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Adaptive Actuator Nonlinearity Compensation and Disturbance Rejection with an Aircraft Application

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***Abstract*—In this paper, three adaptive control designs for systems with unknown dynamics and uncertain actuator nonlinearities and/or actuation disturbances are presented. First, an adaptive design utilizing an adaptive inverse is used to compensate for systems with actuator nonlinearities is developed. Then, an adaptive design to reject actuation disturbances is developed. Finally, an adaptive design is developed to compensate both actuator nonlinearities and actuation disturbances simultaneously. This adaptive control design is applied to an aircraft flight control system model with synthetic jet actuator nonlinearities and wind gust disturbances and simulation results are presented to show the desired adaptive system responses.**

1. Introduction

For simple control problems, classical control techniques are often adequate. However, in real applications, unknown system dynamics, including actuator nonlinearities and actuation disturbances, may make it impossible to control a system using classical control techniques alone. Even when the dynamics of a system are not entirely known, adaptive control techniques may be used to achieve control goals in certain systems with unknown system dynamics [9]. Adaptive control is a hot research topic, showing promise to overcome many challenges present in real control systems.

Adaptive algorithms have been used to compensate for actuator nonlinearities in control systems. In [1], [2], [3], and [4], adaptive algorithms are developed for flight control systems to compensate for synthetic jet actuator nonlinearities. An adaptive inverse is used to compensate for a deadzone nonlinearity in [11]. Neural networks and backstepping techniques are used to compensate nonsmooth nonlinearities in uncertain systems in [10].

Disturbance rejection can also be achieved with adaptive control schemes. In [8], adaptive control for rejection of sinusoidal disturbances to unknown systems is studied. An algorithm is experimentally shown to attenuate sinusoidal disturbances of known frequency in time-varying systems. In [5], an adaptive estimator is designed to estimate system states and the frequencies and magnitudes of unknown sinusoidal disturbances for mininum phase and non-minimum phase MIMO systems. In [7], an adaptive control design is presented to achieve state tracking for MIMO systems with system uncertainties and bounded disturbances. Sufficient gain conditions are derived for the controller to yield semi-global asymptotic stability.

In Section 2, we introduce the system models to be controlled. In Section 3, we develop adaptive designs for actuator nonlinearity compensation. In Section 4, we develop an adaptive design for disturbance rejection. In Section 5, we develop an adaptive design for both actuator nonlinearity compensation and disturbance rejection. In Section 6, we study adaptive compensation for a flight control system model with the synthetic jet actuator nonlinearity and actuation disturbances. In Section 7, we present simulation results to demonstrate the desired adaptive compensation control system performance.

2. Problem Statement

The goal of this paper is to expand upon the results in [6]. In [6], adaptive designs were presented to compensate for actuator nonlinearities and actuation disturbances in systems with known dynamics. In this paper, we will expand these designs to include systems with unknown dynamics (a less restrictive case). This enables us to solve the problem of actuator nonlinearities and actuation disturbances for a more general class of systems.

In this study, we consider a single-input single-output linear time invariant system described by

$$\dot{x} = Ax + Bu + B_d d(t),\ u = N(v(t)),\ y = Cx, \quad (2.1)$$

where $A \in \Re^{n\times n}$, $B \in \Re^{n\times 1}$ and $C \in \Re^{1\times n}$ are unknown constant matrices, $B_d \in \Re^{n\times 1}$ is a disturbance vector, $d(t)$ is a disturbance signal, and $N(\cdot)$ is an actuator nonlinearity.

We will first develop adaptive designs for two special cases of this system. The first case is the system with an actuator nonlinearity without disturbance, described by

$$\dot{x} = Ax + Bu,\ u = N(v(t)), y = Cx. \quad (2.2)$$

The second case is the system without the actuator nonlinearity but with disturbance, described by

$$\dot{x} = Ax + Bu + B_d d(t), y = Cx. \quad (2.3)$$

In developing these adaptive designs, we refer to the nominal case system described by

$$\dot{x}(t) = Ax(t) + Bu(t), y(t) = Cx(t). \quad (2.4)$$

For such a nominal system, a nominal feedback law

$$u(t) = k_1^T(t)x(t) + k_2(t)r(t), \tag{2.5}$$

is used, where $k_1(t) \in \Re^n$ and $k_2(t) \in \Re$ are parameters to be updated with an adaptive law. Such a standard adaptive control system has been well understood, and adaptive methods have already been developed in the literature. However, there are still open issues for systems with unknown actuator nonlinearities and actuation disturbances, such as those for a state feedback output tracking adaptive control design which has a simple controller structure for aircraft flight control system applications, and those for adaptive disturbance using state feedback for output tracking.

