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Sliding Mode Mean-Module Filter Design for Polynomial Systems

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***Abstract*— This paper addresses the mean-module filtering problem for a stochastic polynomial system with Gaussian white noises. The obtained solution contains a sliding mode term, signum of the innovations process. It is shown that the designed sliding mode filter generates the mean-module estimate, which yields a better value of the mean-module criterion in comparison to the mean-square polynomial filter. The theoretical result is complemented with an illustrative example verifying performance of the designed filter, which is compared to the mean-square polynomial filter. The simulation results confirm an advantage in favor of the designed sliding mode filter.**

I. Introduction

Since the sliding mode control was invented in the beginning of 1970s (see a historical review in [1], [2], [3]), it has been applied to solve several classes of problems. For instance, the sliding mode control methodology has been used in stabilization [4], [5], tracking [6], [7], observer design [8], [9], frequency domain analysis [10], and other control problems. Further modifications of the original sliding mode concept, such as integral sliding mode [11] and higher order sliding modes [12], [3], have been developed. The sliding mode optimal regulators have been recently designed for linear systems with non-quadratic Bolza-Meyer criteria [13], [14]. Application of the sliding mode method is extended to stochastic systems [15], [16], [17], [18] and stochastic filtering problems [19], [20], [21], [22], [23]. The last two papers present mean-square and mean-module sliding mode filters for stochastic linear systems.

This paper presents the solution to the mean-module filtering problem for a stochastic polynomial system, which contains a sliding mode term, signum of the innovations process. The mean-module sliding mode filter is obtained in a closed form, which includes the equations for a mean-module estimate and a filter gain matrix. It is shown that the designed sliding mode filter generates the mean-module estimate, which yields a better value of the mean-module criterion in comparison to the mean-square polynomial filter [24]. To the best of our knowledge, this is the first designed sliding mode filter for stochastic polynomial systems that is optimal with respect to the mean-module criterion. The theoretical result is complemented with an illustrative example verifying performance of the designed filter, which is compared to the conventional mean-square polynomial filter. The simulation results confirm an advantage in favor of the designed sliding mode filter.

The paper is organized as follows. Section 2 states the mean-module filtering problem for stochastic polynomial systems with Gaussian white noises. The sliding mode solution to the mean-module filtering problem is given in Section 3. The proof of the obtained results is given in Appendix. Section 4 contains an illustrative example.

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II. Mean-Module Filtering Problem Statement

Let (Ω, F, P) be a complete probability space with an increasing right-continuous family of σ-algebras $F_t, t \geq t_0$, and let $(W_1(t), F_t, t \geq t_0)$ and $(W_2(t), F_t, t \geq t_0)$ be independent standard Wiener processes. The F_t-measurable random process $(x(t), y(t))$ is described by a nonlinear differential equation with a polynomial drift term for the system state

$$dx(t) = f(x,t)dt + b(t)dW_1(t), \quad x(t_0) = x_0, \tag{1}$$

and a linear differential equation for the observation process

$$dy(t) = (A_0(t) + A(t)x(t))dt + B(t)dW_2(t). \tag{2}$$

Here, $x(t) \in R^n$ is the state vector and $y(t) \in R^m$ is the linear observation vector, $m \leq n$. The initial condition $x_0 \in R^n$ is a Gaussian vector such that x_0, $W_1(t) \in R^p$, and $W_2(t) \in R^q$ are independent. The observation matrix $A(t) \in R^{m\times n}$ is not supposed to be invertible or square. It is assumed that $B(t)B^T(t)$ is a positive definite matrix, therefore, $m \leq q$. All coefficients in (1)–(2) are deterministic functions of appropriate dimensions.

