta}_{k-1}(t),\hat{\psi}_k(t-1)\hat{\vartheta}_{k-1}(t),\cdots,\\ &\hat{\psi}_k(t-p+1)\hat{\vartheta}_{k-1}(t)]\}^{\mathrm{T}}.\end{aligned} \tag{14}$$

Or

$$\begin{aligned}\hat{\vartheta}_k(t) &= [I-\mu_k(t)\hat{\Psi}_k^{\mathrm{T}}(t)\hat{\Psi}_k(t)]\hat{\vartheta}_{k-1}(t)-\mu_k(t)\hat{\Psi}_k^{\mathrm{T}}(t)\\ &\quad\times\left(Y_1(t)-\begin{bmatrix}\hat{\theta}_{k-1}^{\mathrm{T}}(t)\varphi(t)\\ \hat{\theta}_{k-1}^{\mathrm{T}}(t)\varphi(t-1)\\ \vdots\\ \hat{\theta}_{k-1}^{\mathrm{T}}(t)\varphi(t-p+1)\end{bmatrix}\right),\\ \hat{\theta}_k(t) &= [I-\mu_k(t)\Phi(t)\Phi^{\mathrm{T}}(t)]\hat{\theta}_{k-1}(t)\\ &\quad+\mu_k(t)\Phi(t)\{Y_2(t)+[\hat{\psi}_k(t)\hat{\vartheta}_{k-1}(t),\\ &\quad\hat{\psi}_k(t-1)\hat{\vartheta}_{k-1}(t),\cdots,\hat{\psi}_k(t-p+1)\hat{\vartheta}_{k-1}(t)]\}^{\mathrm{T}}.\end{aligned}$$

The above two equations may be regarded as two discrete-time systems and the necessary condition of the convergence for the parameter estimation $\hat{\theta}_k(t)$ and $\hat{\vartheta}_k(t)$ is that the matrices $[I-\mu_k(t)\hat{\Psi}_k^{\mathrm{T}}(t)\hat{\Psi}_k(t)]$ and $[I-\mu_k(t)\Phi(t)\Phi^{\mathrm{T}}(t)]$ have all eigenvalues inside the unit circle. So the convergence factor $\mu_k(t)$ must satisfy

$$\mu_k(t)\leqslant\frac{2}{\lambda_{\max}[\hat{\Psi}_k^{\mathrm{T}}(t)\hat{\Psi}_k(t)]},\quad \mu_k(t)\leqslant\frac{2}{\lambda_{\max}[\Phi(t)\Phi^{\mathrm{T}}(t)]}.$$

Their intersection is

$$\mu_k(t)\leqslant 2\left\{\max\{\lambda_{\max}[\hat{\Psi}_k^{\mathrm{T}}(t)\hat{\Psi}_k(t)],\lambda_{\max}[\Phi(t)\Phi^{\mathrm{T}}(t)]\}\right\}^{-1}.$$

One conservative choice of $\mu_k(t)$ is

$$\mu_k(t)\leqslant 2\left\{\lambda_{\max}[\hat{\Psi}_k^{\mathrm{T}}(t)\hat{\Psi}_k(t)]+\lambda_{\max}[\Phi(t)\Phi^{\mathrm{T}}(t)]\right\}^{-1},$$

or

$$0\leqslant\mu_k(t)\leqslant 2\{\|\hat{\Psi}_k(t)\|^2+\|\Phi(t)\|^2\}^{-1}. \tag{15}$$

Substituting $Y_1(t)$, $\hat{\Psi}_k(t)$, $Y_2(t)$ and $\Phi(t)$ into (13), (14) and (15) and summarizing the above expressions give the following hierarchical gradient-based iterative parameter estimation algorithm for multivariable OEMA systems (the OEMA-HGI algorithm for short):

$$\begin{aligned}\hat{\vartheta}_k(t) &= \hat{\vartheta}_{k-1}(t)-\mu_k(t)\sum_{i=t-p+1}^{t}\hat{\psi}_k^{\mathrm{T}}(i)\\ &\quad\times[y(i)+\hat{\psi}_k(i)\hat{\vartheta}_{k-1}(t)-\hat{\theta}_{k-1}^{\mathrm{T}}(t)\varphi(i)],\end{aligned} \tag{16}$$