Actuator Nonlinearities. In real-life control systems, actuators used to control systems are often nonlinear. These nonlinearities must be compensated. In our paper, the actuator nonlinearities $N(\cdot)$ will be parameterizable as

$$u(t) = N(v(t)) = -\theta_a^{*T}\omega_a(t). \tag{2.6}$$

We will specifically study the synthetic jet actuator nonlinearity used for aircraft flight control [1], which for low angles of attack of an aircraft may be parameterizable as

$$N(v(t)) = \theta_{a2}^* - \frac{\theta_{a1}^*}{v(t)}, \tag{2.7}$$

where θ_{a1}^* and θ_{a2}^* are some unknown parameters. We will use this model as a benchmark example in our actuator nonlinearity compensation study.

Actuation Disturbances. Actuation disturbances can come in many forms. For our main topic of interest, flight control, actuation disturbances are usually in the form of wind gusts. We denote the actuation disturbances $\delta(t)$ as

$$\delta(t) = B_d d(t), \tag{2.8}$$

where B_d is the disturbance matrix and $d(t)$ is the disturbance function. We will consider disturbances of the form

$$d(t) = d_0 + \sum_{j=1}^{q} d_j f_j(t), \tag{2.9}$$

where $d_i, i = 0, 1, 2, \ldots, q$, are real constants, and $f_j(t), j = 1, 2, \ldots, q$, are real continuous functions.

Control Objective. The control objective is to make the system output $y(t)$ track the output $y_m(t)$ of a reference model system

$$y_m(t) = W_m(s)[r](t), W_m(s) = \frac{1}{P_m(s)}, \tag{2.10}$$

where $P_m(s)$ is the desired closed-loop characteristic polynomial whose zeros are all stable.

To state the system assumptions, we consider the system (2.4) in the input-output form

$$y(s) = C(sI - A)^{-1}Bu(s) = \frac{Z(s)}{P(s)}u(s) \tag{2.11}$$

where $Z(s) = z_m s^m + \cdots + z_1 s + z_0,\ z_m \neq 0$, and $P(s) = \det(SI - A)$ is a monic polynomial. We can design adaptive control schemes to achieve output tracking with the following assumptions:

Assumption 2.1: $Z(s)$ is a stable polynomial;
Assumption 2.2: The degree of $Z(s)$ is known; and
Assumption 2.3: The sign of z_m is known.

It can be shown that under assumptions 2.1-2.2 there exist matching parameters $k_1^* \in \Re^n$ and $k_2^* \in \Re$ that satisfy

$$\det(sI - A - Bk_1^*) = P_m(s)Z(s)k_2^*, k_2^* = \frac{1}{z_m}. \tag{2.12}$$

This implies that there will always be matching parameters k_1^* and k_2^* that when used in place of $k_1(t)$ and $k_2(t)$, respectively, in the feedback law (2.5), the closed-loop system will match the reference model system.

For the nominal system (2.4), we can design adaptive laws to update the estimates $k_1(t)$ and $k_2(t)$ for k_1^* and k_2^* when unknown, to ensure output tracking is achievable.

For the system (2.2) with actuator nonlinearities, the system (2.3) with actuation disturbances, and the system (2.1) with both actuator nonlinearities and disturbances, adaptive control designs will be developed next, with new features and properties associated with system structures.

3. Adaptive Design for Actuator Nonlinearity Compensation

An adaptive actuator nonlinearity compensation scheme for a system with an actuator nonlinearity and known (A, B, C) was proposed in [6]. We will develop this scheme the case with unknown (A, B, C), in a state feedback control framework useful for aircraft flight control. First, we derive a compensation algorithm, then we apply the algorithm to a system with the synthetic jet actuator nonlinearity.

A system with an actuator nonlinearity is described by (2.2). The control objective is to make the output of this system track the output of a reference model system. Since this system to be controlled has actuator nonlinearity $N(v(t))$, parameterizable as (2.6), We employ an adaptive inverse $v(t) = \widehat{NI}(u_d(t))$ that is parameterizable as

$$u_d(t) = -\theta_a^T(t)\omega_a(t), \tag{3.1}$$

where $u_d(t)$ is a control signal from a feedback design, $\omega_a(t)$ is some known signal which contains the signal $v(t)$, and $\theta_a(t)$ is an adaptively updated estimate of θ_a^* from the actuator nonlinearity $N(v(t))$ parameterized in (2.6).

For the synthetic jet nonlinearity (2.7), the adaptive inverse is

$$v(t) = \widehat{NI}(u_d(t)) = \frac{\theta_{a1}(t)}{\theta_{a2}(t) - u_d(t)} \tag{3.2}$$

where $\theta_{a1}(t)$ and $\theta_{a2}(t)$ are the estimates of the unknown parameters θ_{a1}^* and θ_{a2}^*, and $u_d(t)$ is a design feedback signal. This inverse can be expressed in the form of $u_d(t) = -\theta_a^T(t)\omega_a(t)$ for $\theta_a = [\theta_{a1}, \theta_{a2}]^T$ and $\omega_a(t) = [\frac{1}{v(t)}, -1]^T$.

