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Robust Iterative Learning Control: $\mathcal{L}_1$ Adaptive Feedback Control in an ILC Framework

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Abstract— **This paper presents a novel Iterative Learning Control (ILC) scheme for linear systems in the presence of parametric uncertainty. The developed ILC architecture is comprised of an $\mathcal{L}_1$ adaptive feedback controller combined with an ILC feedforward controller. The learning controller is designed to compensate for repetitive system uncertainties, while the adaptive controller compensates for non-repetitive uncertainties. Simulation results for a simplified motion control system illustrate the potential benefits of the architecture.**

I. Introduction

The recent increase in technological requirements for advanced manufacturing continues to drive research in precision motion control (PMC). PMC processes are controlled through feedback or feedforward control schemes or a combination of the two in a 2-degree-of-freedom (2DOF) design. 2DOF designs are particularly effective for systems with distinct combinations of repetitive and non-repetitive exogenous signals. In these systems, feedback control addresses the non-repetitive signal content, while feedforward control addresses the repetitive signal content.

Iterative learning control (ILC) is a feedforward control design technique for repetitive processes [1]. ILC algorithms use information from earlier trials of a repetitive process to improve performance in the current trial. The key design feature of ILC is the efficient use of past information to improve tracking performance within a small number of trials, while ensuring robustness of the process to system uncertainty. Stability and performance of iterative learning algorithms in the iteration domain highly depends on the behavior of the process to be controlled in the time domain. In this sense, the ability of the feedback controller to compensate for non-repetitive disturbances and changes in the system dynamics plays an important role in the design of robust iterative learning controllers. Substantial research has been focused on the design of robust ILC methods for systems with high-frequency modeling uncertainties (see [1], [2] and references therein). There has also been a considerable research effort focused on the design of iterative learning algorithms for processes with large parametric uncertainty (see, for example, [3]–[6]). In particular, the development of ILC schemes providing improved stability robustness and fast convergence for systems with large parametric variations is of particular interest, as it may benefit many applications, such as pick and place robotic systems, precision motion control systems, and manufacturing systems with modular components.

In this paper, we propose the use of an inner feedback control law based on $\mathcal{L}_1$ adaptive control in an ILC framework. The $\mathcal{L}_1$ adaptive controller is designed to compensate for non-repetitive, low-frequency (parametric) uncertainty in the time domain, while the iterative learning controller compensates for repetitive system uncertainties in the iteration domain. The key feature of $\mathcal{L}_1$ adaptive control is the decoupling of adaptation and robustness, which enables fast adaptation with guaranteed robustness. In this sense, the *fast* and *robust* adaptation of $\mathcal{L}_1$ adaptive controllers is able to compensate for the undesirable effects of significant low-frequency uncertainty in the system dynamics and provide a predictable closed-loop response. From an ILC perspective, the $\mathcal{L}_1$ adaptive control law ensures that the transfer function from the feedforward ILC input to the plant output remains close to this nominal-plant sensitivity function regardless of the change in parameters.

The paper is organized as follows. Section II presents an $\mathcal{L}_1$ adaptive controller for a class of uncertain LTI systems. Section III gives a brief overview of the ILC problem. In section IV, the combined feedback-feedforward control system is introduced. A preliminary stability analysis of the proposed structure is included in section V. Simulation results and conclusions are presented in sections VI and VII.

II. $\mathcal{L}_1$ Adaptive Control

$\mathcal{L}_1$ Adaptive Control Theory [7] appeared recently as a method for the design of *robust* adaptive control architectures using *fast* estimation schemes. The key feature of $\mathcal{L}_1$ adaptive control is the decoupling of adaptation and robustness to unmodeled dynamics, which enables fast adaptation with guaranteed robustness margins. In $\mathcal{L}_1$ adaptive control architectures, the speed of adaptation is limited only by the available hardware (computational power and high-frequency sensor noise), while the trade-off between $\mathcal{L}_1$ performance and robustness can be addressed via conventional methods from classical and robust control. The separation of adaptation from robustness is achieved by appropriately inserting a bandwidth-limited filter into the control structure, which ensures that the control signal stays in the desired frequency range and within the bandwidth of the control channel. The bandwidth and structure of this filter define the trade-off between $\mathcal{L}_1$ performance and robustness. The combination of the bandwidth-limited filter and high adaptation rates ensures uniform performance bounds for both system signals, input and output, without enforcing persistency of excitation or

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resorting to high-gain feedback. The benefits of $\mathcal{L}_1$ adaptive control theory have been verified –consistently with the theory– in a large number of flight tests and in mid- to high-fidelity simulation environments (see for example [8]–[10]).

