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A hybrid real-time supervisory scheme for nonlinear systems

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Abstract— **In this paper we develop a hybrid supervisory control architecture in a real-time environment for constrained nonlinear systems. The strategy is based on Command Governor (CG) ideas that are here specialized in order to take into account both time-varying set-points and constraints. Experimental results on a laboratory four-tank test-bed are presented.**

I. Introduction

The complexity of current computer control systems arises from the engineering requirement to integrate computers, actuators and sensors for control, signal processing and data networks, visualization and display with the technology of the application domain which ranges from manufacturing control, temperature control, cruise control in cars and planes, monitoring and regulation of various parameters etc. [1].

The resulting feedback setup is often thus quite "large" and is expected to adapt in a timely, rapid and correct fashion to frequently changing environment variables and conditions. Also, modern computer architectures, usually (highly) parallel, often distributed, using many heterogeneous resources need to be designed to properly operate for decades, due in part to the tremendous cost of their development. As a consequence, the control system needs to incorporate, in accordance with the requirements of the chosen application, a wide variety of often conflicting functional/non-functional objectives and it is then natural to characterize all these setups within a real-time control system framework [2].

It is well known in literature that control schemes and/or paradigms which are based on predictive control ideas may efficiently handle all these requirements within a real-time framework [3], [4].

Given these premises, the aim of this paper is to describe a real-time implementation of a command governor (CG) control strategy for the supervision of nonlinear dynamical systems subject to sudden switchings amongst operating points and whose constraints structure varies with time due to unpredictable events [5]. The proposed scheme prescribes that any changing in the plant structure affects the CG design and for each modification affecting the plant structure a different CG unit should be in principle designed complying with the new conditions. The idea is then that a suitable supervisory unit must be designed to take care of orchestrating the switching among the CG candidates during the online operations.

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Finally, to test the benefits of the proposed hybrid strategy, experiments carried out on a laboratory interconnected four tanks model will be discussed. All the numerical set up has been implemented via a Real-Time software platform.

II. Basic Command Governor (CG) design

Consider the following linear time-invariant system:

$$\begin{cases} x(t+1) &= \Phi x(t) + Gg(t) + G_d d(t) \\ y(t) &= H_y x(t) \\ c(t) &= H_c x(t) + Lg(t) + L_d d(t) \end{cases} \tag{1}$$

where $t \in \mathbb{Z}_+$, $x(t) \in \mathbf{R}^n$ is the overall state which includes the plant and primal controller states ; $g(t) \in \mathbf{R}^m$ is the CG action, which would be typically $g(t) = r(t)$ if no CG were present, viz. $g(t)$ is a suitably modified version of the reference signal $r(t) \in \mathbf{R}^m$; $d(t) \in \mathbf{R}^{n_d}$ is an exogenous disturbance satisfying $d(t) \in \mathcal{D}$, $\forall t \in \mathbb{Z}_+$, with $\mathcal{D}$ a specified convex and compact set such that $0_{n_d} \in \mathcal{D}$; $y(t) \in \mathbf{R}^m$ is the output, viz. a performance related signal which is required to track $r(t)$; $c(t) \in \mathbf{R}^{n_c}$ is the vector to be constrained, i.e. $c(t) \in \mathcal{C}$, $\forall t \in \mathbb{Z}_+$ with $\mathcal{C}$ a specified convex and compact set. It is also assumed that
A1 - Φ is a stability matrix and the system (1) is offset-free, i.e. $H_c(I_n - \Phi)^{-1}G = I_m$..
The CG design problem is that of generating, at each time instant t, the set-point $g(t)$ as a function of the current state $x(t)$ and reference $r(t)$

$$g(t) := \overline{g}(x(t), r(t)) \tag{2}$$

in such a way that, under suitable conditions and regardless of disturbances, the constraints are always fulfilled along the system trajectories generated by the application of the modified set-points $g(t)$ and possibly $y(t) \approx r(t)$. Moreover, it is required that: 1) $g(t) \to \hat{r}$ whenever $r(t) \to r$, where $\hat{r}$ is either r or its best feasible approximation; and 2) the CG has a finite settling time, viz. $g(t) = \hat{r}$ for a possibly large but finite t whenever the reference stays constant after a finite time. By linearity, one is allowed to separate the effects of the initial conditions and input from those of disturbances, e.g. $x(t) = \overline{x}(t) + \tilde{x}(t)$, where $\overline{x}(t)$ is the disturbance-free component and $\tilde{x}(t)$ depends only on disturbances. It has been proved in [5] that one can consider only disturbance-free evolutions of the system and adopt a "worst-case" approach. Moreover, it is convenient to introduce the following sets for a given vanishing $\delta > 0$

$$\mathcal{C}^{\delta} := \mathcal{C}_\infty \sim \mathcal{B}_\delta, \quad \mathcal{W}^{\delta} := \left\{ w \in \mathcal{R}^m : \overline{c}_w \in \mathcal{C}^{\delta} \right\} \tag{3}$$

where $\mathcal{B}_\delta$ is a ball of radius δ centered at the origin. We shall assume that $\mathcal{W}_\delta$ is the non-empty closed and convex set of all commands whose corresponding steady-state solutions satisfy the constraints with margin δ.

