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Stability of Feedback Linearization under Intermittent Information: A Target-Pursuit Case

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***Abstract*— Managing networks of Autonomous Vehicles (AVs) for accomplishing a common goal, such as target pursuit, is very challenging due to the limited processing, sensing and communication capabilities of the agents. The effects of these limitations on stability of control systems are investigated in this paper. Having the performance of a *target-pursuit* controller provided with limited information about the target as an incentive, we develop a complete methodology for analyzing robustness of nonlinear controllers under intermittent information. As long as new information arrive within *Maximum Allowable Transfer Intervals* (MATIs), stability of the closed-loop system is guaranteed. Considering networks of AVs as spatially distributed systems, we adopt a Network Control Systems (NCSs) approach. Using Lyapunov techniques and the small-gain theorem, we are able to analyze stability of internal dynamics in feedback linearized systems within the same framework, and not as a separate problem. Finally, based on the *target*'s maneuver, we provide MATIs leading to different types of stability for the investigated *target-pursuit* policy, and provide corroborating numerical simulations.**

I. INTRODUCTION

With the advent of more skillful and sophisticated Autonomous Vehicles (AVs) in recent years, novel methods are investigated in order to exploit the full potential of multi-agent networks performing a coordinated task. AVs in such groups are inevitably coupled by sensing and communication. As technology pushes sensing, communication and processing capabilities of AVs further, the appetites and expectations of the industrial and civil sectors grow accordingly. Therefore, it will always be important to consider effects that non-perfect communication and sensing have on the quality of the group's performance.

A tendency in the field of robotics networks is to fully exploit accomplished agents via distributed control laws and estimation schemes resulting in more robust and reliable performance of the group [1]. Many open problems in the area of distributed control and estimation are tackled within the area of Networked Control Systems (NCSs) [2]. NCSs are spatially distributed systems for which the communication between sensors, actuators, and controllers is supported by a shared communication network.

Among many interesting problems and approaches presented in [2], the work that grabbed our attention is presented in [3] and [4]. The authors in [3] present a framework in which one first designs a controller without taking into account the network and then, in the second step, one determines a design parameter called the *Maximum Allowable Transfer Interval* (MATI) so that the closed loop remains stable when control and sensor signals are transmitted via the network. This framework models NCSs as hybrid (or jump) systems ([5], [6]), and utilizes the small-gain theorem [7] to study stability. Notice that MATI is an upper bound between two consecutive sampling times, and the source of the MATI is not specified (e.g., communication delays and collisions, sampling period, processing time, network throughput, occlusions of agents, limited communication/sensing range, etc.). A limitation of considering only Lyapunov Uniformly Globally Exponentially Stable (UGES) scheduling protocols in [3] is circumvented in [4] where Persistently Exciting Protocols (PEPs) are covered. Informally, PEPs are scheduling protocols that always visit all links of a NCS within every $T < \infty$ consecutive transmissions. A very educative example where the fast sampling and processing are essential in order to achieve desirable performance is the control of UAVs [8].

One of the driving forces for the research reported in this paper is the Marco Polo game introduced in [9] and formalized further in more recent work ([10], [11], and [12]). The game mimics a pursuit-evasion game often played by children in swimming pools. Marco Polo is a pursuer that receives information about the evaders' location in random time intervals. The goal of the Marco Polo game is to capture a group of intelligent evaders as quickly as possible using a team of autonomous pursuers that have intermittent knowledge of the evaders' locations. Our former work successfully brings together surveillance of an area-of-interest, sensor placement, collision avoidance and false alarms. Estimation of targets' locations and velocities is investigated in [13]. In [13], we compare performance of Unscented Kalman Filter (UKF) and Particle Filter (PF) under intermittent (or limited) information. In addition, we develop a novel filter whose performance is comparable to UKF and PF given limited and noisy information of targets and targets' locations.

Once a sensor is deployed to capture a target, a question that is still not adequately addressed is: "Given some control law, how often a pursuer has to obtain information about an evader in order to assure that the evader will be captured?" In this manner, the pursuer rationalizes the use of expensive resources needed to work within a group and energy at disposition. The controller investigated in this paper is the *leader-follower* controller designed via input-output feedback linearization, and is presented in [14]. We choose this controller due to versatility of its applications, its complexity and our familiarity with it. For instance, in the case where the desired separation between the *leader* and *follower* is zero, one practically deals with a *target-pursuit* problem.

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The contributions of this paper are threefold: a) all steps needed to analyze robustness of nonlinear controllers under intermittent information are provided; b) resulting in a novel approach for analyzing stability of internal dynamics in feedback linearized systems; and c) based on the *target*'s maneuver, MATIs leading to different types of stability for the *target-pursuit* policy are provided. In addition, to the best of our knowledge, the work in [4] has not been applied to nonlinear settings yet.

Other works related to this paper are found in [15], [16], [17] and [18]. However, these works appear to be more restrictive and less general than the approach proposed herein in terms of types of stability reached under intermittent information (e.g., exponential stability, a user defined performance), requirements on the system in the absence of communication network (e.g., exponential stability, state feedback, error-to-state stability, availability of Lyapunov functions, homogeneous/polynomial systems, controllers without dynamics), and no consideration of external inputs (or disturbances).

The rest of the paper is organized as follows. Section II presents the notation and previous results utilized in this work. A methodology for analyzing robustness of nonlinear controllers under intermittent information utilizing NCSs modeling is presented in Section III. In the exposition of Section III, an emphasis is put on controllers designed via feedback linearization method. The methodology is exemplified on the *target-pursuit* controller and stability analysis is provided in Section IV. Simulations and numerical results are presented in Section V. Finally, conclusions are drawn and the future work is discussed in Section VI.

