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Observer synthesis for a class of descriptor LPV systems

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***Abstract*— A method to synthesize an observer for continuous-time Descriptor Linear Parameter Variant (D-LPV) systems is presented in this paper. The main contribution consists of an observer synthesis developed for descriptor linear time invariant extended to LPV systems. The conditions for the existence of LPV observer are given. Such conditions guarantee the observer convergence and they are proved through a Lyapunov-like analysis based on Linear Matrix Inequality (LMI) formulation. The observer is evaluated through numerical simulations.**

I. INTRODUCTION

The observer synthesis for descriptor systems (also known as singular systems) has been widely investigated in the literature, see for instance [1], [2], [3], [4]. Furthermore, in the last years, more researchers have paid attention to the problem of observer synthesis for Linear Parameter Variant systems (LPV). LPV systems can be considered to approximate nonlinear systems and hence systematic and generic available theoretical results for LPV systems can be then applied to derive nonlinear control laws. Some examples of practical processes modeled as descriptor LPV systems are: aircrafts [13], [14], mechanical systems [15] and chemical processes [16]. For instance, [5] consider a LPV representation to model and to control diesel engines. Fault diagnosis methods dedicated to LPV systems are presented in [6]. Fault tolerant control design for polytopic LPV systems is treated in [7], whereas in [8], both fault detection and control of LPV systems is presented. Unknown input reconstruction for LPV systems to design fault detection filters is presented in [9]. However, to the best of our knowledge, few works are only studied observers and controllers for descriptor LPV systems. Although the idea of merging descriptor and LPV systems is not new (see for instance [10], [11] and more recently [12]), but there is not a general approach of observer synthesis for affine, multi-affine, polynomial or rational descriptor LPV systems.

The main contribution of this paper consists of the observer design for descriptor LPV systems where the regularity assumption is not required. The proposed approach extends the results obtained in [2] where a method to design full-order observers for non-regular descriptor linear time invariant systems has been considered. Then the synthesis of an observer for polytopic descriptor LPV systems is presented. The existence conditions of a LPV observer synthesized with appropriate transformations are given. Such conditions guarantee the observer convergence proved through a combined method based on the original approach proposed in [2] and a Lyapunov-like analysis. The LPV observer is evaluated through numerical simulations. This paper complements the ideas presented in [12] where the authors proposed an observer for a class of descriptor LPV systems where gains are computed by constant matrices according to from a system transformation.

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II. OBSERVER SYNTHESIS

A. Problem formulation

Consider the following descriptor LPV system:

$$\begin{aligned} E\dot{x}(t) &= A(\rho(t))x(t) + B(\rho(t))u(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^q$, $y(t) \in \mathbb{R}^p$ are the state, the measured input and the measured output, respectively, $E \in \mathbb{R}^{m\times n}$, $C \in \mathbb{R}^{p\times n}$ are constant matrices and $A(\rho(t)) \in \mathbb{R}^{m\times n}$, $B(\rho(t)) \in \mathbb{R}^{m\times q}$ are time varying matrices. It should be noticed that E and $A(\rho)$ are two square matrices if $m = n$ or two non-square matrices if $m \neq n$.

Two main classes of LPV systems can be identified: affine LPV systems where the parameter-varying matrices depend affinely on $\rho(t)$ and polytopic LPV systems where the parameter $\rho(t)$ varies in a convex polytope whose vertices are denoted by ρ_i, i.e. $\rho(t) \in Co\{\rho_1, \rho_2, \ldots, \rho_\sigma\}$, where σ is the number of vertices included in the polytope [11]. In the sequel the second class is considered. The polytopic D-LPV system (1) have the following form:

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{M} \varepsilon_i(\rho(t))\left(A_i x(t) + B_i u(t)\right) \\ y(t) &= Cx(t) \end{aligned} \tag{2}$$

where

$$\sum_{i=1}^{M} \varepsilon_i(\rho(t)) = 1, \;\; \varepsilon_i(\rho(t)) \geq 0 \tag{3}$$

with $i = 1, \ldots, M$, and M is the total number of scheduling functions $\varepsilon_i(\rho(t))$.

