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Output Feedback Adaptive Stabilization and Command Following for Minimum Phase Dynamical Systems with Unmatched Uncertainties

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Abstract— **In this paper, we develop an output feedback adaptive control framework for continuous-time minimum phase multivariable dynamical systems for output stabilization and command following. The approach is based on a nonminimal state space realization that generates an expanded set of states using the filtered inputs and filtered outputs and their derivatives of the original system. Specifically, a direct adaptive controller for the nonminimal state space model is constructed using the expanded states of the nonminimal realization and is shown to be effective for multi-input, multi-output linear dynamical systems with unmatched system uncertainties and unstable dynamics. Two illustrative numerical examples are provided to demonstrate the efficacy of the proposed approach.**

I. Introduction

Mathematical models are critical in capturing and studying physical phenomena that undergo spatial and temporal evolution arising in most applications of science and engineering. These models are often based on first-principles of physics and are derived using fundamental physical laws. However, due to system complexity, nonlinearities, uncertainty, and disturbances, first-principle models are often based on simplifying approximations resulting in system modeling errors. For systems where the system model does not adequately capture the physical system due to idealized assumptions, model simplification, and model parameter uncertainty, adaptive control methods can be used to achieve system performance without excessive reliance on system models.

Direct adaptive controllers require less system modeling information than robust controllers and can address system uncertainties and system failures. These controllers adapt feedback gains in response to system variations without requiring a parameter estimation algorithm. This property distinguishes them from indirect adaptive controllers that employ an estimation algorithm to estimate the unknown system parameters and adapt the controller gains. Direct adaptive controllers can be classified as either full state feedback or output feedback designs.

Full state feedback designs assume knowledge of the state variables, and this assumption leads to computationally simpler adaptive controller algorithms as compared to output feedback algorithms. Output feedback direct adaptive controllers, however, are required for most applications that involve high-dimensional models such as active noise suppression, active control of flexible structures, fluid flow control systems, and combustion control processes. Models for these applications vary from (reasonably) accurate low frequency models in the case of structural control problems, to less accurate low-order models in the case of active control of noise, vibrations, flows, and combustion processes.

There has been a number of results in recent decades focused on output feedback direct adaptive controllers (see [1]–[12], and references therein). These results require an observer for unknown state variables, an observer for output tracking errors, an output predictor, and/or estimation of Markov parameters that lead to adaptive control algorithms with varying sets of assumptions. These assumptions include knowledge of the relative degree of the regulated system output and the dimension of the system, as well as the requirement that the system be minimum phase or passive. The main reason for the minimum phase assumption is because direct adaptive controllers employ high gain feedback that can drive nonminimum systems to instability.

In this paper, we develop an output feedback adaptive control framework for continuous-time minimum phase multivariable dynamical systems for output stabilization and command following. The approach is based on a nonminimal state space realization that generates an expanded set of states using the filtered inputs and filtered outputs and their derivatives of the original system. Specifically, a direct adaptive controller for the nonminimal state space model is constructed using the expanded states of the nonminimal realization and is shown to be effective for multi-input, multi-output linear dynamical systems with unmatched uncertainties and unstable dynamics. The proposed adaptive control architecture can be viewed as a continuous-time framework that complements the discrete-time adaptive command following results proposed in [11]. Two illustrative numerical examples are provided to demonstrate the efficacy of the proposed approach.

The notation used in this paper is fairly standard. Specifically, $\mathbb{R}^n$ (resp., $\mathbb{C}^n$) denotes the set of $n \times 1$ real (resp., complex) column vectors, $\mathbb{R}^{n\times m}$ (resp., $\mathbb{C}^{n\times m}$) denotes the set of $n \times m$ real (resp., complex) matrices, $(\cdot)^{\mathrm{T}}$ denotes transpose, $(\cdot)^{-1}$ denotes inverse, and $\triangleq$ denotes equal by definition. Furthermore, we write $\lambda_{\min}(A)$ (resp., $\lambda_{\max}(A)$) for the minimum (resp., maximum) eigenvalue of the Hermitian matrix A, $\|\cdot\|_2$ for the Euclidian norm, $\|\cdot\|_{\mathrm{F}}$ for the Frobenius matrix norm, $\mathrm{tr}(\cdot)$ for the trace operator, $\mathrm{id}(A)$ for I_n (resp., $-I_n$) if $A \in \mathbb{R}^{n\times n}$ is positive-definite (resp., negative-definite), and $\mathrm{pd}(A)$ for A (resp., $-A$) if $A \in \mathbb{R}^{n\times n}$ is positive-definite (resp., negative-definite).

II. Nonminimal State Space Realization Formulation

In this section, we present a nonminimal state space realization architecture for continuous-time, linear multivariable uncertain dynamical systems. The nonminimal state space realization involves an expanded system state that consists entirely of the system filtered inputs and filtered outputs

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and their derivatives, which allows us to cast an output feedback control problem as a full-state feedback problem. Specifically, consider the controllable and observable linear uncertain dynamical system given by

$$
\begin{aligned}
\dot{x}_{\mathrm{p}}(t) &= A_{\mathrm{p}} x_{\mathrm{p}}(t) + B_{\mathrm{p}} u(t), \quad x_{\mathrm{p}}(0) = x_{\mathrm{p}_0}, \quad t \geq 0, && (1)\\
y(t) &= C_{\mathrm{p}} x_{\mathrm{p}}(t), && (2)
\end{aligned}
$$

where $x_{\mathrm{p}}(t) \in \mathbb{R}^n$, $t \geq 0$, is the state vector, $u(t) \in \mathbb{R}^m$, $t \geq 0$, is the control input, $y(t) \in \mathbb{R}^l$, $t \geq 0$, is the system output, and $A_{\mathrm{p}} \in \mathbb{R}^{n \times n}$, $B_{\mathrm{p}} \in \mathbb{R}^{n \times m}$, and $C_{\mathrm{p}} \in \mathbb{R}^{l \times n}$ are *unknown* system matrices. An input-output equivalent nonminimal observer canonical state space model of (1) and (2) for $l > 1$ is given by ([13])

$$
\begin{aligned}
\dot{x}_{\mathrm{o}}(t) &= A_{\mathrm{o}} x_{\mathrm{o}}(t) + B_{\mathrm{o}} u(t), \quad x_{\mathrm{o}}(0) = x_{\mathrm{o}_0}, \quad t \geq 0, && (3)\\
y(t) &= C_{\mathrm{o}} x_{\mathrm{o}}(t), && (4)
\end{aligned}
$$

