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Robust Filtering for Networked Systems with Random Transmission Delays and Packet Dropouts

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Abstract — **This paper addresses a robust H_∞ filtering problem for networked systems that are subject to both random transmission delays and packet dropouts. To start with, a data transmission model is established by employing random series with Bernoulli distributions. A sufficient condition for robust stability with H_∞ constraints is derived for the filtering error system. The robust filter is designed in terms of the feasibility of a linear matrix inequality (LMI). The numerical examples are provided to show the effectiveness of the data transmission model and the proposed filtering method.**

I. Introduction

WITH the rapid advances in networking and communication technologies, networked systems are becoming ubiquitous across an increasing number of fields including industry, environment, economic, military, and so on. State estimation over networks plays a key role in applications such as remote sensing, space exploration, and sensor networks [1-3]. However, the use of a shared network in contrast to using several dedicated independent connections present some new challenges: the inevitable transmission delays such as network-induced delays, packet dropouts, and missing measurements. The transmission delays and dropouts are of random nature. In this case, the data in networked systems lose causality and are uncertain. Moreover, transmission delays and dropouts are two major factors leading to the degradation of system performance. Thus, to investigate the modeling and filter designing for networked systems is of great significance.

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Different aspects of networked filter design have been studied extensively in the literature. The modeling of the transmission delays and data dropouts has been investigated in a number of recent studies ([4-7]). In general, two common approaches are employed to develop stochastic models for time delay and data dropout. One is Markov chains. The other is binary random series. Some studies utilized Markov chains to model packet dropouts (see [4][6] and the references therein). The main problem of the Markov chain approach is to how to identify the number of states in a Markov chain and how to get the transient probability in the hidden Markov models. The binary random series approach has received much attention due to its practicality and simplicity. A common practice is to use Bernoulli distribution to model data transmission process in networked systems. Most related studies have considered the situation that only time delay or packet dropout is present.

There are some recent works that have considered the problem of state estimation in networked systems. References [8] and [9] formulated the multiple random packet dropouts by representing the dropout as a Bernoulli distributed parameter in the system model. Stochastic H_2 and H_∞ norms of the estimation error systems were defined and robust filters were proposed. Reference [10] studied optimal estimators, which included filter, predictor and smoother via an innovation analysis of a stochastic parameter uncertainty model. A recent reference [11] studied a type of linear-minimum-variance filter for packet dropouts in the case that packet dropouts are Bernoulli distributed. The aforementioned studies have focused on data dropout, without consideration of transmission delay. References [8][9] stated that transmission delays and packet dropouts could be treated within the framework proposed therein, though no explicit results were presented. Furthermore, reference [12] proposed a robust filtering scheme for nonlinear networked systems with delays and packet dropouts, but the communication delays and dropouts were incorporated in the system model, instead of in the measurement. Reference [13] introduced a weighted H_∞ performance index to investigate the filter design problem for networked systems with multiple sensors, which focused more on the dropouts and bounded the transmission delays.

In this paper a robust H_∞ filtering problem for networked systems that are subject to both random transmission delays

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and packet dropouts is considered. The main contributions of this paper are summarized as follows: 1) a new data transmission model in networked systems is proposed to take into account random delays and packet dropouts, via Bernoulli process; 2) stochastic analysis is conducted to enforce H_∞ performance for the proposed model; 3) the robust stability condition is derived with the filter parameters designed by LMI technique.

Notation: R^n denotes the n-dimensional Euclidean space and $R^{n\times m}$ the set of all $n\times m$ real matrices. A' denotes the transpose of matrix A. $X\le Y$ or $X<Y$, respectively, where X and Y are symmetric matrices, means that $Y-X$ is positive (semi-) definite. I is the identity matrix with a compatible dimension. In symmetric block matrices, $*$ is used as an ellipsis for terms induced by symmetry. $E\{\cdot\}$ stands for the mathematical expectation of $\{\cdot\}$. $\mathrm{var}\{\cdot\}$ stands for the variance of $\{\cdot\}$.

I. DATA TRANSMISSION MODEL

Figure 1 depicts the relationship between the plant and its filter in a networked environment. The plant is a discrete-time linear system subject to random disturbances and the sensor data are contaminated with noise. The aim of robust filtering is that designing the filter to minimize the H_∞-norm for the transfer function from the noise signals to the filtering errors.