$$\begin{aligned}\hat{\theta}_k(t) &= \hat{\theta}_{k-1}(t)+\mu_k(t)\sum_{i=t-p+1}^{t}\varphi(i)\\ &\quad\times[y(i)+\hat{\psi}_k(i)\hat{\vartheta}_{k-1}(t)-\hat{\theta}_{k-1}^{\mathrm{T}}(t)\varphi(i)]^{\mathrm{T}},\end{aligned} \tag{17}$$

$$\varphi(t) = [u^{\mathrm{T}}(t-1),u^{\mathrm{T}}(t-2),\cdots,u^{\mathrm{T}}(t-n)]^{\mathrm{T}}, \tag{18}$$

$$\hat{\psi}_k(t) = [\hat{\psi}_{\mathrm{s},k}(t),\hat{\psi}_{\mathrm{n},k}(t)], \tag{19}$$

$$\hat{\psi}_{\mathrm{s},k}(t) = [\hat{x}_{k-1}(t-1),\hat{x}_{k-1}(t-2),\cdots,\hat{x}_{k-1}(t-n)], \tag{20}$$

$$\hat{\psi}_{\mathrm{n},k}(t) := [-\hat{v}(t-1),-\hat{v}(t-2),\cdots,-\hat{v}(t-n_d)], \tag{21}$$

$$\hat{\vartheta}_k(t) = \begin{bmatrix}\hat{\vartheta}_{\mathrm{s},k}(t)\\ \hat{\vartheta}_{\mathrm{n},k}(t)\end{bmatrix}, \tag{22}$$

$$\begin{aligned}\hat{x}_k(t-i) &= -\hat{\psi}_{\mathrm{s},k}(t-i)\hat{\vartheta}_{\mathrm{s},k}(t)\\ &\quad+\hat{\theta}_k^{\mathrm{T}}(t)\varphi(t-i),\ i=1,2,\cdots,n,\end{aligned} \tag{23}$$

$$\begin{aligned}\hat{v}_k(t-i) &= y(t-i)+\hat{\psi}_k(t-i)\hat{\vartheta}_k(t)\\ &\quad-\hat{\theta}_k^{\mathrm{T}}(t)\varphi(t-i),\ i=1,2,\cdots,n_d,\end{aligned} \tag{24}$$

$$\mu_k(t)\leqslant 2\left(\sum_{i=t-p+1}^{t}[\|\hat{\psi}_k(t)\|^2+\|\varphi(t)\|^2]\right)^{-1}. \tag{25}$$

IV. The Case with Finite Measurement Data

If we set $p=L$ and $t=L$ (L: the data length) in the OEMA-HGI algorithm, then we have

$$\begin{aligned}Y_1(L) &:= \begin{bmatrix}y(L)\\ y(L-1)\\ \vdots\\ y(1)\end{bmatrix},\quad \Psi(L):=\begin{bmatrix}\psi(L)\\ \psi(L-1)\\ \vdots\\ \psi(1)\end{bmatrix},\\ B_1(L) &:= \begin{bmatrix}b_1(L)\\ b_1(L-1)\\ \vdots\\ b_1(1)\end{bmatrix}=\begin{bmatrix}\theta^{\mathrm{T}}\varphi(L)\\ \theta^{\mathrm{T}}\varphi(L-1)\\ \vdots\\ \theta^{\mathrm{T}}\varphi(1)\end{bmatrix},\end{aligned} \tag{26}$$

$$\begin{aligned}Y_2(L) &:= [y(L),y(L-1),\cdots,y(1)],\\ \Phi(L) &:= [\varphi(L),\varphi(L-1),\cdots,\varphi(1)],\\ B_2(L) &:= [b_2(L),b_2(L-1),\cdots,b_2(1)]\\ &= [\psi(L)\vartheta,\psi(L-1)\vartheta,\cdots,\psi(1)\vartheta].\end{aligned} \tag{27}$$