In addition to the parameters $\theta_a(t)$ of the actuator nonlinearity inverse, the parameters $k_1(t)$ and $k_2(t)$ are adaptively updated. We construct the feedback control law to generate the signal $u_d(t)$:

$$u_d(t) = k_1^T(t)x(t) + k_2(t)r(t), \tag{3.3}$$

where $r(t)$ is a bounded signal and the parameters $k_1(t)$ and $k_2(t)$ are updated with the adaptive law described later in this Section. Using (2.6) and (3.2), the control error can be parameterized as

$$u(t) - u_d(t) = -\theta_a^{*T}\omega_a^*(t) + \theta_a^T(t)\omega_a(t). \tag{3.4}$$

Using (3.4) and (3.3), we can rewrite $u(t)$ as

$$u(t) = k_1^{*T}x + k_2^* r + (\theta(t) - \theta^*)^T\omega(t) + d_a(t), \tag{3.5}$$

where error caused by the uncertain actuator nonlinearity is

$$d_a(t) = -\theta_a^{*T}(\omega_a(t) - \omega_a^*(t)),; \tag{3.6}$$
$$\theta(t) = [k_1^T(t), k_2(t), \theta_a(t)]^T, \tag{3.7}$$
$$\theta^* = [k_1^{*T}, k_2^*, \theta_a^*]^T, \tag{3.8}$$
$$\omega(t) = [x^T(t), r(t), \omega_a(t)]^T. \tag{3.9}$$

The system (2.2) with unknown parameters (A, B, C) and $u(t)$ from (3.5) can be expressed as

$$y(t) = W_m(s)[r](t) + \rho^* W_m(s)[(\theta-\theta^*)^T\omega_a + d_a](t) + \epsilon_0(t), \tag{3.10}$$

where $\rho^* = z_m$ and ϵ_0 is an exponentially decaying term due to the initial conditions. With $\bar{d_a}(t) = \rho^* W_m(s)[d_a](t)$, and using the controlled system output (3.10) and the reference model output (2.10), we have the output error as

$$y(t) - y_m(t) = \rho^* W_m(s)[(\theta - \theta^*)^T\omega](t) + \bar{d_a}(t), \tag{3.11}$$

where the term ϵ_0 has been ignored. We then define the estimation error

$$\epsilon(t) = y(t) - y_m(t) + \rho(t)\xi(t), \tag{3.12}$$

where

$$\xi(t) = \theta^T(t)\zeta(t) - W_m(s)[\theta^T\omega](t), \tag{3.13}$$
$$\zeta(t) = W_m(s)[\omega](t). \tag{3.14}$$

With (3.11)-(3.14), the estimation error simplifies to

$$\epsilon(t) = \rho^*(\theta(t)-\theta^*)^T\zeta(t) + (\rho(t)-\rho^*)\xi(t) + \bar{d_a}(t). \tag{3.15}$$

Adaptive Laws. We choose initial parameters that satisfy

$$\theta_i(0) \in [\theta_i^a, \theta_i^b], \tag{3.16}$$

for $i = 1, 2, ..., n + 1 + n_\theta$. We define the adaptive laws

$$\dot{\theta}(t) = -\frac{\text{sign}[\rho^*]\Gamma\epsilon(t)\zeta(t)}{m^2(t)} + f(t), \tag{3.17}$$

$$\dot{\rho}(t) = -\frac{\gamma_\rho\xi(t)\epsilon(t)}{m^2(t)},\ \gamma_\rho > 0, \tag{3.18}$$

where $m(t) = \sqrt{1 + \zeta^T(t)\zeta(t) + \xi^2(t)}$, and

$$f_i(t) = \begin{cases} 0 & \text{if } \theta_i(t) \in (\theta_i^a, \theta_i^b) \text{ or} \\ & \text{if } \theta_i(t) = \theta_i^a \text{ and } g_i(t) \geq 0 \text{ or} \\ & \text{if } \theta_i(t) = \theta_i^b \text{ and } g_i(t) \leq 0, \\ -g_i(t) & \text{otherwise,} \end{cases}$$

$$\Gamma = \text{diag}\{\gamma_1, \ldots, \gamma_{n+1+n_\theta}\} = \Gamma^T > 0. \tag{3.19}$$