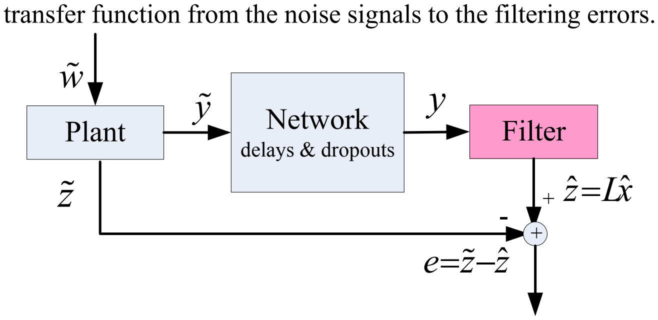

Fig 1. Plant and Filter in Networked Systems

The plant can be represented by the following equations:

$$\begin{cases}\tilde{x}_{k+1}=a_0\tilde{x}_k+b_0\tilde{w}_k\\ \tilde{y}_k=c_0\tilde{x}_k+d_0\tilde{w}_k\\ \tilde{z}_k=L_0\tilde{x}_k\end{cases}\tag{1}$$

where $\tilde{x}_k\in R^n$ is the state vector; $\tilde{y}_k\in R^m$ is the system output; $\tilde{z}_k\in R^n$ is the signal needed to be estimated; $y_k\in R^m$ is the result which $\tilde{y}_k$ is transmitted through network, i.e. the filter input; $\tilde{w}_k$ is system noise which include dynamic noise and measurement noise; a_0, b_0, c_0, d_0, L_0 are constant matrices with appropriate dimensions.

The system output $\tilde{y}_k$ passes through the network and there may be random delays and dropouts. We adopt two random series $\{\delta_k\}$ and $\{\gamma_k\}$ to describe delay and dropout happening situation, moreover assume that δ_k and γ_k are independent of each other, w_k, and the initial state values.

Remark 1: The stochastic series $\{\delta_k\}$ and $\{\gamma_k\}$, $k=0,1,2,\ldots$, consist of independent and identically distributed Bernoulli random variable, taking the values of 0 or 1 with probabilities:

$$\begin{cases}prob\{\gamma_k=1\}=E\{\gamma_k\}=\alpha, 0\le\alpha\le 1\\ prob\{\delta_k=1\}=E\{\delta_k\}=\beta, 0\le\beta\le 1\end{cases}\tag{2}$$

From the statistical view, the data transmission model in networked systems is established based Table I.

TABLE I
THE REALATIONSHIP BETWEEN
THE SYSTEM OUPUT $\tilde{y}_k$ AND THE FILTER INPUT y_k

Probabilities	Dropout ($1-\gamma_k$)	No dropout (γ_k)
Delay ($1-\delta_k$)	$y_k=(1-\gamma_k)y_{k-1}$	$y_k=\gamma_k(1-\delta_k)\tilde{y}_{k-1}$
No delay (δ_k)	$y_k=(1-\gamma_k)y_{k-1}$	$y_k=\gamma_k\delta_k\tilde{y}_k$

The dropout and the delay are modeled as follows: The probability of no dropout is α, i.e. $prob\{\gamma_k=1\}=\alpha$, and the probability of no delay is β, i.e. $prob\{\delta_k=1\}=\beta$. If there is no dropout and delay happened, $y_k=\gamma_k\delta_k\tilde{y}_k$, otherwise, if there is no dropout but delay happened, $y_k=\gamma_k(1-\delta_k)\tilde{y}_{k-1}$; if there is dropout happened, whether delay is happened or not, $y_k=(1-\gamma_k)y_{k-1}$.

According to this model, the current observation y_k is obtained from the system output $\tilde{y}_k$, so the estimator input is

$$y_k=\delta_k\gamma_k\tilde{y}_k+\gamma_k(1-\delta_k)\tilde{y}_{k-1}+(1-\gamma_k)y_{k-1}\tag{3}$$

Combining Equation (1) and (3), we get the system dynamics formulation with delays and dropouts as follows:

$$\begin{cases}\tilde{x}_{k+1}=a_0\tilde{x}_k+b_0\tilde{w}_k\\ \tilde{y}_k=c_0\tilde{x}_k+d_0\tilde{w}_k\\ y_k=\delta_k\gamma_k\tilde{y}_k+\gamma_k(1-\delta_k)\tilde{y}_{k-1}+(1-\gamma_k)y_{k-1}\\ \tilde{z}_k=L_0\tilde{x}_k\end{cases}\tag{4}$$

Remark 2: In order to get the compact formulations and design the filter, we will augment the system states twice.