where $x_{\mathrm{o}}(t) \in \mathbb{R}^{ln}$, $t \geq 0$, is the state vector,

$$
A_{\mathrm{o}} = \begin{bmatrix} 0 & I_l & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & I_l \\ -a_0 I_l & -a_1 I_l & \cdots & -a_{n-1} I_l \end{bmatrix} \in \mathbb{R}^{ln \times ln}, \quad (5)
$$

$$
B_{\mathrm{o}} = \begin{bmatrix} C_{\mathrm{p}} B_{\mathrm{p}} \\ C_{\mathrm{p}} A_{\mathrm{p}} B_{\mathrm{p}} \\ \vdots \\ C_{\mathrm{p}} A_{\mathrm{p}}^{n-1} B_{\mathrm{p}} \end{bmatrix} \in \mathbb{R}^{ln \times m}, \quad (6)
$$

and

$$
C_{\mathrm{o}} = \begin{bmatrix} I_l & 0 & \cdots & 0 \end{bmatrix} \in \mathbb{R}^{l \times ln}. \quad (7)
$$

Note that a_i, $i = 0, 1, \ldots, n-1$, in (5) are the coefficients of the characteristic polynomial of the matrix A_{p} in (1).

Next, let

$$
\begin{aligned}
\bar{B}_0 &= C_{\mathrm{o}}(a_1 I_{ln} + \cdots + a_{n-1} A_{\mathrm{o}}^{n-2} + A_{\mathrm{o}}^{n-1}) B_{\mathrm{o}}, && (8)\\
\bar{B}_1 &= C_{\mathrm{o}}(a_2 I_{ln} + \cdots + a_{n-1} A_{\mathrm{o}}^{n-3} + A_{\mathrm{o}}^{n-2}) B_{\mathrm{o}}, && (9)\\
&\;\;\vdots\\
\bar{B}_{n-1} &= C_{\mathrm{o}} B_{\mathrm{o}}. && (10)
\end{aligned}
$$

Now, an alternative input-output equivalent nonminimal *controllable* state space realization of (1) and (2) is given by

$$
\begin{aligned}
\dot{x}_{\mathrm{f}}(t) &= A_{\mathrm{f}} x_{\mathrm{f}}(t) + B_{\mathrm{f}} u(t), \quad x_{\mathrm{f}}(0) = x_{\mathrm{f}_0}, \quad t \geq 0, && (11)\\
y(t) &= C_{\mathrm{f}} x_{\mathrm{f}}(t), && (12)
\end{aligned}
$$

where $x_{\mathrm{f}}(t) \in \mathbb{R}^{n_{\mathrm{f}}}$, $t \geq 0$, $n_{\mathrm{f}} \triangleq (m + l)n$, is the *known* filtered expanded state vector given by

$$
x_{\mathrm{f}}(t) = \left[q_1^{\mathrm{T}}(t), \ \ldots, \ q_n^{\mathrm{T}}(t), \ v_1^{\mathrm{T}}(t), \ \ldots, \ v_n^{\mathrm{T}}(t) \right]^{\mathrm{T}}, \quad (13)
$$

where $q_i(t) \triangleq y_{\mathrm{f}}^{(i-1)}(t)$, $v_i(t) \triangleq u_{\mathrm{f}}^{(i-1)}(t)$, $i = 1, 2, \ldots, n$, $z^{(n)}(t) \triangleq \mathrm{d}^n z(t)/\mathrm{d}t^n$, and where $x_{\mathrm{f}}(t)$ is obtained by filtering $u(t)$ and $y(t)$ through the filter $1/\Lambda(s)$, where

$$
\begin{aligned}
\Lambda(s) = (s + \lambda)^n &= \sum_{k=0}^{n} \binom{n}{k} s^{n-k} \lambda^k \\
&= s^n + n\lambda s^{n-1} + \cdots + \lambda^n, && (14)
\end{aligned}
$$

is a monic Hurwitz polynomial of degree n with $\lambda > 0$,

$$
A_{\mathrm{f}} = \begin{bmatrix}
0 & I_l & 0 & \cdots & \cdots & & \cdots & 0 \\
\vdots & & \ddots & \cdots & \cdots & & & \vdots \\
0 & \cdots & 0 & I_l & 0 & \cdots & \cdots & 0 \\
-a_0 I_l & \cdots & \cdots & -a_{n-1} I_l & \bar{B}_0 & \cdots & \cdots & \bar{B}_{n-1} \\
0 & \cdots & & \cdots & 0 & I_m & 0 & 0 \\
\vdots & & & \cdots & \cdots & \ddots & \vdots & \\
\vdots & & & \cdots & \cdots & \cdots & 0 & I_m \\
0 & \cdots & \cdots & 0 & -\lambda^n I_m & \cdots & \cdots & -n\lambda I_m
\end{bmatrix} \in \mathbb{R}^{n_{\mathrm{f}} \times n_{\mathrm{f}}}, \quad (15)
$$

$$
B_{\mathrm{f}} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ I_m \end{bmatrix} \in \mathbb{R}^{n_{\mathrm{f}} \times m}, \quad (16)
$$

and

$$
\begin{aligned}
C_{\mathrm{f}} = \big[\; -a_0 I_l + \lambda^n I_l \quad \cdots \quad \cdots \quad -a_{n-1} I_l + n\lambda I_l \\
\bar{B}_0 \quad \cdots \quad \cdots \quad \bar{B}_{n-1} \; \big] \in \mathbb{R}^{l \times n_{\mathrm{f}}}. \quad (17)
\end{aligned}
$$

Theorem 2.1 ([14]). System (1) and (2) is input-output equivalent to system (11) and (12).

Remark 2.1. The proof of Theorem 2.1 presents a construction of a nonminimal, albeit controllable, state space realization of (1) and (2) involving the expanded state $x_{\mathrm{f}}(t)$, $t \geq 0$, comprising of filtered versions of the inputs and outputs and their derivatives of the original system, without requiring differentiation of the actual input and output signals. It is important to note here that even though the original system is *unknown*, the expanded state vector $x_{\mathrm{f}}(t)$, $t \geq 0$, is *known*.