The vector $\varepsilon(\rho(t))$ evolves over the convex set defined by [17]:

$$\Gamma = \left\{ \underset{i}{\text{col}} : \sum_{i=1}^{M} \varepsilon_i(\rho(t)) = 1,\ \varepsilon_i(\rho(t)) \geq 0 \right\}$$

The following assumptions are considered [2]:

(**A1**) The rank r of the matrix E is smaller than the number of states n, i.e. $\text{rank}\, E = r < n$.

(**A2**) $\text{rank} \begin{pmatrix} E \\ C \end{pmatrix} = n$.

Considering assumption **A2**, there exists a nonsingular matrix Δ:

$$\Delta = \begin{pmatrix} \alpha & \beta \\ \gamma & \xi \end{pmatrix}$$

such that

$$\alpha E + \beta C = I_n \tag{4}$$

$$\gamma E + \xi C = 0 \tag{5}$$

where $\alpha \in \mathbb{R}^{n\times m}$, $\beta \in \mathbb{R}^{n\times p}$, $\gamma \in \mathbb{R}^{(m+p-n)\times m}$ and $\xi \in \mathbb{R}^{(m+p-n)\times p}$ are constant matrices which can be found by the Singular Value Decomposition of $\begin{pmatrix} E \\ C \end{pmatrix}$. This is accomplished by computing two orthogonal matrices X and Y such that

$$X \begin{bmatrix} E \\ C \end{bmatrix} = \begin{bmatrix} \Xi Y \\ 0 \end{bmatrix} \tag{6}$$

where $\Xi = \text{diag}(\varsigma_1, \ldots, \varsigma_j, \ldots, \varsigma_n)$, $\varsigma_j > 0$, $j = 1, \ldots, n$. The matrix Δ is then given by

$$\Delta = \begin{bmatrix} Y\Xi^{-1} & 0 \\ 0 & I_{m+p-n} \end{bmatrix} X^T \tag{7}$$

Based on [2], an observer associated to system (2) is proposed such as:

$$\mathcal{O} : \left\{ \begin{array}{rcl} \dot{z}(t) & = & \sum_{i=1}^{M} \varepsilon_i(\rho(t)) \left[N_i z(t) + L_{1i} y(t) \right. \\ & + & \left. G_i u(t) + L_{2i} y(t) \right] \\ \hat{x}(t) & = & z(t) + \beta y(t) + K\xi y(t) \end{array} \right. \tag{8}$$

where $z(t) \in \mathbb{R}^n$ is the state vector and $\hat{x}(t) \in \mathbb{R}^n$ is the estimated state vector. The inputs are the measured outputs $y(t)$ and the process inputs $u(t)$. The matrices N_i, L_{1i}, L_{2i}, G_i and K, $\forall i = 1, \ldots, M$ should be determined such that the estimates of the state variables $\hat{x}(t) \in \mathbb{R}^n$ converge asymptotically to $x(t)$. For the sake of simplicity, the following notation is used:

$$\Omega(\rho(t)) = \Omega(\rho) = \sum_{i=1}^{M} \varepsilon_i(\rho(t)) \Omega_i \ \forall i = 1, \ldots, M$$

Thus, the system (8) is rewritten as follows:

$$\begin{array}{rcl} \dot{z}(t) & = & N(\rho) z(t) + L_1(\rho) y(t) \\ & + & G(\rho) u(t) + L_2(\rho) y(t) \\ \hat{x}(t) & = & z(t) + \beta y(t) + K\xi y(t) \end{array} \tag{9}$$