The nonlinear function $f(x,t)$ is considered polynomial of n variables, components of the state vector $x(t) \in R^n$, with time-dependent coefficients. Since $x(t) \in R^n$ is a vector, this requires a special definition of the polynomial for $n > 1$. In accordance with [24], a p-degree polynomial of a vector $x(t) \in R^n$ is regarded as a p-linear form of n components of $x(t)$

$$f(x,t) = a_0(t) + a_1(t)x + a_2(t)xx^T + \ldots + a_p(t)x \ldots_{p\ times} \ldots x,$$

where $a_0(t)$ is a vector of dimension n, a_1 is a matrix of dimension $n \times n$, a_2 is a 3D tensor of dimension $n \times n \times n$, a_p is an $(p+1)$D tensor of dimension $n \times \ldots_{(p+1)\ times} \ldots \times n$, and $x \times \ldots_{p\ times} \ldots \times x$ is a pD tensor of dimension $n \times \ldots_{p\ times} \ldots \times n$ obtained by p times spatial multiplication of the vector $x(t)$ by itself. Such a polynomial can also be expressed in the summation form

$$f_k(x,t) = a_{0\ k}(t) + \sum_i a_{1\ ki}(t)x_i(t) + \sum_{ij} a_{2\ kij}(t)x_i(t)x_j(t) + \ldots$$

$$+\sum_{i_1\dots i_p} a_{p\ ki_1\dots i_p}(t)x_{i_1}(t)\dots x_{i_p}(t), \quad k,i,j,i_1\dots\ i_p=1,\dots,n.$$

Here, $x(t)\in R^n$ is the state vector and $y(t)\in R^m$, $m\le n$, is the observation process. The initial condition $x_0\in R^n$ is a Gaussian vector such that x_0, $W_1(t)$, and $W_2(t)$ are independent. It is assumed that $B(t)B^T(t)$ is a positive definite matrix. All coefficients in (1)–(2) are deterministic functions of time of appropriate dimensions.

The state and observation equations can also be written in an alternative form

$$\dot{x}(t)=f(x,t)dt+b(t)\psi_1(t), \quad x(t_0)=x_0, \qquad (1^*)$$

$$y(t)=A(t)x(t)+B(t)\psi_2(t), \qquad (2^*)$$

where $y(t)=\dot{Y}(t)$, and $\psi_1(t)$ and $\psi_2(t)$ are white Gaussian noises, which are the weak mean-square derivatives of standard Wiener process $W_1(t)$, and $W_2(t)$ (see [25]). The representations (1),(2) and (1^*),(2^*) are equivalent ([26]). The equations (1^*),(2^*) present the conventional form for the equations (1),(2), which is actually used in practice.

The mean-square filtering problem is to find the estimate $\hat{x}(t)$ of the system state $x(t)$, based on the observation process $Y(t)=\{y(s),t_0\le s\le t\}$, that minimizes the mean-square norm

$$J=E[(x(t)-\hat{x}(t))^T(x(t)-\hat{x}(t))\mid F_t^Y]$$

at every time moment t. Here, $E[z(t)\mid F_t^Y]$ means the conditional expectation of a stochastic process $z(t)=(x(t)-\hat{x}(t))^T(x(t)-\hat{x}(t))$ with respect to the σ - algebra F_t^Y generated by the observation process $Y(t)$ in the interval $[t_0,t]$. The solution to this filtering problem for polynomial systems is given by the mean-square polynomial filter [24] generalizing the optimal Kalman-Bucy filter [27] for linear systems.

This paper addresses the mean-module filtering problem to find the estimate $\hat{x}(t)$ of the system state $x(t)$, based on the observation process $Y(t)=\{y(s),t_0\le s\le t\}$, that minimizes the mean-module norm

$$J=E[(\mid x(t)-\hat{x}(t)\mid)\mid F_t^Y] \qquad (3)$$

at every time moment t. Here, $\mid x\mid=[\mid x_1\mid,\dots,\mid x_n\mid]\in R^n$ is defined as the vector of absolute values of the components of the vector $x\in R^n$

The solution to the stated filtering problem, involving the sliding mode term, is given in the next section and then proved in Appendix. As demonstrated, the obtained sliding mode filter is optimal with respect to the criterion (3).