Remark 2.2. Since the *controllable* nonminimal state space realization of (11) and (12) is defined by a state that consists entirely of filtered inputs and outputs and their derivatives of the original system, an output feedback stabilization problem for (1) and (2) can be converted into a full-state feedback control design problem by equivalently considering (11) and (12). Furthermore, for an output feedback control design of the form (1) and (2) we typically require that $(A_{\mathrm{p}}, B_{\mathrm{p}})$ be controllable (or stabilizable) and $(A_{\mathrm{p}}, C_{\mathrm{p}})$ be observable (or detectable). In contrast, for a feedback control design using the input-output equivalent nonminimal state space model (11) and (12) we only require controllability of the pair $(A_{\mathrm{f}}, B_{\mathrm{f}})$, which is automatic. Finally, it is important to note that only the system matrix A_{f} in (11) is *partially unknown* for full-state feedback control design, whereas the triple $(A_{\mathrm{p}}, B_{\mathrm{p}}, C_{\mathrm{p}})$ is *unknown* in (1) and (2) for an output feedback control design.

Remark 2.3. Nonminimal state space realizations for discrete-time adaptive control have been extensively developed in the literature, see [11], [15], [16] and the references therein. The proposed nonminimal state space realization for continuous-time adaptive control developed in this section was first used in [17], [18] for active noise blocking and robust control and [14] for adaptive control.

III. Adaptive Control for the Nonminimal State Space Model

In this section, we introduce a direct adaptive state feedback control architecture for the nonminimal state space model (11) and (12) that guarantees adaptive output stabilization for the original system (1) and (2), as well as boundedness of the original system state $x_{\mathrm{p}}(t)$, $t \geq 0$.

Assumption 3.1. The system given by (1) and (2) is minimum phase and the smallest positive integer i such that the ith Markov parameter of (1) and (2) given by $C_{\mathrm{p}}A_{\mathrm{p}}^{i-1}B_{\mathrm{p}}$ is nonzero and known.

Letting d denote the smallest positive integer i in Assumption 3.1, it follows from (8)–(10) that

$$\begin{aligned} \bar{B}_{n-1} &= C_{\mathrm{o}}B_{\mathrm{o}} = C_{\mathrm{p}}B_{\mathrm{p}} = 0, && (18)\\ \bar{B}_{n-2} &= C_{\mathrm{o}}(a_1 I_{ln} + A_{\mathrm{o}})B_{\mathrm{o}} \\ &= a_1 C_{\mathrm{p}}B_{\mathrm{p}} + C_{\mathrm{p}}A_{\mathrm{p}}B_{\mathrm{p}} = 0, && (19)\\ &\ \vdots \\ \bar{B}_{n-d+1} &= 0, && (20)\\ \bar{B}_{n-d} &= C_{\mathrm{p}}A_{\mathrm{p}}^{d-1}B_{\mathrm{p}} \neq 0, && (21) \end{aligned}$$

where (21) is the first nonzero Markov parameter of the original system (1) and (2).

Assumption 3.2. The first nonzero Markov parameter given by (21) can be parameterized as

$$C_{\mathrm{p}}A_{\mathrm{p}}^{d-1}B_{\mathrm{p}} \quad = \quad \bar{B}\Lambda, \tag{22}$$

where $\bar{B} \in \mathbb{R}^{l\times m}$ is a *known* matrix and $\Lambda \in \mathbb{R}^{m\times m}$ is an *unknown* matrix given by

$$\Lambda \quad = \quad \text{block}-\text{diag}[\Lambda_{m_1}, \ldots, \Lambda_{m_s}], \tag{23}$$

where $\Lambda_{m_1} \in \mathbb{R}^{m_1\times m_1}$, …, $\Lambda_{m_s} \in \mathbb{R}^{m_s\times m_s}$, and $m_1 + \cdots + m_s = m$. Furthermore, for each $i \in \{1, \ldots, s\}$, Λ_{m_i} is either positive definite or negative definite.

Note that it follows from Assumption 3.2 that Λ given by (23) can be written as $\Lambda = \mathrm{id}(\Lambda)\mathrm{pd}(\Lambda)$, where $\mathrm{id}(\Lambda) \triangleq \text{block}-\text{diag}[\mathrm{id}(\Lambda_{m_1}), \ldots, \mathrm{id}(\Lambda_{m_s})]$ is known and $\mathrm{pd}(\Lambda) \triangleq \text{block}-\text{diag}\ [\mathrm{pd}(\Lambda_{m_1}), \ldots, \mathrm{pd}(\Lambda_{m_s})]$ is unknown and positive-definite. For single-input, single-output dynamical systems without loss in generality letting $\bar{B} = 1$ in (22) gives $\Lambda = \mathrm{id}(C_{\mathrm{p}}A_{\mathrm{p}}^{d-1}B_{\mathrm{p}})\mathrm{pd}(C_{\mathrm{p}}A_{\mathrm{p}}^{d-1}B_{\mathrm{p}}) = \mathrm{sgn}(C_{\mathrm{p}}A_{\mathrm{p}}^{d-1}B_{\mathrm{p}})|C_{\mathrm{p}}A_{\mathrm{p}}^{d-1}B_{\mathrm{p}}|$, where $\mathrm{sgn}(y) \triangleq y/|y|$, $y \neq 0$, and $\mathrm{sgn}(0) \triangleq 0$. In this case, Assumption 3.2 implies that the sign of the first nonzero Markov parameter denoted by $\mathrm{id}(C_{\mathrm{p}}A_{\mathrm{p}}^{d-1}B_{\mathrm{p}})$ is known.

