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A Nonlinear Optimal Human Postural Regulator

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***Abstract*—A postural control model for a quietly standing human is proposed. The musculoskeletal dynamics of the human is modeled as a triple inverted pendulum in the sagittal plan, including ankle, knee and hip joints. A nonlinear optimal control problem is defined to study the postural sway. It has a performance measure with quadratic terms in the controls and a quartic term in either the center of pressure (COP) or the horizontal projection of the center of mass (COM). This objective function provides a trade-off between the allowed deviations from the nominal value and the neuromuscular energy required to correct for these deviations. By using the Model Predictive Control (MPC) technique, the discrete-time approximation to each of these problems is converted into a nonlinear programming problem and then solved. The solution gives a control scheme that demonstrates qualitative agreement to the main features of the joint kinematics and coordination observed experimentally.**

I. Introduction

CONTROL theory has been used for many years to study human motor behavior and provide insight into its neurophysiological mechanisms. A properly designed biomechanics model and its control implementation can help us better understand how humans control their muscles to generate movements and how their joints interact during motor activity.

The regulation of quiet standing posture is one of the most fundamental human motor control tasks. Bipedal upright stance is inherently unstable without a balance control scheme [1]. One of the simplest and most widely studied posture regulation problems is to maintain a stable upright posture despite random perturbations of the base of support (BOS). There are two important observations from such clinical experiments [7]. The first is that there is much more sway (small amplitude movements in the anterior-posterior direction) than would occur with a linear feedback control without delay. A second notable feature of the human postural control is that the response to perturbations varies with their amplitude. Small disturbances produce motion only at the ankles with the hip and knee angles largely fixed. Larger perturbations evoke ankle and hip angular movement with no knee angular movement. Still larger perturbations result in movement of all three joint angles. In all of these experiments the feet are kept motionless. This biological behavior is likely to be optimal with respect to some performance measure that involves energy. Inspired by these features, we propose a biomechanical optimal control model resembling human balance control.

The proposed model consists of three main components: body dynamics, sensory estimator, and an optimal control scheme. In our earlier work [10][11][12], the human body was modeled as either a single or double inverted pendulum in the sagittal plane and controlled by ankle torque only or ankle plus hip torques. A series of nonlinear optimal control problems were devised as mathematical models of human postural control during quiet standing. Several performance criteria that are quartic in the body states or functions of those states, such as the Center of Pressure (COP), and quadratic in the controls were utilized. In this work, we address the question of sway by improving the biomechanical dynamics into a triple inverted pendulum and using this model to study the coordinated control of ankle, knee and hip at different perturbation levels.

The human body has the sensors necessary to provide the central nervous system (CNS) with measurements of both the COP and the center of mass (COM). Both the COP and the COM are good indicators of stability. We hypothesize that the human is (unconsciously) trying to keep COP or COM in the sagittal plane (x coordinate) close to its nominal location at the center of the foot. Thus, we propose a performance criterion that is quartic in the COP or COM[12].

$$J = \frac{1}{2}\int_0^\infty [ql_x^4(t) + \sum_{j=1}^{3} r_j u_j^2(t)]dt \tag{1}$$

where q and r_j are cost coefficients, and l_x is deviation from the nominal equilibrium values of either the COP or COM. The u_j are the control torques at each joint. Note that this performance measure reduces the actuator energy used by penalizing small postural errors very lightly. The crucial test for this hypothesis is whether it replicates the experimentally observed change in the response to the size of the perturbations. That is, does our optimal control produce only ankle angle movement for small disturbances and coordinated movement for larger perturbations?

The paper is organized as follows: after this brief introduction, the mathematical model of the human and the precise formulation of the performance measure are given. The resulting optimal control problem is then solved nu-

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merically by the Model Predictive Control (MPC) technique. That is followed by a series of simulations with different amounts of noise perturbing the system. The simulation results show more three-joint collaborative movement for large perturbations and more solely ankle movement for small perturbations. This is encouraging evidence in favor of our conjecture.