III. Sliding Mode Mean-Module Filter Design

The solution to the mean-module filtering problem for the linear system (1) and the criterion (3) is given as follows. The mean-module estimate satisfies the differential equation with the sliding mode term (the proof is given in Appendix)

$$\dot{m}(t)=E(f(x,t)\mid F_t^Y)dt+Q(t)A^T(t)(B(t)B^T(t))^{-1}\times \qquad (4)$$
$$A(t)Sign[A^T(t)(A(t)A^T(t))^{-1}y(t)-m(t)].$$

with the initial condition $m(t_0)=E(x(t_0)\mid F_{t_0}^Y)$. Here, the Signum function of a vector $x=[x_1,\dots,x_n]\in R^n$ is defined as $Sign[x]=[sign(x_1),\dots,\ sign(x_n)]\in R^n$, and the signum function of a scalar x is defined as $sign(x)=1$, if $x>0$, $sign(x)=0$, if $x=0$, and $sign(x)=-1$, if $x<0$ ([28]).

The matrix function $Q(t)$ satisfies the matrix equation with time-varying coefficients

$$\dot{Q}(t)=b(t)b^T(t)+E(f(x,t)(x(t)-m(t))^T)\mid F_t^Y), \qquad (5)$$

with the initial condition $Q(t_0)=E[(x(t_0)-m(t_0))(Sign(A^T(t_0)(A(t_0)A^T(t_0))^{-1}A(t_0)x(t_0)-m(t_0)))^T\mid F_{t_0}^Y]$.

Note that the equations (4) and (5) do not form a closed system of equations due to the presence of polynomial terms depending on x, $E(f(x,t)\mid F_t^Y)$, and $E(f(x,t)(x(t)-m(t))^T)\mid F_t^Y)$, which are not expressed yet as functions of the filter variables, $m(t)$ and $Q(t)$ (or $P(t)$). However, as shown in [29], the closed system of the filtering equations can be obtained for any polynomial state (1) over linear observations (2), using the technique of representing of superior moments of the conditionally Gaussian random variable $x(t)-m(t)$ as functions of only two its lower conditional moments, $m(t)$ and $P(t)$ (see [29] for more details of this technique). Apparently, the polynomial dependence of $f(x,t)$ and $f(x,t)(x(t)-m(t))^T$ on x is the key point making this representation possible.

Next, a closed form of the filtering equations is obtained from (4) and (5) for a third-order function $f(x,t)$ in the equation (1), as follows. It should be noted, however, that application of the same procedure would result in designing a closed system of the filtering equations for any polynomial function $f(x,t)$ in (1).

Let the function

$$f(x,t)=a_0(t)+a_1(t)x+a_2(t)xx^T+a_3(t)xxx^T \qquad (6)$$

be a third-order polynomial, where x is an n-dimensional vector, $a_0(t)$ is an n-dimensional vector, $a_1(t)$ is a $n\times n$-dimensional matrix, $a_2(t)$ is a 3D tensor of dimension $n\times n\times n$, $a_3(t)$ is a 4D tensor of dimension $n\times n\times n\times n$. In this case, the following filtering equations for the optimal estimate $m(t)$ and the filter gain matrix $Q(t)$ are obtained

$$\dot{m}(t)=a_0(t)+a_1(t)m(t)+a_2(t)m(t)m^T(t)+ \qquad (7)$$
$$a_2(t)Q(t)*\mid A^T(t)(A(t)A^T(t))^{-1}y(t)-m(t)\mid+$$
$$3a_3(t)m(t)Q(t)*\mid A^T(t)(A(t)A^T(t))^{-1}y(t)-m(t)\mid+$$
$$a_3(t)m(t)m(t)m^T(t)+Q(t)A^T(t)(B(t)B^T(t))^{-1}\times$$
$$A(t)Sign[A^T(t)(A(t)A^T(t))^{-1}y(t)-m(t)],$$
$$m(t_0)=E(x(t_0)\mid F_t^Y)),$$
$$\dot{Q}(t)=a_1(t)Q(t)+2a_2(t)m(t)Q(t)+ \qquad (8)$$
$$a_3(t)[Q(t)Q(t)*\mid A^T(t)(A(t)A^T(t))^{-1}y(t)-m(t)\mid+$$
$$3m(t)m^T(t)Q(t)])+b(t)b^T(t),$$
$$Q(t_0)=E[(x(t_0)-m(t_0))\times$$
$$(Sign(A^T(t_0)(A(t_0)A^T(t_0))^{-1}A(t_0)x(t_0)-m(t_0)))^T\mid F_{t_0}^Y].$$

Consequently, this result is formulated in the following theorem and proved in Appendix.