Next, consider the nonminimal state space model (11), where the *known* state vector $x_{\mathrm{f}}(t)$, $t \geq 0$, is given by (13), the *partially unknown* matrix A_{f} is given by (15), and the *known* input matrix B_{f} is given by (16), and note that (11) can be equivalently written as

$$\dot{q}(t) \ = \ A_0 q(t) + B_0 v_0(t) + B_1\Lambda\phi(t),\ q(0) = q_0,\ t \geq 0, \tag{24}$$

$$\dot{v}(t) \ = \ A_v v(t) + B_v u(t),\ v(0) = v_0, \tag{25}$$

where $q(t) \triangleq [q_1^{\mathrm{T}}(t), \ \ldots, \ q_n^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{ln}$, $v_0(t) \triangleq [v_1^{\mathrm{T}}(t), \ \ldots, \ v_{n-d}^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{m(n-d)}$, $\phi(t) \triangleq v_{n-d+1}(t) \in \mathbb{R}^m$, $v(t) \triangleq [v_1^{\mathrm{T}}(t), \ \ldots, \ v_n^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{mn}$,

$$A_0 \triangleq \begin{bmatrix} 0 & I_l & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & I_l \\ -a_0 I_l & -a_1 I_l & \cdots & -a_{n-1} I_l \end{bmatrix} \in \mathbb{R}^{ln\times ln}, \tag{26}$$

$$B_0 \triangleq \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \\ \bar{B}_{\mathrm{o}} & \cdots & \bar{B}_{n-d-1} \end{bmatrix} \in \mathbb{R}^{ln\times m(n-d)}, \tag{27}$$

$$B_1 \triangleq \begin{bmatrix} 0 & \cdots & 0 & \bar{B}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{ln\times m}, \tag{28}$$

$$A_v \triangleq \begin{bmatrix} 0 & I_m & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & I_m \\ -\zeta_1 I_m & \cdots & \cdots & -\zeta_n I_m \end{bmatrix} \in \mathbb{R}^{mn\times mn}, \tag{29}$$

and

$$B_v \triangleq \begin{bmatrix} 0 & \cdots & 0 & I_m \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{mn\times m}, \tag{30}$$

where $\zeta_1 \triangleq \lambda^n$, …, $\zeta_n \triangleq n\lambda$. Note that A_0, B_0, and Λ in (24) are unknown, and hence, the dynamics in (24) are unknown, whereas the dynamics in (25) are completely known with A_v being Hurwitz. Hence, we use a two-stage design framework wherein we first design a *virtual* control signal $\phi(t)$, $t \geq 0$, that stabilizes the unknown dynamics in (24), and then design the *actual* control signal $u(t)$, $t \geq 0$, using the known dynamics in (25). The existence of such a virtual control signal $\phi(t)$, $t \geq 0$, is guaranteed under the following assumption.

Assumption 3.3. There exists $K_q \in \mathbb{R}^{ln\times m}$ and $K_v \in \mathbb{R}^{m(n-d)\times m}$ such that $A_{\mathrm{m}} \triangleq A_0 + B_1\Lambda K_q^{\mathrm{T}}$ is Hurwitz and $B_0 = B_1\Lambda K_v^{\mathrm{T}}$ holds.

Remark 3.1. It is important to note that if (1) and (2) is *square* (i.e., $m = l$) and $\bar{B}$ is nonsingular, then Assumption 3.3 is automatically satisfied.

Next, we write (24) as

$$\begin{aligned} \dot{q}(t) &= A_{\mathrm{m}}q(t) - B_1\Lambda K_q^{\mathrm{T}}q(t) + B_0 v_0(t) + B_1\Lambda\phi(t) \\ &= A_{\mathrm{m}}q(t) + B_1\Lambda\tilde{K}_q^{\mathrm{T}}(t)q(t) - B_1\Lambda\tilde{K}_v^{\mathrm{T}}(t)v_0(t) \\ &\quad + B_1\Lambda\big[\phi(t) - \hat{K}_q^{\mathrm{T}}(t)q(t) + \hat{K}_v^{\mathrm{T}}(t)v_0(t)\big], \\ &\qquad q(0) = q_0, \quad t \geq 0, \end{aligned} \tag{31}$$

where $\tilde{K}_q(t) \triangleq \hat{K}_q(t) - K_q \in \mathbb{R}^{ln\times m}$, $t \geq 0$, $\tilde{K}_v(t) \triangleq \hat{K}_v(t) - K_v \in \mathbb{R}^{m(n-d)\times m}$, $t \geq 0$, and $\hat{K}_q(t) \in \mathbb{R}^{ln\times m}$, $t \geq 0$, and $\hat{K}_v(t) \in \mathbb{R}^{m(n-d)\times m}$, $t \geq 0$, are the estimates of K_q and K_v, respectively, and $\hat{K}_q(t)$, $t \geq 0$, and $\hat{K}_v(t)$, $t \geq 0$, satisfy

$$\dot{\hat{K}}_q(t) \ = \ -\Gamma_q q(t)q^{\mathrm{T}}(t)P_{\mathrm{m}}B_1\mathrm{id}(\Lambda),\ \hat{K}_q(0) = \hat{K}_{q0},\ t \geq 0, \tag{32}$$

$$\dot{\hat{K}}_v(t) \ = \ \Gamma_v v_0(t)q^{\mathrm{T}}(t)P_{\mathrm{m}}B_1\mathrm{id}(\Lambda),\ \hat{K}_v(0) = \hat{K}_{v0}, \tag{33}$$

where $\Gamma_q \in \mathbb{R}^{ln\times ln}$ and $\Gamma_v \in \mathbb{R}^{m(n-d)\times m(n-d)}$ are positive-definite gain matrices and P_{m} is a positive-definite solution of the Lyapunov equation

$$0 \quad = \quad A_{\mathrm{m}}^{\mathrm{T}}P_{\mathrm{m}} + P_{\mathrm{m}}A_{\mathrm{m}} + R_{\mathrm{m}}, \tag{34}$$

where $R_{\mathrm{m}} \in \mathbb{R}^{ln\times ln}$ is a symmetric positive-definite matrix. Note that since A_{m} is Hurwitz, it follows from converse Lyapunov theory [19] that there exists a unique symmetric positive-definite matrix P_{m} satisfying (34) for a given symmetric positive definite matrix R_{m}.

Proposition 3.1. Consider the uncertain dynamical system given by (24) and the virtual control signal

$$\phi(t) = \hat{K}_q^{\mathrm{T}}(t)q(t) - \hat{K}_v^{\mathrm{T}}(t)v_0(t), \quad t \geq 0, \tag{35}$$

with update laws (32) and (33), and assume that Assumptions 3.1, 3.2, and 3.3 hold. Then, the solution $(q(t),\ \hat{K}_q(t),\ \hat{K}_v(t))$ of the system (31)–(33) is Lyapunov stable for all $(q_0,\ \hat{K}_{q0},\ \hat{K}_{v0}) \in \mathbb{R}^{ln}\times\mathbb{R}^{ln\times m}\times\mathbb{R}^{m(n-d)\times m}$ and $t \geq 0$, and $q(t) \to 0$ as $t \to \infty$.