Let the observer error be defined as:

$$e(t) = x(t) - \hat{x}(t) \tag{10}$$

Replacing $\hat{x}(t)$ from (9) into (10):

$$e(t) = x(t) - z(t) - \beta C x(t) - K\xi C x(t) \tag{11}$$

Considered βC and ξC from (4) and (5), (11) becomes:

$$\begin{array}{rcl} e(t) & = & x(t) - z(t) - (I_n - \alpha E) x(t) \\ & - & K(-\gamma E) x(t) \\ & = & (\alpha + K\gamma) E x(t) - z(t) \end{array} \tag{12}$$

Thus

$$\dot{e}(t) = (\alpha + K\gamma) E \dot{x}(t) - \dot{z}(t) \tag{13}$$

Replacing $E\dot{x}(t)$ and $\dot{z}(t)$ from (2) and (9), respectively, into (13):

$$\begin{array}{rcl} \dot{e}(t) & = & (\alpha + K\gamma)\left(A(\rho) x(t) + B(\rho) u(t) \right) \\ & - & N(\rho) z(t) - L_1(\rho) y(t) \\ & - & L_2(\rho) y(t) - G(\rho) u(t) \end{array} \tag{14}$$

By grouping common terms in $x(t)$, $u(t)$ and $z(t)$, (14) becomes:

$$\begin{array}{rcl} \dot{e}(t) & = & [K\gamma A(\rho) + \alpha A(\rho) - L_1(\rho) C \\ & - & L_2(\rho) C]\, x(t) - N(\rho) z(t) \\ & + & [K\gamma B(\rho) + \alpha B(\rho) - G(\rho)]\, u(t) \end{array} \tag{15}$$

By adding and subtracting the term $N(\rho)(\alpha + K\gamma) E x(t)$ in (15), it follows that:

$$\begin{array}{rcl} \dot{e}(t) & = & [(\alpha + K\gamma) A(\rho) - N(\rho)(\alpha + K\gamma) E - L_1(\rho) C \\ & - & L_2(\rho) C]\, x(t) + N(\rho) \underbrace{[(\alpha + K\gamma) E x(t) - z(t)]}_{e(t)} \\ & + & [(\alpha + K\gamma) B(\rho) - G(\rho)]\, u(t) \end{array} \tag{16}$$

It can be seen in (16) that if the following conditions are fulfilled

$$\begin{array}{rl} & (\alpha + K\gamma) A(\rho) - N(\rho)(\alpha + K\gamma) E \\ - & L_1(\rho) C - L_2(\rho) C = 0 \end{array} \tag{17}$$

and

$$G(\rho) = (\alpha + K\gamma) B(\rho) \tag{18}$$

then (16) reduces to

$$\dot{e}(t) = N(\rho)e(t) \tag{19}$$

.

Now, replacing αE and γE from (4) and (5), respectively, in (17), it is easy to deduce:

$$\begin{aligned} N(\rho) &= (\alpha + K\gamma)\, A(\rho) - L_2(\rho)C \\ &+ \left[N(\rho)\,(\beta + K\xi) - L_1(\rho)\right] C \end{aligned} \tag{20}$$

Under the assumption that the second term in the right hand side of (20) is zero

$$L_1(\rho) = N(\rho)\,(\beta + K\xi) \tag{21}$$

then, (20) becomes

$$N(\rho) = K\gamma A(\rho) + \alpha A(\rho) - L_2(\rho)C \tag{22}$$

.

B. *Observer stability*

In order to ensure the stability of the observer error (19), the following theorem is proposed:

***Theorem** 1: The system* (9) *is an observer for the system* (2) *if there exist appropriate matrices P, Q and R_i such that*

$$\begin{aligned} &\sum_{i=1}^{M} \varepsilon_i(\rho(t)) \left(A_i^T \alpha^T P + A_i^T \gamma^T Q^T - C^T R_i^T + \right. \\ &\left. P\alpha A_i + Q\gamma A_i - R_i C\right) < 0 \end{aligned} \tag{23}$$

and consequently $\hat{x}(t)$ will asymptotically converge to $x(t)$.