Theorem 1. The mean-module filter for the third degree polynomial system state (6) over the linear observations (2) is given by the equation (7) for the estimate $m(t)$ and the equation (8) for the filter gain matrix $Q(t)$.

IV. EXAMPLE

This section presents an illustrative example of designing the mean-module sliding mode filter for a second degree polynomial state (6) over linear observations (2), using the filtering equations (7),(8).

Consider a scalar linear unmeasured state

$$\dot{x}(t) = 0.1x^2(t) + \psi_1(t), \quad x(0) = x_0, \tag{9}$$

and the scalar linear observation process

$$y(t) = x(t) + \psi_2(t), \tag{10}$$

where $\psi_1(t)$ and $\psi_2(t)$ are white Gaussian noises, which are the weak mean-square derivatives of standard Wiener processes (see [25]). The equations (6),(7) correspond to the alternative conventional form (1^*),(2^*) for the equations (1),(2).

The filtering problem is to find the mean-module estimate for the second degree polynomial state (9), using linear observations (10) confused with independent and identically distributed disturbances modeled as white Gaussian noises.

The filtering equations (7),(8) take the following particular form for the system (9),(10)

$$\dot{m}(t) = 0.1m^2(t) + \tag{11}$$

$$0.1Q(t) \mid y(t) - m(t) \mid + Q(t)sign[y(t) - m(t)],$$

with the initial condition $m(0) = E(x(0) \mid y(0)) = m_0$,

$$\dot{Q}(t) = 0.2m(t)Q(t) + 1, \tag{12}$$

with the initial condition $Q(0) = E((x(0) - m(0))(Sign(x(0) - m(0)))^T \mid y(0))$.

The estimates obtained upon solving the equations (11),(12) are also compared to the estimates satisfying the mean-square filtering equations [24] for the second degree polynomial system (9),(10)

$$\dot{m}_P(t) = 0.1m_P^2(t) + 0.1P(t) + P(t)[y(t) - m_P(t)], \tag{13}$$

with the initial condition $m(0) = E(x(0) \mid y(0)) = m_0$,

$$\dot{P}(t) = 1 + 0.4m_P(t)P(t) - P^2(t), \tag{14}$$

with the initial condition $P(0) = E((x(0) - m(0))(x(0) - m(0))^T \mid y(0))$.

Numerical simulation results are obtained solving the systems of filtering equations (11),(12) and (13),(14). The obtained values of the estimates $m(t)$ and $m_P(t)$ satisfying the equations (11) and (13), respectively, are compared to the real values of the state variables $x(t)$ in (9).

For each of the two filters (11),(12) and (13),(14) and the reference system (9),(10), involved in simulation, the following initial values are assigned: $x_0 = 1$, $m_0 = 4$, $P(0) = Q(0) = 100$. The filtering horizon is set to $T = 0.4$. Gaussian disturbances $\psi_1(t)$ and $\psi_2(t)$ in (9),(10) are realized using the built-in MatLab white noise function.

Note that the initial conditions $P(0)$ and $Q(0)$ are assigned equal for simulation purposes, since the results should be compared with respect to the mean-module criterion (3). If the initial value for Q is assigned as $Q(0) = 10$, the mean-square polynomial filter of [24] would yield a better result as the mean-square polynomial filter.

The following graphs are obtained: graphs of the reference state $x(t)$, satisfying the equation (9), the mean-module sliding mode filter estimate $m(t)$, satisfying the equations (11), and the mean-square polynomial filter estimate $m_P(t)$, satisfying the equation (13), are shown in the entire simulation interval $[0, 0.4]$ in Fig. 1.

It can be observed that the mean-module sliding mode filter (11),(12) yields a certainly better value of the mean-module criterion (3) in comparison to the mean-square polynomial filter (13),(14).

Note that the comparison of the designed mean-module sliding mode filter (11),(12) to the mean-square polynomial filter (13),(14) with respect to the criterion (3) is conducted for illustration purposes, since the filter (11),(12) should theoretically yield a better result, as follows from Theorem 1.