II. Mathematical Preliminaries

A. Notation

For brevity, we use (x,y) for $[x^T \quad y^T]^T$. Let $f:\mathbb{R}\to\mathbb{R}^n$ be a Lebesgue measurable function on $[a,b]\subset\mathbb{R}$. We use

$$\|f[a,b]\|_2 := \left(\int_{[a,b]} \|f(s)\|^2 \mathrm{d}s\right)^{1/2}$$

to denote the $\mathcal{L}_2$ norm of f on $[a,b]$. In the above expression, $\|\cdot\|$ refers to Euclidean norm of a vector. If the argument of $\|\cdot\|$ is a matrix, then it denotes the induced matrix 2-norm. Let us define an operator $\bar{x}$ where $x\in\mathbb{R}^n$ such that $\bar{x}=(|x_1|,|x_2|,\ldots,|x_n|)$ where $|\cdot|$ denotes the absolute value of a real number. For a function $f:\mathbb{R}^n\to\mathbb{R}^m$, $\Gamma(f)$ denotes the function that takes components of the image of f to their absolute values, i.e., $\Gamma(f): x\to\bar{f}(x)$. $\mathbb{R}^{n_x}$ denotes Euclidean space of dimension n_x where n_x equals the dimension of x, and $\mathbb{R}^{r\times c}$ denotes the vector space of matrices with dimension $r\times c$.

The partial order $\preceq$ on vectors x and y is defined by

$$x\preceq y \iff (\forall i\in\{1,\cdots,n\})\; x_i\le y_i.$$

Finally, $\mathcal{A}_n^+$ denotes the set of all $n\times n$ matrices that are positive semidefinite, symmetric, and have nonnegative entries, and $*_{\mathcal{E}}$ is the element-wise matrix multiplication.

B. Definitions and Earlier Results

Let $\{t_i\}_{i=0}^{\infty}$ be a sequence of increasing time instants such that $0<t_{i+1}-t_i<\tau<\infty, \forall i\in\mathbb{N}$. Consider the hybrid system

$$\Sigma_j \ldots \begin{cases} \dot{x}_j = f_j(t,x_j,\omega),\ t\in[t_i,t_{i+1}] \\ x_j(t_i^+) = h_j(i,x_j(t_i)) \\ y_j = g_j(t,x_j) \end{cases}$$

initialized at (t_0,x_{j0}), with input (or disturbance) ω and output y_j. We assume enough regularity on f_j and h_j to guarantee existence of the solution $x_j(\cdot,t_0,x_{j0},\omega)$ on the interval of interest.

Definition 1: Let $\gamma\ge 0$ be given and $p=2$. We say that Σ_j is $\mathcal{L}_2$-stable from ω to y_j with (linear) gain γ if $\exists K\ge 0$ such that $\|y_j[t_0,t]\|_2\le K\|x_{j0}\|+\gamma\|\omega[t_0,t]\|_2$ for all $t\ge t_0$.

Definition 2: Let $\gamma\ge 0$ be given and $p=2$. We say that state x_j of Σ_j is $\mathcal{L}_2$ to $\mathcal{L}_2$ detectable from (y_j,ω) to x_j with (linear) gain γ if $\exists K\ge 0$ such that $\|x_j[t_0,t]\|_2\le K\|x_{j0}\|+\gamma\|y_j[t_0,t]\|_2+\gamma\|\omega[t_0,t]\|_2$ for all $t\ge t_0$.

Next, we state the theorems of [4] utilized herein in order to make this paper self-contained.

Consider a feedback interconnection, denoted with Σ, of two hybrid systems Σ_1 and Σ_2 with a common input ω. The small-gain theorem for hybrid systems is stated in the following theorem.

Theorem 1: Suppose that the feedback interconnection system Σ of two hybrid system satisfies:

1) hybrid system Σ_1 is $\mathcal{L}_2$-stable from (y_2,ω) to y_1 with gain γ_1;

2) state x_1 is $\mathcal{L}_2$ to $\mathcal{L}_2$ detectable from (y_1,ω);

3) hybrid system Σ_2 is $\mathcal{L}_2$-stable from (y_1,ω) to y_2 with gain γ_2;

4) state x_2 is $\mathcal{L}_2$ to $\mathcal{L}_2$ detectable from (y_2,ω); and

5) $\gamma_1\gamma_2<1$ (the small-gain condition).

Then, Σ is $\mathcal{L}_2$-stable from ω to (x_1,x_2).

Theorem 2: In addition, suppose that the interconnected system Σ satisfies the following:

$$\begin{aligned} \exists L_1\ge 0: &\quad \|f_1(t,x_1,x_2,0)\|\le L_1(\|x_1\|+\|x_2\|) \\ \exists L_2\ge 0: &\quad \|f_2(t,x_1,x_2,0)\|\le L_2(\|x_1\|+\|x_2\|) \\ \exists L_3\ge 0: &\quad \|h_1(i,x_1)\|\le L_3\|x_1\| \quad (1) \\ \exists L_4\ge 0: &\quad \|h_2(i,x_2)\|\le L_4\|x_2\| \quad (2) \end{aligned}$$

for all $x_1\in\mathbb{R}^{n_{x_1}}$, $x_2\in\mathbb{R}^{n_{x_2}}$, all $t\ge t_0$ and all $i\in\mathbb{N}$. Then, Σ with $\omega\equiv 0$ is UGES.