For our stability analysis, we consider a positive definite function $V(\tilde{\theta}, \tilde{\rho}) = |\rho^*|\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} + \gamma_\rho^{-1}\tilde{\rho}^2$, where $\tilde{\theta}(t) = \theta(t) - \theta^*$ and $\tilde{\rho}(t) = \rho(t) - \rho^*$. Using the above adaptive laws, we find the time derivative

$$\dot{V} = -\frac{\epsilon^2(t)}{m^2(t)},\ t \geq 0. \tag{3.20}$$

This ensures that $\theta_i(t) \in [\theta_i^a, \theta_i^b]$ for $i = 1, 2, ..., n + 1 + n_\theta$, and that $\int_{t_1}^{t_2}\frac{\epsilon^2(t)}{m^2(t)}\,dt \leq a_1 + b_1\int_{t_1}^{t_2}\frac{\bar{d_a}^2(t)}{m^2(t)}\,dt$, $\int_{t_1}^{t_2}\|\dot{\theta}(t)\|_2^2\,dt \leq a_2 + b_2\int_{t_1}^{t_2}\frac{\bar{d_a}^2(t)}{m^2(t)}\,dt$ and $\int_{t_1}^{t_2}\|\dot{\rho}(t)\|_2^2\,dt \leq a_2 + b_2\int_{t_1}^{t_2}\frac{\bar{d_a}^2(t)}{m^2(t)}\,dt$ for constants $a_1, a_2, b_1, b_2 > 0$ and all $t_2 > t_1 \geq 0$. The use of parameter projection is to ensure a desired implementation of the adaptive inverse (3.2).

4. Adaptive Design for Disturbance Rejection

To design adaptive algorithms for actuation disturbance rejection, we first develop a disturbance reject parametrization, and then derive an adaptive algorithm for compensating for the actuation disturbances in a system when the system matrices A, B and C are unknown. We will apply this algorithm to a flight control system model.

For output tracking to be achievable, the same assumptions as Assumptions 2.1-2.3 for the system dynamics are needed. Additionally, when disturbances are nonconstant, the following matching equation also needs to be satisfied:

$$\frac{1}{k_2^*}W_m(s)u_0^* + C(sI - A - Bk_1^{*T})^{-1}B_d = 0 \tag{4.1}$$

for some $u_0^* \in \Re$. The following proposition is used to determine whether or not the above matching condition is satisfied and will be used for the aircraft model evaluation.

Proposition 4.1: [9] *Assuming (A, B) is controllable, there exist constants $k_1^* \in \Re^n$, k_2^*, $u_0^* \in \Re$ such that*

$$C(sI - A - Bk_1^{*T})^{-1}Bk_2^* = W_m(s) = \frac{1}{P_m(s)}, \tag{4.2}$$

and matching equation 4.1 is satisfied, where $P_m(s)$ is a stable monic polynomial of degree n^*, if and only if (C, A, B) and (C, A, B_d) have the same relative degree n^*.

When the above condition is true, we develop the compensation term

$$u_0(t) = u_{00}(t) + \sum_{j=1}^{q} u_{0j}(t)f_j(t), \tag{4.3}$$

which is an estimate of

$$u_0^*(t) = u_{00}^* + \sum_{j=1}^{q} u_{0j}^* f_j(t), \quad (4.4)$$

where $u_{0j}^* = u_0^* d_j,\ j = 0,\ 1,\ 2,\ \ldots,\ q$, or alternatively

$$u_0^*(t) = u_0^* d(t). \quad (4.5)$$

We can express $u_0(t)$ and $u_0^*(t)$ as

$$u_0(t) = \theta_d^T(t)\omega_d(t), \quad (4.6)$$
$$u_0^*(t) = \theta_d^{*T}\omega_d(t), \quad (4.7)$$

respectively, where

$$\theta_d(t) = [u_{00}(t),\ u_{01}(t),\ \ldots,\ u_{0q}(t)]^T, \quad (4.8)$$
$$\theta_d^* = [u_{00}^*,\ u_{01}^*,\ \ldots,\ u_{0q}^*]^T, \quad (4.9)$$
$$\omega_d(t) = [1\ , f_1(t),\ \ldots,\ f_q(t)]^T. \quad (4.10)$$

We then choose the adaptive control law

$$u(t) = k_1^T(t)x(t) + k_2(t)r(t) + u_0(t), \quad (4.11)$$

for disturbance rejection, where $u_0(t)$ is for either a constant or nonconstant disturbance $d(t)$, as defined in (4.6).

The tracking error is

$$y - y_m = \rho^* W_m(s)[(k_1(t) - k_1^*)^T x + (k_2(t) - k_2^*)r + (\theta_d - \theta_d^*)\omega_d](t) + \epsilon_0(t),$$

where $W_m(s) = \frac{1}{P_m(s)}$, where $P_m(s)$ is a stable monic polynomial of degree n^* and $\epsilon_0(t) = Ce^{A+bk_1^{*T}t}x(0)+\delta_d(t)$ converges to zero exponentially.