The main idea is to choose at each time step a constant virtual command $v(\cdot) \equiv w$, with $w \in \mathcal{W}_\delta$ such that the corresponding virtual evolution fulfills the constraints over a semi-infinite horizon and its "distance" from the constant reference of value $r(t)$ is minimal. Such a command is applied, a new state is measured and the procedure is repeated. In this respect we define the set $\mathcal{V}(x)$ as

$$\mathcal{V}(x) = \left\{ w \in \mathcal{W}_\delta : \bar{c}(k,x,w) \in \mathcal{C}_k, \ \forall k \in \mathbb{Z}_+ \right\} \tag{4}$$

where

$$\bar{c}(k,x,w) = H_c \left(\Phi^k x(t) + \sum_{i=0}^{k-1} \Phi^{k-i-1} G w \right) + Lw \tag{5}$$

is the disturbance-free virtual evolution at time k of the constrained vector c from the initial condition x at time zero under the constant command $v(\cdot) \equiv w$. As a consequence $\mathcal{V}(x) \subset \mathcal{W}_\delta$, and, if non-empty, it represents the set of all constant virtual sequences in $\mathcal{W}_\delta$ whose evolutions starting from x satisfies the constraints also during transients. Thus the CG output is chosen according to the solution of the following constrained optimization problem

$$g(t) = \arg \min_{w \in \mathcal{V}(x(t))} \|w - r(t)\|_\Psi \tag{6}$$

where $\Psi = \Psi^T > 0_p$ and $\|w\|_\Psi := x^T \Psi x$.
A detailed discussion about the CG approach and its main properties can be found in [5].

III. Hybrid Command Governors

In this section the previous basic CG scheme is generalized to time-varying set-points and constraints configurations. The proposed scheme is termed Hybrid CG (HCG) and, for the sake of simplicity, we will refer to the following plant description

$$x(t+1) = f(x(t), u(t)) \tag{7}$$

where $x(t) \in X \subseteq \mathbf{R}^n$ and $u(t) \in U \subseteq \mathbf{R}^m$ are the system state and control input, respectively. It is worth to note that the disturbance effects $d(t) \in \mathcal{D}$ can be also taken into account without compromising all the next developments. Let us assume that $f(x,u)$ is continuously differentiable and that the nonlinear plant (7) could operate in N different and pre-specified working regions, characterized by N equilibrium points denoted as (x_i^{eq}, u_i^{eq}), $i = 1, \ldots, N$.

For each equilibrium (x_i^{eq}, u_i^{eq}) a linearized model of (7) can be derived

$$\begin{aligned} x(t+1) - x_{eq} = \quad & A_i(x(t) - x_i^{eq}) + B_i(u(t) - u_i^{eq}) \\ & + F_i(x(t) - x_i^{eq}, u(t) - u_i^{eq}) \end{aligned}$$

where $A_i = \frac{\partial f}{\partial x}(x,u) \mid_{x=x_i^{eq}, u=u_i^{eq}}$ and $B_i = \frac{\partial f}{\partial u}(x,u) \mid_{x=x_i^{eq}, u=u_i^{eq}}$ are Jacobian matrices. Let $\delta_x = x - x_i^{eq}$ and $\delta_u = u - u_i^{eq}$, $F(\delta_x, \delta_u)$ contains all the higher order terms of the Taylor series. By defining the vector $z = [\delta_x^T, \delta_u^T]^T$ and by continuity arguments, we have that

$$\|F_i(z)\|_2 / \|z\|_2 \to 0 \text{ as } \|z\|_2 \to 0 \tag{8}$$

Therefore, for any $\gamma_z^i > 0$ there exists $r_z^i > 0$ such that

$$\|F_i(z)\|_2 < \gamma_z^i \|z\|_2, \ \forall \|z\|_2 < r_z^i \tag{9}$$

In principle, for each linearized model a corresponding CG unit, hereafter termed CG_i, can then be properly designed.

A. Time-varying set-points

The ideas behind the following CG framework are mainly taken from [6]. Consider the following finite family of reference set-points $\{r_1, \ldots, r_q\} \in \mathbf{R} \subset \mathbf{R}^m$ and w.l.o.g. let us assume that

A2 -

$$\mathbf{R} \subset \bigcup_{i=1}^{N} \mathcal{W}_i^\delta \tag{10}$$

where $\mathcal{W}_i^\delta$, $i \in \mathcal{N} := \{1, 2, \ldots, N\}$ are the command sets each one computed w.r.t. the $i-th$ linearized model, $\bigcup_{i=1}^{N} \mathcal{W}_i^\delta$ is a connected set and

$$\forall i, j \in \mathcal{N} \ \ \mathcal{W}_i^\delta \cap \mathcal{W}_j^\delta \neq \emptyset \Rightarrow Int\{\mathcal{W}_i^\delta \cap \mathcal{W}_j^\delta\} \neq \emptyset \tag{11}$$

where $Int\{\cdot\}$ denotes the interior set of $\mathcal{W}_i^\delta \cap \mathcal{W}_j^\delta$.
It is well known that a single CG_i unit can be efficiently used if the initial and final set-points belong to $\mathcal{W}_i^\delta$. Otherwise, if the final set-point belongs to a different set $\mathcal{W}_j^\delta$, a procedure for switching between CG_i and CG_j needs to be defined. To this end, we consider the following statement:

Definition 1: The output admissible set for the generic CG_i is given by

$$Z_i^\delta := \left\{ [r^T, x^T]^T \in \mathbf{R}^m \times \mathbf{R}^n \,|\, c_i(k,x,r) \in \mathcal{C}, \forall k \in \mathcal{Z}_+ \right\} \tag{12}$$

Hence, we characterize the set of all states, which can be steered to feasible equilibrium points without constraints violation as

$$\mathcal{X}_j^\delta := \left\{ x \in \mathbf{R}^n \mid \begin{bmatrix} w \\ x \end{bmatrix} \in Z_i^\delta \text{ for at least one } w \in \mathbf{R}^m \right\} \tag{13}$$

and the following property holds true:

Property 1: Let $i, j \in \mathcal{N}$, then

$$Int\{\mathcal{W}_i^\delta \cap \mathcal{W}_j^\delta\} \neq \emptyset \Rightarrow Int\{\mathcal{X}_i^\delta \cap \mathcal{X}_j^\delta\} \neq \emptyset \tag{14}$$

As a consequence, a convenient transition reference $\hat{r} \in Int\{\mathcal{W}_i^\delta \cap \mathcal{W}_j^\delta\}$, with $\hat{x} \in Int\{\mathcal{X}_i^\delta \cap \mathcal{X}_j^\delta\}$ and $\hat{x}$ the equilibrium steady-state corresponding to $\hat{r}$, can be defined such that $[\hat{r}^T, \hat{x}^T]^T \in Z_i^\delta \cap Z_j^\delta$.

Then, by assuming that CG_i is in use at $\bar{t}$, $r(\bar{t}) \in \mathcal{W}_i^\delta \cap \mathcal{W}_j^\delta$ and $r(\bar{t}+1) \in \mathcal{W}_j^\delta$, a possible switching logic is as follows:
Switching procedure -

1) Solve and apply

$$g(\bar{t}+k) := \arg \min_{w \in \mathcal{V}_i(x(\bar{t}+k))} \|w - r(\bar{t})\|_\Psi, k = 1, \ldots, \bar{k}$$

2) At $t = \bar{t} + \bar{k}$ as soon as

$$x(t) \in Int\{\mathcal{X}_i^\delta \cap \mathcal{X}_j^\delta\} \tag{15}$$

switch to CG_j and solve

$$g(\bar{t}+k) := \arg \min_{w \in \mathcal{V}_j(x(t))} \|w - r(\bar{t}+1)\|_\Psi, \ t \geq \bar{t} + \bar{k} + 1$$

The rationale of the illustrated scheme is that for any $x \in \mathbf{R}^n$, the state evolution will enter into $Int\{\mathcal{X}_i^{\delta} \cap \mathcal{X}_j^{\delta}\}$ after a finite number of $\bar{k}$ time instants. In conclusion, an HCG scheme, named **SV-HCG**, can be adopting an adequate selection criterion to characterize the set $Int\{\mathcal{X}_i^{\delta} \cap \mathcal{X}_j^{\delta}\}$ in (15).

B. Time-varying constraints

In what follows, for ease of notation, we will retain fixed the equilibrium, the corresponding linearized plant structure and we will assume that a number of L constraints configurations may occur. Let us denote with $\mathcal{C}_i$, $i \in \mathcal{I} := \{1, 2, \ldots, L\}$, the $i-th$ constraints configuration and introduce

$$\mathcal{W}_i^{\delta_i} := \{w \in \mathbf{R}^m : \bar{c}_w \in \mathcal{C}_i^{\delta_i}\}, \ \forall i \in \mathcal{I} \tag{16}$$

where $\mathcal{W}_i^{\delta_i}$ is the set of all commands w whose steady-state evolutions of the vector c satisfy the $i-th$ constraints configuration $\mathcal{C}_i$ with a margin of tolerance δ_i. The sets $\mathcal{C}_i^{\delta_i}$ and $\mathcal{W}_i^{\delta_i}$ satisfy the following properties:

Property 2: Let $(i, j) \in \mathcal{I}$, then $\mathcal{C}_i^{\delta_i} \bigcap \mathcal{C}_j^{\delta_j} \neq \emptyset \Leftrightarrow \mathcal{W}_i^{\delta_i} \bigcap \mathcal{W}_j^{\delta_j} \neq \emptyset$

Property 3: $\bigcap_{i=1}^{L} \mathcal{C}_i^{\delta_i} \neq \emptyset \Leftrightarrow \bigcap_{i=1}^{L} \mathcal{W}_i^{\delta_i} \neq \emptyset$

Hereafter, we suppose that the sets $\mathcal{W}_i^{\delta_i}, \forall i \in \mathcal{I}$ are nonempty, closed and convex. Let us introduce the following definitions.

Definition 2: Let $i \in \mathcal{I}$, the state $x(t)$ is defined $\mathcal{C}_i^{\delta_i}$-*admissible* if $c(k, x(t), w) \in \mathcal{C}_i^{\delta_i}, \forall k \in \mathbb{Z}_+$, with $w \in \mathcal{W}_i^{\delta_i}$. Moreover, the pair $(x(t), w)$ is said $\mathcal{C}_i^{\delta_i}$-*executable*.