II. NEUROMUSCULOSKELETAL MODEL

In this work, we present a computational model of a quietly standing human body which uses four rigid and connected segments to represent the foot, shank, thigh, and torso as depicted in Fig 1.

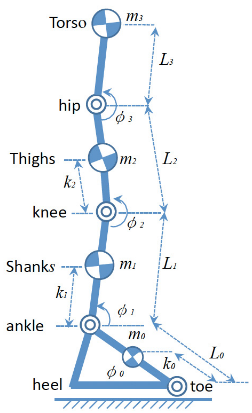

Figure 1. The model is composed of four rigid links. The term k_i for $i = 0, 1, 2, 3$ is the distance from the bottom of link i to the center of mass of link i. The term L_i is the length of link i. Link 0, the foot, is stationary

We first derived the equations of motion using the Euler-Lagrange method for this three joint, four segment model controlled by torques on the ankle, knee and hip joints. The complete expressions for the body dynamics with ankle, knee and hip torque are:

$$\begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{21} & Q_{22} & Q_{23} \\ Q_{31} & Q_{32} & Q_{33} \end{bmatrix} \begin{bmatrix} \ddot{\phi}_1 \\ \ddot{\phi}_2 \\ \ddot{\phi}_3 \end{bmatrix} + \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} = \begin{bmatrix} u_a \\ u_k \\ u_h \end{bmatrix} \quad (2)$$

We can also write the expression for u_t and for f_v as

$$u_t = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}_0}\right) - \frac{\partial \mathcal{L}}{\partial \phi_0}$$

$$f_v = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{h}}\right) - \frac{\partial \mathcal{L}}{\partial h}$$

where the ϕ_i are defined in Fig 1, u_a is ankle torque, u_k is knee torque, u_h is hip torque, u_t is toe torque, and f_v is vertical component of the ground reaction force. The Q_{ij} and R_i are defined in the following page.

Experimental data during quiet standing has consisted primarily of force platform measurements of the *COP*, estimated from the weighted average of pressure distributed over the surface of the area in contact with the ground. The *COP* is precisely defined by u_t and f_v. One can replace u_t by applying the force f_v at the distance from the toe that creates u_t. That distance is the COP. Mathematically, denoting the position of the COP by

$$l_{cop} = \frac{u_t}{f_v} \quad (3)$$

The foot does not move. Hence, ϕ_0 is a fixed constant, $\dot{\phi}_0 = \ddot{\phi}_0 = 0$ and $h = \dot{h} = \ddot{h} = 0$. These constraints are then substitued into the equations for u_t and f_v. The resutls are then used in Eqn (3) to compute l_{cop}.

It is generally good practice to work with dimensionless quantities in the mathematical models. We introduce the quantities $t = \tau/\beta$ and the normalization factor $\beta = \sqrt{L/g}$, which has dimension $[\beta] = T$ (time). Given $\frac{d\tau}{dt} = \beta$ then $\phi_i(\tau) = \phi_i(\beta t)$, for $i = 0, 1, 2, 3$, (for simplicity, we use ϕ_i as the normalized variable in the rest of the paper). Define M to be the total body mass and L to be the height of the upright body. Then, each segment is proportional to these two quantities. We have used typical numerical values for simplicity as shown in Table I. In reality, the fractions would have to be measured or estimated for a specific individual.

Table I
BODY SEGMENT LENGTH AND WEIGHT EXPRESSED AS A FRACTION OF THE ENTIRE BODY

Foot	*Shank*	*Thigh*	*Torso*
$m_0 = \frac{2}{35}M$	$m_1 = \frac{6}{35}M$	$m_2 = \frac{9}{35}M$	$m_3 = \frac{18}{35}M$
$k_0 = 0.094L$	$k_1 = 0.464L$	$k_2 = 0.45L$	$L_3 = 0.42L$
$L_0 = 0.117L$	$L_1 = 0.28L$	$L_2 = 0.3L$	