First we define a new state vector

$$x_{k+1}=\begin{bmatrix}\tilde{x}_{k+1}\\ \tilde{x}_k\\ y_k\end{bmatrix}\tag{5}$$

and

$$w_k = \begin{bmatrix} \widetilde{w}_k \\ \widetilde{w}_{k-1} \end{bmatrix} \quad (6)$$

Then an augmented state-space model can be expressed as

$$\begin{cases} x_{k+1} = \bar{a}x_k + \bar{b}w_k \\ y_k = \bar{c}x_k + \bar{d}w_k \\ z_k = Lx_k \end{cases} \quad (7)$$

where $z_k = \tilde{z}_k$, $\bar{a},\bar{b}$, $\bar{c},\bar{d}$ are the augment system matrices which are random and are as follows:

$$\bar{a} = \begin{bmatrix} a_0 & 0 & 0 \\ I & 0 & 0 \\ \lambda_k\delta_k c_0 & \gamma_k(1-\delta_k)c_0 & (1-\gamma_k)I \end{bmatrix},$$

$$\bar{b} = \begin{bmatrix} b_0 & 0 \\ 0 & 0 \\ \gamma_k\delta_k d_0 & \gamma_k(1-\delta_k)d_0 \end{bmatrix},$$

$$\bar{c} = \begin{bmatrix} \gamma_k\delta_k c_0 & \gamma_k(1-\delta_k)c_0 & (1-\gamma_k)I \end{bmatrix},$$

$$\bar{d} = \begin{bmatrix} \gamma_k\delta_k d_0 & \gamma_k(1-\delta_k)d_0 \end{bmatrix},$$

$$L = \begin{bmatrix} L_0 & 0 & 0 \end{bmatrix}$$

Based on the system (7), the estimator for $\widetilde{x}_k$ by finding the estimation $\hat{x}_k$ should be designed, such that the H$_\infty$ -norm of the filtering error dynamics is minimized. The filer is as follows:

$$\begin{cases} \hat{x}_{k+1} = \hat{a}\hat{x}_k + \hat{b}y_k \\ \hat{z}_k = \hat{L}\hat{x}_k \end{cases} \quad (8)$$

where $\hat{a}$, $\hat{b}$, $\hat{L}$ are the filter parameters which need to be obtained. The filtering error is defined as $e_k = z_k - \hat{z}_k$.

Furthermore, define

$$\xi_k = \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix},$$

the augmented state-space model combing system (7) and filter (8) can be expressed as:

$$\Sigma := \begin{cases} \xi_{k+1} = A\xi_k + Bw_k \\ e_k = C\xi_k \end{cases} \quad (9)$$

where

$$A = \begin{bmatrix} \bar{a} & 0 \\ \hat{b}\bar{c} & \hat{a} \end{bmatrix}, B = \begin{bmatrix} \bar{b} \\ \hat{b}\bar{d} \end{bmatrix}, \quad (10)$$

$$C = \begin{bmatrix} L & -\hat{L} \end{bmatrix}$$

According to the definition of system's norm, the system's H$_\infty$ -norm is defined as in [14]:

$$\|\Sigma\|_\infty^2 = \frac{\sum_{k=0}^{\infty}\|e_k\|^2}{\sum_{k=0}^{\infty}\|w_k\|^2} \quad (11)$$

Consider the robust stability with H$_\infty$ constraints for system Σ .

Theorem 1: Given the system (1) and the filer (8), for $\gamma > 0$, the filtering error system (9) is asymptotically stable and satisfies $\|\Sigma\|_\infty^2 < \gamma$, if there exist positive matrices P , such that

$$\begin{bmatrix} E\{A'PA + C'C\} - P & E\{A'PB\} \\ * & E\{B'PB\} - \gamma^2 I \end{bmatrix} < 0 \quad (12)$$

Proof: Choose the Lyapnuov candidate function $V_k = E\{\xi_k' P\xi_k\}$.