Definition 3: Let $i \in \mathcal{I}$, $x(t)$ a state $\mathcal{C}_i^{\delta_i}$-*admissible* and $\mathcal{C}_j^{\delta_j}$, $j \neq i$, a constraints configuration to be fulfilled. The state $x(t)$ is defined *switching-*$\mathcal{C}_i^{\delta_i}$*-admissible* if $c(k, x(t), w) \in \mathcal{C}_j^{\delta_j}, \forall k \in \mathbb{Z}_+$, with $w \in \mathcal{W}_j^{\delta_j}$. Moreover, the pair $(x(t), w)$ is said *switching-*$\mathcal{C}_i^{\delta_i}$*-executable* and the constraints configuration $\mathcal{C}_j^{\delta_j}$ *switchable*.

Definition 4: Let $i \in \mathcal{I}$, $x(t)$ a state $\mathcal{C}_i^{\delta_i}$-*admissible*. The state $x(t)$ is defined *full-switching-admissible* if $c(k, x(t), w) \in \mathcal{C}_j^{\delta_j}, \forall j \in \mathcal{I}, \forall k \in \mathbb{Z}_+$, with $w \in \mathcal{W}_j^{\delta_j}$. Moreover, the pair $(x(t), w)$ is said *full-switching-executable* and the constraints configuration $\mathcal{C}_i^{\delta_i}$ *full-switchable*.

Definition 5: Let $i \in \mathcal{I}$, $x(t)$ a state $\mathcal{C}_i^{\delta_i}$-*admissible* but not *switching-*$\mathcal{C}_i^{\delta_i}$*-admissible* and $\mathcal{C}_j^{\delta_j}$, $j \neq i$, a constraints configuration to be fulfilled. $\mathcal{C}_j^{\delta_j}$ is defined *reachable* if it belongs to the finite sequence of constraints configurations $\mathcal{S}_{sw} := \{\mathcal{C}_{i_1}^{\delta_{i_1}}, \mathcal{C}_{i_2}^{\delta_{i_2}}, \ldots, \mathcal{C}_{j_1}^{\delta_{j_1}}, \mathcal{C}_j^{\delta_j}\}$, where $(\mathcal{C}_{i_1}^{\delta_{i_1}}, \mathcal{C}_{i_2}^{\delta_{i_2}}) \ldots (\mathcal{C}_{j_1}^{\delta_{j_1}}, \mathcal{C}_j^{\delta_j})$ are couples *switchable* amongst them. Moreover, the state $x(t)$ is said *indirectly-switching-admissible*.

Finally, the sets

$$\mathcal{V}_i(x) := \{w \in \mathcal{W}_i^{\delta_i} : c(k, x, w) \in \mathcal{C}_i^{\delta_i}, \forall k \in \mathbb{Z}_+\}, \forall i \in \mathcal{I} \tag{17}$$

represent all constant virtual sequences in $\mathcal{W}_i^{\delta_i}$ whose c-evolutions, starting from a $\mathcal{C}_i^{\delta_i}$-*admissible* state x, satisfy the constraints configuration $\mathcal{C}_i^{\delta_i}$ also during transients. As a consequence, for a fixed $i \in \mathcal{I}$, $\mathcal{V}_i(x) \subset \mathcal{W}_i^{\delta_i}$.

Then, whenever the supervisory unit selects the $i-th$ CG candidate (CG_i), a command $g(t)$ is computed as a solution of the following constrained optimization problem

$$g(t) = \arg \min_{w \in \mathcal{V}_i(x(t))} \|w - r(t)\|_{\Psi} \tag{18}$$

Hence, an admissible HCG strategy can be developed if at each switching time instant $\bar{t}$, the current state $x(\bar{t})$ is *switching-admissible* or, alternatively, *full* or *indirectly-switching-admissible*. Finally, we introduce

$$\mathcal{X}_i^{\delta_i} := \begin{array}{l}\{x \in \mathbf{R}^n : c(k, x, w) \in \mathcal{C}_i^{\delta_i}, \\ \text{for at least one } w \in \mathcal{W}_i^{\delta_i}, \forall k \in \mathbb{Z}_+\}, \forall i \in \mathcal{I}\end{array} \tag{19}$$

that represents the set of all $\mathcal{C}_i^{\delta_i}$-*admissible* states, i.e. each state $x \in \mathcal{X}_i^{\delta_i}$ can be steered to an equilibrium point without constraints violation.

One crucial point is to prove that the proposed time-varying constraints CG strategy enjoys the viability property.

Proposition 1: Let us consider (1) and a family of constant command sequences $w \in \mathcal{W}_i^{\delta_i}$, $i \in \mathcal{I}$. Let $\bar{x}_{\bar{w}}$ be an equilibrium point reached under a constant virtual command $\bar{w} \in \mathcal{W}_i^{\delta_i}$ from a whatever $\mathcal{C}_i^{\delta_i}$-*admissible* state. Let the assumptions (A1) be fulfilled, the sets $\mathcal{C}_i^{\delta_i}$, $\forall i \in \mathcal{I}$ compact and convex, and the sets $\mathcal{W}_i^{\delta_i}$, $\forall i \in \mathcal{I}$, non-empty, closed and convex. Then, there exists a concatenation of finite virtual constant commands $\bar{w} \in \mathcal{W}_i^{\delta_i}$, with $i \in \mathcal{I}$, and of constraints configurations $\mathcal{C}_i$, chosen by a supervisory unit at switching time instants $\bar{t}$, capable of asymptotically driving the system (1) from $x_{\bar{w}}$ to x_w, any other $w \in \mathcal{W}_j^{\delta_j}$.