The next step is to linearize the problem about the nominal vertical posture. This is reasonable because the perturbations of the upright posture that are being considered are small. Thus, we linearize the multi-segment inverted pendulum model around the unstable equilibrium point $\underline{p}^*$ and also define the small angular deviations from the vertical equilibrium $\underline{p} = \underline{p}^* + \Delta\underline{p}$:

$$\underline{p} = \begin{pmatrix} \phi_1^* \\ \phi_2^* \\ \phi_3^* \\ \dot{\phi}_1^* \\ \dot{\phi}_2^* \\ \dot{\phi}_2^* \\ u_a^* \\ u_k^* \\ u_h^* \end{pmatrix} + \begin{pmatrix} \Delta\phi_1 \\ \Delta\phi_2 \\ \Delta\phi_3 \\ \Delta\dot{\phi}_1 \\ \Delta\dot{\phi}_2 \\ \Delta\dot{\phi}_3 \\ \Delta u_a \\ \Delta u_k \\ \Delta u_h \end{pmatrix}$$

where $\phi_1^* = \frac{\pi}{2}$, $\phi_2^* = \pi$, $\phi_3^* = \pi$ and all the other nominal values are zero. We define the state space variables $\underline{z}$ and convert the dynamic equations into:

$$\dot{\underline{z}} = A\underline{z} + B\underline{u} \quad (4)$$

$$Q_{11} = I_1 + m_1k_1^2 + m_2L_1^2 + m_2L_2^2 + m_3L_1^2 + m_3L_2^2 + m_3L_3^2 + 2m_2L_1k_2 + 2m_3L_1L_2 + 2m_3L_1L_3 + 2m_3L_2L_3$$
$$Q_{12} = m_2k_2^2 + m_2L_1k_2 + m_3L_2^2 + m_3L_3^2 + m_3L_1L_2 + m_3L_1L_3 + 2m_3L_2L_3$$
$$Q_{13} = m_3L_3^2 + m_3L_1L_3 + m_3L_2L_3$$
$$Q_{21} = m_2k_2^2 + m_2L_1k_2 + m_3L_2^2 + m_3L_3^2 + m_3L_1L_2 + m_3L_1L_3 + 2m_3L_2L_3$$
$$Q_{22} = I_2 + m_2k_2^2 + m_3L_2^2 + m_3L_3^2 + 2m_3L_2L_3$$
$$Q_{23} = m_3L_3^2 + m_3L_2L_3$$
$$Q_{31} = m_3L_3^2 + m_3L_1L_3 + m_3L_2L_3$$
$$Q_{32} = m_3L_3^2 + m_3L_2L_3$$
$$Q_{33} = I_3 + m_3L_3^2$$
$$R_1 = (m_1k_1 + m_2L_1 + m_3L_1)(cos\phi_0 cos\phi_1 + sin\phi_0 sin\phi_1)g - (m_2k_2 + m_3L_2)[cos\phi_0 cos(\phi_2 + \phi_1) + sin\phi_1 sin(\phi_2 + \phi_1)]g + m_3L_3[cos\phi_0 cos(\phi_3 + \phi_2 + \phi_1) + sin\phi_1 sin(\phi_3 + \phi_2 + \phi_1)]g$$
$$R_2 = -(m_2k_2 + m_3L_2)cos(\phi_2 + \phi_1 - \phi_0)g + m_3L_3cos(\phi_3 + \phi_2 + \phi_1 - \phi_0)g$$
$$R_3 = m_3L_3cos(\phi_3 + \phi_2 + \phi_1 - \phi_0)g$$
$$u_t = [-(m_1k_1^2 + m_2L_1^2 + m_2k_2^2 + m_3L_1^2 + m_3L_2^2 + m_3L_3^2) + 2m_2L_1k_2cos\phi_2 + (m_1L_0k_1 + m_2L_0L_1)cos\phi_0 cos(\phi_1 - \phi_0) - m_2L_0k_2cos\phi_0 cos(\phi_2 + \phi_1 - \phi_0)]\ddot{\phi}_1 + [-(m_2k_2^2 + m_3L_2^2 + m_3L_3^2) + m_2L_1k_2cos\phi_2 - m_2L_0k_2cos\phi_0 cos(\phi_2 + \phi_1 - \phi_0)]\ddot{\phi}_2 - m_3L_3^2\ddot{\phi}_3$$
$$f_v = (m_1k_1 + m_2L_1 + m_3L_1)cos(\phi_1 - \phi_0)\ddot{\phi}_1 - (m_2k_2 + m_2L_2)cos(\phi_2 + \phi_1 - \phi_0)(\ddot{\phi}_2 + \ddot{\phi}_1) + m_3L3cos(\phi_3 + \phi_2 + \phi_1 - \phi_0)(\ddot{\phi}_3 + \ddot{\phi}_2 + \ddot{\phi}_1) + Mg$$