Proof: Consider the following Lyapunov function candidate $V(e(t)) = e^T(t)Pe(t)$ with $P = P^T > 0$ ($(\cdot)^T$ denotes matrix transposition). The time derivative of the Lyapunov function along the system trajectories (19) is:

$$\begin{aligned} \dot{V}(e(t)) &= \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) \\ &= e^T(t)\left(N^T(\rho)P + PN(\rho)\right)e(t) \\ &= e^T(t)\sum_{i=1}^{M} \varepsilon_i(\rho(t))\left(N_i^T P + PN_i\right)e(t) \end{aligned} \tag{24}$$

Quadratic stability [19] is guaranteed if $\dot{V}(e(t)) < 0$, $\forall e(t) \neq 0$. This condition is satisfied if

$$\sum_{i=1}^{M} \varepsilon_i(\rho(t))\left(N_i^T P + PN_i\right) < 0 \tag{25}$$

If there exists an appropriate symmetric matrix P to achieve that $\left(N_i^T P + PN_i\right) < 0$ holds $\forall\ i = 1, \ldots, M$, then it is obvious that (25) holds for any $\varepsilon_i(\rho(t))$. Since Eq. (25) must be true for every value of $\varepsilon_i(t)$ then it must be true at every vertices of the polytope and this implies [17]:

$$\left(N_i^T P + PN_i\right) < 0\ \ \forall\, i = 1, \ldots, M \tag{26}$$

N_i is deduced from (22) and defined such as:

$$N_i = K\gamma A_i + \alpha A_i - L_{2i}C\ \ \forall\, i = 1, \ldots, M \tag{27}$$

Replacing N_i from (27) in (25), the following BMI is obtained:

$$\begin{aligned} &A_i^T \alpha^T P + A_i^T \gamma^T K^T P - C^T L_{2i}^T P + \\ &P\alpha A_i + PK\gamma A_i - PL_{2i}C < 0 \forall\, i = 1, \ldots, M \end{aligned} \tag{28}$$

The BMI conditions (28) can be transformed into LMI conditions by considering $Q = PK$ and $R_i = PL_{2i}$. Eq. (28) becomes:

$$\begin{aligned} &A_i^T \alpha^T P + A_i^T \gamma^T Q^T - C^T R_i^T + \\ &P\alpha A_i + Q\gamma A_i - R_i C < 0\ \forall\, i = 1, \ldots, M \end{aligned} \tag{29}$$

By multiplying each LMI (29) by $\sum_{i=1}^{M} \varepsilon_i(\rho(t))$ and adding them all together, the following inequalities are defined:

$$\begin{aligned} &\sum_{i=1}^{M} \varepsilon_i(\rho(t)) \left(A_i^T \alpha^T P + A_i^T \gamma^T Q^T - C^T R_i^T + \right. \\ &\left. P\alpha A_i + Q\gamma A_i - R_i C\right) < 0 \end{aligned} \tag{30}$$

Finally, if there exist appropriate matrices P, Q and R_i, then it is obvious that (30) holds and consequently the system (19) is stable. □

As suggested by [7], the observer gains can be designed through the poles assignment of the system (19) in a subregion of the complex left half-plane [20]. It can be accomplished by defining a LMI region $\mathcal{D}$, for instance a circle with an affix $(-\lambda, 0)$ and a radius δ. The values of λ and δ determine a specific region $\mathcal{D}$ to place the eigenvalues of each N_i. In this context, the pole placement of the system (19) in the LMI region $\mathcal{D}$ can be expressed as:

$$\begin{pmatrix} -\delta P & \lambda P + W_i^T \\ \lambda P + W_i & -\delta P \end{pmatrix} < 0 \tag{31}$$

where $W_i = P\alpha A_i + Q\gamma A_i - R_i C$, $\forall\, i = 1, \ldots, M$.