V. APPENDIX

Proof of Theorem 1. According to the general filtering theory based on the innovations process [25], the optimal estimate is a linear function of the minimized residual criterion. For instance, the mean-square polynomial estimate linearly depends on the integral of $x(t) - E(x(t) \mid F_t^Y)$, which is the derivative of the minimized mean-square residue $(1/2)(x(t) - E(x(t) \mid F_t^Y))^T(x(t) - E(x(t) \mid F_t^Y))$, given that the right-side of the mean-square polynomial filter estimate equation linearly includes the derivative term $x(t) - E(x(t) \mid F_t^Y)$ (see [24]). Similarly, the mean-module estimate equation linearly includes the derivative $Sign(x(t) - E(x(t)) \mid F_t^Y)$ of the minimized mean-module residue $\mid x(t) - E(x(t) \mid F_t^Y) \mid$ in the criterion (3). Therefore, the mean-module estimate can be represented by the equation

$$\dot{m}(t) = a_0(t) + a_1(t)m(t) + a_2(t)m(t)m^T(t) + a_2(t)P(t) +$$

$$3a_3(t)m(t)P(t) + a_3(t)m(t)m(t)m^T(t) +$$

$$+Q(t)A^T(t)(B(t)B^T(t))^{-1} \times$$

$$A(t)Sign[A^T(t)(A(t)A^T(t))^{-1}y(t) - m(t)].$$

with the initial condition $m(t_0) = E(x(t_0) \mid F_{t_0}^Y)$. Here, the gain matrix $Q(t)$ should be selected to minimize the conditional variance of the estimation error produced by the estimate $m(t)$. According to the Ito formula (see, for example, [25]), the equation for the estimation error conditional variance $P(t) = E[(x(t) - m(t))(x(t) - m(t))^T \mid F_t^Y]$, produced by the estimate $m(t)$, takes the form

$$\dot{P}(t) = (a_1(t)P(t) + P(t)a_1^T(t) +$$

$$2a_2(t)m(t)P(t)+2(a_2(t)m(t)P(t))^T+$$
$$3(a_3(t)[P(t)P(t)+m(t)m^T(t)P(t)])+$$
$$3(a_3(t)[P(t)P(t)+m(t)m^T(t)P(t)])^T+$$
$$+b(t)b^T(t)-Q(t)A^T(t)(B(t)B^T(t))^{-1}A(t)\times$$
$$E(Sign(A^T(t)(A(t)A^T(t))^{-1}A(t)x(t)-m(t))\times$$
$$\times(x(t)-m(t))^T \mid F_t^Y)-E((x(t)-m(t))\times$$
$$\times(Sign(A^T(t)(A(t)A^T(t))^{-1}A(t)x(t)-m(t))^T \mid F_t^Y)\times$$
$$A^T(t)(B(t)B^T(t))^{-1}A(t)Q^T(t)+$$
$$Q(t)A^T(t)(B(t)B^T(t))^{-1}A(t)Q^T(t).$$

As follows from the preceding equation, the variable $P(t)$ is minimized, if the gain matrix $Q(t)$ is assigned as $Q(t)=E((x(t)-m(t))(Sign(A^T(t)(A(t)A^T(t))^{-1}A(t)x(t)-m(t)))^T \mid F_t^Y)$. In view of the definition of $Q(t)$, the equation for $m(t)$ is represented as (7) and, in view of the Ito formula [25], the equation for $Q(t)$ is given by (8), with the initial condition $Q(t_0)=E[(x(t_0)-m(t_0))(Sign(A^T(t_0)(A(t_0)A^T(t_0))^{-1}A(t_0)x(t_0)-m(t_0)))^T \mid F_{t_0}^Y]$. The theorem is proved. ■

VI. Conclusions

This paper presents a mean-module filtering problem and designs, as a solution, a filter based on a sliding mode gain. The mean-module filtering problem is considered for a stochastic polynomial system with Gaussian white noises. It is shown that the designed sliding mode filter generates the mean-module estimate, which yields a better value of the mean-module criterion in comparison to the mean-square polynomial filter. This conclusion is theoretically proved and numerically verified in an illustrative example. The proposed approach based on involving a sliding mode innovations term is expected to be applicable to other non-mean-square filtering problems for nonlinear systems, where the conventional mean-square polynomial filter would not work.