Theorem 3: Suppose that the NCS scheduling protocol is uniformly persistently exciting in time T (see Definition 4.4. from [4]), and there exist $A\in\mathcal{A}_{n_e}^+$ and a continuous $\tilde{y}:\mathbb{R}^{n_x}\times\mathbb{R}^{n_\omega}\to\mathbb{R}_+^{n_e}$ so that the error dynamics satisfies

$$\bar{\dot{e}} = \bar{g}(t,x,e,\omega)\preceq A\bar{e}+\tilde{y}(x,\omega) \quad (3)$$

for all $(x,e,\omega)\in\mathbb{R}^{n_x}\times\mathbb{R}^{n_e}\times\mathbb{R}^{n_\omega}$, all $t\in(t_i,t_{i+1})$, for all $i\in\mathbb{N}$. Further suppose that MATI satisfies $\tau\in(\kappa,\tau^*),\kappa\in(0,\tau^*)$ where

$$\tau^* = \frac{\ln(2)}{\|A\|T}. \quad (4)$$

Then, the NCS error subsystem is $\mathcal{L}_2$-stable from $\tilde{y}(x,\hat{\omega})$ to e for $p\in[1,\infty]$ with gain

$$\gamma_s(\tau) = \frac{T exp(\|A\|(T-1)\tau)(exp(\|A\|\tau)-1)}{\|A\|(2-exp(\|A\|T\tau))}. \quad (5)$$

Theorem 4: Consider interconnected NCS Σ and suppose that

1) the hypothesis of Theorem 3 hold with $\tilde{y} = G(x) + \omega$;

2) $(x_1, x_2) = f(t, x_1, x_2, e, \omega)$ is $\mathcal{L}_2$-stable from (e, ω) to $G(x)$ with gain γ;

3) and MATI satisfies $\tau \in (\epsilon, \tau^*), \epsilon \in (0, \tau^*)$, where $\tau^* = \ln(z)/\|A\|T$, A comes from (3), and z solves

$$z(\|A\| + \gamma T) - \gamma T z^{1-1/T} - 2\|A\| = 0. \quad (6)$$

Then, Σ is $\mathcal{L}_2$-stable from ω to $(G(x), e)$ with linear gain.

III. Methodology

A. Feedback Linearization Method

In this subsection we restate only the most necessary details of the feedback linearization method needed to understand the approach presented herein. For a comprehensive treatment of feedback linearization techniques, refer to [7] and [19].

Consider a "square" (number of inputs is equal to the number of outputs, i.e., $n_u = n_y$) Multi-Input-Multi-Output (MIMO) system that is affine in control:

$$\dot{x} = f(x) + \sum_{i=1}^{n_u} g_i(x) u_i, \quad y = h(x) \quad (7)$$

where $f : \mathcal{D} \to \mathbb{R}^{n_x}$, $g_i : \mathcal{D} \to \mathbb{R}^{n_x}$, and $h : \mathcal{D} \to \mathbb{R}^{n_y}$ are sufficiently smooth on a domain $\mathcal{D} \in \mathbb{R}^{n_x}$.

Using substitution $z = D(x)$ such that $D(x)$ is a diffeomorphism (smooth, invertible and has the smooth inverse) on a domain $\mathcal{D}_0 \subset \mathcal{D}$, we can write (7) in the *normal form*

$$\dot{\eta} = f_0(\eta, \xi), \quad (8a)$$

$$\dot{\xi} = A_c \xi + B_c \gamma(x)[u(x) - \alpha(x)], \quad (8b)$$

$$y = C_c \xi \quad (8c)$$

where $\eta \in \mathbb{R}^{n_x - \rho}$, $\xi \in \mathbb{R}^{\rho}$, (A_c, B_c, C_c) is the Brunovsky canonical form of a chain of ρ integrators, ρ is the relative degree of (7), and $z = (\xi, \eta)$. The Brunovsky canonical form is both controllable and observable. Next, by choosing the control law $u(x) = \alpha(x) + \gamma^{-1}(x)v$, we linearize (8b) with v as the auxiliary control input, while (8a) is made unobservable. Hence, the linearized part of the system (7) becomes

$$\dot{\xi} = A_c \xi + B_c v, \quad y = C_c \xi. \quad (9)$$

While analyzing stability of a feedback linearized system, internal dynamics (8a) play an important role and have to be treated carefully.

In order for the feedback linearization to succeed, state x has to be known with great accuracy so that the corresponding terms cancel out. Due to the inevitable existence of intermittent knowledge of the state x, the cancellation might not occur in real-life applications. As a result, the linearization process might not yield the expected behavior, and the closed-loop system can become unstable.

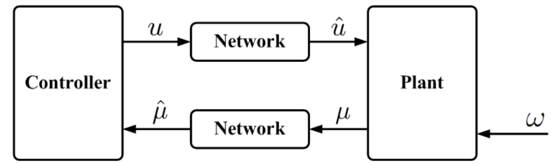


Fig. 1. A diagram of a control system from NCS perspective.

B. Networked Controlled Systems Modeling

Let us introduce communication networks between the controller and plant, and model nonlinear control system (8) as an NCS. The communication networks considered in this paper introduce intermittent knowledge of signals being exchanged between the controller and plant, but do not distort the signals (i.e., measurement noise is not considered).

The effect of the communication networks is comprised in the information error vector e defined as

$$e(t) := \begin{bmatrix} \hat{\mu}(t) - \mu(t) \\ \hat{u}(t) - u(t) \end{bmatrix} = \begin{bmatrix} e_\mu \\ e_u \end{bmatrix} \quad (10)$$

where $\hat{\mu}$ ($\hat{u}$) is an estimate of μ (u) performed from the perspective of the controller (plant). In scenarios where no estimation is performed, $\hat{\mu}$ ($\hat{u}$) is the most recently transmitted measurement (control signal). An illustration of the obtained NCS is provided in Figure 1.