Proof. Consider the Lyapunov function candidate

$$\begin{aligned} V(q, \tilde{K}_q, \tilde{K}_v) &= q^{\mathrm{T}}P_{\mathrm{m}}q + \mathrm{tr}\ \tilde{K}_q^{\mathrm{T}}\Gamma_q^{-1}\tilde{K}_q\mathrm{pd}(\Lambda) \\ &\quad + \mathrm{tr}\ \tilde{K}_v^{\mathrm{T}}\Gamma_v^{-1}\tilde{K}_v\mathrm{pd}(\Lambda), \end{aligned} \tag{36}$$

where $P_{\mathrm{m}} > 0$ satisfies (34). Differentiating (36) along the trajectories of (31)–(33) yields

$$\begin{aligned} &\dot{V}(q(t), \tilde{K}_q(t), \tilde{K}_v(t)) \\ &= -q^{\mathrm{T}}(t)R_{\mathrm{m}}q(t) + 2q^{\mathrm{T}}(t)P_{\mathrm{m}}B_1\Lambda\tilde{K}_q^{\mathrm{T}}(t)q(t) \\ &\quad -2q^{\mathrm{T}}(t)P_{\mathrm{m}}B_1\Lambda\tilde{K}_v^{\mathrm{T}}(t)v_0(t) \\ &\quad -2\mathrm{tr}[\tilde{K}_q^{\mathrm{T}}(t)q(t)q^{\mathrm{T}}(t)P_{\mathrm{m}}B_1\mathrm{id}(\Lambda)\mathrm{pd}(\Lambda)] \\ &\quad +2\mathrm{tr}[\tilde{K}_v^{\mathrm{T}}(t)v_0(t)q^{\mathrm{T}}(t)P_{\mathrm{m}}B_1\mathrm{id}(\Lambda)\mathrm{pd}(\Lambda)] \\ &= -q^{\mathrm{T}}(t)R_{\mathrm{m}}q(t) \\ &\leq 0, \quad t \geq 0. \end{aligned} \tag{37}$$

Hence, the solution $(q(t),\ \hat{K}_q(t),\ \hat{K}_v(t))$ of the system (31)–(33) is Lyapunov stable for all $(q_0,\ \hat{K}_{q0},\ \hat{K}_{v0}) \in \mathbb{R}^{ln}\times\mathbb{R}^{ln\times m}\times\mathbb{R}^{m(n-d)\times m}$ and $t \geq 0$. Now, by the LaSalla-Yoshizawa theorem [19], $\lim_{t\to\infty} q^{\mathrm{T}}(t)R_{\mathrm{m}}q(t) = 0$ and, hence, $q(t) \to 0$ as $t \to \infty$. □

Proposition 3.1 shows that the virtual control signal $\phi(t)$, $t \geq 0$, given by (35) ensures that $q(t) \to 0$ as $t \to \infty$. Next, we construct the actual control signal $u(t)$, $t \geq 0$, using the known dynamics in (25). For this case, it follows from (25) that

$$\begin{aligned} u(t) &= \dot{v}_n(t) + \zeta_{n-1}v_{n-1}(t) + \zeta_{n-2}v_{n-2}(t) + \cdots \\ &\quad + \zeta_{n-d+2}v_{n-d+2}(t) + \zeta_{n-d+1}v_{n-d+1}(t) \\ &\quad + \zeta_{n-d}v_{n-d}(t) + \cdots + \zeta_2v_2(t) + \zeta_1v_1(t),\ \ t \geq 0. \end{aligned} \tag{38}$$

Using $\phi(t)$, $t \geq 0$, (38) can be equivalently rewritten as

$$\begin{aligned} u(t) &= \phi^{(d)}(t) + \zeta_{n-1}\phi^{(d-1)}(t) + \zeta_{n-2}\phi^{(d-2)}(t) + \cdots \\ &\quad + \zeta_{n-d+2}\dot{\phi}(t) + \zeta_{n-d+1}\phi(t) + \zeta_{n-d}\left[\int_0^t \phi(\sigma_1)\mathrm{d}\sigma_1\right] \\ &\quad + \cdots + \zeta_2\left[\int_0^t \cdots \int_0^t \left[\int_0^t \phi(\sigma_1)\mathrm{d}\sigma_1\right]\mathrm{d}\sigma_2 \cdots \right. \\ &\quad \left. \times\mathrm{d}\sigma_{n-d-1}\right] + \zeta_1\left[\int_0^t \cdots \int_0^t \left[\int_0^t \phi(\sigma_1)\mathrm{d}\sigma_1\right.\right. \\ &\quad \left.\left. \times\mathrm{d}\sigma_2\cdots\mathrm{d}\sigma_{n-d}\right], \quad t \geq 0. \end{aligned} \tag{39}$$

The following theorem presents the main result of this section.

Theorem 3.1. Consider the uncertain dynamical system given by (11) and the control signal (39) with (35), (32), and (33), and assume that Assumptions 3.1, 3.2, and 3.3 hold. Then, $x_{\mathrm{p}}(t)$, $t \geq 0$, satisfying (1) is bounded for all $x_{\mathrm{p}}(0) \in \mathbb{R}^n$ and $y(t) \to 0$ as $t \to \infty$.

Proof. It follows from Proposition 3.1 that the solution $(q(t),\ \hat{K}_q(t),\ \hat{K}_v(t))$ to (31)–(33) is Lyapunov stable for all $(q_0,\ \hat{K}_{q0},\ \hat{K}_{v0}) \in \mathbb{R}^{ln}\times\mathbb{R}^{ln\times m}\times\mathbb{R}^{m(n-d)\times m}$ and $t \geq 0$, and $q(t) \to 0$ as $t \to \infty$. Since the first l components of $q(t)$, $t \geq 0$, correspond to the filtered output of the original system, it follows that $y_{\mathrm{f}}(t) \to 0$ as $t \to \infty$. Now, since the filter given by (14) is asymptotically stable, it follows that $y(t) \to 0$ as $t \to \infty$.