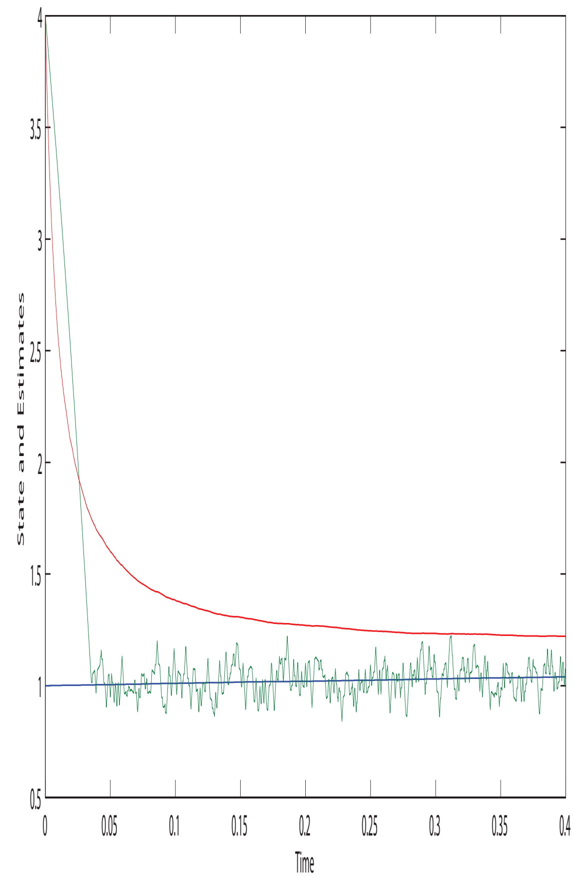

Fig. 1. Graphs of the unmeasured state (9) $x(t)$ (blue), the mean-module sliding mode estimate (11) $m(t)$ (green), and the mean-square polynomial filter estimate (13) $m_P(t)$ (red) in the interval $[0,0.4]$.

References

[1] V. I. Utkin, *Sliding Modes in Control and Optimization*, Springer, 1992.

[2] C. Edwards and S. K. Spurgeon, *Sliding Mode Control: Theory and Applications*, Taylor and Francis, London, 1998.

[3] L. Fridman and A. Levant, "Higher order sliding modes," In *Sliding Mode Control in Engineering* (W. Perruquetti, J. P. Barbot, Eds.), Marcel Dekker, Inc., New York, 2002, pp. 53–101.

[4] V.I. Utkin, J. Guldner, J. Shi, *Sliding Mode Control in Electromechanical Systems*, Taylor and Francis, London, 1999.

[5] S. Suzuki, Y. Pan, K. Furuta, and S. Hatakeyama, "VS-control with time-varying sliding sector: Design and application to pendulum," *Asian Journal of Control*, Vol. 6, 2004, pp. 307–316.

[6] F. Castaños and L. Fridman, "Analysis and design of integral sliding manifolds for systems with unmatched perturbations," *IEEE Trans. Automatic Control*, Vol. 51, 2006, pp. 853–858.

[7] S. Baev, Y. B. Shtessel, C. Edwards, and S. K. Spurgeon, "Output feedback tracking in causal nonminimum-phase nonlinear systems using HOSM techniques," *Proc. 10th International Workshop on Variable Structure Systems*, Antalya, Turkey, 2008, pp. 209–214.

[8] A. Azemi and E. Yaz, "Sliding mode adaptive observer approach to chaotic synchronization," *ASME Transactions. J. Dynamic Systems, Measurements and Control*, Vol. 122, 2000, pp. 758–765.

[9] S.K. Spurgeon, "Sliding mode observers: A survey," *Intern. Journal of Systems Science*, Vol. 39, 2008, pp. 751–764.

[10] I. Boiko, L. Fridman, A. Pisano, and E. Usai, "Analysis of chattering in systems with second order sliding modes," *IEEE Trans. Automatic Control*, Vol. 52, 2007, pp. 2085–2102.

[11] V. I. Utkin and L. Shi, "Integral sliding mode in systems operating under uncertainty conditions," *Proc. 35th Conference on Decision and Control*, Kobe, Japan, 1996, pp. 4591–4596.