Next, the $\mathcal{L}_1$ adaptive control architecture is presented for a class of single-input single-output uncertain LTI systems in the presence of uncertain system input gain and unknown constant parameters. $\mathcal{L}_1$ adaptive control theory has been developed for a broader class of systems, and the reader is referred to [7] for an account of other $\mathcal{L}_1$ adaptive control architectures and a detailed explanation of the $\mathcal{L}_1$ adaptive controller described in this paper.

A. $\mathcal{L}_1$ Adaptive Control for LTI SISO Systems

1) Problem Formulation: Consider the class of systems:

$$\dot{x}(t) = A_m\, x(t) + b\,\left(\omega\, u(t) + \theta^\top x(t)\right), \quad x(0) = x_0,$$
$$y(t) = c^\top x(t),$$

where $x(t) \in \mathbb{R}^n$ is the system state vector (measured); $u(t) \in \mathbb{R}$ is the control signal; $y(t) \in \mathbb{R}$ is the regulated output; A_m is a known Hurwitz $n \times n$ matrix that defines the desired dynamics for the closed-loop system; b, $c \in \mathbb{R}^n$ are known constant vectors, (A_m, b) controllable, $(A_m, c^\top)$ observable; $\omega \in \mathbb{R}$ is an unknown constant; and $\theta \in \mathbb{R}^n$ is a vector of constant unknown parameters.

The system above verifies the following assumptions:

Assumption 1 (Boundedness of θ): The parameter vector θ belongs to a given compact convex set $\theta \in \Theta \subset \mathbb{R}^n$.

Assumption 2 (Partial knowledge of ω): The system input gain is assumed to be an unknown constant with known sign (wihtout loss of generality, we assume $\omega > 0$). Also, we assume that there exist known conservative bounds ω_ℓ and ω_u such that $\omega \in [\omega_\ell, \omega_u] \triangleq \Omega$, where $0 < \omega_\ell < \omega_u$.

The control objective is to design an adaptive state feedback controller to ensure that $y(t)$ tracks the output response of a *desired system* $M(s)$ defined as

$$M(s) \triangleq c^\top\,(s\mathbb{I}_n - A_m)^{-1}\, b\, k_g(s)$$

where $k_g(s)$ is a feedforward prefilter, to a given bounded reference signal $r(t)$ *both in transient* and *steady-state*, while all other signals remain bounded.

2) $\mathcal{L}_1$ Adaptive Controller: The $\mathcal{L}_1$ adaptive controller used in this paper consists of a *fast estimation scheme* and a *control law*. The fast estimation scheme includes a *state predictor* and an appropriately designed *adaptation law*, which are used to generate estimates of the uncertainties present in the plant. Based on these estimates, the control law generates the control signal as the output of bandwidth-limited filter. The fast estimation scheme and the control law of the $\mathcal{L}_1$ adaptive controller are introduced below:

State predictor: Consider the following state predictor:

$$\dot{\hat{x}}(t) = A_m\hat{x}(t) + b\big(\hat{\omega}(t)u(t) + \hat{\theta}^\top(t)x(t)\big) + K_s\tilde{x}(t), \quad (1)$$

with initial condition $\hat{x}(0) = x_0$, and where $\hat{\omega}(t) \in \mathbb{R}$ and $\hat{\theta}(t) \in \mathbb{R}^n$ are the adaptive estimates, $K_s \in \mathbb{R}^{n\times n}$ is such that $A_s \triangleq A_m + K_s$ is a Hurwitz matrix, and $\tilde{x}(t) \triangleq \hat{x}(t) - x(t)$ is the prediction error.