We define the estimation error

$$\epsilon(t) = y(t) - y_m(t) + \rho(t)\xi(t), \quad (4.12)$$

where

$$\xi(t) = \theta^T(t)\zeta(t) - W_m(s)[\theta^T\omega](t), \quad (4.13)$$
$$\zeta(t) = W_m(s)[\omega](t), \quad (4.14)$$
$$\theta(t) = [k_1^T(t), k_2(t), \theta_d^T(t)]^T, \quad (4.15)$$
$$\omega(t) = [x(t), r(t), \omega_d^T(t)]^T. \quad (4.16)$$

Adaptive Laws. We then choose the adaptive laws

$$\dot{\theta}(t) = -\frac{\text{sign}[\rho^*]\Gamma\zeta(t)\epsilon(t)}{m^2(t)},\ \Gamma = \Gamma^T > 0, \quad (4.17)$$

$$\dot{\rho}(t) = -\frac{\gamma_\rho\xi(t)\epsilon(t)}{m^2(t)},\ \gamma_\rho > 0, \quad (4.18)$$

where $m(t) = \sqrt{1 + \zeta^T(t)\zeta(t) + \xi^2(t)}$.

For our stability analysis, we consider a positive definite function $V(\tilde{\theta}, \tilde{\rho}) = |\rho^*|\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} + \gamma_\rho^{-1}\tilde{\rho}^2$, where $\tilde{\theta}(t) = \theta(t) - \theta^*$ and $\tilde{\rho}(t) = \rho(t) - \rho^*$. Using the above adaptive laws, we find the time derivative

$$\dot{V} = -\frac{\epsilon^2(t)}{m^2(t)},\ t \geq 0. \quad (4.19)$$

This ensures that $\theta(t) \in L^\infty,\ \rho(t) \in L^\infty,\ \frac{\epsilon(t)}{m(t)} \in L^2 \cap L^\infty$, $\dot{\theta}(t) \in L^2 \cap L^\infty$, and $\dot{\rho}(t) \in L^2 \cap L^\infty$. We have satisfied the objectives that all closed-loop signals are bounded, $y(t) - y_m(t) \in L^2$ and $\lim_{t\to\infty}(y(t) - y_m(t)) = 0$.

5. Adaptive Designs for Actuator Nonlinearity Compensation and Disturbance Rejection

In this Section, we will develop algorithms for the system (2.1) that has both actuator nonlinearities and actuation disturbances. We will first develop an algorithm for the case with unknown $(A,\ B,\ C)$. We will then apply the algorithm to a flight control system model with the synthetic jet actuator nonlinearity and nonconstant disturbance. For output tracking to be achievable, the same assumptions as Assumptions 2.1-2.3 for disturbance rejection or actuator nonlinearity compensation are needed. Additionally, when disturbances are nonconstant, the relative degree condition from Proposition 4.1 should also be satisfied.

In this case, we choose the control law as

$$u_d(t) = k_1^T(t)x(t) + k_2(t)r(t) + u_0(t), \quad (5.1)$$

where $u_0(t)$ is the disturbance rejection signal. The signal $u_d(t)$ is used for the adaptive inverse (3.2) to generate the control signal $v(t)$ for the system (2.1). The tracking error can be expressed as

$$y - y_m = \rho^* W_m(s)[(\theta - \theta^*)^T\omega](t) + \bar{d}_a(t) + \epsilon_0(t), \quad (5.2)$$

with

$$\theta(t) = [k_1^T(t),\ k_2(t),\ \theta_a^T(t),\ \theta_d^T(t)]^T, \quad (5.3)$$
$$\theta^* = [k_1^{*T},\ k_2^*,\ \theta_a^{*T},\ \theta_d^{*T}]^T, \quad (5.4)$$
$$\omega(t) = [x^T(t),\ r(t),\ \omega_a^T(t),\ \omega_d^T(t)]^T. \quad (5.5)$$

We also choose the disturbance rejection signal (4.3).

We define the estimation error

$$\epsilon(t) = y(t) - y_m(t) + \rho(t)\xi(t), \quad (5.6)$$

where

$$\xi(t) = \theta^T(t)\zeta(t) - W_m(s)[\theta^T\omega](t), \quad (5.7)$$
$$\zeta(t) = W_m(s)[\omega](t). \quad (5.8)$$