When $w_k = 0$,

$$\Delta V_k = V_{k+1} - V_k = \xi'(E\{A'PA\} - P)\xi_k$$

Because $C'C \geq 0$, we get

$E\{A'PA\} - P < 0$.

Thus the filtering error system is asymptotically stable.

When $w_k \neq 0$,

$$\Delta V_k = \xi_k'(E\{A'PA\} - P)\xi_k + 2\xi_k' E\{A'PB\}w_k + w_k' B' PBw_k$$

$$= \begin{bmatrix} \xi_k' & w_k' \end{bmatrix} \begin{bmatrix} E\{A'PA\} - P & E\{A'PB\} \\ * & E\{B'PB\} \end{bmatrix} \begin{bmatrix} \xi_k \\ w_k \end{bmatrix}$$

Let

$$\Delta V_k = \Delta V_k + E\{\gamma^2(\|w_k\|^2 - \|w_k\|^2) + \|\hat{z}_k\|^2 - \|\hat{z}_k\|^2\},$$

and we get

$$\Delta V_k = \begin{bmatrix} \xi_k' & w_k' \end{bmatrix} \Phi \begin{bmatrix} \xi_k \\ w_k \end{bmatrix} + \gamma^2\|w_k\|^2 - \|\hat{z}_k\|^2$$

where Φ equals the left side of (12) , i.e. $\Phi < 0$.

$\Delta V_k - \gamma^2(\|w_k\|^2 - \|\hat{z}_k\|^2) < 0$ i.e.

$$\|\hat{z}_k\|^2 < \gamma^2\|w_k\|^2 - \Delta V_k \quad (13)$$

To sum up both sides of (13) for $k = 0, \ldots, \infty$,

$$\sum_{k=0}^{\infty}\|\hat{z}_k\|^2 < \gamma^2\sum_{k=0}^{\infty}\|w_k\|^2 + V_0 - V_\infty \quad (14)$$

Considering zero initial conditions, we can conclude that

$$\sum_{k=0}^{\infty}\|\hat{z}_k\|^2 < \gamma^2\sum_{k=0}^{\infty}\|w_k\|^2 \quad (15)$$

The proof is completed.

II. FILTER DESIGN

This section is devoted to the design of robust H$_\infty$ filter parameters, $\hat{a}$, $\hat{b}$ and $\hat{L}$ according to the result in Theorem 1.

The robust filtering problem can be stated as follows: Design a filter as in (8), such that the filtering error dynamics in (9) is asymptotically stable and the H_∞ criterion in (11) is satisfied.

Consider the filtering error dynamics defined in (9). By using Theorem 1, the H_∞ filtering problem can be formulated as

$$\min_{\hat{a},\hat{b},\hat{L},P} \gamma$$

$$s.t. \quad E\left\{\begin{bmatrix} A'PA+C'C & A'PB \\ * & B'PB-\gamma^2 I \end{bmatrix}\right\}<0 \qquad (16)$$

In order to deal with the random system parameters, we introduce two random series $\{\lambda_{\gamma k}\}$, $\{\lambda_{\delta k}\}$, $k=1,2...$, s.t.

$$\gamma_k = \alpha + \lambda_{\gamma k}$$

$$\delta_k = \beta + \lambda_{\delta k}$$

where,

$$E\{\lambda_{\gamma k}\}=0, \operatorname{var}\{\lambda_{\gamma k}\}=q_1^2$$

$$E\{\lambda_{\delta k}\}=0, \operatorname{var}\{\lambda_{\delta k}\}=q_2^2$$

For the first augment system (7), we obtain

$$\bar{a} = a_{00} + \lambda_{\gamma k} a_1 + \lambda_{\delta k} a_2 + \lambda_{\gamma k}\lambda_{\delta k} a_{12},$$

a_{00}, a_1, a_2 and a_{12} are constant matrices as follows:

$$a_{00} = \begin{bmatrix} a_0 & 0 & 0 \\ I & 0 & 0 \\ \alpha\beta c_0 & \alpha(1-\beta)c_0 & (1-\alpha)I \end{bmatrix},$$

$$a_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \beta c_0 & (1-\beta)c_0 & -I \end{bmatrix},$$

$$a_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \alpha c_0 & -\alpha c_0 & 0 \end{bmatrix}, a_{12} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ c_0 & -c_0 & 0 \end{bmatrix}$$