The measurement model considered herein is

$$\mu = H(z) = (H_1(\xi), H_2(\eta)) \quad (11)$$

where $H : D(\mathcal{D}) \to \mathbb{R}^{n_\mu}$ has the following property that allows us to analyze internal dynamics within the same framework:

Assumption 1: State x can be reconstructed from μ (and possibly u) using H, and H is continuously differentiable.

Remark 1: Since $x = D^{-1}(z) = D^{-1}(H^{-1}(\mu))$, we require that H is invertible on the set $D(\mathcal{D})$. Recall that D is invertible being a diffeomorphism. Continuous differentiability is required in (14).

Remark 2: The conclusion of the previous remark can be loosen under the following conditions. In scenarios where $\hat{u} = u$ or an estimator collocated with the controller can access $\hat{u}$, one can take $H_1(\xi) = C_c \xi$ and perform estimation of ξ using Kalman filtering under intermittent observation [20] for the linear part of the system. In scenarios including AVs where $\hat{u}$ is not accessible to the estimator collocated with the controller, the estimation methodology presented in [13] can be used. Obviously, H_2 has to be invertible in all scenarios.

Assuming no estimation is performed ($\dot{\hat{\mu}} = \dot{\hat{u}} = 0$), we have everything to rewrite the closed loop system as a system with jumps that is more amenable for analysis:

$$\dot{z} = f_{big}(z, e), \quad \forall t \in [t_i, t_{i+1}] \quad (12a)$$

$$\dot{e} = g_{big}(z, e), \quad \forall t \in [t_i, t_{i+1}] \quad (12b)$$

$$e(t_i^+) = e(t_i) *_{\mathcal{E}} \Delta(t_i), \quad (12c)$$

where $\Delta(t_i)$ is a $n_e \times 1$ matrix of zeros and ones such that

$$\Delta_j(t_i) = \begin{cases} 0, & j^{th} \text{ component of } e \text{ is refreshed at } t_i \\ 1, & j^{th} \text{ component of } e \text{ is not refreshed at } t_i, \end{cases}$$

and $f_{big}(z, e)$ and $g_{big}(z, e)$ are given with (13) and (14), respectively. After a novel signal value is received, $\hat{\mu}$ and

$$f_{big}(z,e) = \begin{bmatrix} A_c\xi + B_c\gamma(D^{-1}(z))[u(D^{-1}(H^{-1}(H(z)+e_\mu))) + e_u - \alpha(D^{-1}(z))] \\ f_0(z) \end{bmatrix}, \tag{13}$$

$$g_{big}(z,e) = -H'(z)f_{big}(z,e) \tag{14}$$

$\hat{u}$ experience jumps as indicated in (12c). For more details regarding the above step, refer to [4].

Having f_{big} and g_{big}, we are ready to utilize the results of Subsection II-B. Theorem 3 is applied to g_{big}, while Theorem 4 is applied to both f_{big} and g_{big}. In order to further specify stability, we apply Theorems 1 and 2.

Although the aforementioned theorems give conditions of existence and sufficient conditions for achieving different types of stability (as the majority of results in nonlinear control), the actual implementation presents a significant challenge. For example, upper bounding a nonlinear vector-valued function with a function of a certain form (in Theorem 3), turns out to be quite a difficult task. The tighter the upper bound is, the less conservative and more useful results become. Another difficult problem is to estimate the $\mathcal{L}_2$-gain (or H_∞ norm) of nonlinear systems (Theorem 4). This problem is reportedly very hard to solve ([21], [22], [4]). While solving the problem, we use the algorithm of [21] with initial points being signals characteristic to the protocol used in our setup. A comprehensive discussion about $\mathcal{L}_2$-gain techniques in nonlinear settings can be found in [23] and [24].

In what follows, we present a complete approach to find MATIs for the *target-pursuit* controller designed via the input-output feedback linearization method. In other words, we provide all steps needed to successfully solve any similar problem, and fill in gaps between the theory presented in [4] and its actual implementation in nonlinear settings. Moreover, all the steps are exemplified with the above controller.

IV. Target-Pursuit Example

The AVs in this work are modeled as velocity-controlled unicycles. Hence, the kinematics of the i^{th} robot are given with

$$\dot{x}_i = v_i\cos\theta_i, \quad \dot{y}_i = v_i\sin\theta_i, \quad \dot{\theta}_i = \omega_i \tag{15}$$

where $(x_i, y_i, \theta_i) \in SE(2)$, and v_i and ω_i are the linear and angular velocity, respectively. Since we consider a *target-pursuit* problem, we have $i \in \{1,2\}$.

The nonlinear control system is given with:

$$\dot{\eta} = \omega_1 - \omega_2, \quad \dot{\xi} = G(\xi,\eta)u + \Upsilon(\xi)\omega \tag{16}$$

where $\xi = (l_{12}, \psi_{12})$ is the system output, $\eta = \theta_1 - \theta_2$ is the relative orientation, $\omega = (v_1, \omega_1)$ is the input (or disturbance) to the system, $u = (v_2, \omega_2)$ is the control input to the system, and $G(\eta,\xi)$ and $\Upsilon(\xi)$ are the following matrices

$$G(\eta,\xi) = \begin{bmatrix} \cos\gamma_{12} & d\sin\gamma_{12} \\ \frac{-\sin\gamma_{12}}{l_{12}} & \frac{d\cos\gamma_{12}}{l_{12}} \end{bmatrix}, \Upsilon(\xi) \quad = \begin{bmatrix} -\cos\psi_{12} & 0 \\ \frac{\sin\psi_{12}}{l_{12}} & -1 \end{bmatrix}$$

where $\gamma_{12} = \beta_{12} + \psi_{12}$, and d is the offset to an off-axis reference point on the *pursuer*.