Adaptive Laws. We then use the adaptive laws for updating the parameter estimates $\theta(t)$ and $\rho(t)$:

$$\dot{\theta}(t) = -\frac{\text{sign}[\rho^*]\Gamma\zeta(t)\epsilon(t)}{m^2(t)} + f(t),\ \Gamma = \Gamma^T > 0,$$
$$\dot{\rho}(t) = -\frac{\gamma_\rho\xi(t)\epsilon(t)}{m^2(t)},\ \gamma_\rho > 0, \quad (5.9)$$

where $m(t) = \sqrt{1 + \zeta^T(t)\zeta(t) + \xi^2(t)}$, and

$$f_i(t) = \begin{cases} 0 & \text{if } \theta_i(t) \in (\theta_i^a, \theta_i^b) \text{ or} \\ & \text{if } \theta_i(t) = \theta_i^a \text{ and } g_i(t) \geq 0 \text{ or} \\ & \text{if } \theta_i(t) = \theta_i^b \text{ and } g_i(t) \leq 0, \\ -g_i(t) & \text{otherwise,} \end{cases}$$

with $\theta_i^a, i = n+1, \ldots, n+n_a$ and $\theta_i^b, i = n+1, \ldots, n+n_a$ as the lower and upper bounds, respectively, to ensure there are no singularities in the adaptive inverse.

For our stability analysis, we consider a positive definite function $V(\tilde{\theta}, \tilde{\rho}) = |\rho^*|\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} + \gamma_\rho^{-1}\tilde{\rho}^2$, where $\tilde{\theta}(t) =$

$\theta(t) - \theta^*$ and $\tilde{\rho}(t) = \rho(t) - \rho^*$. Using the above adaptive laws, we find the time derivative

$$\dot{V} = -\frac{\epsilon^2(t)}{m^2(t)}, t \geq 0. \tag{5.10}$$

This verifies that that $\theta_i(t) \in [\theta_i^a, \theta_i^b]$ for $i = 1, 2, ..., n_a + n_d$, and that $\int_{t_1}^{t_2} \frac{\epsilon^2(t)}{m^2(t)}\, dt \leq a_1 + b_1 \int_{t_1}^{t_2} \frac{\bar{d_a}^2(t)}{m^2(t)}\, dt$, $\int_{t_1}^{t_2} \|\dot{\theta}(t)\|_2^2\, dt \leq a_2 + b_2 \int_{t_1}^{t_2} \frac{\bar{d_a}^2(t)}{m^2(t)}\, dt$ and $\int_{t_1}^{t_2} \|\dot{\rho}(t)\|_2^2\, dt \leq a_2 + b_2 \int_{t_1}^{t_2} \frac{\bar{d_a}^2(t)}{m^2(t)}\, dt$ for constants $a_1, a_2, b_1, b_2 > 0$ and all $t_2 > t_1 \geq 0$. For the synthetic jet actuator nonlinearity (2.7) and its inverse (3.2), the effect of d_a can be further reduced.

6. An Aircraft Flight Control System Study

Consider a linearized lateral flight control system model, described by (2.1) with

$$A = \begin{bmatrix} -0.0134 & 48.5474 & -632.3724 & 32.0756 \\ -0.0199 & -0.1209 & 0.1628 & 0 \\ -0.0024 & -0.0526 & -0.0252 & 0 \\ 0 & 1 & 0.0768 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ -0.0431 \\ -0.0076 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}^T,$$

from [4] are unknown. The states are lateral velocity $x_1(t)$, roll rate $x_2(t)$, yaw rate $x_3(t)$, and roll angle $x_4(t)$. The input $u(t)$ is the equivalent aileron angular position (generated by a set of synthetic jet actuators). The input $v(t)$ is used to control a synthetic jet actuator to change the output $y(t) = x_4(t)$, or roll angle. The system has an actuation (wind gust) disturbance $\delta(t) = B_d d(t)$ and the synthetic jet actuator nonlinearity (2.7). The transfer function of $(C,\ A,\ B)$ has relative degree 2, so we choose the reference model with a $P_m(s)$ of degree 2:

$$W_m(s) = \frac{1}{P_m(s)} = \frac{1}{s^2 + s + 1}. \tag{6.1}$$

We will now study the nature of the disturbance and ensure that it can be rejected. The disturbance function $d(t)$ represents the effect of the wind forces on $\dot{x}(t)$. The disturbance may have a physical meaning on the accelerations, $\dot{x}_1(t)$, $\dot{x}_2(t)$ and $\dot{x}_3(t)$. A disturbance component $\delta_i(t)$ to each $\dot{x}_i(t)$, is of the form $\delta_i(t) = B_{di} d_i(t)$, $i = 1, 2, 3$, where $B_{d1} = [1\ 0\ 0\ 0]^T$, $B_{d2} = [0\ 1\ 0\ 0]^T$, and $B_{d3} = [0\ 0\ 1\ 0]^T$. Then the disturbance to the aircraft can be described as $\delta(t) = \sum_{i=1}^{3} B_{di}\delta_i(t)$. The lateral velocity x_1 and lateral acceleration $\dot{x}_1$ do not affect the output $y = x_4$, therefore the disturbance $\delta_1(t)$ does not need to be rejected for output tracking. Transfer functions $(C,\ A,\ B_{d2})$ and $(C,\ A,\ B_{d3})$ both have relative degree 2. This implies that u_0^* from (4.4) exists for the system with disturbances $d_2(t)$ and $d_3(t)$, that is, $d_2(t)$ and $d_3(t)$ can be rejected. Since the condition of Proposition 4.1 is satisfied, we conclude that disturbance rejection is achievable for this aircraft control system. For this system, we use the adaptive inverse (3.2) to compensate for the actuator nonlinearity $N(\cdot)$, and the control law (5.1). We notice a singularity to avoid when $u_d(t) = \theta_{a2}$. We continue developing the algorithm under the assumption that u_d is smaller than θ_{a2}, then modify the algorithm for when the assumption does not hold. The estimate $\theta_d(t)$ of θ_d^* and the signal $\omega_d(t)$ of the disturbance rejection signal $u_0(t)$ are chosen from (4.3).