$Y_1(L)$, $Y_2(L)$, $\Phi(L)$ and $B_1(L)$ contain all the measured data $\{u(t),y(t):t=1,2,3,\cdots,L\}$. Similarly, define two criterion functions:

$$\begin{aligned}J_1(\vartheta) &:= \|Y_1(L)+\Psi(L)\vartheta-B_1(L)\|^2,\\ J_2(\theta) &:= \|Y_2(L)-\theta^{\mathrm{T}}\Phi(L)+B_2(L)\|^2.\end{aligned}$$

According to the derivation of the OEMA-HGI algorithm, we yield the following OEMA-HGI algorithm with finite measurement data:

$$\hat{\vartheta}_k = \hat{\vartheta}_{k-1} - \mu_k \sum_{i=1}^{L} \hat{\psi}_k^{\mathrm{T}}(i) \times [y(i) + \hat{\psi}_k(i)\hat{\vartheta}_{k-1} - \hat{\theta}_{k-1}^{\mathrm{T}}\varphi(i)], \quad (28)$$

$$\hat{\theta}_k = \hat{\theta}_{k-1} + \mu_k \sum_{i=1}^{L} \varphi(i) \times [y(i) + \hat{\psi}_{k-1}(i)\hat{\vartheta}_{k-1} - \hat{\theta}_{k-1}^{\mathrm{T}}\varphi(i)]^{\mathrm{T}}, \quad (29)$$

$$\varphi(t) = [u^{\mathrm{T}}(t-1), u^{\mathrm{T}}(t-2), \cdots, u^{\mathrm{T}}(t-n)]^{\mathrm{T}}, \quad (30)$$

$$\hat{\psi}_k(t) = [\hat{\psi}_{\mathrm{s},k}(t), \hat{\psi}_{\mathrm{n},k}(t)], \quad (31)$$

$$\hat{\psi}_{\mathrm{s},k}(t) = [\hat{x}_{k-1}(t-1), \hat{x}_{k-1}(t-2), \cdots, \hat{x}_{k-1}(t-n)], \quad (32)$$

$$\psi_{\mathrm{n},k}(t) = [-\hat{v}(t-1), -\hat{v}(t-2), \cdots, -\hat{v}(t-n_d)], \quad (33)$$

$$\hat{\vartheta}_k = \begin{bmatrix} \hat{\vartheta}_{\mathrm{s},k} \\ \hat{\vartheta}_{\mathrm{n},k} \end{bmatrix}, \quad (34)$$

$$\hat{x}_k(t) = -\hat{\psi}_{\mathrm{s},k}(t)\hat{\vartheta}_{\mathrm{s},k} + \hat{\theta}_k^{\mathrm{T}}\varphi(t), \quad (35)$$

$$\hat{v}_k(t) = y(t) + \hat{\psi}_k(t)\hat{\vartheta}_k - \hat{\theta}_k^{\mathrm{T}}\varphi(t), \quad (36)$$

$$\mu_k \leqslant 2\left(\sum_{i=1}^{L} [\|\hat{\psi}_k(t)\|^2 + \|\varphi(t)\|^2]\right)^{-1}. \quad (37)$$

The steps involved in the algorithm in (28)–(37) are listed in the following.

1) Collect the input/output data $\{u(t), y(t)\colon t = 1, 2, \cdots, L\}$ (L: the data length), form $\varphi(t)$ by (30).
2) To initialize, let $k = 1$, $\hat{\vartheta}_0(t) = \mathbf{1}_{n+n_d}/p_0$, $\hat{\theta}_0^{\mathrm{T}}(t) = \mathbf{1}_{m\times(nr)}/p_0$, $\hat{x}_0(t) = \mathbf{1}_{m\times 1}/p_0$, $\hat{v}_0(t) = \mathbf{1}_{m\times 1}/p_0$, $p_0 = 10^6$.
3) Form $\hat{\psi}_{\mathrm{s},k}(t)$ by (32), $\hat{\psi}_{\mathrm{n},k}(t)$ by (33), and $\hat{\psi}_k(t)$ by (31).
4) Choose a large convergence factor μ_k satisfying (37) and update $\hat{\vartheta}_k$ and $\hat{\theta}_k$ by (28) and (29), respectively.
5) Compute $\hat{x}_k(t)$ by (35) and $\hat{v}_k(t)$ by (36).
6) Compute the errors $\|\hat{\vartheta}_k - \hat{\vartheta}_{k-1}\|$ and $\|\hat{\theta}_k - \hat{\theta}_{k-1}\|$, if