Notice that (16) already has the form of (8); therefore, $D(x) = x$. Assuming that we know ω, we can cancel the term including ω using the following control input

$$u(\xi,\eta) = G(\xi,\eta)^{-1}[v(\xi) - \Upsilon(\xi)\omega], \tag{17}$$

where v is an auxiliary control input given by

$$v(\xi) = \begin{bmatrix} k_1(l_{12}^d - l_{12}) \\ k_2(\psi_{12}^d - \psi_{12}) \end{bmatrix} = k(\xi^d - \xi).$$

Positive real constants k_1 and k_2 are user defined controller gains, and $k = [k_1 \quad k_2]$. A desired separation between robots is l_{12}^d while a desired relative bearing is ψ_{12}^d. Finally, the closed-loop linearized system is simply given by

$$\dot{\eta} = \omega_1 - \omega_2, \quad \dot{\xi} = v(\xi) = k(\xi^d - \xi). \tag{18}$$

The equilibrium point of the closed-loop system is $z_{eq} = (l_{12}^d, \psi_{12}^d, 0)$. In order to proceed further, we have to place the equilibrium at the origin. Therefore, we introduce substitution $z^{new} = z - z_{eq}$. For brevity, z is used instead of z^{new} in the rest of the paper unless specified otherwise, and the state z before the above substitution is denoted z^{old}. Consequently, the components of z are $[l_{12} \quad \psi_{12} \quad \eta]$ instead of $(l_{12}^{new}, \psi_{12}^{new}, \eta^{new})$, and γ_{12} is used instead of γ_{12}^{new}.

When dealing with AVs, the communication network for transmitting control input u can be neglected due to on-board controllers. Therefore, we have $e = \hat{\mu} - \mu = (e_1, e_2, e_3)$. Furthermore, for the sake of simplicity, we choose

$$\mu = H(z) = z. \tag{19}$$

It is implicitly assumed that we deal with an adversarial target. In adversarial scenarios, v_1 and ω_1 can be estimated using the scheme presented in [14] or [13]. In the case of a cooperative scenario, ω is communicated to the *pursuer* at the same time instances when the measurements are obtained. These information are denoted $\hat{\omega}$.

Having said that, we proceed as follows

$$\begin{aligned} \dot{e} = -\dot{z} = &- G_{ext}(z)G^{-1}(z+e)[v(z+e) - \\ &\Upsilon(z+e)\hat{\omega}] - \Upsilon_{ext}(z)\omega \end{aligned} \tag{20}$$

where

$$G_{ext}(z) = \begin{bmatrix} \cos(\gamma_{12}+\psi_{12}^d) & d\sin(\gamma_{12}+\psi_{12}^d) \\ \frac{-\sin(\gamma_{12}+\psi_{12}^d)}{l_{12}+l_{12}^d} & \frac{-\cos(\gamma_{12}+\psi_{12}^d)}{l_{12}+l_{12}^d} \\ 0 & -1 \end{bmatrix}$$

and

$$\Upsilon_{ext}(z) = \begin{bmatrix} -\cos(\psi_{12}+\psi_{12}^d) & 0 \\ \frac{\sin(\psi_{12}+\psi_{12}^d)}{l_{12}+l_{12}^d} & -1 \\ 0 & 1 \end{bmatrix}$$

are extended matrices of the system (16). After some calculation we obtain equation (21).

Let us proceed further by putting constraints on values of $l_{12} + l_{12}^d$ and ψ_{12} that stem from physical properties of AVs in our control problem. Since the AVs are not points in $\mathbb{R}^2$, we put $\min\{l_{12} + l_{12}^d\} = m > 0$. In addition, we constrain $\max\{|\psi_{12}|\} = M > 0$ as a characteristic of the given problem. In order to make the further calculations tractable and easier to follow, we assume that input ω is (approximately) constant between two measurements, i.e., $\hat{\omega} = \omega$. This is not a restrictive assumption since the worst-case scenario, based on finite maximal absolute accelerations $a_{v,\max}$ and $a_{\omega,\max}$ of the *target*, can easily be implemented.

$$
\begin{aligned}
g_{big}(z,e) = &\begin{bmatrix} k_1 l_{12}\cos e_2 + k_2(l_{12}+e_1+l_{12}^d)\psi_{12}\sin e_2 \\ \frac{1}{l_{12}+l_{12}^d}[k_1 l_{12}\sin e_2 + k_2(l_{12}+e_1+l_{12}^d)\psi_{12}\cos e_2] \\ \frac{-k_1 l_{12}\sin(\gamma_{12}+e_2+\psi_{12}^d)}{d} - \frac{k_2(l_{12}+e_1+l_{12}^d)\psi_{12}}{d}\cos(\gamma_{12}+e_2+\psi_{12}^d) \end{bmatrix} + \\
&+ \begin{bmatrix} -\cos(\psi_{12}+2e_2+\psi_{12}^d) & -(l_{12}+e_1+l_{12}^d)\sin e_2 \\ \frac{1}{l_{12}+l_{12}^d}\sin(\psi_{12}+\psi_{12}^d) & -\frac{l_{12}+e_1+l_{12}^d}{l_{12}+l_{12}^d}\cos e_2 \\ \frac{1}{d}\sin(\theta_1-\theta_2) & \frac{l_{12}+e_1+l_{12}^d}{d}\cos(\gamma_{12}+e_2+\psi_{12}^d) \end{bmatrix} \begin{bmatrix} \hat{v}_1 \\ \hat{\omega}_1 \end{bmatrix} + \begin{bmatrix} \cos(\psi_{12}+\psi_{12}^d) & 0 \\ \frac{-\sin(\psi_{12}+\psi_{12}^d)}{l_{12}+l_{12}^d} & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ w_1 \end{bmatrix}
\end{aligned} \tag{21}
$$