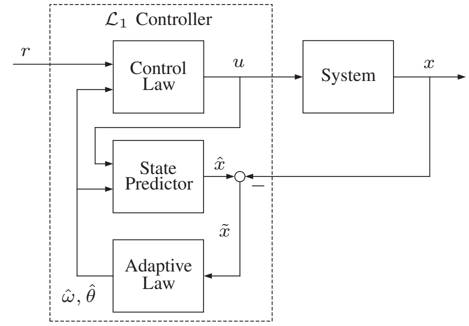


Fig. 1: Closed-loop system with the $\mathcal{L}_1$ adaptive controller

Adaptation laws: The adaptation laws for $\hat{\omega}(t)$ and $\hat{\theta}(t)$ are given by

$$\begin{aligned}\dot{\hat{\omega}}(t) &= \Gamma\mathrm{Proj}(\hat{\omega}(t), -\tilde{x}^\top(t)Xb\,u(t)), \quad \hat{\omega}(0) = \hat{\omega}_0,\\ \dot{\hat{\theta}}(t) &= \Gamma\mathrm{Proj}(\hat{\theta}(t), -\tilde{x}^\top(t)Xb\,x(t)), \quad \hat{\theta}(0) = \hat{\theta}_0,\end{aligned} \quad (2)$$

where $\Gamma > 0$ is the adaptation rate, $\mathrm{Proj}(\cdot,\cdot)$ denotes the projection operator defined in [11], and $X = X^\top > 0$ is the solution to the algebraic Lyapunov equation $A_s^\top X + XA_s = -Y$ for arbitrary $Y = Y^\top > 0$. In the implementation of the projection operator, we use the compact convex sets Θ and Ω as given in Assumptions 1 and 2.

Control law: The control signal is generated as the output of the following (feedback) system:

$$u(s) = -k_{\mathcal{L}_1} D(s)\big(\hat{\eta}(s) - k_g(s)r(s)\big)\,, \quad (3)$$

where $\hat{\eta}(s)$ is the Laplace transform of the signal

$$\hat{\eta}(t) \triangleq \hat{\omega}(t)u(t) + \hat{\theta}^\top(t)x(t)\,, \quad (4)$$

while $k_{\mathcal{L}_1} > 0$ and $D(s)$ are a feedback gain and a strictly proper transfer function leading to a strictly proper stable

$$C(s) \triangleq \frac{\omega\, k_{\mathcal{L}_1}\, D(s)}{1 + \omega\, k_{\mathcal{L}_1}\, D(s)}\,, \qquad \forall \omega \in \Omega,$$

with DC gain $C(0) = 1$. One simple choice is $D(s) = \frac{1}{s}$, which yields a first-order strictly proper $C(s)$ of the form

$$C(s) = \frac{\omega\, k_{\mathcal{L}_1}}{s + \omega\, k_{\mathcal{L}_1}}\,.$$

The complete $\mathcal{L}_1$ adaptive state-feedback controller consists of (1), (2), and (3)-(4), and its design is subject to the following $\mathcal{L}_1$-norm stability condition:

$$\|G(s)\|_{\mathcal{L}_1}\,\theta_{\max} < 1\,, \quad (5)$$

where $G(s)$ and $\theta_{\max}$ are defined as

$$G(s) \triangleq (s\mathbb{I}_n - A_m)^{-1}b\,(1 - C(s))\,, \quad \theta_{\max} \triangleq \max_{\theta\in\Theta}\|\theta\|_1\,.$$

The $\mathcal{L}_1$ adaptive control architecture with its main elements is represented in Figure 1.

If the stability condition in (5) is satisfied, the closed-loop adaptive system is stable and, moreover, one can derive computable uniform performance bounds for both system input and output, $y(t)$ and $u(t)$ respectively, with respect to the signals $y_{\rm ref}(t)$ and $u_{\rm ref}(t)$ of a *closed-loop reference system*, which is defined in terms of the ideal nonadaptive version of the adaptive controller in (3)-(4):

$$\|y - y_{\rm ref}\|_{\mathcal{L}_\infty} \leq \bar{\gamma}_y \, , \qquad \|u - u_{\rm ref}\|_{\mathcal{L}_\infty} \leq \bar{\gamma}_u \, , \qquad (6)$$

where

$$\lim_{\Gamma\to\infty} \bar{\gamma}_y = \lim_{\Gamma\to\infty} \bar{\gamma}_u = 0 \, .$$

This implies that, both in transient and steady-state, one can achieve arbitrary close tracking performance for both signals simultaneously by increasing the adaptation rate Γ. Moreover, one can show that the output of the closed-loop adaptive system tracks the desired response $y_{\rm des}(t)$ both in transient and steady state with uniform performance bounds that can be systematically improved by increasing the adaptation gain Γ and properly selecting the low-pass filter $C(s)$. Details on the stability proof, definition of the reference system, and derivation of the performance bounds can be found in [7].