$$\|\hat{\vartheta}_k - \hat{\vartheta}_{k-1}\| + \|\hat{\theta}_k - \hat{\theta}_{k-1}\| \leqslant \varepsilon,$$

then terminate the procedure and obtain the iteration times k and estimates $\hat{\vartheta}_k$ and $\hat{\theta}_k$; otherwise, increase k by 1 and go to step 3.

V. Example

Consider the two-input two-output OEMA-like system,

$$y(t) = \frac{Q(z)}{\alpha(z)}u(t) + D(z)v(t),$$

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}, \quad u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}, \quad v(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix},$$

$$\alpha(z) = 1 - 0.80z^{-1}, \quad D(z) = 1 + 0.20z^{-1},$$

$$Q(z) = \begin{bmatrix} 3.00 & 1.00 \\ 1.00 & 3.00 \end{bmatrix} z^{-1}.$$

In simulation, the inputs $\{u_1(t)\}$ and $\{u_2(t)\}$ are taken as two persistent excitation signal sequences with zero mean and unit variance, and $\{v_1(t)\}$ and $\{v_2(t)\}$ as two white noise sequences with zero mean and variances $\sigma_1^2 = \sigma_2^2 = 0.50^2$.

Apply the proposed OEMA-HGI algorithm in (28)–(37) to estimate the parameters of this example system, the parameter estimates and their errors with different data length $t = L = 1000$, 2000 and 3000 are shown in Tables I–III and the parameter estimation errors

$$\delta := \sqrt{[\|\hat{\vartheta}_k - \vartheta\|^2 + \|\hat{\theta}_k - \theta\|^2]/[\|\vartheta\|^2 + \|\theta\|^2]}$$

versus k are shown in Figures 1–3.

TABLE I

THE PARAMETER ESTIMATES AND ERRORS ($L = 1000$)

k	α_1	$Q_1(1,1)$	$Q_1(1,2)$	$Q_1(2,1)$	$Q_1(2,2)$	d_1	δ (%)
1	-0.77434	0.04880	0.02107	0.01341	0.05504	0.00000	96.74256
2	-0.78779	0.09793	0.04219	0.02701	0.11043	0.76721	95.74504
5	-0.89625	0.35366	0.14215	0.10820	0.37616	0.84545	87.53196
10	-0.68860	0.56589	0.22470	0.17323	0.59879	0.76941	80.23171
50	-0.77264	2.10985	0.77016	0.67879	2.15267	0.80791	31.38214
100	-0.79424	2.67189	0.93301	0.88656	2.69431	0.62252	13.85567
200	-0.78060	2.93035	1.00399	0.99175	2.94570	0.44377	5.72097
500	-0.77627	2.97890	1.01944	1.01255	2.99794	0.40589	4.60946
True values	-0.80000	3.00000	1.00000	1.00000	3.00000	0.20000	

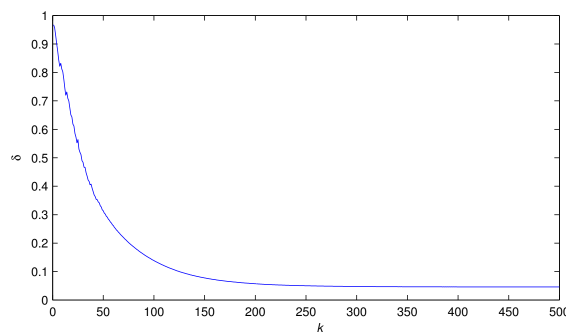

Fig. 1. The parameter estimation errors δ versus k ($L = 1000$)