Define $a_{q1} = q_1 a_1$, $a_{q2} = q_2 a_2$, and $a_{q12} = q_1 q_2 a_{12}$, we get

$$E\{\bar{a}'\bar{a}\} = a_{00}' a_{00} + a_{q1}' a_{q1} + a_{q2}' a_{q2} + a_{q12}' a_{q12};$$

Similarly, we can get

$$E\{\bar{b}'\bar{b}\} = b_{00}' b_{00} + b_{q1}' b_{q1} + b_{q2}' b_{q2} + b_{q12}' b_{q12}$$

$$E\{\bar{c}'\bar{c}\} = c_{00}' c_{00} + c_{q1}' c_{q1} + c_{q2}' c_{q2} + c_{q12}' c_{q12}$$

$$E\{\bar{d}'\bar{d}\} = d_{00}' d_{00} + d_{q1}' d_{q1} + d_{q2}' d_{q2} + d_{q12}' d_{q12}$$

where, b_{00}, b_1, b_2, b_{12}, c_{00}, c_1, c_2, c_{12} and d_{00}, d_1, d_2, d_{12} can be easily computed based system parameters.

For the filtering error dynamics (9), we get

$$A = A_0 + \lambda_{\gamma k} A_1 + \lambda_{\delta k} A_2 + \lambda_{\gamma k}\lambda_{\delta k} A_{12}$$

A_0, A_1, A_2 and A_{12} are constant matrices and can be computed via the similar way with a_{00}, a_1, a_2 and a_{12}, so we omit them here.

Define $A_{q1} = q_1 A_1$, $A_{q2} = q_2 A_2$, $A_{q12} = q_1 q_2 A_{12}$, then for *A* we have

$$E\{A'A\} = A_0' A_0 + A_{q1}' A_{q1} + A_{q2}' A_{q2} + A_{q12}' A_{q12}$$

Using using similar approaches, we have

$$E\{B'B\} = B_0' B_0 + B_{q1}' B_{q1} + B_{q2}' B_{q2} + B_{q12}' B_{q12}$$

$$E\{C'C\} = C_0' C_0 + C_{q1}' C_{q1} + C_{q2}' C_{q2} + C_{q12}' C_{q12}$$

where, B_0, B_{q1}, B_{q2}, B_{q12} C_0, C_{q1}, C_{q2}, C_{q12} can be obtained by the similar way with $A_0, A_{q1}, A_{q2}, A_{q12}$.

Let

$$\Psi = \begin{bmatrix} A_0' & A_{q1}' & A_{q2}' & A_{q12}' & C' \\ B_0' & B_{q1}' & B_{q2}' & B_{q12}' & 0 \end{bmatrix},$$

$$\Pi = diag\left(-P^{-1}, -P^{-1}, -P^{-1}, -P^{-1}, -I\right),$$

$$\Gamma = diag(-P, -\gamma^2 I)$$

According to Schur complement:

$$\Gamma - \Psi'\Pi^{-1}\Psi < 0$$

is equivalent to

$$\begin{bmatrix} \Pi & \Psi \\ \Psi' & \Gamma \end{bmatrix} < 0 \qquad (17)$$

Remark 3: The above inequality is not an LMI style. In order to design the filter parameter, we should depart them from the compound matrix A_i, B_i and $C_i, i=1,2,12$, then transform (17) into LMI.

As P^{-1} exists, we put $Q = P^{-1}$, separate *P* and *Q* as

$$P = \begin{bmatrix} X & U \\ U' & X_2 \end{bmatrix}, Q = \begin{bmatrix} Y & V \\ V' & Y_2 \end{bmatrix}$$

where *X,Y* are $(m+n)\times(m+n)$ and X_2, Y_2 are $(n*n)$ symmetric and positive definite matrices. Define the matrix

$$T = \begin{bmatrix} Z & Y \\ 0 & V' \end{bmatrix}$$

Applying the congruence transformation twice on the (17), the first transformation matrix is $diag(I,I,I,I,I,Q,I)$, the second is $diag(T,T,T,T,I,T,I)$. For descriptions convenience, define,