Proof - The proof uses similar arguments of [5] and is here omitted for the sake of space. □

Moreover, the next result proves the asymptotic stability property of the overall system.

Proposition 2: Let the assumption **A1** be fulfilled and the sets $\mathcal{V}_i(x(\bar{t}))$, $\forall i \in \mathcal{I}$ be nonempty for any switching time instant $\bar{t}$. Moreover, it is assumed that there exists an instant time t^* under which the switching of constraints configurations terminates and the reference signal $r(t)$ stays constant, $r(t) \equiv r$, for all $t \geq \hat{t}$, with $\hat{t} \geq t^*$. Then:

$$\lim_{t \to \infty} [g(t+1) - g(t)] = 0_m \quad \lim_{t \to \infty} [x(t) - x_{g(t)}] = 0_n \tag{20}$$

where $x_{g(t)} = (I - \Phi)^{-1} G g(t)$. There exits a finite time $t_f > \hat{t}$ such that

$$g(t) = \bar{g} := \arg \min_{w \in \mathcal{W}_i^{\delta_i}} \|w - r(t)\|_{\Psi}, \ \forall t \geq t_f \tag{21}$$

where i being identified as the last constraints configuration activated by the supervisory from t_f onward.

Proof - The proof follows *mutatis mutandis* the same lines used for the basic CG scheme. For details see [5]. □

Finally by resorting to the *Switching procedure* a hybrid scheme, hereafter termed **CV-HCG**, comes out.

IV. REAL-TIME HYBRID COMMAND GOVERNOR (RT-HCG) SCHEME

In this section, a supervisory CG-based real-time architecture for constrained dynamical systems subject to time-varying set-points and/or constraints is detailed. The following assumptions hold true:

B1 - A set of working points $\left\{(x_i^{eq}, u_i^{eq})\right\}_{i=1}^{N}$ is given;

B2 - At each time instant t, the supervisory unit is informed on the plant structure (set-point and constraints configuration) to be fulfilled exactly at time $t+1$;

B3 - An off-line computed CG unit acts as the initialization controller, e.g. CG_i;

B4 - The maximum requested time T_{ON} for the on-line computation of the CG action is such that $T_{ON} < T_s$, with T_s the sampling time.

The supervisor logic retains valid the CG_i unit as long as the distance between the equilibrium x_i^{eq} and the actual state $x(t)$ is minimal. On the contrary, the supervisor switches to the CG_j unit when

$$j := \min_j \|x_j^{eq} - x(t)\| \tag{22}$$

Based on $r(t,t+1)$ (the reference known at the time instant t and to be tracked at $t+1$), $c_{t,t+1}$ (the constraints configuration known at the time instant t and to be fulfilled at $t+1$), $r(t)$ and $x(t)$ (the actual reference and state measurements), one of the following events could happen:

Set-point change - If the new set-point $r(t,t+1) \notin \mathcal{W}_i^{\delta_i}$, a switching to the $j-th$ linearized model selected by (22) is prescribed. Then, the time interval $[t,\, t+1]$, is split in two fractions: the first fraction is used to compute the new command $g(t)$ via the nominal CG_i, while the remaining available time is used to start the computation of the new CG_j unit.

Constraints configuration change - Because the on-line design of the CG_j unit could require more than one sampling time and the action of the CG_i is no longer admissible, to ensure the constraint fulfillment at each time instant t a new controller must be considered. Such a controller, in place of the primal control law K_i and the CG_i device, should be capable to satisfy all the constraints regardless set-point tracking properties until the computation of CG_j is accomplished. Hereafter, we denote it as the *safe controller* K_{safe}. The above means that the CG_i unit and its corresponding primal controller K_i will be disconnected from the plant and, before the next time instant $t+1$ the plant is connected to K_{safe}. Therefore within the time interval $[t,\, t+1]$, the K_{safe} is first achieved, then the command $g(t)$ is computed based on CG_i unit and finally, if possible, the design of the CG_j unit starts.

Equilibrium change - By checking (22) it results that a switching to the $j-th$ model is more adequate to describe the nonlinear dynamics of (7). Then, the time interval $[t,\, t+1]$ is split in two fractions: the first portion is used to compute the new command $g(t)$ via the CG_i device, while the remaining available time is used to start the computation of the CG_j unit.

A. Supervisor finite state automaton

The aim of this section is to describe the hybrid structure of the proposed real-time architecture. To this end, the *Supervisor* discrete behaviour is described by means of a three state automaton (see Fig. 1):

- **HOME**: normal operating condition under a CG action;
- **EQ-SW**: set-point or equilibrium point change event;
- **CNF-SW**: constraints configuration change event.

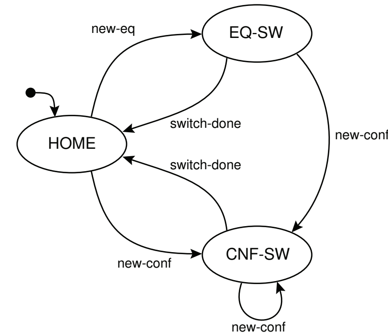

Fig. 1. *Supervisor* automaton

Initially, the *Supervisor* is set to the **HOME** state where the control action is carried out by a single periodic task τ_{CG_i},, which runs at the highest priority level and executes all the standard CG operations. In particular, the τ_{CG_i} actions are: reference and state measurements acquisition, on-line computation of the CG output $g(t)$, primal controller execution and application of $g(t)$.

