

Incremental Closed-loop Identification of Linear Parameter Varying Systems

Jan Bendtsen Klaus Trangbaek

Abstract—This paper deals with system identification for control of linear parameter varying systems. In practical applications, it is often important to be able to identify small plant changes in an incremental manner without shutting down the system and/or disconnecting the controller; unfortunately, closed-loop system identification is more difficult than open-loop identification. In this paper we prove that the so-called Hansen Scheme, a technique known from linear time-invariant systems theory for transforming closed-loop system identification problems into open-loop-like problems, can be extended to accommodate linear parameter varying systems as well.

I. INTRODUCTION

Industrial control systems are typically in operation for extensive periods of time, amongst other things due to the fact that once a functioning system has been commissioned and brought into operation, it is very costly in terms of engineering manpower and loss of production output (and hence income) to take the system out of action in order to maintain and update it. On the other hand, most large-scale industrial systems are subject to frequent changes and modifications, which may change the dynamics of various subsystems of the overall plant. Thus, it is often the case that a control system can be improved after initial commissioning, as more actual operation data becomes available.

Assuming that a good, or at least acceptable, model for the original system already exists, however, it seems wasteful to estimate the total model from scratch in case of limited structural modifications. Motivated by this observation, we look at incremental modelling for control of plants running in closed loop in this paper.

In particular, we look at the so-called *Hansen scheme* [1], [2], [3], which, given a nominal system model and controller, allows open-loop-like system identification of any ‘missing’ dynamics parameterised by a stable system in a particular feedback structure with the nominal system and controller, the so-called *dual Youla-Kucera factorisation*—see the survey paper [4] and the references therein for further details.

In this paper, we show how the Hansen scheme can be reformulated to deal with *linear parameter varying* (LPV) systems [5], [6].

There are already a number of methods for identification of LPV systems available in the literature. [7] presents a simple ARX method; [8] proposes a control-oriented identification

framework that relies on solution of a set of Linear Matrix Inequalities. [9] considers robust invalidation of candidate LPV models. [10] discusses an approach where linear local models in a number of operating points are found by applying standard identifications procedures for linear systems in time domain. Next, an LPV model with linear fractional dependency on the measured variables is fitted with the condition of containing all the linear models identified in the previous step (differential inclusion). The fit is carried out using nonlinear least squares algorithms. [11] takes a non-parametric approach to the LPV identification problem. [12] examines interpolation methods for SISO LPV models. [13] shows that one can achieve bias-free estimation by using an instrumental variables-based approach, at least in the SISO case. [14] refines the instrumental variables method for Box-Jenkins-type models. [15], [16], [17] and [18] propose various subspace-based approaches to identification of LPV systems. Finally, [19] examines how to choose optimal orthonormal basis functions for LPV system identification.

The main contribution of the present paper is to show that the Hansen scheme can be formulated for LPV systems in a non-conservative setting using the notions of *LPV stability* shown via *polyhedral Lyapunov functions* [20]. The work presented here is related to results presented in [21] and [22], which presented similar results in a quite general, nonlinear setting. However, by restricting the class of systems under consideration here, we are able to present an explicit methodology for the identification and control design, which is suitable for controller updating as it focuses on incremental modelling. In principle, any of the above-mentioned methods can be employed for LPV identification of the dual Youla-Kucera parameter and avoid some of the specific closed-loop difficulties.

The outline of the rest of the paper is as follows. Section II provides some important preliminary results on the notion of LPV stability employed in the rest of the paper. Section III then presents a Youla-Kucera parametrisation of LPV systems, after which Section IV shows how the Hansen scheme is cast in this framework. Section V illustrates the applicability of the method on a simple simulation example. Finally, Section VI sums up the conclusions of the work.

Our notation is standard; in particular, 0 and I denote zero and identity matrices and $\mathcal{F}_u(G, \Delta)$ denotes the *upper linear fractional transformation* of G wrt. Δ , see e.g., [23, Chap. 10]. Furthermore, for $x \in \mathbb{R}^n$ $\|\cdot\|_\infty$ denotes the infinity norm defined by $\|x\|_\infty = \max_{1 \leq i \leq n} |x_i|$. $(\cdot)_\theta$ indicates that (\cdot) depends on the parameter θ .