We express the output of the controlled system as

$$y(t) = y_m(t) + \frac{1}{k_2^*} W_m(s)[(\theta - \theta^*)^T \omega](t) + \epsilon_0(t), \tag{6.2}$$

where $\theta_a(t) = [\theta_{a1}(t), \theta_{a2}(t)]^T$, $\theta_a^* = [\theta_{a1}^*,\ \theta_{a2}^*]^T$, and $\omega_a(t) = [\frac{1}{v(t)}, -1]^T$ are derived in Section 3, $u_0 \in \Re^{n_d}, u_0^* \in \Re^{n_d}$, $\theta_d(t) \in \Re^{n_d}$, $\theta_d^* \in \Re^{n_d}$ and $\omega_d(t) \in \Re^{n_d}$ are derived in Section 4, $\theta(t) = [k_1^T(t),\ k_2(t),\ \theta_a^T(t),\ \theta_d^T(t)]^T$, $\omega(t) = [x(t),\ r(t),\ \omega_a^T(t),\ \omega_d^T(t)]^T$ and $\epsilon_0(t)$ is the exponentially decaying term.

We use the adaptive laws (5.9), ensuring that $\theta(t) \in L^\infty$, $\rho(t) \in L^\infty$, $\frac{\epsilon(t)}{m(t)} \in L^2 \cap L^\infty$, $\dot{\theta}(t) \in L^2 \cap L^\infty$, and $\dot{\rho}(t) \in L^2 \cap L^\infty$. It can be shown that under the condition that $u_d(t) < \theta_{a2}$, that is, the singularity is avoided, all closed-loop signals are bounded, $y(t) - y_m(t) \in L^2$ and $\lim_{t\to\infty}(y(t) - y_m(t)) = 0$.

We now modify our scheme to compensate for the case in which $u_d(t)$ approaches $\theta_{a2}(t)$. We choose a small δ_a, and use the new feedback control law

$$u_d(t) = \begin{cases} \theta_{a2}(t) - \delta_a & \text{if } \bar{u}_d(t) \geq \theta_{a2}(t) - \delta_a \\ k_1^T(t)x(t) + k_2(t)r(t) + u_0(t) & \text{otherwise,} \end{cases}$$

where $\bar{u}_d(t) = k_1^T(t)x(t) + k_2(t)r(t) + u_0(t)$. In a local system operation, this could prevent the condition $v(t) = \infty$ which would cause a failure to reach our control goals.

7. Simulation Study

The use of an adaptive algorithm to overcome actuator nonlinearities and actuation disturbance was demonstrated in [6] for systems with known dynamics. In this simulation, we verify the adaptive algorithm developed in Section 5 for the more general case of systems with unknown dynamics.

In our simulation study, we will examine the performance of the adaptive control scheme. We will use the system (2.1) with unknown system matrices, the synthetic jet actuator nonlinearity (2.7), and a sinusoidal disturbance.

We use the reference model (6.1) and the system matrices from Section 6, we calculate k_1^* and k_2^* to be $[-.4597\ \ \frac{\text{deg}\cdot\text{sec}}{\text{ft}},\ \ 20.0309\ \text{sec},\ \ 5.4434\ \text{sec},\ \ 22.8914]^T$ and -22.8912 respectively. The parameter θ_a^* is arbitrarily chosen to be $[33.33\ \text{volt}\cdot\text{deg},\ 20\ \text{deg}]^T$.