To show that $x_{\mathrm{p}}(t)$, $t \geq 0$, satisfying (1) is bounded, note that since the solution $(q(t),\ \hat{K}_q(t),\ \hat{K}_v(t))$ to (31)–(33) is Lyapunov stable for all $(q_0,\ \hat{K}_{q0},\ \hat{K}_{v0}) \in \mathbb{R}^{ln}\times\mathbb{R}^{ln\times m}\times\mathbb{R}^{m(n-d)\times m}$ and $t \geq 0$, and $q(t) \to 0$ as $t \to \infty$, it follows from the dynamics in (31) with the virtual control signal defined in (35) that $v_0(t)$, $t \geq 0$, is bounded. In addition, since the first m components of $v_0(t)$, $t \geq 0$, correspond to the filtered input of the original system, it follows that $u_{\mathrm{f}}(t)$, $t \geq 0$, is bounded. Now, since the filter given by (14) is asymptotically stable, it follows that $u(t)$, $t \geq 0$, is bounded. Furthermore, since $u(t)$, $t \geq 0$, is bounded and A_v is Hurwitz, it follows from (25) that $v(t)$, $t \geq 0$, is bounded. Similarly, $\dot{y}(t), \ldots, y^{(n-1)}(t)$, and $\dot{u}(t), \ldots, u^{(n-1)}(t)$, $t \geq 0$, are bounded, and hence, uniformly continuous. Hence, it follows from the minimality of $(A_{\mathrm{p}}, B_{\mathrm{p}}, C_{\mathrm{p}})$ that $x_{\mathrm{p}}(t)$, $t \geq 0$, is bounded. □

To elucidate the structure of the control architecture (39), consider a second-order, single-input, single-output system with $d = 1$. In this case, the actual control signal given by (39) becomes

$$\begin{aligned} u(t) &= \dot{\phi}(t) + \zeta_2\phi(t) + \zeta_1\int_0^t \phi(\sigma)\mathrm{d}\sigma \\ &= \dot{\phi}(t) + 2\lambda\phi(t) + \lambda^2\int_0^t \phi(\sigma)\mathrm{d}\sigma, \end{aligned} \tag{40}$$

which involves a proportional-integral-derivative control architecture. To further elucidate the controller structure (40), assume that the adaptive gains $\hat{K}_q(t)$, $t \geq 0$, and $\hat{K}_v(t)$, $t \geq 0$, converge to $\hat{K}_{q\infty} = \left[\hat{k}_{q1},\ \hat{k}_{q2}\right]^{\mathrm{T}}$ and $\hat{K}_{v\infty} = \hat{k}_v$, respectively. In this case, using (35) with $q(t) = \left[q_1(t),\ q_2(t)\right]^{\mathrm{T}} = \left[y_{\mathrm{f}}(t),\ \dot{y}_{\mathrm{f}}(t)\right]^{\mathrm{T}}$ and $v_0(t) = v_1(t) = u_{\mathrm{f}}(t)$, it follows that

$$u(s) = \frac{\hat{k}_{q2}s + \hat{k}_{q1}}{s + \hat{k}_v}y(s), \tag{41}$$

which involves a lead/lag-type compensator. Note that *unstable pole-zero cancelation* in (41) is precluded by Assumption 3.1 since (1) and (2) is assumed to be minimum phase.

IV. Adaptive Command Following for the Nonminimal State Space Model

In this section, we extend the adaptive control architecture developed in Section 3 to the case of command following. To address system tracking, consider the additional integrator state satisfying

$$\dot{q}_{\mathrm{int}}(t) = -y_{\mathrm{f}}(t) + r_{\mathrm{f}}(t) = -q_1(t) + r_{\mathrm{f}}(t),\ \ t \geq 0, \tag{42}$$

where $r_{\mathrm{f}}(t) \in \mathbb{R}^l$, $t \geq 0$, is a filtered (through the filter $\Lambda(s)$ defined by (14)) command of a given bounded piecewise continuous reference command $r(t) \in \mathbb{R}^l$, $t \geq 0$. Now, (24) can be augmented with the integrator state (42) to give

$$\dot{q}_{\mathrm{a}}(t) = A_{\mathrm{a0}} q_{\mathrm{a}}(t) + B_{\mathrm{a0}} v_0(t) + B_{\mathrm{a1}} \Lambda \phi(t) + B_{\mathrm{am}} r_{\mathrm{f}}(t), \quad q_{\mathrm{a}}(0) = q_{\mathrm{a0}}, \quad t \geq 0, \quad (43)$$

where $q_{\mathrm{a}}(t) \triangleq [q^{\mathrm{T}}(t),\ q_{\mathrm{int}}^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{l(n+1)}$,

$$A_{\mathrm{a0}} \triangleq \begin{bmatrix} 0 & I_l & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & I_l & \vdots \\ -a_0 I_l & -a_1 I_l & \cdots & -a_{n-1} I_l & 0 \\ -I_l & 0 & \cdots & 0 & 0 \end{bmatrix} \in \mathbb{R}^{l(n+1)\times l(n+1)}, \quad (44)$$

$$B_{\mathrm{a0}} \triangleq \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \\ \bar{B}_{\mathrm{o}} & \cdots & \bar{B}_{\mathrm{n-d-1}} \\ 0 & \cdots & 0 \end{bmatrix} \in \mathbb{R}^{l(n+1)\times m(n-d)}, \quad (45)$$

$$B_{\mathrm{a1}} \triangleq \begin{bmatrix} 0 & \cdots & 0 & \bar{B}^{\mathrm{T}} & 0 \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{l(n+1)\times m}, \quad (46)$$

and

$$B_{\mathrm{am}} \triangleq \begin{bmatrix} 0 & \cdots & 0 & 0 & I_l \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{l(n+1)\times m}. \quad (47)$$

Note that in this case (25) remains unchanged. Analogous to Assumption 3.3, we have the following assumption.

Assumption 4.1. There exists $K_{\mathrm{aq}} \in \mathbb{R}^{l(n+1)\times m}$ and $K_{\mathrm{av}} \in \mathbb{R}^{m(n-d)\times m}$ such that $A_{\mathrm{am}} \triangleq A_{\mathrm{a0}} + B_{\mathrm{a1}} \Lambda K_{\mathrm{aq}}^{\mathrm{T}}$ is Hurwitz and $B_{\mathrm{a0}} = B_{\mathrm{a1}} \Lambda K_{\mathrm{av}}^{\mathrm{T}}$ holds.

Remark 4.1. Once again, note that if (1) and (2) is square (i.e., $m = l$) and $\bar{B}$ is nonsingular, then Assumption 4.1 is automatically satisfied.