TABLE II

THE PARAMETER ESTIMATES AND ERRORS ($L = 2000$)

k	α_1	$Q_1(1,1)$	$Q_1(1,2)$	$Q_1(2,1)$	$Q_1(2,2)$	d_1	δ (%)
1	-0.77073	0.05155	0.01717	0.01631	0.05332	0.00000	96.73271
2	-0.78418	0.10346	0.03447	0.03273	0.10701	0.76235	95.70980
5	-0.88926	0.36102	0.12186	0.11709	0.37446	0.84633	87.50258
10	-0.69852	0.58214	0.19640	0.18907	0.60346	0.76279	79.95360
50	-0.91204	2.12135	0.71080	0.70168	2.17877	0.77642	30.81644
100	-0.83991	2.67834	0.89172	0.90049	2.72832	0.55788	12.60501
200	-0.82212	2.93780	0.96978	0.99718	2.96625	0.37379	4.20814
500	-0.81927	2.98865	0.98265	1.01547	3.00653	0.33704	3.09921
True values	-0.80000	3.00000	1.00000	1.00000	3.00000	0.20000	

From Tables I–III and Figures 1–3, we can draw the following conclusions.

- The parameter estimation errors given by the OEMA-HGI algorithm become small as the iteration k increases.
- The parameter estimation errors given by the OEMA-HGI algorithm become small with the data length L increasing.

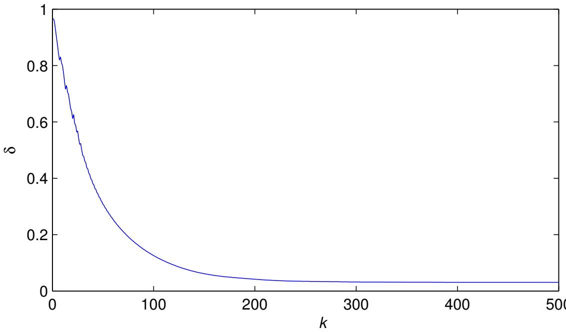

Fig. 2. The parameter estimation errors δ versus k ($L = 2000$)

TABLE III

THE PARAMETER ESTIMATES AND ERRORS ($L = 3000$)

k	α_1	$Q_1(1,1)$	$Q_1(1,2)$	$Q_1(2,1)$	$Q_1(2,2)$	d_1	δ (%)
1	-0.75756	0.05486	0.01674	0.01873	0.05226	0.00000	96.69220
2	-0.77030	0.11001	0.03362	0.03755	0.10489	0.75136	95.59634
5	-0.86931	0.37985	0.11936	0.12590	0.37113	0.84723	87.24067
10	-0.69642	0.61566	0.19407	0.20419	0.60242	0.75222	79.38341
100	-0.83404	2.72828	0.89563	0.90657	2.72842	0.52209	11.47093
200	-0.81998	2.95635	0.97550	0.98710	2.96574	0.34852	3.56643
500	-0.81734	2.99521	0.98844	1.00092	3.00622	0.31290	2.53066
True values	-0.80000	3.00000	1.00000	1.00000	3.00000	0.20000	

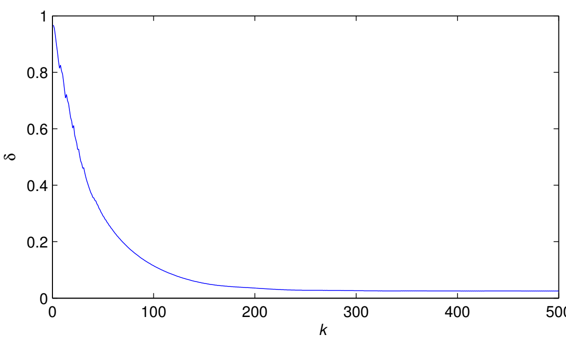

Fig. 3. The parameter estimation errors δ versus k ($L = 3000$)

VI. Conclusions

This paper presents a hierarchical gradient-based iterative algorithm for multivariable output error moving average systems. The parameter estimation errors given by the OEMA-HGI algorithm become small as the iteration increases. The OEMA-HGI algorithm can be extended to Hammerstein nonlinear systems [55]–[61]. The proposed parameter estimation method can be applied to predict the melt index for coupled distillation columns [62] and to identify dual-rate/multirate or non-uniformly sampled-data systems [63]–[74].

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