We arbitrarily choose the initial state $x(0) = [0.01\ \frac{\text{ft}}{\text{sec}},\ -0.001\ \frac{\text{deg}}{\text{sec}},\ 0.015\ \frac{\text{deg}}{\text{sec}},\ 0.01\ \text{deg}]^T$, the initial actuator nonlinearity parameter $\theta_a(0) = [32\ \text{volt}\cdot\text{deg},\ 27\ \text{deg}]^T$ with $\omega_a = [\frac{1}{v(t)}, -1]^T$, initial disturbance rejection parameter $\theta_d(0) = [0\ \text{deg},\ 0\ \text{deg}]^T$ with $\omega_d = [\sin(3t),\ \ 1]^T$, and initial parameter estimates $k_1(0) =$

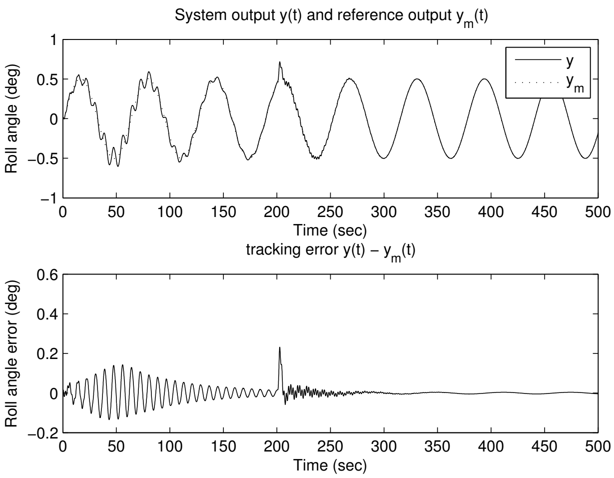

Fig. 1. System output and tracking error.

$[-0.5\ \frac{\text{deg}\cdot\text{sec}}{\text{ft}},\ 20\ \text{sec},\ 5.5\ \text{sec},\ 23]^T$ and $k_2(0) = -25$, and $\rho(0) = -0.04$. We choose a reference signal $r(t) = 0.5\sin(.1t)$ degrees. We choose $\gamma_\rho = 10$ and $\Gamma = \text{diag}\{5000, 5000, 250000, 30000, 100, 100, 100, 100, 100\}$.

The system has sinusoidal disturbance (in degrees):

$$d(t) = \begin{cases} 0 & \text{for } t < 200 \\ 0.25(t-200)(\sin(3t)+1) & \text{for } 200 \le t \le 201 \\ 0.25(\sin(3t)+1) & \text{for } t > 201, \end{cases}$$

with disturbance matrix $B_d = [0\ 1\ 0\ 0]^T$.

From Figure 1 we see that output tracking is achieved. The adaptive parameters compensate for the initial errors in estimates of k_1^*, k_2^* and θ_a^*, and adjust when the disturbance at $t = 200$ is applied to achieve output tracking. The effect of the disturbance is still present in $x_1(t)$ and $x_3(t)$, as shown in Figure 2. This shows that disturbance rejection is achieved at the chosen output whose tracking is a design goal.

8. Conclusions

In this paper, we have recognized the challenges caused by actuator nonlinearities and actuation disturbances in systems with unknown (A, B, C). We have derived adaptive algorithms for actuator nonlinearity compensation and actuation disturbance rejection that use state feedback in systems with unknown (A, B, C). We have applied these algorithms to realistic models for flight control systems and found that they can achieve output tracking. We have developed an expanded adaptive algorithm to simultaneously compensate for actuator nonlinearities and reject disturbances and shown their potential for application to real-life system models. We have illustrated the advantages of adaptive control schemes in the presence of disturbances, system dynamics uncertainties and actuator nonlinearities in simulation.

In future work, we will expand our methods for use in multi-input multi-output systems.

Acknowledgments

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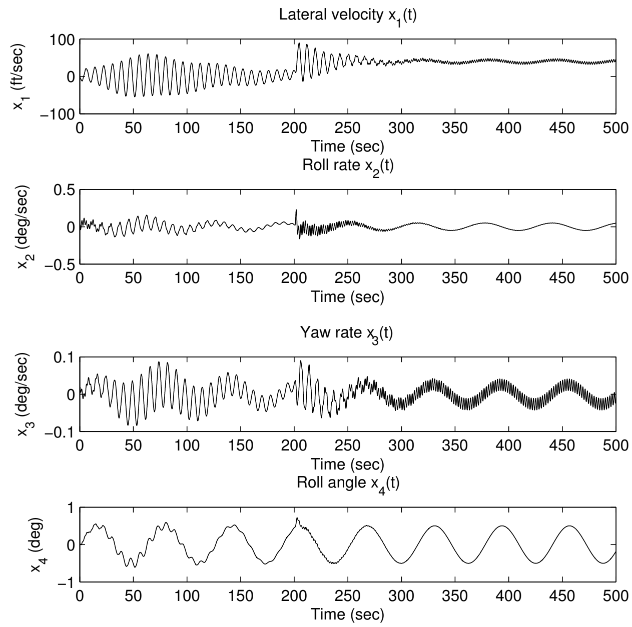

Fig. 2. State variables.

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