For example, the *pursuer* can use the following sampling period

$$T_s = \inf_{\substack{v_1 \in [\hat{v}_1 - \tau(\hat{\omega})a_{v,\max}, \hat{v}_1 + \tau(\hat{\omega})a_{v,\max}] \\ \omega_1 \in [\hat{\omega}_1 - \tau(\hat{\omega})a_{\omega,\max}, \hat{\omega}_1 + \tau(\hat{\omega})a_{\omega,\max}]}} \tau(v_1, \omega_1).$$

From (21) and taking $m \leq 1$, we are now are now able to write (22) and (23). The case with $m > 1$ is very similar; therefore, not included. Positive real constants ϵ_1 and ϵ_2 in (22) are chosen to minimize $\|A\|$ such that $A \in \mathcal{A}_n^+$ provided $\hat{\omega}_1$. Notice that all the conditions of Theorem 3 are satisfied.

In order to obtain stability results regarding the closed-loop NCS in Figure 1, according to Theorem 4, we have to find $\mathcal{L}_2$-gain γ from $(e, \hat{\omega})$ to $\tilde{y}(z, \omega)$. Using (20), (23) and the approach of [21], we are able to estimate the corresponding gain γ for every MATI τ (i.e., over a finite horizon). Basically, we are solving the following maximization problem

$$[\gamma(\tau, \hat{\omega})]^2 = \sup_{e \in \mathcal{L}_2[0,\tau]} \left\{ \frac{\int_{[0,\tau]} \|\tilde{y}\|^2 \mathrm{d}t}{\int_{[0,\tau]} \|(e, \hat{\omega})\|^2 \mathrm{d}t} \right\} \tag{24}$$

with zero initial conditions. A good initial guess for components of the input e are segments of lines going through the origin since we use the protocol in which the components of input e are equal to zero after obtaining measurements (illustrated in Figure 5(c)). We choose such MATIs that result in the $\mathcal{L}_2$-stable closed-loop system from ω to $(e, \tilde{y}(z, \hat{\omega}))$ according to the small-gain theorem, i.e., $\gamma_s(\tau)\gamma(\tau, \hat{\omega}) < 1$. Notice that expression (6) cannot be used since γ is not constant any more.

A. Stability of the Pursuit Policy

The stability analysis for the pursuit policy is comprised in the following theorem.

Theorem 5: Consider the *target-pursuit* problem given with (16), control law (17), and measurement model (19). If MATIs τ are obtained with the previously presented methodology, then the pursuit policy is:

i) stable for the case $v_1 = \omega_1 = 0$ with exponential convergence to a point dependent on the initial condition,

ii) UGES for the case $v_1 \neq 0$ and $\omega_1 = 0$, and

iii) stable for the case $v_1 \neq 0$ and $\omega_1 \neq 0$.

Proof: Due to the space limitation, and long and cumbersome expressions involved in this proof, we provide only an outline of the proof.

By checking whether (20), (21) and the corresponding jump equations satisfy the conditions of Theorems 1 and 2, we are able to further specify $\mathcal{L}_2$-stability obtained for an adequate MATI τ. The communication protocol considered herein results in the following jump equations:

$$z(t_i^+) = z(t_i), \quad e(t_i^+) = 0.$$

By choosing $L_3 = 1$ and $L_4 > 0$, conditions (1) and (2) of Theorem 2 are satisfied.

i) In the case of $v_1 = \omega_1 = 0$, the conditions of the aforementioned theorems are satisfied, but $\gamma(\tau) \equiv 0$ as illustrated in Figure 2. Therefore, we have a stable system that exponentially converges to l_{12}^d and ψ_{12}^d provided a non-zero initial condition, but relative orientation β_{12} converges to a finite value dependent on the initial condition.

ii) If we consider a constant linear velocity $v_1 \neq 0$ with $\omega_1 = 0$, the conditions of Theorems 1 and 2 are satisfied as well; therefore, the pursuit policy is UGES (as long as $\beta_{12} \neq \pi$ as stated in [14]).

iii) In the case of $\omega_1 \neq 0$, the theorems are no longer satisfied due to the second and third row in (21). Stability of the pursuit policy follows from $\omega = \hat{\omega}$ and $\mathcal{L}_2$-stability of the closed-loop system.

The second and third case are illustrated in Figure 5(a). ■

Remark 3: The small-gain theorem ensures boundedness of β_{12} in all the above cases. Hence, the internal dynamics are analyzed within the same framework, and not as a separate problem as in [14]. This integrated stability analysis makes the presented approach really appealing.

Remark 4: Apparently, we have reached the same stability results as in [14], but under intermittent information.

V. Simulations and Numerical Results

In order to provide a better analysis, the control error $e_c = z - z^d$ is introduced. Since we do not consider bus communication, we set $T = 1$. The parameters of the *target-pursuit* system are chosen as follows: $d = 0.25$ m, $m = 0.8$ m, $l_{12}^d = 3$ m, $\psi_{12}^d = \frac{13\pi}{18}$ rad, $k_1 = 1$, $k_2 = 0.01$, and $M = \max\{|2\pi - \psi_{12}^d|, |-\psi_{12}^d|\}$ rad. The "physical" parameters d, m, and M are chosen based on the characteristics of our testbed [25], while the controller gains represent a trade-off between the dynamics of e_c and e. Note that the greater k_1 and k_2 become, the faster e_c converges in the ideal case (without intermittent information). It is straightforward to conclude that such an increase causes e to grow faster leading to an increase in $\gamma_s(\tau, \hat{\omega})$, i.e., a decrease in MATI τ. Hence, conclusions made about parameters of "continuous" controllers have to be carefully transferred to settings including communication networks.