III. Iterative Learning Control

ILC is a plug-in type controller in which the ILC input can be added to an existing control loop, either in a parallel or series type architecture [1]. While either approach is equally valid, the work in this paper utilizes the parallel structure illustrated in Figure 2. The reader should note that the learning input signal is combined with the feedback signal in a format often used with feedforward control signals.

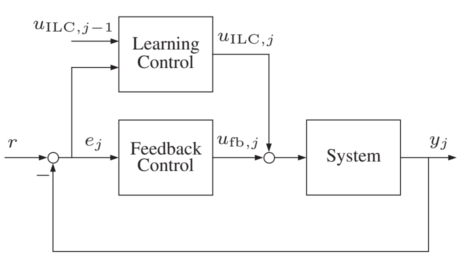


Fig. 2: Block diagram of a parallel ILC process

There are several dominant design paradigms in ILC, linear repetitive process design, internal model design, norm optimal design, and frequency-domain design. The learning controller implemented in this work is designed using the frequency domain framework. One of the advantages of this approach is that many frequency domain learning controllers use tunable designs such as proportional controllers, thereby requiring little a priori knowledge of the system.

A. ILC Update Law

The common continuous-time frequency-domain update law for ILC is of the form,

$$u_{ILC,j+1}(s) = Q(s)(u_{ILC,j}(s) + L(s)e_j(s)) \, , \qquad (7)$$

where

$$e_j(s) = r(s) - y_j(s) \, , \qquad (8)$$

$$y_j(s) = T(s)r(s) + P_S(s)u_{ILC,j}(s) \, . \qquad (9)$$

In (7)-(9), $e_j(s)$ is the tracking error signal between the reference $r(s)$ and the system output $y_j(s)$, $T(s)$ is the complementary sensitivity function, and $P_S(s)$ is the plant sensitivity function defined as the relationship between the current ILC control signal $u_{ILC,j}(s)$ and the system output $y_j(s)$, and $j = 0, 1, \ldots$ is the iteration index. The design elements in the update law include the Q-filter and L-filter. $Q(s)$ is generally designed as a low-pass filter to limit the learning bandwidth and provide robustness to the system. $L(s)$, also known as the learning filter, is designed to maximize the learnable bandwidth and convergence rate.

B. Convergence

The goal in designing $Q(s)$ and $L(s)$ is to ensure contraction mapping or monotonic convergence of the control signal. Substituting (8) and (9) into (7) and rearranging the terms results in the following control iteration dynamics:

$$u_{ILC,j+1}(s) = Q(s)(1 - L(s)P_S(s))u_{ILC,j}(s) + Q(s)L(s)r(s) \, .$$

A contraction mapping from $u_{ILC,j+1}(s)$ to $u_{ILC,j}(s)$ is accomplished by ensuring,

$$\|Q(s)(1 - L(s)P_S(s)\|_\infty < 1 \, , \qquad (10)$$

where the infinity norm $\| \bullet (s)\|_\infty$ is defined as

$$\|\bullet(s)\|_\infty \triangleq \max_{\omega\in\mathbb{R}} \bar{\sigma}[\bullet(i\omega)|.$$

Satisfying (10) guarantees monotonic convergence of the asymptotic control $u_\infty(s)$ and error signals $e_\infty(s)$, which are calculated as

$$u_\infty(s) \triangleq \lim_{j\to\infty} u_{ILC,j}(s) = \frac{Q(s)L(s)}{1 - Q(s)(1 - L(s)P_S(s))} r(s)$$

$$e_\infty(s) \triangleq r(s) - P_S(s)u_\infty(s) \, .$$

The authors refer the reader to [1] for more details on frequency based ILC designs.