$$\Theta_{11}=\begin{bmatrix} -T'QT & 0 & 0 & 0 & 0 \\ * & -T'QT & 0 & 0 & 0 \\ * & * & -T'QT & 0 & 0 \\ * & * & * & -T'QT & 0 \\ * & * & * & * & -I \end{bmatrix},$$

$$\Theta_{12}=\begin{bmatrix} T'A_0QT & T'B_0 \\ T'A_{q1}QT & T'B_{q1} \\ T'A_{q2}QT & T'B_{q2} \\ T'A_{q12}QT & T'B_{q12} \\ CQT & 0 \end{bmatrix},$$

$$\Theta_{22}=\begin{bmatrix} -T'QT & 0 \\ * & -\gamma^2 I \end{bmatrix}$$

Then we get the following LMI:

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} \\ * & \Theta_{22} \end{bmatrix}<0 \qquad (18)$$

Let $F=V\hat{b}, J=V\hat{a}U'Z$ and $G=\hat{L}U'Z$, we get:

$$T'QT=\begin{bmatrix} Z & Z \\ Z & Y \end{bmatrix},$$

$$T'A_0QT=\begin{bmatrix} Za_{00} & Za_{00} \\ Ya_{00}+Fc_{00}+J & Ya_{00}+Fc_{00} \end{bmatrix},$$

$$T'A_{qi}QT=\begin{bmatrix} Za_{qi} & Za_{qi} \\ Ya_{qi}+Fc_{qi} & Ya_{qi}+Fc_{qi} \end{bmatrix},$$

$$T'B_0=\begin{bmatrix} Zb_{00} \\ Yb_{00}+Fd_{00} \end{bmatrix},$$

$$T'B_{qi}=\begin{bmatrix} Zb_{qi} \\ Yb_{qi} \end{bmatrix}, CQT=\begin{bmatrix} L-G & L \end{bmatrix}$$

where i=1,2,12. Thus, we can get the robust filtering design theorem in networked environments with random time delay and packet dropout as follow:

Theorem 2: The H_∞ filter design for system (1) is equivalent to the following convex programming problem:

$$\min_{\hat{a},\hat{b},\hat{L},P} \gamma \quad \text{s.t. (16)}$$

and the filter parameters are given by

$$\hat{a}=(V')^{-1}J(Z'U)^{-1}, \hat{b}=(V')^{-1}F, \qquad (19)$$
$$\hat{L}=G*(Z'U)^{-1}$$

Remark 4: In order to find the filter parameters, $\hat{a},\hat{b},\hat{L}$, we need to know two matrices U and V, which do not appear in the LMI. Although the nonsingular matrices U and V can be found from the fact $PW=I$, they are not square matrices. We give V and use generalized inverse matrix, then get $U=(Z^{-1}Y-I)V(V'V)^{-1}$.

III. SIMULATION

In this section, we aim to demonstrate the effectiveness and applicability of the proposed method.

Simulation 1: Considering a discrete -time LTI system represented by (1) with the following coefficients:

$$a_0=\begin{bmatrix} 1.7240 & -0.7788 \\ 1 & 0 \end{bmatrix}; b_0=\begin{bmatrix} 0.5 \\ 1 \end{bmatrix};$$

$$c_0=\begin{bmatrix} -0.0286 & 0.0264 \end{bmatrix}; d_0=1; L_0=0.01I_2$$

Given the dropout rate and delays probability are 0.8 and 0.2, we come up with the design of a robust filer with the H_∞ norm of the filtering error system less than 1. Set the initial condition

$x_0=\begin{bmatrix} 0 & 0 \end{bmatrix}'$, $\tilde{z}_0=\begin{bmatrix} 0 & 0 \end{bmatrix}'$ and $\hat{x}_0=\begin{bmatrix} -1 & -1 \end{bmatrix}'$, $\hat{z}_0=\begin{bmatrix} -0.1 & -1 \end{bmatrix}'$,

and the measurement noise is normally distributed white noise.