Next, consider the reference system given by

$$\dot{q}_{\mathrm{am}}(t) = A_{\mathrm{am}} q_{\mathrm{am}}(t) + B_{\mathrm{am}} r_{\mathrm{f}}(t),\ q_{\mathrm{am}}(0) = q_{\mathrm{am_0}},\ t \geq 0, \quad (48)$$

where $q_{\mathrm{am}}(t) \in \mathbb{R}^{l(n+1)}$, $t \geq 0$, is the reference system state vector. Since A_{am} is Hurwitz, it follows from converse Lyapunov theory that there exist a positive-definite matrix $R_{\mathrm{am}} \in \mathbb{R}^{l(n+1)\times l(n+1)}$ and a positive-definite matrix $P_{\mathrm{m}} \in \mathbb{R}^{l(n+1)\times l(n+1)}$ such that

$$0 = A_{\mathrm{am}}^{\mathrm{T}} P_{\mathrm{am}} + P_{\mathrm{am}} A_{\mathrm{am}} + R_{\mathrm{am}}. \quad (49)$$

Finally, note that since $r(t)$ is bounded for all $t \geq 0$ and the filter given by (14) is asymptotically stable, it follows that $r_{\mathrm{f}}(t)$ is bounded for all $t \geq 0$. Furthermore, $q_{\mathrm{am}}(t)$ is uniformly bounded for all $q_{\mathrm{am_0}} \in \mathbb{R}^{l(n+1)}$ and $t \geq 0$.

Next, define $e(t) \triangleq q_{\mathrm{a}}(t) - q_{\mathrm{am}}(t)$ and note that it follows from the augmented dynamics (43) and the reference system (48) that

$$\begin{aligned} \dot{e}(t) &= A_{\mathrm{am}} e(t) + B_{\mathrm{a1}} \Lambda \tilde{K}_{\mathrm{a}q}^{\mathrm{T}}(t) q_{\mathrm{a}}(t) - B_{\mathrm{a1}} \Lambda \tilde{K}_{\mathrm{a}v}^{\mathrm{T}}(t) v_0(t) \\ &\quad + B_{\mathrm{a1}} \Lambda \big[\phi(t) - \hat{K}_{\mathrm{a}q}^{\mathrm{T}}(t) q_{\mathrm{a}}(t) + \hat{K}_{\mathrm{av}}^{\mathrm{T}}(t) v_0(t)\big], \\ &\qquad e(0) = e_0, \quad t \geq 0, \end{aligned} \quad (50)$$

where $\tilde{K}_{\mathrm{a}q}(t) \triangleq \hat{K}_{\mathrm{a}q}(t) - K_{\mathrm{a}q} \in \mathbb{R}^{l(n+1)\times m}$, $t \geq 0$, $\tilde{K}_{\mathrm{av}}(t) \triangleq \hat{K}_{\mathrm{av}}(t) - K_{\mathrm{av}} \in \mathbb{R}^{m(n-d)\times m}$, $t \geq 0$, and $\hat{K}_{\mathrm{a}q}(t) \in \mathbb{R}^{l(n+1)\times m}$ and $\hat{K}_{\mathrm{av}}(t) \in \mathbb{R}^{m(n-d)\times m}$ are the estimates of $K_{\mathrm{a}q}$ and K_{av}, $t \geq 0$, respectively, and $\hat{K}_{\mathrm{a}q}(t)$, $t \geq 0$, and $\hat{K}_{\mathrm{av}}(t)$, $t \geq 0$, satisfy

$$\dot{\hat{K}}_{\mathrm{a}q}(t) = -\Gamma_{\mathrm{a}q} q_{\mathrm{a}}(t) e^{\mathrm{T}}(t) P_{\mathrm{am}} B_{\mathrm{a1}} \mathrm{id}(\Lambda),\ \hat{K}_{\mathrm{a}q}(0) = \hat{K}_{\mathrm{a}q0}, \quad t \geq 0, \quad (51)$$

$$\dot{\hat{K}}_{\mathrm{av}}(t) = \Gamma_{\mathrm{av}} v_0(t) e^{\mathrm{T}}(t) P_{\mathrm{am}} B_{\mathrm{a1}} \mathrm{id}(\Lambda),\ \hat{K}_{\mathrm{av}}(0) = \hat{K}_{\mathrm{av0}}, \quad (52)$$

where $\Gamma_{\mathrm{a}q} \in \mathbb{R}^{l(n+1)\times l(n+1)}$ and $\Gamma_{\mathrm{av}} \in \mathbb{R}^{m(n-d)\times m(n-d)}$ are positive-definite gain matrices.

Theorem 4.1. Consider the uncertain dynamical system given by (11) and the control signal (39) with

$$\phi(t) = \hat{K}_{\mathrm{a}q}^{\mathrm{T}}(t) q_{\mathrm{a}}(t) - \hat{K}_{\mathrm{a}v}^{\mathrm{T}}(t) v_0(t), \quad t \geq 0, \quad (53)$$

and with update laws (51) and (52), and assume that Assumptions 3.1, 3.2, and 4.1 hold. Then, the solution $(e(t),\ \hat{K}_{\mathrm{aq}}(t),\ \hat{K}_{\mathrm{av}}(t))$ to (50)–(52) is Lyapunov stable for all $(e_0,\ \hat{K}_{\mathrm{aq0}},\ \hat{K}_{\mathrm{a}v0}) \in \mathbb{R}^{l(n+1)} \times \mathbb{R}^{l(n+1)\times m} \times \mathbb{R}^{m(n-d)\times m}$ and $t \geq 0$, and $e(t) \to 0$ as $t \to \infty$. Furthermore, $x_{\mathrm{p}}(t)$, $t \geq 0$, satisfying (1) is bounded for all $x_{\mathrm{p}}(0) \in \mathbb{R}^n$.