$$\underline{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \end{bmatrix} = \begin{bmatrix} \Delta\phi_1 \\ \Delta\phi_2 \\ \Delta\phi_3 \\ \frac{d\Delta\phi_1}{dt} \\ \frac{d\Delta\phi_2}{dt} \\ \frac{d\Delta\phi_3}{dt} \end{bmatrix}, \underline{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \Delta u_a \\ \Delta u_k \\ \Delta u_h \end{bmatrix}$$

Eqn (4) has a completely dimensionless format:

$$A = \begin{bmatrix} 0_{3\times3} & I_{3\times3} \\ \beta^2(\frac{\mathbf{Q}}{ML^2})^{-1}\frac{\mathbf{R}}{ML^2} & 0_{3\times3} \end{bmatrix}$$

$$B = \begin{bmatrix} 0_{3\times3} \\ \frac{\beta^2}{ML^2}(\frac{\mathbf{Q}}{ML^2})^{-1} \end{bmatrix}$$

The center of pressure is a function of all states and controls, $l_{cop}(\underline{p}) = f(\phi_1, \phi_2, \phi_3, \dot{\phi}_1, \dot{\phi}_2, \dot{\phi}_3, u_a, u_k, u_h)$. For small perturbations, such as those with which we are concerned, we can linearize this about the nominal $\underline{p}^*$. The linear approximation $\triangle l_{cop}$ is then defined by:

$$\triangle l_{cop}(\underline{p}) = \nabla l_{cop}(\underline{p}^*)\Delta\underline{p}$$

where

$$\nabla l_{cop}(\underline{p}^*) = \left[\frac{\partial l_{cop}}{\partial \phi_1}\bigg|_{\underline{p}^*}, \frac{\partial l_{cop}}{\partial \phi_2}\bigg|_{\underline{p}^*}, ..., \frac{\partial l_{cop}}{\partial u_{hip}}\bigg|_{\underline{p}^*} \right]$$

III. Solving The Optimal Control Problem

The cost function for the quiet standing postural regulation problem is then defined as:

$$J = \frac{1}{2}\int_0^\infty d_1 \triangle l_{cop}^{2m}(t)dt + \frac{1}{2}\int_0^\infty [d_2 \triangle u_{ankle}^2(t) + d_3 \triangle u_{hip}^2(t) + d_4 \triangle u_{hip}^2(t)]dt$$

where d_1, d_2, d_3, d_4 are cost coefficients, m is an integer (m is 2 in this paper), and $\triangle l_{cop}$, $\triangle u_a$, $\triangle u_k$ and $\triangle u_h$ are deviations from the nominal equilibrium values of the COP and controls respectively.