When a set-point or an equilibrium point change occurrence is detected (**new-eq** event) the *Supervisor* moves to the **EQ-SW** state where the operation mode is instructed for the computation of the new CG. The design of the CG is assigned to an aperiodic task τ_{SW} which is released when **EQ-SW** is entered and which runs at a lower priority level with respect to τ_{CG_i}. In the general case, τ_{SW} is not able to complete its task (CG design) within a single sampling period because a fraction of it must be used for the execution of τ_{CG_i}. Therefore, τ_{SW} is pre-empted by τ_{CG_i} for a finite number of time instants. As soon as τ_{SW} accomplishes its job (**switch-done** event), the *Supervisor* switches to the new CG_j and the system operation mode is set to **HOME**.

A different mode transition occurs when a constraints configuration change is detected (**new-conf** event). In this case, the *Supervisor* applies a transition to the **CNF-SW** state where the CG design is accomplished by an instance of the aperiodic task τ_{SW}. On the other hand, at the actual time instant t, the actions provided by τ_{CG_i} are not adequate because the fulfillment of the new constraints set is no longer guaranteed. Then, within $[t,\, (t+1)]$ the *Supervisor* establishes the execution of a new task, denoted as τ_{CS}, whose actions are: computation of the switching controller K_{safe} and its application in order to ensure at least constraints satisfaction. Then, at each future $[(t+i),\, (t+1+i)]$, $i \geq 1$, a periodic task τ_{safe} applies the control action due to K_{safe}. Note that both τ_{CS} and τ_{safe} tasks inherit the priority level of τ_{CG_i}. Finally, when τ_{SW} ends its job (**switch-done** event), the *Supervisor* de-schedules the task τ_{safe}, restarts the task τ_{CG_i}, now equipped with the new on-line computed CG, and the system operation mode is re-set up to the **HOME** state. We have also taken into consideration the chance that the event **new-conf** could occur while an instance of τ_{SW} is still running (i.e. the system operation mode is defined by the **EQ-SW** or **CNF-SW** states). In such a situation, the *Supervisor* is instructed to execute the following steps: the instance of τ_{SW} is aborted, a system mode transition to **CNF-SW** is (re-)applied and the scheduled actions (K_{safe} computation, τ_{CS} and τ_{safe} tasks executions) are carried out.

B. Real-time computational aspects and main results

In this paragraph all the real-time events will be investigated by detailing the conditions under which set-point tracking and viability properties can be ensured for the proposed supervisory scheme. Let CG_i be the unit used at the current time instant t and the system mode operation set to the **HOME** state. Then, the following system mode operation transitions will take place:

HOME $\rightarrow$ **EQ-SW**: Two events could occur:

- $x(t) \notin \mathcal{T}_i$
- $r(t,t+1) \neq r(t)$ and $r(t,t+1) \notin \mathcal{W}_i^\delta$

Then if $j = \min_j \|x_j^{eq} - x(t)\|$, $j \neq i$, a switching to the $j-th$ plant linearized model has to be considered and the **SV-HCG** policy has to be carried out. As a consequence, a primal controller K_j is computed and the CG_j design starts by imposing $Int\{\mathcal{W}_i^\delta \cap \mathcal{W}_j^\delta\} \neq \emptyset$. Once the CG_j design is accomplished at the time instant $\bar{t}$, the *Supervisor* disconnects K_i and CG_i, and from $\bar{t}$ onward the new control architecture (K_j, CG_j) is used;

HOME $\rightarrow$ **CNF-SW**: The one-step ahead constraint configuration $\mathcal{C}_{t,t+1} \nsubseteq \mathcal{C}_i$, such that $\mathcal{C}_{t,t+1} \cap \mathcal{C}_i \neq \emptyset$, requires the computation of the new unit CG_j. At $t+1$ the action of CG_i is no longer admissible because it cannot ensure the satisfaction of the constraints configuration $\mathcal{C}_{t,t+1}$. Therefore, until the CG_j computation is not completed, the set-point tracking mode is suspended and the constraints fulfilled by using the *safe controller* K_{safe}. Hence the actions of the *Supervisor* are: first, at $t+1$ the system switches from the control architecture (K_i, CG_i) to the K_{safe} controller complying with $\mathcal{C}_{t,t+1}$; then the **CV-HCG** policy is carried out and, once the CG_j design is accomplished at the time instant $\bar{t}$, the *Supervisor* disconnects K_{safe} and from $\bar{t}$ onward the couple (K_j, CG_j) is applied;

EQ-SW $\rightarrow$ **CNF-SW**: At a certain time $\hat{t} \geq (t+1)$, while the *Supervisory* is executing the tasks due to **HOME** $\rightarrow$ **EQ-SW** and computing the CG_j unit, it is detected that $\mathcal{C}_{\hat{t}|\hat{t}+1} \nsubseteq \mathcal{C}_j$, then the CG_j design is aborted and a new K_{safe}^j controller is computed to be applied at $\hat{t}+1$. Hence, the same actions of the transition **HOME** $\rightarrow$ **CNF-SW** will take place;

CNF-SW $\rightarrow$ **CNF-SW**: While the *Supervisory* is executing the tasks due to **HOME** $\rightarrow$ **CNF-SW**, it is detected that $\mathcal{C}_{\hat{t}|\hat{t}+1} \nsubseteq \mathcal{C}_j$, therefore the same steps of **EQ-SW** $\rightarrow$ **CNF-SW** have to be carried out.