IV. $\mathcal{L}_1$ Adaptive Control with Parallel ILC

As mentioned in Section III, there are several dominant design paradigms in ILC. This paper presents a novel ILC design paradigm in which an $\mathcal{L}_1$ adaptive feedback controller is combined with an ILC feedforward controller. The learning controller is designed to compensate for repetitive disturbances within the system *in the iteration domain*, while the adaptive controller ensures compensation for low-frequency non-repetitive disturbances and parametric uncertainties *in the time domain*. The ILC architecture, comprised of the $\mathcal{L}_1$ adaptive feedback controller combined with the ILC feedforward controller, is shown in Figure 3. We notice that, from an architectural viewpoint, this control approach is similar to the one reported in [5]. From a design perspective, however, the use of a feedback control law based on

$\mathcal{L}_1$ adaptive control ensures that the transfer function from the feedforward ILC input to the plant output remains close to a nominal plant-sensitivity function, which facilitates the design of the learning algorithm.

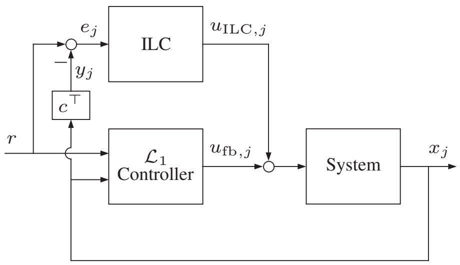


Fig. 3: Parallel ILC architecture, augmenting the (state-feedback) adaptive control signal

Given the importance of the filters $C(s)$ and $Q(s)$ on the trade-off between performance and robustness, it is important to evaluate the effects of combining $\mathcal{L}_1$ and ILC controllers into the same design framework. The next section presents a preliminary stability analysis of the proposed controller.

V. Preliminary Stability Analysis

Assume the learning law (7) and let $e_{\mathrm{ref},j}(s) \triangleq r(s) - y_{\mathrm{ref},j}(s)$. From $\mathcal{L}_1$ adaptive control theory, the error $y_{\mathrm{ref},j}(s) - y_j(s)$ can be rendered arbitrarily small by increasing the adaptive gain Γ (see the bound in (6)). This implies that, for sufficiently high adaptation rates, $e_j(s) \approx e_{\mathrm{ref},j}(s)$, and thus it seems reasonable to assume that the ILC update law can be designed for the *closed-loop reference signal*. Then, the update law in (7) can be rewritten as

$$u_{ILC,j+1}(s) = Q(s)\left(u_{ILC,j}(s) + L(s)e_{\mathrm{ref},j}(s)\right) . \quad (11)$$

The transfer function from the feedforward ILC input, $u_{ILC,j}(s)$, to the output of the reference system, $y_{\mathrm{ref}}(s)$, is given by

$$P_S(s) = c^\top \left[\mathbb{I}_n - G(s)\theta^\top\right]^{-1} \left[s\mathbb{I}_n - A_m\right]^{-1} b\,\omega ,$$

where θ and ω are the uncertain parameters, and $G(s) \triangleq (s\mathbb{I}_n - A_m)^{-1} b\,(1 - C(s))$ as illustrated in Section II. Define a nominal plant model $G_m(s) \triangleq c^\top (s\mathbb{I}_n - A_m)^{-1} b$, and let $\omega = 1+\delta_\omega$. For low-pass filters $C(s)$ with sufficiently high bandwidth, $P_S(s)$ can be approximated by

$$\begin{aligned} P_S(s) &\approx c^\top \left[\mathbb{I}_n + G(s)\theta^\top\right] \left[s\mathbb{I}_n - A_m\right]^{-1} b\,(1+\delta_\omega) \\ &\approx c^\top \left[s\mathbb{I}_n - A_m\right]^{-1} b + c^\top \left[s\mathbb{I}_n - A_m\right]^{-1} b\delta_\omega \\ &\quad + c^\top G(s)\theta^\top \left[s\mathbb{I}_n - A_m\right]^{-1} b\,(1+\delta_\omega) \\ &= G_m(s)\,(1 - C(s))\,\theta^\top \left[s\mathbb{I}_n - A_m\right]^{-1} b\,(1+\delta_\omega) \\ &\quad + G_m(s) + G_m(s)\delta_\omega . \end{aligned}$$