Proof. The proof is similar to the proofs of Proposition 3.1 and Theorem 3.1 with the Lyapunov function given by

$$\begin{aligned} V(e, \tilde{K}_{\mathrm{aq}}, \tilde{K}_{\mathrm{av}}) &= e^{\mathrm{T}} P_{\mathrm{am}} e + \mathrm{tr}\ \tilde{K}_{\mathrm{a}q}^{\mathrm{T}} \Gamma_{\mathrm{a}q}^{-1} \tilde{K}_{\mathrm{a}q} \mathrm{pd}(\Lambda) \\ &\quad + \mathrm{tr}\ \tilde{K}_{\mathrm{av}}^{\mathrm{T}} \Gamma_{\mathrm{a}v}^{-1} \tilde{K}_{\mathrm{a}v} \mathrm{pd}(\Lambda), \end{aligned} \quad (54)$$

where $P_{\mathrm{am}} > 0$ satisfies (49). □

Remark 4.2. Theorem 4.1 shows that $x_{\mathrm{p}}(t)$, $t \geq 0$, is bounded and $q_{\mathrm{a}}(t) \to q_{\mathrm{am}}(t)$ as $t \to \infty$. Since the first l components of $q_{\mathrm{a}}(t)$, $t \geq 0$, correspond to the filtered output of the original system $y_{\mathrm{f}}(t)$, $t \geq 0$, we can always choose an appropriate reference system for (48) that captures a desired tracking behavior for $y_{\mathrm{f}}(t)$, $t \geq 0$. Hence, Theorem 4.1 guarantees adaptive command following for the original uncertain dynamical system (1) and (2), as well as boundedness of the original system state $x_{\mathrm{p}}(t)$, $t \geq 0$.

V. Illustrative Numerical Examples

In this section, we present two numerical examples to illustrate the efficacy of the proposed adaptive control architectures for adaptive output stabilization and command following.

Example 5.1 (Adaptive command following of an unstable plant). Consider the plant given by

$$\dot{x}(t) = \begin{bmatrix} 0.5 & 1 \\ -2 & 1 \end{bmatrix} x(t) + \begin{bmatrix} -0.1 \\ 1 \end{bmatrix} u(t), \ t \geq 0, \quad (55)$$

$$y(t) = \begin{bmatrix} 1 & 2 \end{bmatrix} x(t), \quad (56)$$

with $x^{\mathrm{T}}(0) = [0.5, -0.5]$, and poles $\{0.75 \pm 1.39\jmath\}$ and zero $\{-0.26\}$. Let $\lambda = 5$ and

$$A_{\mathrm{am}} = \begin{bmatrix} 0 & 1 & 0 \\ -0.69 & -1.22 & 0.15 \\ -1 & 0 & 0 \end{bmatrix}. \quad (57)$$

Furthermore, let $R_{\mathrm{am}} = 10 I_3$, $\Gamma_{\mathrm{a}q} = 50 I_3$, $\Gamma_{\mathrm{av}} = 10$, and $\bar{B} = 1$. Finally, assume $\mathrm{id}(\Lambda) = \mathrm{id}(C_{\mathrm{p}} B_{\mathrm{p}}) = 1$. Here, our aim is to track a given square-wave reference command $r(t)$, $t \geq 0$. The closed-loop response along with the control signal and adaptive gains is shown in Figure 1. △

Example 5.2 (Adaptive command following of an unstable plant). Consider the plant given by

$$\dot{x}(t) = \begin{bmatrix} 0.5 & 5 \\ 2 & 0.5 \end{bmatrix} x(t) + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(t), \ t \geq 0, \quad (58)$$

$$y(t) = \begin{bmatrix} 1 & 2 \end{bmatrix} x(t), \quad (59)$$

with $x^{\mathrm{T}}(0) = [0.5, -0.5]$, and poles $\{3.66, \ -2.66\}$ and zero $\{-2.75\}$. Here, we use the same control design as in Example 5.1 and assume $\mathrm{id}(\Lambda) = \mathrm{id}(C_{\mathrm{p}}B_{\mathrm{p}}) = 1$. Once again, our aim is to track a given square-wave reference command $r(t)$, $t \geq 0$. The closed-loop response along with the control signal and adaptive gains is shown in Figure 2. $\triangle$

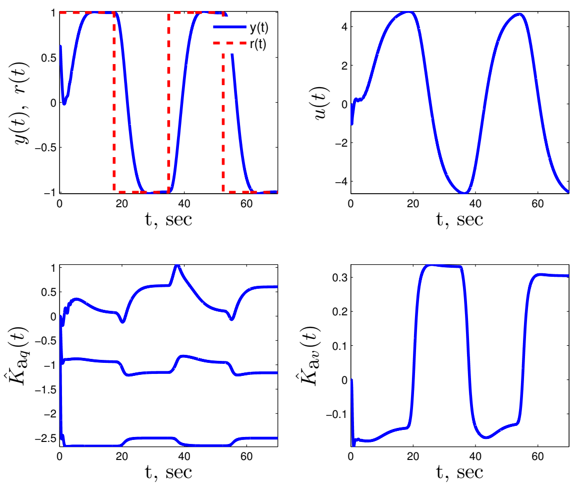

Fig. 1. Closed-loop response of the unstable plant in Example 5.1. The adaptive controller (39) with (53), (51), and (52) with $\Gamma_{\mathrm{a}q} = 50I_3$ and $\Gamma_{\mathrm{a}v} = 10$ tracks the reference $r(t)$.

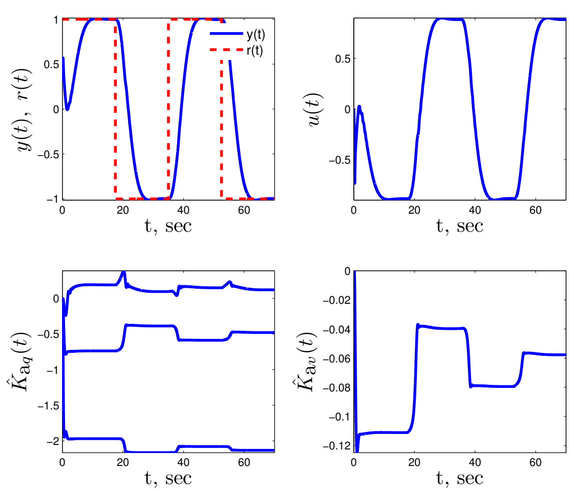

Fig. 2. Closed-loop response of the unstable plant in Example 5.2. The adaptive controller (39) with (53), (51), and (52) with $\Gamma_{\mathrm{a}q} = 50I_3$ and $\Gamma_{\mathrm{a}v} = 10$ tracks the reference $r(t)$.

VI. Conclusion

In this paper, we presented an output feedback direct adaptive control architecture for minimum phase multivariable uncertain systems with unmatched uncertainties and unstable dynamics. The proposed adaptive control algorithm is predicated on a nonminimal state space realization involving an expanded set of states with filtered versions of the system inputs and outputs and their derivatives. Future work will include extensions to nonminimum phase systems, systems with unmatched disturbances, and nonlinear uncertain dynamical systems.

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