Inspired by the way in which Model Predictive Control (MPC) problems are solved, we discretized the entire optimal control problem with respect to time and replaced the infinite time horizon of the original problem by the limited time duration N, and wrote the resulting discrete time optimal control problem as:

$$\min_{\underline{u}} \sum_{n=0}^N d_1(\triangle l_{cop})^4[k] + d_2u_1^2[k] + d_3u_2^2[k] + d_4u_3^2[k]$$
$$\textbf{s.t.}\ \underline{x}[k+1] = \overline{A}\underline{x}[k] + \overline{B}\underline{u}[k]$$

where $\overline{A} = \sum_{n=0}^\infty \frac{A^n(\delta)^n}{n!}$ and $\overline{B} = A^{-1}(\overline{A} - I)^{-1}B$ denote the system matrices of the discrete time system. Define

$$\underline{s} = [\underline{z}(0), \underline{u}(0), \underline{z}(1), \underline{u}(1), \ldots, \underline{z}(N), \underline{u}(N)]^T$$

The constraint then becomes $A\underline{s} = b$ where,

$$A_s = \begin{bmatrix} I & 0 & \cdots & \cdots & \cdots & O \\ -\overline{A} & -\overline{B} & I & 0 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ O & \cdots & -\overline{A} & -\overline{B} & I & 0 \end{bmatrix}, b = \begin{bmatrix} \underline{z}(0) \\ O \\ \vdots \\ O \end{bmatrix}$$

This nonlinear programming problem can be solved by the Newton-KKT algorithm[2], the key step of which is the repeated solution of the following system of linear equations involving the gradient and the Hessian of $J(\underline{s})$.

$$\begin{bmatrix} \nabla^2 J(\underline{s}^{(i)}) & A_s^T \\ A_s & O \end{bmatrix} \begin{bmatrix} \triangle\underline{s}_{nt}^{(i)} \\ \underline{w} \end{bmatrix} = \begin{bmatrix} -\nabla J(\underline{s}^{(i)}) \\ O \end{bmatrix}$$

Here $\triangle\underline{s}_{nt}^{(i)}$ is the Newton step at the i^{th} iteration. We could solve the open-loop optimal control problem for every initial condition in a grid of initial conditions near the vertical, storing only the first value of the two controls. This would give us a nonlinear, approximately optimal, full-state feedback regulator for posture. In fact, all of the elements of the state of this system are measured by sensors in the human body. Biologically, this nonlinear controller can be learned over time and would not impose any computational burden on the human nervous system.

In reality, the human postural control system includes significant delays [4]. These would require inclusion of a predictor in the feedback controller. This is discussed in some detail in our earlier paper [11]. However, we ignore the delays here because the overall control problem can be separated into two parts by imposing certainty equivalence and the full state feedback problem can be solved first. Inclusion of the delays and the predictor will complicate the controller and the exposition but add little to our understanding of the coordination.

IV. The Results

We have successfully solved the constrained nonlinear optimal control problem using the method described in the preceding section. In this section, we demonstrate that the proposed control could automatically adjust and coordinate different balance strategies according to the disturbance level.

The parameters and coefficients in the simulations are based on the simplified sway model defined in Eqn (4) using body parameters from Peterka [8] as shown in Table II.

Table II
BODY CHARACTERISTICS, DIMENSIONLESS MODEL PARAMETERS AND SIMULATION VARIABLES

M	$76\,kg$	I_1	$0.6\,kg \cdot m^2$	δ	$0.1\,s$
L	$1.76\,m$	I_2	$0.45\,kg \cdot m^2$	N	200
g	$9.81\,m/s^2$	I_3	$0.35\,kg \cdot m^2$	N^d	40

The approximately optimal control, with a look-ahead time of 4 seconds and a sampling interval of 0.1 seconds, makes the sampled horizon $N^d = 40$. The duration of the simulation is 20 seconds, which gives total number of *N* = *200* sample points The dimensionless results are then converted back to the real units in order to have a fair comparison with the experimental measurements.

A. Small Perturbation

Since the postural motion is normally a small amplitude sway around the equilibrium position, it is reasonable to simulate the optimally controlled system during steady state. The system will start at an initial position in which its COP is set to its equilibrium value as $l_{cop}[0] = 0\ cm$. In order to investigate the coordinated control of ankle and hip under different perturbations, we first tested the system's stability under small disturbance – white Gaussian noise with a standard deviation of 0.01.