Separating $G_m(s)$ from each term, the plant dynamics can be approximated by,

$$P_S(s) \approx G_m(s) \cdot W$$

where W is defined as

$$W \triangleq 1 + \delta_\omega + (1 - C(s))\,\theta^\top \left[s\mathbb{I}_n - A_m\right]^{-1} b\,(1+\delta_\omega) .$$

Note that $G_m(s)$ is the nominal plant about which the learning algorithm is to be designed. In the absence of any parametric uncertainty ($\theta = 0; \omega = 1$), the monotonic stability condition in the frequency domain for the learning law in (11) is

$$\left\| Q(s)\left(1 - L(s)G_m(s)\right)\right\|_\infty < 1 .$$

In the presence of parametric uncertainty, the stability condition is modified to (12).

$$\left\| Q(s)\left(1 - L(s)G_m(s)W\right)\right\|_\infty < 1 \quad (12)$$

The weight W is comprised of two uncertain terms and 1. The low-pass filter $C(s)$ can be used to negate the effect of the parametric uncertainty θ, i.e., by increasing the bandwidth of the filter $C(s)$ the effect of the uncertainty θ in (12) can be arbitrarily reduced. It is critical, however, to remember that this results in a reduction of the stability margins of the closed-loop adaptive system. The effect of the input-gain uncertainty δ_ω on the stability of the learning law can be mitigated by choosing a conservative (smaller) learning gain. This again results in a trade-off against slower convergence rates.

VI. Simulation Results

To demonstrate the effectiveness of the proposed ILC architecture on systems with large parametric uncertainty, we consider a simplified model of a motion control system. In particular, the plant is modeled as an *uncertain* mass-spring-damper system with dynamics

$$\begin{aligned} \dot{x}_p(t) &= A_p x_p(t) \;+\; b_p u(t) , \quad x_p(0) = x_0 , \\ y(t) &= c_p^\top x_p(t) , \end{aligned}$$

where the state vector is $x_p(t) = [x(t), \dot{x}(t)]^\top$, with $x(t)$ being the position of the mass, and

$$A_p = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix}, \quad b_p = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}, \quad c_p = \begin{bmatrix} 1 \\ 0 \end{bmatrix} .$$

The nominal values for the parameters m, k, and c are:

$$m_0 = 1 , \quad c_0 = 0.6 , \quad k_0 = 1 .$$

The control objective is to design an ILC architecture that ensures accurate position-trajectory tracking in the presence of uncertainty in the system parameters.

For this purpose, we consider the following adaptive state-feedback controller:

$$u_{\mathrm{fb}}(t) = u_\ell(t) + u_{\mathcal{L}_1}(t) ,$$

where $u_\ell(t)$ is a nonadaptive component of the form

$$u_\ell(t) = -k_\ell^\top x_p(t) ,$$

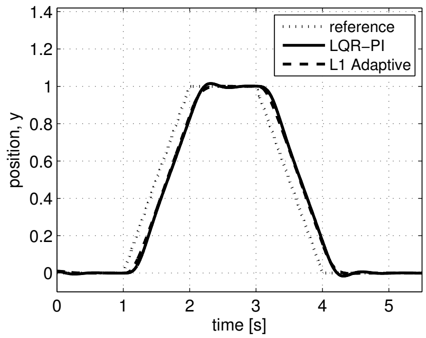


Fig. 4: Closed-loop response for the nominal plant with the two feedback controllers

while $u_{\mathcal{L}_1}(t)$ is the $\mathcal{L}_1$ adaptive control signal, generated as detailed in Section II. For the design of this feedback controller, we select:

$$k_\ell = \begin{bmatrix} 99.0 \\ 16.4 \end{bmatrix}, \quad A_m = \begin{bmatrix} 0.0 & 1.0 \\ -100.0 & -17.0 \end{bmatrix},$$
$$K_s = \begin{bmatrix} 0.0 & 0.0 \\ -201.0 & -2.6 \end{bmatrix}, \quad \Gamma = 50,000\,, \quad Y = \mathbb{I}_2\,,$$
$$k_{\mathcal{L}_1} = 40\,, \quad D(s) = \frac{1}{s\left(\frac{s}{50}+1\right)}\,, \quad k_g(s) = 100\frac{\frac{s}{40}+1}{\frac{s}{100}+1}\,,$$

which guarantees a satisfactory tracking performance for the nominal system with a time-delay margin of 50 msec.