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The authors are with the Department of Electronic Systems, Automation and Control, Aalborg University, Denmark; email: {dimon, ktr}@es.aau.dk

II. LPV STABILITY

In this work, we consider discrete-time linear parameter-varying (LPV) systems G_θ with a minimal state space realisation given by matrix functions $A_\theta \in \mathbb{R}^{n \times n}$, $B_\theta \in \mathbb{R}^{n \times m}$, $C_\theta \in \mathbb{R}^{p \times n}$ and $D_\theta \in \mathbb{R}^{p \times m}$, mapping an input signal vector $u \in \mathbb{R}^m$ to an output measurement signal $y \in \mathbb{R}^p$. Specifically, we deal with systems of the form

$$G_\theta : \quad x_{k+1} = A_{\theta(k)}x_k + B_{\theta(k)}u_k \quad (1)$$

$$y_k = C_{\theta(k)}x_k + D_{\theta(k)}u_k \quad (2)$$

where $\theta(k) \in \mathbb{R}^q$ is an external scheduling parameter, which is allowed to vary as a function of time but not as a function of the system states x . Since we only allow θ to depend on k , we will simply write θ rather than $\theta(k)$ in the following. We require that θ belongs to the bounded compact set

$$\Theta = \left\{ \theta \in \mathbb{R}^q \mid \theta_i \geq 0, \sum_{i=1}^q \theta_i = 1 \right\}$$

and that $A_\theta, B_\theta, C_\theta$ and D_θ are continuous, bounded functions of $\theta \in \Theta$ (only).

For notational convenience, we will use the shorthand

$$G_\theta = \left[\begin{array}{c|c} A_\theta & B_\theta \\ \hline C_\theta & D_\theta \end{array} \right]$$

for the LPV system (1)–(2) in the sequel.¹

If D_θ is nonsingular, i.e., D_θ^{-1} is well defined for all θ , the LPV system G_θ has an inverse operator

$$G_\theta^{-1} = \left[\begin{array}{c|c} A_\theta + B_\theta D_\theta^{-1} C_\theta & B_\theta D_\theta^{-1} \\ \hline D_\theta^{-1} C_\theta & D_\theta^{-1} \end{array} \right]$$

in the sense that $G_\theta G_\theta^{-1} = G_\theta^{-1} G_\theta = I$ for any trajectory of θ . We will ensure invertibility by construction whenever necessary in the sequel.

With this notion of inverse LPV system in place, the *upper fractional transformation* can be naturally extended from LTI theory – see [23, Chap. 10] – to linear time varying operators.

Next, consider the autonomous LPV system $x_{k+1} = A_\theta x_k$ along with the Lyapunov function candidate $V(x) = \|Wx\|_\infty$, where $W \in \mathbb{R}^{n \times n}$ is a constant matrix of rank n . $V(x)$ is obviously a positive definite function with $V(0) = 0$. Computing the sample-to-sample difference yields

$$\begin{aligned} V(x_{k+1}) - V(x_k) &= \|Wx_{k+1}\|_\infty - \|Wx_k\|_\infty \\ &= \|WA_\theta x_k\|_\infty - \|Wx_k\|_\infty \end{aligned}$$

which is negative if A_θ is sufficiently small; this can be tested via algebraic means. If the autonomous part of an LPV system admits such a Lyapunov function for all $\theta \in \Theta$, we say that it is *LPV stable*.

In particular, it is known that a *polytopic* LPV system, i.e., a system where $A_\theta, B_\theta, C_\theta$ and D_θ are given as convex combinations of fixed matrices A_i, B_i, C_i and $D_i, i = 1, \dots, q$, admits a polyhedral Lyapunov function if the associated

¹Please note that this notation should not be confused with “transfer functions”; throughout the paper we strictly consider operators defined in state space, as given by (1)–(2), with $x_0 = 0$ unless otherwise noted.

matrix equalities hold for each vertex system. Furthermore, it is shown in [20] that the existence of a polyhedral Lyapunov function is in fact *equivalent* to LPV stability for polytopic LPV systems. That is, this class of Lyapunov functions is non-conservative, as opposed to e.g. quadratic Lyapunov functions in the sense that one may find examples of stable polytopic LPV systems that do not permit a quadratic Lyapunov function, but it is not possible to find stable polytopic LPV systems that do not permit a polyhedral Lyapunov function. We require the following technical result:

Lemma 1: $V(x) = \|Wx\|_\infty$ is a (*polyhedral*) Lyapunov function for the polytopic autonomous LPV system $x_{k+1} = A_\theta x_k$ if and only if there exist matrices $Q_i \in \mathbb{R}^{n \times n}$ such that $WA_i = Q_i W$ and $\|Q_i\|_\infty < 1$ for $i = 1, \dots, q$.