The parameters for the steady state response simulation are listed in Table III. Here d_1 is the weight on the COP deviation. The parameters d_2, d_3, d_4 are the weights on the control torques at the ankle knee and hip joints. We choose d_2 = d_3 = d_4 =10 in our simulation.

Table III
SIMULATION PARAMETERS FOR STEADY STATE RESPONSE

d_1	100	$l_{cop}[0]$	$0\,cm$
d_2	10	$u_1[0]$	$0\,Nm$
d_3	10	$u_2[0]$	$0\,Nm$
d_4	10	$u_3[0]$	$0\,Nm$

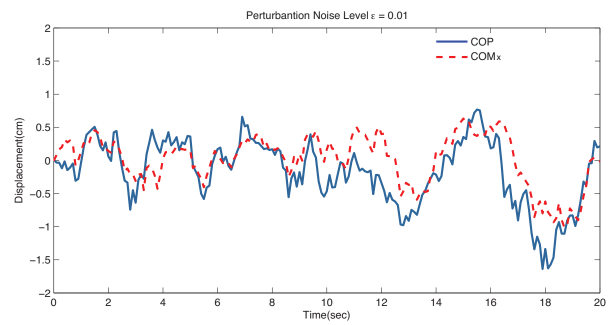

Figure 2. Trajectories of COM and COP under small perturbation

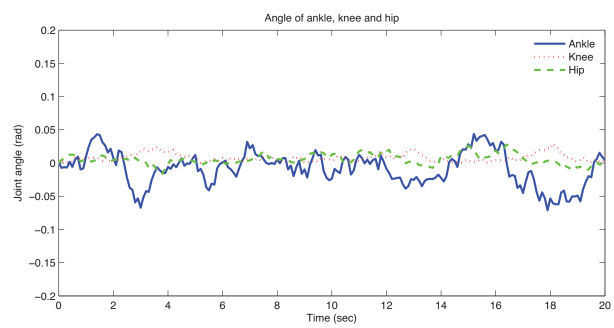

Figure 3. Trajectories of ankle and hip angle under small perturbation

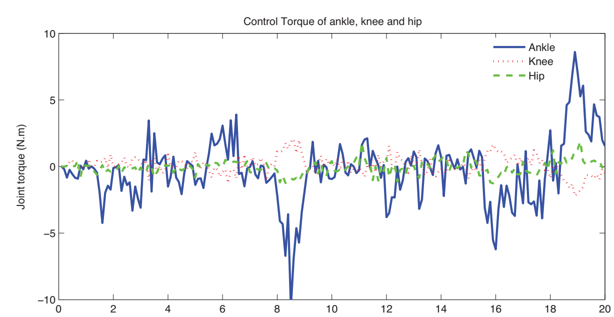

Figure 4. Control torque of ankle and hip joint under small perturbation

In Figure 2, the COM_x and COP are plotted. The trajectories of the ankle and hip angles are depicted in Figure 3; The corresponding control torques are shown in Figure 4.

Observe that the ankle response is larger than that of the knee and hip. To better demonstrate this feature, we have compared the 1, 2, and ∞ norms for sway movement at the ankle, knee and hip as shown in the following Table IV:

Table IV
DIFFERENT NORMS OF THE BODY SWAY UNDER NOISE LEVEL $\|\epsilon\| = 0.01$

	$\|\phi_*\|_1$	$\|\phi_*\|_2$	$\|\phi_*\|_\infty$
$ankle$	0.0250	0.007	0.0922
$knee$	0.0206	0.001	0.0781
hip	0.0205	0.001	0.0711

It is also interesting to compute the consumed energy $\sum_0^N u_i[k]\triangle\dot{\phi}_i[k]$ at each joint during the steady state response, where i = 1, 2, 3 for ankle, knee and hip respectively. The results are listed in the following Table V. One can see the ankle provides the most control effort while the hip barely contributes.