To improve tracking performance, this feedback controller is then augmented with a *parallel* ILC control input of the form presented in (7). The learning controller consists of $L(s)$, a P-type frequency domain design with a gain of 100, and $Q(s)$, a low-pass Q-filter of the following form:

$$Q(s) = \frac{1}{\left(\frac{s}{30}+1\right)}\,.$$

To illustrate the benefits and disadvantages of using an $\mathcal{L}_1$ adaptive controller in the ILC architecture, we also consider a nonadaptive LQR-PI feedback controller:

$$\dot{x}_I(t) = r(t) - y(t)\,, \qquad x_I(0) = 0\,,$$
$$u_{\mathrm{PI}}(t) = k_I x_I(t) + k_P^\top x_p(t)\,.$$

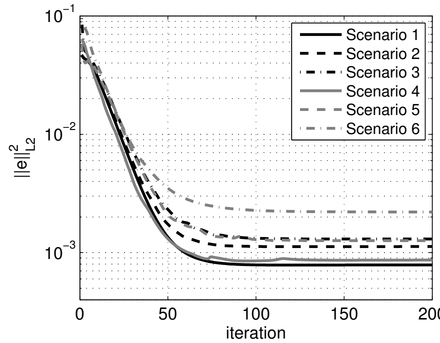
For the design of this LQR-PI controller, we choose

$$k_P^\top = \begin{bmatrix} -270.4 & -22.7 \end{bmatrix}, \quad k_I = 1581.1\,,$$

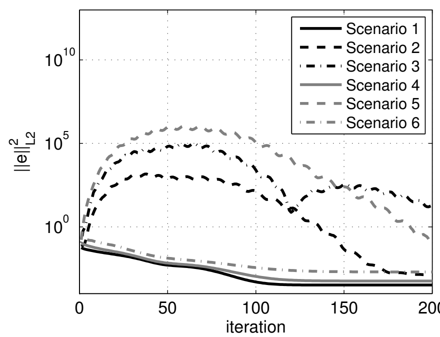
which provides a similar tracking performance for the nominal system as the $\mathcal{L}_1$ adaptive controller, and with the same time-delay margin of 50 msec. Figure 4 shows the closed-loop response for the nominal plant with the two feedback controllers to a trapezoidal position trajectory.

For comparison purposes, a series of design scenarios with varying system dynamics were implemented for both control architectures, the $\mathcal{L}_1$ adaptive and LQR-PI feedback controllers combined with the same ILC feedforward controller. The different scenarios were designed to evaluate the effect of parametric uncertainty on the performance and robustness of the combined controllers. The design scenarios presented in Figure 5 include the following:

- *Scenario 1:* $m = 1$, $c = 0.6$, $k = 1$.
- *Scenario 2:* $m = 3$, $c = 10$, $k = 1$.
- *Scenario 3:* $m = 2$, $c = -10$, $k = -100$.
- *Scenario 4:* $m = 0.5$, $c = 20$, $k = 100$.
- *Scenario 5:* $m = 3$, $c = -20$, $k = -50$.
- *Scenario 6:* $m = 2$, $c = 0$, $k = 350$.



(a) ILC architecture with the $\mathcal{L}_1$ adaptive controller



(b) ILC architecture with the LQR-PI controller

Fig. 5: Simulation results for different plant uncertainties

Figure 5a presents the converged Root Mean Squared (RMS) error signals for the ILC architecture with the $\mathcal{L}_1$ adaptive feedback controller. One can see that the combined ILC and $\mathcal{L}_1$ controller is able to maintain monotonic convergence in the presence of different parametric uncertainties. On the other hand, Figure 5b shows that the ILC with the LQR-PI controller can have poor performance in the iteration domain for some values of the uncertain parameters. The presence of such large transients is not acceptable in most physical systems.