III. EXAMPLE

Consider a continuous-time LPV descriptor system (2) described by:

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} -5 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4.5 \end{pmatrix}\ A_2 = \begin{pmatrix} -6 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -5 \end{pmatrix}$$

$$B_1 \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}\ B_1 \begin{pmatrix} 0 \\ -0.5 \\ 2 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

According to the fullfilled condition rank $\begin{pmatrix} E \\ C \end{pmatrix} = 3$, then the matrices α, β, γ and ξ satisfying (4) and (5) are computed using Eqs. (6) and (7):

$$\alpha = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0 \\ -1 & 0 & 0 \end{pmatrix} \beta = \begin{pmatrix} 0 & 0 \\ 0 & 0.5 \\ 1 & 0 \end{pmatrix}$$

$$\gamma = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -0.707 & 0 \end{pmatrix} \xi = \begin{pmatrix} 0 & 0 \\ 0 & 0.707 \end{pmatrix}$$

The design of the LPV observer has been achieved by considering the pole placement in LMI region $\mathcal{D}$ with the parameters λ=7 and δ=5.5. These values of λ and δ associated to a disk region are chosen from the nominal dynamic behavior of the system in order to stabilize and to guarantee an efficient state estimation. The following matrices are calculted from (31) through the YALMIP toolbox [21] :

$$P = \begin{pmatrix} 5.6424 & 0 & 0.0307 \\ 0 & 5.6731 & 0 \\ 0.0307 & 0 & 5.6424 \end{pmatrix}$$

$$Q = \begin{pmatrix} 0.0017 & 0 \\ 0 & -1.0880 \\ -0.0017 & 0 \end{pmatrix} \times 10^3$$

$$R_1 = \begin{pmatrix} 0.0097 & 0 \\ 0 & -3.0490 \\ 0.0300 & 0 \end{pmatrix} \times 10^3$$

$$R_2 = \begin{pmatrix} 0.0074 & 0 \\ 0 & -3.0490 \\ 0.0324 & 0 \end{pmatrix} \times 10^3$$

The effectiveness of the proposed observer is illustrated with the system studied in open-loop. The input vector is presented in Fig. 1. The weighting functions $\varepsilon_i(\rho(t))$ $i = 1, 2$ of the LPV system (2) are shown in Fig. 2. It can be seen that the system is evaluated over the entire operating range.

Given the initial conditions $x(0) = [3\ \ 2\ \ 1]^T$ and $\hat{x}(0) = [0\ \ 0\ \ 0]^T$, the simulation result of the state space vector $x(t)$ is depicted in Fig. 3. The observer error $e(t) = x(t) - \hat{x}(t)$ (which is plotted during the first 15000 s of the simulation in order to appreciate better the observer convergence) is illustrated in Fig. 4. It can be appreciated that the observer errors reach rapidly to zero. The observer is insensitive to the input or gain scheduling variations.

IV. CONCLUSIONS

In this paper an observer for polytopic descriptor LPV systems has been proposed. The observer synthesis is an extension of the work presented in [2] for LTI to LPV systems. Sufficient conditions are stated to ensure the existence and the stability of the proposed observer by using a combined Lyapunov-like analysis based on LMI formulation. The observer performance is evaluated via simulations using a numerical example. This approach could be useful to