According to the theorem 2 and by means of the Matlab LMI toolbox, we solve the convex optimization problem (15) with parameters given by (19) and yield:

$$\hat{a}=\begin{bmatrix} -0.1773 & 0.0067 \\ 0.0931 & 0.0012 \end{bmatrix}; \hat{b}=\begin{bmatrix} -0.0346 \\ -0.6079 \end{bmatrix};$$

$$\hat{L}=\begin{bmatrix} 0.0298 & -0.0078 \\ 0.0615 & 0.0156 \end{bmatrix}$$

The H_∞ norm of the filtering error system is 0.2067 and the minimal value of γ is computed to be 0.1938. Fig.2 shows the simulation results of the proposed method.

Remark 6: In order to model practical situation, we generate random numbers to simulate time delays and dropouts in the above program. Based on our transmission model, α is 0.2, β is 0.2, therefore the minimal value of γ is 0.0937.

Simulation 2: We assume there is no disorder of data transition and the filter will adopt the newest data when there are more than one data arriving. We further study the special case, i.e. the transmission delay is longer than one sample interval and less than three sample intervals.

According to the data transmission model given by section II, we get the relationship between system outputs and filter input:

$$y_k=(1-\gamma_k)y_{k-1}+\gamma_k\delta_k\tilde{y}_{k-2}+\gamma_k(1-\delta_k)\tilde{y}_{k-3}$$

Remark 5: When the transmission delay is longer than one sample interval and less than three sample intervals, there will be three possibilities at time k:1). There is no new data arrive, and the filter input is $y_k = y_{k-1}$, with the probability $1-\gamma_k$; 2). There is one data arrive, and the arriving data is $\tilde{y}_{k-3}$, so the filter input is $y_k = \tilde{y}_{k-3}$, with the probability $\gamma_k(1-\delta_k)$; 3). There are some data arrive, and the arriving data is $\tilde{y}_{k-2}$ or two data arrive. In such case, $y_k = \tilde{y}_{k-2}$, with the probability $\gamma_k \delta_k$.

Let the system's parameters are as follows:

$$a_0 = \begin{bmatrix} -0.5 & -0.24 \\ 0.12 & -0.5 \end{bmatrix}; \mathrm{b}_0 = \begin{bmatrix} 0.4 \\ 0.2 \end{bmatrix};$$

$$c_0 = [-0.2 \quad -0.4]; d_0 = 1; L_0 = 0.01 I_2$$

Given the dropout rate and delays probability are 0.8 and 0.2 and using the similar method with section III, we get the robust filter for the system with transmission delay is longer than one sample interval and less than three sample intervals as follows:

$$\hat{a} = \begin{bmatrix} -0.0062 & 0.0993 \\ -0.0013 & -0.3386 \end{bmatrix}; \hat{b} = \begin{bmatrix} -0.0599 \\ 0.0002 \end{bmatrix}$$

$$\hat{L} = \begin{bmatrix} 0.0030 & 0.7845 \\ 0.0081 & 3.5278 \end{bmatrix}$$

The minimal γ is 0.4262. Fig.3. shows the simulation results of the proposed method.

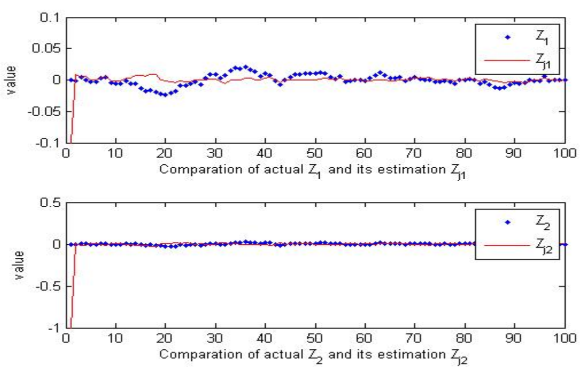

Fig 2. Actual (blue star line) and estimate state (red real line)for robust filtering with α=0.2, β=0.8