In addition to iteration-invariant parametric uncertainty, it is important to observe the effects of iteration-varying parametric uncertainty that is introduced as a disturbance during the iterative learning process. Figure 6 shows the response of the two combined controllers to a sudden change in the mass, which increases by a factor of three with respect to the nominal value. As can be seen from the figure, the ILC architecture with the LQR-PI feedback controller responds to the sudden mass change with a large transient. Although the system eventually converges, the presence of large transients

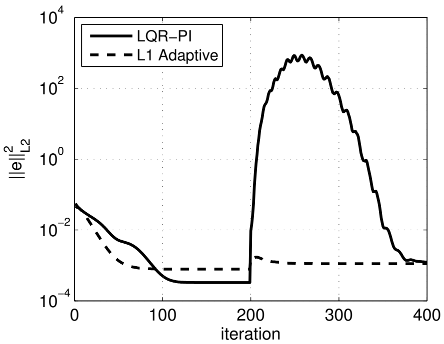


Fig. 6: Transient due to a sudden change in the system mass

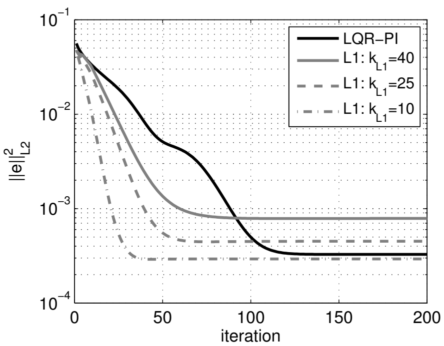


Fig. 7: Effect of the bandwidth of $C(s)$ on the closed-loop performance

may damage the physical system. On the other hand, the ILC scheme using an $\mathcal{L}_1$ feedback controller exhibits a small increase in convergence, but still maintains a converged signal well below the nominal feedback error. These results demonstrate the enhanced performance of the combined ILC and $\mathcal{L}_1$ adaptive controller in systems with large parametric uncertainty.

Lastly, Figure 7 illustrates the effect of the bandwidth of the $\mathcal{L}_1$ low-pass filter $C(s)$ on ILC convergence in the iteration domain. As discussed in Section II, the bandwidth and structure of the low-pass filter $C(s)$ defines the trade-off between performance and robustness in the time domain. In general, a larger filter bandwidth results in improved performance (higher ability to compensate for low-frequency system uncertainty and better tracking of the desired behavior $y_{\text{des}}(t)$), and a reduction in the stability margins of the closed-loop adaptive system. However, we notice that the objectives of the adaptive controller and the ILC law are different; in fact, the adaptive controller tries to track the output response of a given desired system $M(s)$, whereas ILC tries to reduce the reference tracking error. While the ILC law could be easily modified to reduce the tracking error with respect to the desired system output, this might not be acceptable in some applications. It would be the case, for example, of accurate position-trajectory tracking in motion control systems. In such applications, and as a consequence of the different control objectives, a larger bandwidth of the $\mathcal{L}_1$ low-pass filter can lead to larger converged ILC errors. This fact is illustrated in Figure 7, in which the bandwidth of the $\mathcal{L}_1$ filter is reduced by reducing the parameter $k_{\mathcal{L}_1}$. It is important to emphasize that this improvement in converged ILC errors *in the iteration domain* is achieved at the cost of reduced ability to compensate for low-frequency system uncertainty *in the time domain*. The design of the two controllers to minimize the possible interaction between them is an issue that needs further investigation.

VII. Conclusions

The paper proposed the combination of an $\mathcal{L}_1$ adaptive feedback controller with an ILC feedforward controller into a single framework. The $\mathcal{L}_1$ adaptive controller is designed to compensate for non-repetitive, low-frequency (parametric) uncertainty in the time domain, while the iterative learning controller compensates for repetitive system uncertainties in the iteration domain. The use of a feedback control law based on $\mathcal{L}_1$ adaptive control ensures that the transfer function from the feedforward ILC input to the plant output remains close to a nominal plant-sensitivity function, which facilitates the design of the learning algorithm. Simulation results from a simplified motion control system demonstrated the benefits of the proposed ILC architecture for systems with large parametric uncertainty.

Future work will focus on rigorous stability and performance analysis of the $\mathcal{L}_1$-ILC controller, and will explore the benefits of the proposed scheme for nonlinear uncertain systems.

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