Proof: See [20]. ■

Based on Lemma 1 we can show the following simple, yet important result for connection of LPV systems.

Lemma 2: Suppose two autonomous LPV systems $x_{1,k+1} = A_\theta^{11} x_{1,k}$ and $z_{2,k+1} = A_\theta^{22} z_{2,k}$ are LPV stable; then for any continuous and bounded A_θ^{21} of appropriate dimensions, the autonomous LPV system

$$\begin{bmatrix} x_{1,k+1} \\ x_{2,k+1} \end{bmatrix} = \begin{bmatrix} A_\theta^{11} & 0 \\ A_\theta^{21} & A_\theta^{22} \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix} \quad (3)$$

is also LPV stable.

Proof: According to Lemma 1, since the systems $x_{1,k+1} = A_\theta^{11} x_{1,k}$ and $z_{2,k+1} = A_\theta^{22} z_{2,k}$ are LPV stable, there exist matrices $W^1, W^2, Q_\theta^1, Q_\theta^2$ of appropriate dimensions with $\|Q_\theta^1\|_\infty < 1, \|Q_\theta^2\|_\infty < 1$ such that

$$\begin{bmatrix} W^1 & 0 \\ 0 & W^2 \end{bmatrix} \begin{bmatrix} A_\theta^{11} & 0 \\ 0 & A_\theta^{22} \end{bmatrix} = \begin{bmatrix} Q_\theta^1 & 0 \\ 0 & Q_\theta^2 \end{bmatrix} \begin{bmatrix} W^1 & 0 \\ 0 & W^2 \end{bmatrix}$$

for $\theta \in \Theta$. Also, we have

$$\left\| \begin{bmatrix} Q_\theta^1 & 0 \\ 0 & Q_\theta^2 \end{bmatrix} \right\|_\infty < 1.$$

Turning to the combined system (3), if we can find a scalar $\beta > 0$ and a θ -dependent matrix Q_θ^{21} such that

$$\begin{bmatrix} W^1 & 0 \\ 0 & \beta W^2 \end{bmatrix} \begin{bmatrix} A_\theta^{11} & 0 \\ A_\theta^{21} & A_\theta^{22} \end{bmatrix} = \begin{bmatrix} Q_\theta^1 & 0 \\ Q_\theta^{21} & Q_\theta^2 \end{bmatrix} \begin{bmatrix} W^1 & 0 \\ 0 & \beta W^2 \end{bmatrix}$$

and

$$\left\| \begin{bmatrix} Q_\theta^1 & 0 \\ Q_\theta^{21} & Q_\theta^2 \end{bmatrix} \right\|_\infty < 1$$

hold for every $\theta \in \Theta$, then we can conclude that the system is LPV stable by invoking Lemma 1. Rewriting the matrix equality above, we get

$$\begin{bmatrix} W^1 A_\theta^{11} & 0 \\ \beta W^2 A_\theta^{21} & \beta W^2 A_\theta^{22} \end{bmatrix} = \begin{bmatrix} Q_\theta^1 W^1 & 0 \\ Q_\theta^{21} W^1 & \beta Q_\theta^2 W^2 \end{bmatrix}$$

which is satisfied iff $\beta W^2 A_\theta^{21} = Q_\theta^{21} W^1 \forall \theta \in \Theta$.

Since W^1 has full row rank, it has a left pseudo-inverse $W^{1\dagger}$; thus, we may choose $Q_\theta^{21} = \beta W^2 A_\theta^{21} W^{1\dagger}$ with β sufficiently small to satisfy

$$\left\| \begin{bmatrix} Q_\theta^1 & 0 \\ \beta W^2 A_\theta^{21} W^{1\dagger} & Q_\theta^2 \end{bmatrix} \right\|_\infty < 1 \quad \forall \theta \in \Theta$$

which is always possible since A_θ^{21} is bounded. ■

III. BASIC PARAMETRISATION

In the rest of the paper, we will assume that the plant G_θ is strictly proper, i.e.