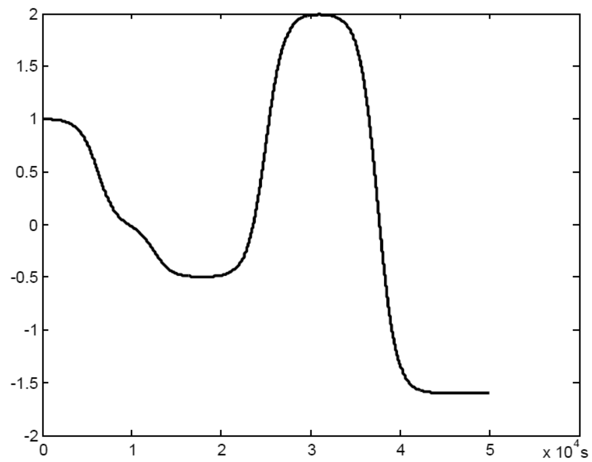

Fig. 1. Dynamic behaviour of the input

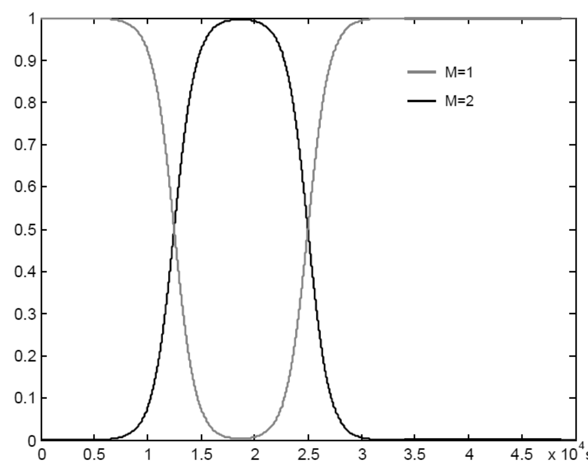

Fig. 2. Dynamic behaviour of the weighting functions

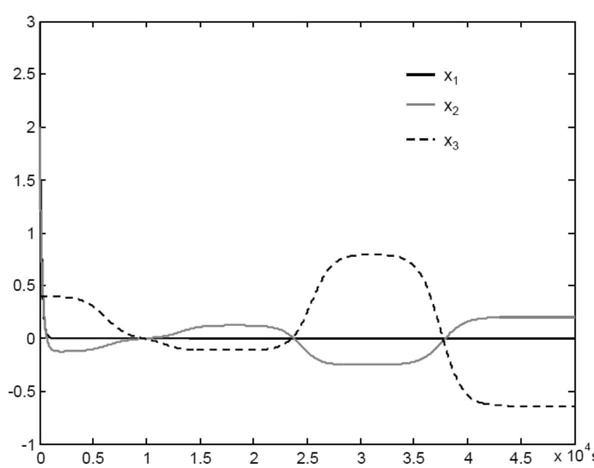

Fig. 3. Dynamic behaviour of the state space vector

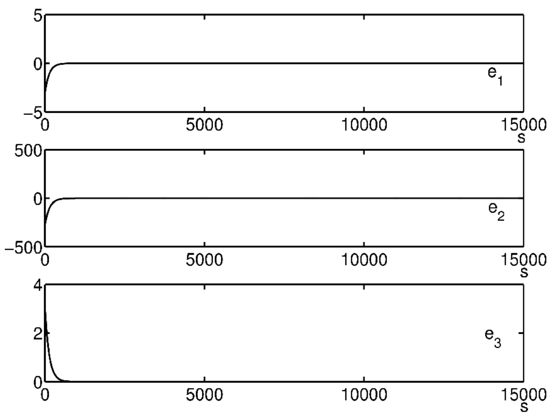

Fig. 4. Dynamic behaviour of the error vector

treat the problem of estimating simultaneously the state and the unknown inputs of several processes modeled in an LPV framework (see for instance [16]). This case will be considered in a future work.

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REFERENCES

[1] P.C. Müller and M. Hou, On the observer design for descriptor systems, *IEEE Trans. on Aut. Control*, vol. 38, 1993, pp 1666–1671.

[2] M. Darouach and M. Boutayeb, Design of observers for descriptor systems, *IEEE Trans. on Aut. Control*, vol. 40, 1995, pp 1323–1327.

[3] L. Zhou and G. Lu, Detection and stabilization for discrete-time descriptor systems via a limited capacity communication channel, *Automatica*, vol. 45, 2009, pp 2272–2277.

[4] M. Darouach and M. Zasadzinski, Optimal unbiased reduced order filtering for discrete-time descriptor systems via LMI, *Systems & Control Letters*, vol. 58, 2009, pp 436–444.