[12] G. Bartolini, A. Ferrara, A. Levant, and E. Usai, "On second order sliding mode controllers," In: *Variable Structure Systems, Sliding Mode and Nonlinear Control* (K. D. Young and U. Ozguner, Eds.), *Lecture Notes in Control and Information Series*, Vol. 247, Springer, 1999, pp. 329–350.

[13] M. V. Basin, A. Ferrara, and D. Calderon-Alvarez, "Sliding mode regulator as solution to optimal control problem," *Proc. 47th Conference on Decision and Control*, Cancun, Mexico, 2008, pp. 2184–2189.

[14] M. V. Basin, A. Ferrara, D. Calderon-Alvarez, and F. Dinuzzo, "Sliding mode optimal regulator for a Bolza-Meyer criterion with non-quadratic state energy terms," *Proc. 2009 American Control Conference*, St. Louis, MO, 2009, pp. 4951–4955.

[15] Y. Xia and Y. Jia, "Robust sliding mode control for uncertain stochastic time-delay systems," *IEEE Trans. Automatic Control*, Vol. 48, 2003, pp. 1086–1092.

[16] Y. Niu, D. W. C. Ho and J. Lam, "Robust integral sliding mode control for uncertain stochastic systems with time-varying delay", *Automatica*, Vol. 41, 2005, pp. 873–880.

[17] P. Shi, Y. Xia, G. P. Liu, and D. Rees, "On designing of sliding mode

control for stochastic jump systems," *IEEE Trans. Automatic Control*, Vol. 51, 2006, pp. 97–103.
[18] Y. Niu and D. W. C. Ho, "Robust H_∞ control for nonlinear stochastic systems: A sliding mode approach", *IEEE Trans. Automatic Control*, Vol. 53, 2008, pp. 1695-1701.
[19] M. V. Basin, L. Fridman, and M. Skliar, "Optimal and robust sliding mode filter for systems with continuous and delayed measurements," *Proc. 41st Conference on Decision and Control*, Las Vegas, NV, 2002, pp. 2594–2599.
[20] M. V. Basin, L. Fridman, J. Rodriguez-Gonzalez, and P. Acosta, "Integral sliding mode design for robust filtering and control of linear stochastic time-delay systems," *Intern. J. Robust Nonlinear Control*, Vol. 15, 2005, pp. 407–421.
[21] Y. Niu and D. W. C. Ho, "Robust observer design for Ito stochastic time-delay systems via sliding mode control", *Systems and Control Letters*, Vol. 55, 2006, pp. 781–793.
[22] M. V. Basin and P. Rodriguez-Ramirez, "Sliding mode mean-square filtering for linear stochastic systems," *Proc. 2010 IEEE International Conference on Industrial Technology*, Viña del Mar, Chile, 2010, pp. 1761–1764.
[23] M. V. Basin and P. Rodriguez-Ramirez, "Sliding mode mean-module filtering for linear stochastic systems," *Proc. 2010 IEEE International Conference on Industrial Technology*, Viña del Mar, Chile, 2010, pp. 1757–1760.
[24] M. V. Basin, D. A. Calderon-Alvarez, and M. Skliar, "Optimal filtering for incompletely measured polynomial states over linear observations," *International J. Adaptive Control and Signal Processing*, Vol. 22, 2008, pp. 482–494.
[25] V. S. Pugachev and I. N. Sinitsyn, *Stochastic Systems: Theory and Applications*, World Scientific, Singapore, 2001.
[26] K. J. Åström, *Introduction to Stochastic Control Theory*, Academic Press, New York, 1970.
[27] R. E. Kalman and R. S. Bucy, "New results in linear filtering and prediction theory," *ASME Trans., Part D (J. of Basic Engineering)*, Vol. 83, 1961, pp. 95–108.
[28] A. F. Filippov, *Differential Equations with Discontinuous Righthand Sides*, Kluwer, Dordrecht, 1988.
[29] M. V. Basin, *New Trends in Optimal Filtering and Control for Polynomial and Time-Delay Systems*, Berlin, Heidelberg: Springer-Verlag, 2008.