Table V
TOTAL ENERGY CONSUMED AT EACH JOINT
NOISE LEVEL: $\|\epsilon\| = 0.01$ UNIT: kg $\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2$

Ankle	Knee	Hip
1.7601	0.5709	0.2904

B. Large Perturbation Case

In this simulation of the steady state response, all the parameters are kept the same as before, except for a larger perturbation – white Gaussian noise with a standard deviation of 0.1. The resulting motion of the COP is plotted in Figure 5 along with the trajectory of the COM_x for the larger noise levels. Not unexpectedly, the COP exhibits greater displacements. This is because it includes the effects of control while the COM_x ignores the applied torques completely.

Figure 6 shows the resulting angular motion of the three joints. We have also compared the 1, 2, and ∞ norms for the sway movement at ankle, knee and hip under the larger perturbation as shown in following Table VI. In contrast to the small perturbation case, when the larger noise level is simulated, the hip angular displacement is larger than that of the ankle angle, and the knee's movement is also increased. This is very promising in that this same difference is observed experimentally, albeit more noticeably.

Table VI
DIFFERENT NORMS OF THE BODY SWAY UNDER NOISE LEVEL $\|\epsilon\| = 0.1$

	$\|\phi_*\|_1$	$\|\phi_*\|_2$	$\|\phi_*\|_\infty$
$ankle$	0.1784	0.0513	0.5986
$knee$	0.2368	0.0954	0.8627
hip	0.2552	0.1112	0.9627

In Figure 7, we plot the control torque of the ankle, knee and hip angles for the large noise levels. It is interesting that the torque applied at the hip is substantially larger than that applied at the ankle, and at the same time, the knee joint starts to contribute substantially.

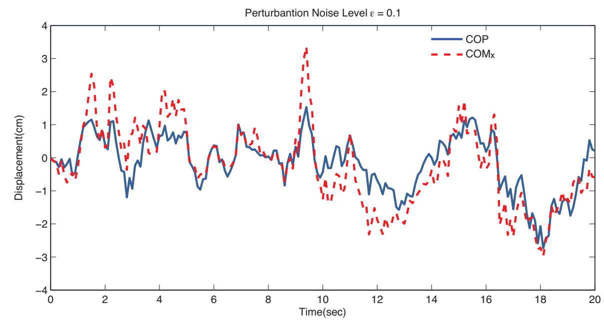

Figure 5. Trajectories of COM and COP under large perturbation

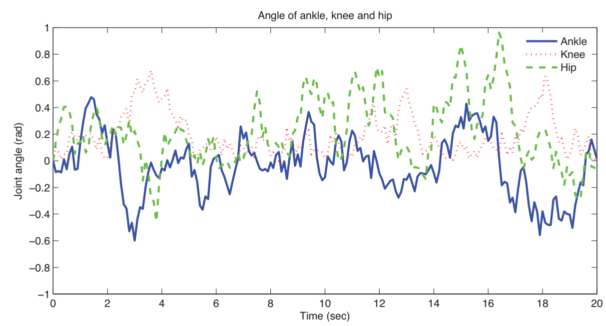

Figure 6. Trajectories of ankle and hip angle under large perturbation

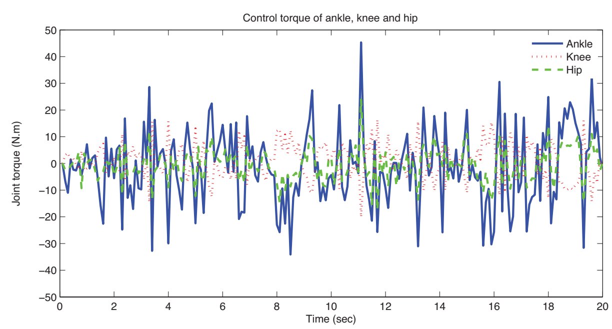

Figure 7. Control torque of ankle and hip joint under large perturbation

The energy consumed at each joint is listed in the following Table VII. The results indicate that the control energy at each joint increased largely due to the increased noise level and the quartic power on the control effort in the objective function. It will be also interesting to compare this with the total energy used by experiment subjects, if such a measurement is feasible.