[5] X. Wei and L. del Re, Gain scheduled H_∞ control for air path systems of diesel engines using LPV techinques, *IEEE Trans. on Control System Technology*, vol. 15, 2007, pp 406–415.

[6] A. Zolghadri, D. Henry and S. Grenaille, *Fault diagnosis for LPV systems*, In: *Proc. of the 16th Mediterranean Conference on Control and Automation*, Ajaccio, France, 2008, pp 261–266.

[7] M. Rodrigues, D. Theilliol, S. Aberkane and D. Sauter, Fault tolerant control design for polytopic LPV systems, *Int. J. Appl. Math. Comput. Sci.*, vol. 17, 2007, pp 27–37.

[8] H. Wang and G.H. Yang, Integrated fault detection and control for LPV systems, *Int. J. of Robust and Nonlinear Control*, vol. 19, 2009, pp 341–363.

[9] B. Kulcsàr, J. Bokor and J. Shinar, Unknown input reconstruction for LPV systems, *Int. J. of Robust and Nonlinear Control*, vol. 20, 2010, pp 579–595.

[10] A. Rehm and F. Allgöwer, Self-scheduled H_∞ output feedback control of descriptor systems, *Comp. Chem. Eng.*, vol. 24, 2000, pp 279–284.

[11] M. Chadli, M. Darouach and J. Daafouz, *Static output stabilisation of singular LPV systems: LMI formulation*, In: *Proc. of the 47th Conference on Decision and Control*, Cancun, Mexico, 2008, pp 4793–4796.

[12] H. Hamdi, M. Rodrigues, C. Mechmeche, D. Theilliol and N. B. Braiek, *State estimation for polytopic LPV descriptor systems: Application to fault diagnosis*, In: *Proc. of the 7th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes*, Barcelona, Spain, 2009, pp 438–443.

[13] A. Fujimori, Descriptor polytopic model of aircraft and gain scheduling state feedback control, *Trans. Japan Soc. Aero. Space Sci.*, vol. 47, 2004, pp 138–145.

[14] I. Masubuchi, J. Kato, M. Saeki and A. Ohara, *Gain-scheduled controller design based on descriptor representation of LPV systems: Application to flight vehicle control*, In: *Proc. of the 43th Conference on Decision and Control*, Atlantis, Paradise Island, Bahamas, 2004, pp 815–820.

[15] I. Polat, E. Eşkinat and I. E. Köse, Dynamic output feedback control of quasi-LPV mechanical systems, *IET Control Theory Appl.*, vol. 1, 2007, pp 1114–1121.

[16] B. Boulkroune, M. Darouach, M. Zasadzinski, S. Gillé and D.Fiorelli, A nonlinear observer design for an activated sludge wastewater treatment process, *J. Proc. Control*, vol. 19, 2009, pp 1558–1565.

[17] Briat C. Commande et observation robustes des Systèmes LPV Retardés. PhD Thesis, Institut Polytechnique de Grenoble, 2008.

[18] L. Dai, *Singular Control Systems*, M. Thoma, A.Wyner, Eds., vol. 118 of Lecture Notes in Control and Information Sciences, Springer-Verlag, The Netherlands, 1989.

[19] F. Amato, *Robust Control of Linear Systems Subject to Uncertain Time-Varying Parameters*, M. Thoma, M. Morari, Eds., vol. 325 of Lecture Notes in Control and Information Sciences, Springer-Verlag, The Netherlands, 2006.

[20] M. Chilali and P. Gahinet, H_∞ design with pole placement constraints: An LMI approach, *IEEE Trans. on Aut. Control*, vol. 41, 1996, pp 358–367.

[21] J. Löfberg, *YALMIP : A Toolbox for Modeling and Optimization in MATLAB*, In: *Proceedings of the CACSD Conference*, Taipei, Taiwan, 2004.