The main properties of the **RT-HCG** are here summarized:

Proposition 3: Suppose that assumptions **A1**, **B1**-**B4**, **A2**, for the time-varying set-point scenario and properties 2-3 for the time-varying constraints scenario hold.

Then: *No constraint configuration change occurrences* - All properties of the CG device (see [6] Theorem 1, pg. 345) are preserved. In particular the constraints are fulfilled for all $t \in \mathbb{Z}_+$, the tracking performance optimized and the overall asymptotic stability ensured.

Constraint configuration change occurrences - The K_{safe} controller ensures persistence of operations: tracking performance may be lost while asymptotic stability and constraints fulfilment are guaranteed.

Proof - It follows from the above discussions. □

C. The safe controller K_{safe}

Let $\Xi_i := \{x \in \mathbf{R}^n \,|x^T P_i x \leq 1\}$ be the invariant ellipsoidal region associated to the control law K_i. Then, because the switching from K_i to K_{safe} must guarantee the asymptotic stability of the closed-loop system and the constraints satisfaction $c_i(t) \in \mathcal{C}_{\hat{t}|\hat{t}+1}, \forall t \geq \hat{t}+1$, the controller K_{safe} is computed by solving the following problem:

K_{safe} **Problem -** *Find a state feedback law K_{safe} such that the following conditions hold true*

$$K_{safe} z \in U, \forall z \in \Xi_{safe} := \{x \in \mathbf{R}^n \,|x^T P_{safe} x \leq 1\} \subset X \tag{23}$$

$$x(\hat{t}) \in \Xi_i \cap \Xi_{safe} \tag{24}$$

where Ξ_{safe} is the ellipsoidal invariant set for the $i-th$ linearized system under the K_{safe} action. □

Note that condition (24) is imposed to ensure the admissibility of the K_i-to-K_{safe} switching and it can be shown by exploiting the same arguments as e.g. in [8], while the requirement $\mathcal{C}_{\hat{t}|\hat{t}+1} \cap \mathcal{C}_i \neq \emptyset$ guarantees that $\Xi_i \cap \Xi_{safe} \neq \emptyset$.

V. Experiments

A four-tank process, fully described and analyzed in [9], is used to validate the performance of the proposed supervisory strategy. The goal is to regulate the water levels $h_3(t)$ and $h_4(t)$ (plant outputs) at given set-points by acting on the incoming water flows via the supply pump voltages $V_1(t)$ and $V_2(t)$ (plant inputs).

We have considered the following three equilibrium points

$$\begin{array}{ll} x_1^{eq} = [0.6065\ 1.3050\ 5\ 5]^T, & u_1^{eq} = [7.1550\ 6.9424]^T, \\ x_2^{eq} = [1.0310\ 2.2185\ 8.5\ 8.5]^T, & u_2^{eq} = [7.2504\ 7.3421]^T, \\ x_3^{eq} = [1.6981\ 3.6540\ 14\ 14]^T, & u_3^{eq} = [7.3664\ 7.8281]^T, \end{array}$$

where $x(t) = [h_1(t)\ h_2(t)\ h_3(t)\ h_4(t)]^T$ and $u(t) = [V_1(t)\ V_2(t)]^T$. Then, the linearized models have been discretized using forward Euler differences with a sampling time $T_s = 0.1\,sec.$ and the physical constraints on maximum water levels and maximum pump supply voltage have been considered: $h_i(t) \leq 16, [cm]$, $i = 1,\ldots,4$ and $6 \leq V_i(t) \leq 8$, $[Volt]$, $i = 1,2$. The following CG parameters, $\delta_i = 10^{-6}, i = 1,2,3$, and $\Psi = I_2$ have been chosen and the constraint horizon $k_0 = 130$ was computed via the numerical procedure proposed in [5]. Further, to characterize the set of admissible disturbances/measurement errors, the following convex and compact region has been considered and used in the CG setting: $\mathcal{D} := \left\{ d \in \mathbf{R}^4 \,|\, Ud \leq h \right\}$, where $U = \begin{bmatrix} I_4 \\ -I_4 \end{bmatrix}$ and $h = 0.3 * [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]^T$ $[cm]$. The primal compensators $K_i, i = 1,2,3$, have been designed as two-degree of freedom LQ controllers.

For comparison purposes a single CG unit has been off-line designed by referring to the equilibrium (x_1^{eq}, u_1^{eq}). The following scenario has been taken into consideration: *Starting from the initial state* $x(0) = [0.5458\ 1.1745\ 4.5\ 4.5]^T$, *first the set-points* $h_{3ref} = h_{4ref} = 14.5\,cm$ *are considered. Then at* $210\,sec.$ *a set-point (*$h_{3ref} = h_{4ref} = 10\,cm$*) and a constraints configuration (*$0.5 \leq h_i(t) \leq 15$, $[cm]$, $i = 1,2$ *and* $9.5 \leq h_i(t) \leq 15$, $[cm]$, $i = 3,4$*) changes jointly occur.* All the experiments are reported in Figs. 2-4. As it results the HGC device is capable to adequately comply both with the tracking requirements of the first phase $[0, 210]$, $sec.$ and the successive time-varying scenario. It can be observed in fact that (Fig. 2) first both the Tanks 3 and 4 settle down to the prescribed set-points and then such a strategy is capable to deal with the new set-point/constraints configuration without constraints violation, see also Fig. 3. This is clearly achieved by means of appropriate CG switchings as highlighted in Fig. 4. Conversely, the single CG_1 action is not capable to settle down to $14.5\,cm$ because $h_{iref} = 14.5\,cm, i = 3,4$, do not belong to $\mathcal{W}_1^\delta$. Moreover, as expected, the time varying scenario cannot be managed by CG_1 and therefore constraints violations occur (Fig. 2).