Graphs of gains $\gamma_s(\tau)$ and $\gamma(\tau, \hat{\omega})$ are provided in Figure 2 for different $\hat{\omega}$. For $\omega_1 = 0.15$ rad/s, values $\epsilon_1 = 0.76$ and $\epsilon_2 = 0.01$ minimize $\|A\|$ in (22), and that case is provided in Figure 2. Having $\gamma(\tau)$ and $\gamma_s(\tau, \hat{\omega})$, we are able to calculate τ for different ω (Figure 3).

Next, we compare MATIs obtained via the presented methodology with "real" MATIs obtained via simulation (Table I). The calculated MATIs are about two or three

$$\dot{\bar{e}} \preceq \underbrace{\begin{bmatrix} |\hat{\omega}_1|+\frac{1}{2}k_2M+\frac{\hat{\omega}_1^2}{2k_2M}+\epsilon_1 & \frac{k_2M+|\hat{\omega}_1|}{m} & \frac{k_2M+|\hat{\omega}_1|}{d} \\ \frac{k_2M+|\hat{\omega}_1|}{m} & 2\frac{k_2M}{m^2} & 0 \\ \frac{k_2M+|\hat{\omega}_1|}{d} & 0 & \frac{2\frac{k_2M}{m^2}\left(\frac{k_2M+|\hat{\omega}_1|}{d}\right)^2}{2\frac{k_2M}{m^2}(|\hat{\omega}_1|+\frac{k_2M}{2}+\frac{\hat{\omega}_1^2}{2k_2M}+\epsilon_1)-\left(\frac{k_2M+|\hat{\omega}_1|}{d}\right)^2}+\epsilon_2 \end{bmatrix}}_{A} \bar{e} + \tilde{y}(z,\hat{\omega}) \tag{22}$$

$$\tilde{y}(z,\hat{\omega}) = \Gamma\left(\begin{bmatrix} k_1 l_{12}\cos e_2+k_2(l_{12}+l_{12}^d)\psi_{12}\sin e_2 \\ \frac{k_1 l_{12}\sin e_2}{l_{12}+l_{12}^d}+k_2\psi_{12}\cos e_2 \\ \frac{-k_1 l_{12}\sin(\gamma_{12}+e_2+\psi_{12}^d)}{d}-\frac{k_2(l_{12}+l_{12}^d)\psi_{12}}{d}\cos(\gamma_{12}+e_2+\psi_{12}^d) \end{bmatrix} + \right.$$
$$\left. + \begin{bmatrix} \cos(\psi_{12}+\psi_{12}^d)-\cos(\psi_{12}+2e_2+\psi_{12}^d) & -(l_{12}+l_{12}^d)\sin e_2 \\ 0 & -\cos e_2+1 \\ \frac{1}{d}(\theta_1-\theta_2) & \frac{l_{12}+l_{12}^d}{d}\cos(\gamma_{12}+e_2+\psi_{12}^d)-1 \end{bmatrix} \begin{bmatrix} \hat{v}_1 \\ \hat{\omega}_1 \end{bmatrix} \right) \tag{23}$$

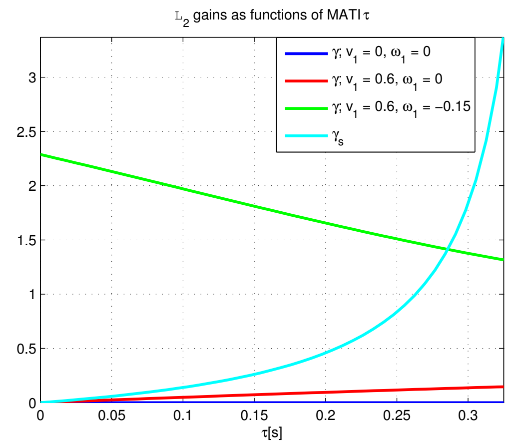


Fig. 2. Gains γ and γ_s (with parameter $\hat{\omega}_1 = 0.15$ rad/s) with respect to different τ.

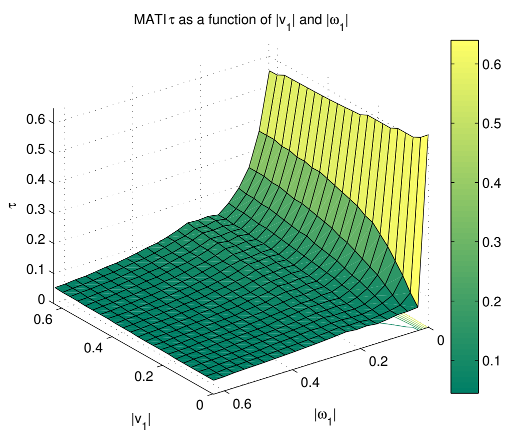


Fig. 3. MATI τ with respect to $|v_1|$ and $|\omega_1|$.

TABLE I

CALCULATION-BASED AND SIMULATION-BASED MATIS FOR DIFFERENT INPUTS $\hat{\omega}$.