Table VII
TOTAL ENERGY CONSUMED AT EACH JOINT
NOISE LEVEL: $\|\epsilon\| = 0.1$ UNIT: kg $\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2$

Ankle	Knee	Hip
3.8079	2.0228	1.7897

To better evaluate the postural sway, Collins and DeLuca proposed an analysis technique called the *Stabilogram Diffusion Function* (SDF) [3]. The SDF measures the similarity of the average COP between different time intervals. The SDF is one way to detect differences in postural sway, and is very sensitive to sway amplitude and velocity. It describes the relationship between the time interval of motion and the average of corresponding changes in position.

$$\langle \triangle l_{cop}^2 \rangle = \langle [l_{cop}(t + \triangle t) - l_{cop}(t)]^2 \rangle$$

where $\langle \cdot \rangle$ denotes the ensemble mean of the time series, and $\triangle t$ ranges from 0 to 10 seconds in the simulation. At $\triangle t = 0$, $\langle \triangle l_{cop}^2 \rangle$ value is zero. As $\triangle t$ increases, $\langle \triangle l_{cop}^2 \rangle$ will also increase. $l_{cop}(t)$ and its time-shifted value, $l_{cop}(t+\triangle t)$, become less similar to each other with increasing Δt.

In Figure 8, we compare a single trial of experimental measurement of quiet standing data with the SDF from our model under the noise level of 0.01. An adult male subject with 76kg weight and 1.76 meter height was tested in a quiet standing posture.

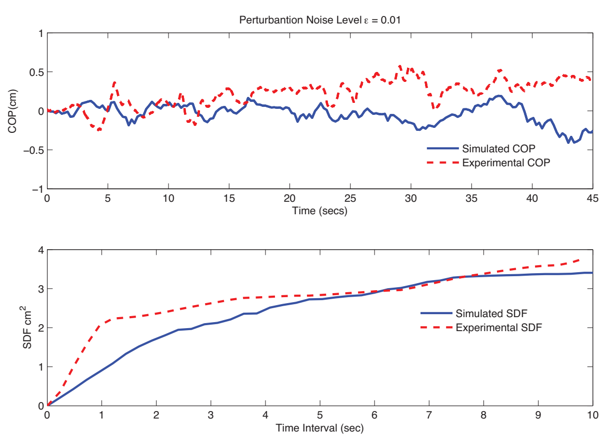

Figure 8. SDF of sway trajectory driven by white Gaussian noise with $\|\epsilon\| = 0.01$ starting from equilibrium point of COP

V. Conclusions

In this paper, we proposed an optimal control scheme for regulation of upright posture in the sagittal plane. The triple inverted pendulum system that approximates the human is controlled by joint torques at the ankle, knee and hip. The proposed optimization criterion is quadratic in the control effort but quartic in the COP, which is a good measurement for assessing the stability of quiet standing. This objective function provides a trade-off between the allowed deviations of the COP from its nominal value and the neuromuscular energy required to correct for these deviations.

This optimal control problem was solved and the optimally controlled system was simulated for both transient and steady-state responses. The results are consistent with those observed experimentally. For small perturbations, the ankle angle motion is larger than that of the knee and hip angle. For larger perturbations, we obtained larger movements at the hip than at the other two joints. The experimental results are more dramatic. That is, the difference between the small and large perturbation cases is larger than in our model. One way that we might achieve better agreement with experiment would be to better match the size of the perturbations to those in the experiments.

We have ignored the delays in the neuronal control system in this work. They can be included easily using the procedure we described in our earlier work. It will not change the basic results of this paper, but it will affect the SDF. In fact, it will be very likely to improve the match between the experimental SDF and our theoretical one.

The control mechanism proposed here is a natural one for the human. The large collection of neurons that provide the input signal to the muscles are threshold devices. They can implement any nonlinear gain by just changing their thresholds. In fact, the size principle[6] suggests that the gain of any feedback controller using muscle as the actuator should increase faster with increasing perturbations than linearly. Thus, our nonlinear feedback controller is as easy, if not easier, for the human central nervous system to implement than any linear one.

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