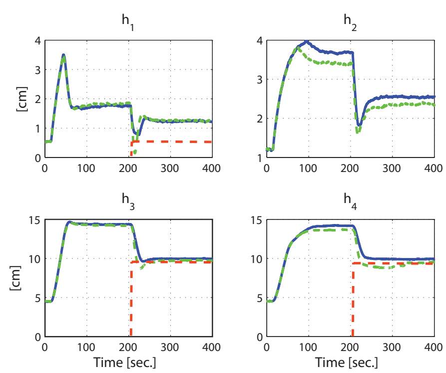

Fig. 2. Water levels behaviors: solid-line with HCG and dot-line with CG_1. The dashed lines represent the prescribed constraints.

VI. Conclusions

A real-time hybrid strategy for orchestrating switching between CG units has been presented. The main feature is the ability to take care of both time-varying set-points and constraints scenarios by on-line computing the proper control architecture. Results on viability, constraints fulfilment and convergence have been derived. Finally, real-time experiments on a laboratory four-tank test-bed subject to voltage saturations and water levels constraints demonstrate the effectiveness of the proposed supervisory scheme.

References

[1] K. J. Aström and M. Lundh, "Lund control program combines theory with hands-on experience", *IEEE Contr. Syst. Mag.*, Vol. 12, No. 3, pp. 22-30, 1992.

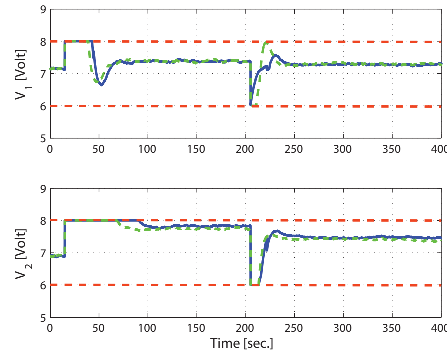

Fig. 3. Voltages provided by the pumps: solid-line with HCG and dot-line with CG_1. The dashed lines represent the prescribed constraints.

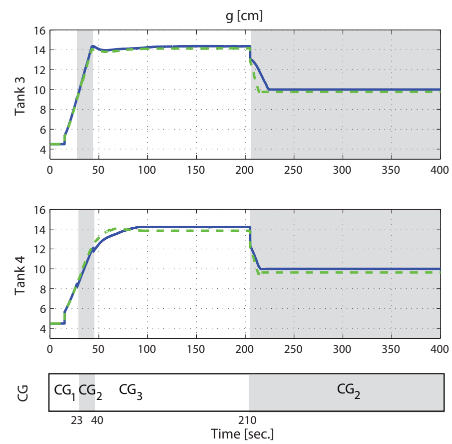

Fig. 4. Outputs of HCG and CG_1 units

[2] S.J. Park and J.M. Yang. "Supervisory control for real-time scheduling of periodic and sporadic tasks with resource constraints", *Automatica*, Vol. 45, No. 11, pp. 2597–2604, 2009.

[3] M. Diehl, H. G. Bock, J. P. Schlder, R. Findeisen, Z. Nagy and F. Allgöwer, "Real-time optimization and nonlinear model predictive control of processes governed by differential-algebraic equations", *Journal of Process Control*, pp. 577–585, Vol. 14, 2002.

[4] R.H. Miller, I. Kolmanovsky, E.G. Gilbert and P.D. Washabaugh, "Control of constrained nonlinear systems: a case study", *IEEE Contr. Syst. Mag.*, Vol. 20, pp. 23-32, 2000.

[5] A. Bemporad, "Reference Governor fo Constrained Nonlinear Systems", *IEEE Trans. Aut. Contr.*, Vol. 43, pp. 415-419, 1998.

[6] F. Bacconi, E. Mosca and A. Casavola, "Hybrid constrained formation flying control of micro-satellites", *IET Control Theory Appl.*, Vol. 1, No. 2, pp. 513–521, 2007.

[7] H.K. Khalil, "Nonlinear Systems", *Prentice Hall*, 1996.

[8] M. S. Branicky,"Multiple Lyapunov Functions and Other Analysis Tools for Switched and Hybrid Systems", *IEEE Trans. Automat. Contr.*, Vol. 43, No. 4, pp. 475-482, 1998.

[9] A. Casavola, D. Famularo, G. Franzè and A. Furfaro. "A fault-tolerant real-time supervisory scheme for an interconneted four tanks process", 2010 ACC, Baltimore, Maryland, 2010.

[10] D. Goldfarb and A. Idnani, "A numerically stable dual method for solving strictly convex quadratic programs", *Math. Program.*, Vol. 27, pp. 1–33, 1983.