	$\tau_{cal}[s]$	$\tau_{sim}[s]$	τ_{sim}/τ_{cal}
$\hat{v}_1 = 0;\ \hat{\omega}_1 = 0$	$\tau^* = 0.64$	1.32	2.1
$\hat{v}_1 = 0.6;\ \hat{\omega}_1 = 0$	0.48	0.68	1.42
$\hat{v}_1 = 0.6;\ \hat{\omega}_1 = -0.15$	0.16	0.47	2.94

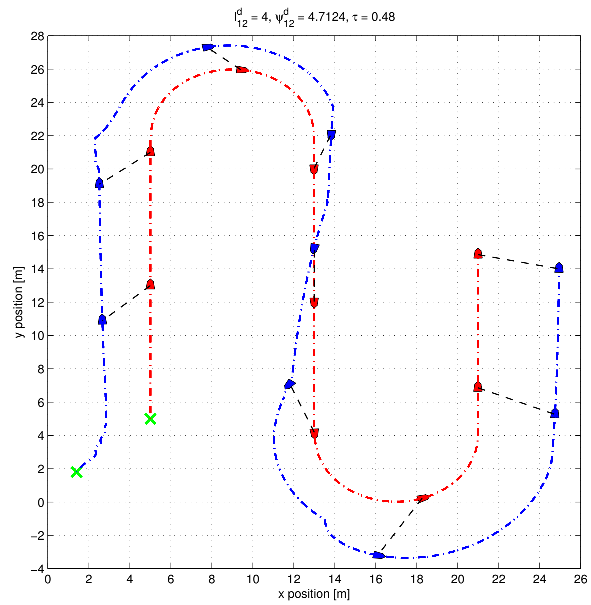


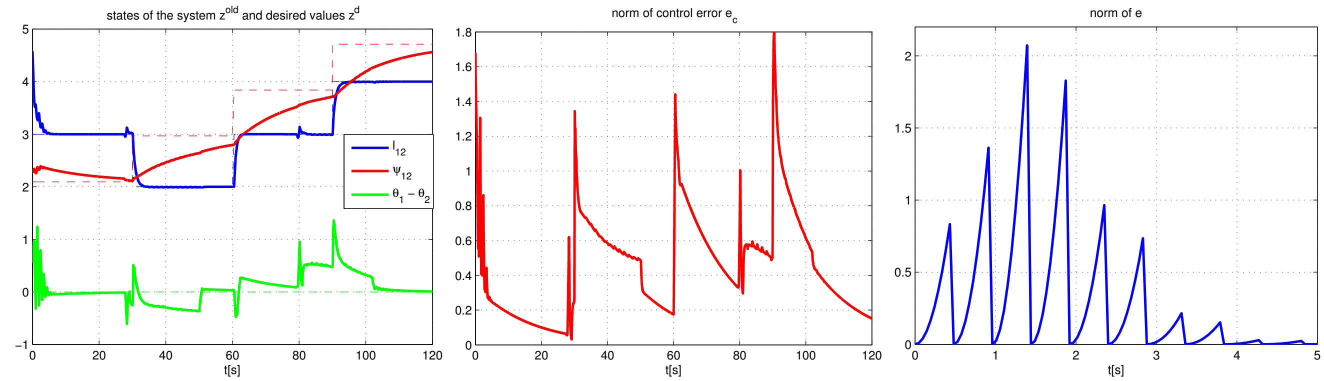
Fig. 4. Trajectories of the *target* and *pursuer* using the adaptive sampling represented by the red and blue dashed lines, respectively. Red triangles represent the *target* in different time instants, while blue triangles represent the *pursuer* in different time instants. Green crosses represent initial positions of the agents. Black dashed lines connect positions of the *target* and *pursuer* at the corresponding time instants.

times smaller than the "real" ones. This observation is highly comparable with the linear scenarios from [4] since MATIs from [4] are 1.6 to 15 times smaller than the "real" ones.

In the scenario included in this paper, the *target* performs maneuvers with $v_1 = 0.6$ m/s and $\omega_1 \in \{0, \pm 0.15\}$ rad/s. During the simulation, MATI τ adaptively switches between 0.48 s and 0.16 s depending on input ω, and l_{12}^d and ψ_{12}^d are changed by a user as indicated in Figure 5(a). Corresponding characteristic signals are provided in Figures 4 and 5, and the results of Theorem 5 are verified. Hence, no matter what maneuver the *target* performs, it will be captured for $l_{12}^d = 0$.

VI. CONCLUSION AND FUTURE WORK

Motivated by the limited processing, sensing and communication capabilities of the agents in mobile agent networks, we analyze stability of agents' control policies provided with intermittent information about the environment. A complete methodology for analyzing robustness of nonlinear controllers under intermittent information is provided. First, one designs a nonlinear control system without taking into account the presence of limited information. Second, the system is modeled as an NCS and an information error vector is introduced. Afterwards, utilizing the small-gain theorem,



(a) (b) (c)

Fig. 5. The adaptive scenario: (a) States of the *target-pursuit* system using the adaptive sampling. Dashed lines represent desired values. For clarity, z^{old} is provided; (b) Control error e_c of the *target-pursuit* system using the adaptive sampling; and, (c) An illustrative detail of information error e of the *target-pursuit* system at the beginning of the simulation.

MATIs leading to closed-loop stability under intermittent information are obtained. Specifically, a *target-pursuit* controller is investigated, and MATIs resulting in different types of stability for the pursuit policy are obtained. The obtained MATIs depend on the *target*'s linear and angular velocity. Finally, a stability analysis of the pursuit policy is provided and illustrated using numerical simulations. The experimental verification of the obtained MATIs has already been conducted, and will be included in upcoming publications.

In the future, we plan to apply the presented methodology to controllers designed via other nonlinear methods. In addition, saturation of actuators (i.e., bounded inputs of pursuers) should also be taken into account. Although preliminary simulations suggest that MATIs increase in that case, a more detailed analysis is needed. Another interesting extension of this work is to consider measurement noise and estimation. Estimation and noise call for a stochastic approach. A number of stochastic approaches are investigated in the area of NCSs. However, this area is still under investigation and is part of our future research agenda.

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