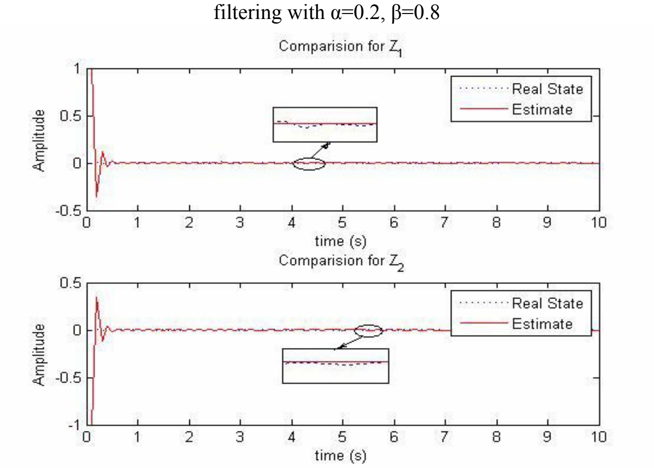

Fig. 3 Actual (blue dot line) and estimate state (red real line)for robust filtering with α=0.2, β=0.8

IV. CONCLUSIONS

In this paper, the problem of data transmission model and robust filtering for networked systems with random delays and packet dropouts has been studied. A data transmission model is established by employing random series with Bernoulli distributions. Stochastic analysis is conducted to enforce H_∞ performance for the proposed model. The robust filtering problem is cast as a convex optimal problem and is solved by the LMI technique. Simulation examples show the applicability and effectiveness of the proposed model and filter design.

REFERENCES

[1] J. Nilsson, B.Bernhardsson, B. Wittenmark, "Stochastic analysis and control of real-time systems with random time delays", Automatica, vol. 34, no.1, pp. 57–64, 1998

[2] T.C. Yang, "Networked control system: A brief survey", IEE Proc. Control Theory Appl, vol. 153, no.4, pp. 403-41, 2006

[3] M. Teddy Cheng, V. Malyavej, V.Andrey Savkin, "Decentralized robust set-valued state estimation in networked multiple sensor systems", Computers & Mathematics with Applications, vol. 59, no.8, pp. 2636-2646, 2010

[4] W. Zhang, L. Yu, H.B. Song, "H_∞ filtering of networked discrete-time systems with random packet losses", Information Sciences, vol.179, no.22, pp. 3944-3955, 2009

[5] Y. Liang, T.Chen, Q.Pan, "Multi-rate stochastic H_∞ filtering for networked multi-sensor fusion", Automatica, vol. 46, no.2, pp. 437-444, 2010

[6] J. P. Hespanha, P. Naghshtabrizi, Y. Xu, "A survey of recent results in networked control systems", Proc. IEEE, Special Issue on Networked Control Systems, vol.95,no.1,pp. 138–162,2007

[7] Z. Wang, F. Yang, D.W.C. Ho, X. Liu, "Robust H_∞ filtering for stochastic time-delay systems with missing measurements", IEEE Trans. Signal Processing, vol.54,no.7,pp. 2579-2587,2006

[8] M. Sahebsara, T.Chen, S.L.Shah, "Optimal H_2 filtering with multiple packet dropout", IEEE Trans. Automatic Control, vol 52, no.8, pp. 1508 -1513, 2007

[9] M. Sahebsara, T.Chen, S.L.Shah, "Optimal H_∞ filtering in networked control systems with multiple packet dropouts", System and Control Letters, vol. 40,no.8,pp.1361-1373,2008

[10] S. Sun, L. Xie, W. Xiao, Y. C. Soh, "Optimal linear estimation for systems with multiple packet dropouts", Automatica, vol. 44, no.5, pp.1333 –1342, 2008

[11] Y. Liang, T.Chen, Q.Pan, "Optimal Linear State Estimator With Multiple Packet Dropouts", IEEE Transactions on Automatic Control, vol. 55,no.6,pp.1428 – 1433, 2010

[12] H.L.Dong, Z. D.Wang, H.J. Gao, "Robust H_∞ filtering of nonlinear networked systems with multiple stochastic communication delays", 7th Asian Control Conference, Hong Kong, 2009, pp. 203 – 208

[13] H. Zhang,Y. Shi, A.S. Mehr, "Robust Weighted H_∞Filtering for Networked Systems with Intermittent Measurements of Multiple Sensors," International Journal of Adaptive Control and Signal Processing, 8, 2010, Available: http://onlinelibrary.wiley.com/doi/10.1002/acs.1200/

[14] J. H.Lee, S.W.Kim, W. H.Kwon, "Memoryless H_∞ controllers for state delayed system", IEEE Trans. on Automat Contr, vol.39, no.1,pp.159 -162, 1994