$$G_\theta = \left[\begin{array}{c|c} A_\theta & B_\theta \\ \hline C_\theta & 0 \end{array} \right] \quad (4)$$

and that it can be stabilised by an observer-based LPV controller of the form

$$K_\theta = \left[\begin{array}{c|c} A_\theta + B_\theta F_\theta + L_\theta C_\theta & -L_\theta \\ \hline F_\theta & 0 \end{array} \right] \quad (5)$$

for all $\theta \in \Theta$, where F_θ and L_θ are such that $\bar{x}_{k+1} = (A_\theta + B_\theta F_\theta)\bar{x}_k$ and $\hat{x}_{k+1} = (A_\theta + L_\theta C_\theta)\hat{x}_k$ are LPV stable.

Any G_θ that satisfies the above assumption for any trajectory of $\theta \in \Theta$, can be written as a right, respectively left, coprime factorisation of the form:

$$G_\theta = N_\theta M_\theta^{-1} = \tilde{M}_\theta^{-1} \tilde{N}_\theta \quad (6)$$

where $N_\theta, M_\theta, \tilde{M}_\theta$ and \tilde{N}_θ are LPV stable operators of a specific form given below. Correspondingly, K_θ can be factorised as

$$K_\theta = U_\theta V_\theta^{-1} = \tilde{V}_\theta^{-1} \tilde{U}_\theta \quad (7)$$

with LPV stable $U_\theta, V_\theta, \tilde{U}_\theta, \tilde{V}_\theta$. The factors are given as

$$\begin{bmatrix} M_\theta & U_\theta \\ N_\theta & V_\theta \end{bmatrix} = \left[\begin{array}{cc|cc} A_\theta + B_\theta F_\theta & B_\theta & -L_\theta & \\ \hline F_\theta & I & 0 & \\ C_\theta & 0 & I & \end{array} \right] \quad (8)$$

$$\begin{bmatrix} \tilde{V}_\theta & -\tilde{U}_\theta \\ -\tilde{N}_\theta & \tilde{M}_\theta \end{bmatrix} = \left[\begin{array}{cc|cc} A_\theta + L_\theta C_\theta & -B_\theta & L_\theta & \\ \hline F_\theta & I & 0 & \\ C_\theta & 0 & I & \end{array} \right] \quad (9)$$

Then, it is possible to check that

$$\begin{aligned} \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} &= \begin{bmatrix} \tilde{V}_\theta & -\tilde{U}_\theta \\ -\tilde{N}_\theta & \tilde{M}_\theta \end{bmatrix} \begin{bmatrix} M_\theta & U_\theta \\ N_\theta & V_\theta \end{bmatrix} \\ &= \begin{bmatrix} M_\theta & U_\theta \\ N_\theta & V_\theta \end{bmatrix} \begin{bmatrix} \tilde{V}_\theta & -\tilde{U}_\theta \\ -\tilde{N}_\theta & \tilde{M}_\theta \end{bmatrix} \end{aligned} \quad (10)$$

holds; this equation is referred to as the *double Bezout identity*.

We are now able to show the following result; see Figure 1.

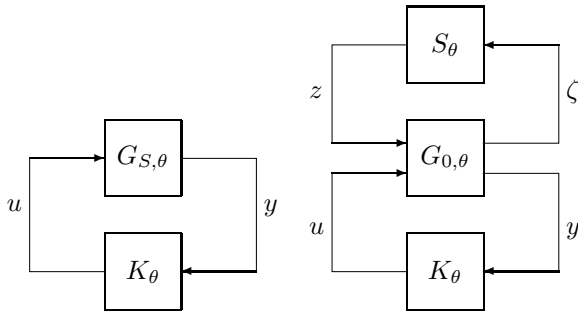


Fig. 1. All LPV systems $G_{S,\theta}$ stabilised by the LPV controller K_θ (left) can be represented by a nominal system $G_{0,\theta}$ stabilised by K_θ and a dual Youla-Kucera parameter S_θ (right).

Theorem 1: Let $G_\theta = N_\theta M_\theta^{-1}$ with state space realisation (4) be LPV stabilised by a feedback controller $K_\theta = U_\theta V_\theta^{-1}$ with state space realisation (5). Let F_θ and L_θ be matrix functions such that $\bar{x}_{k+1} = (A_\theta + B_\theta F_\theta)\bar{x}_k$ and $\hat{x}_{k+1} = (A_\theta + L_\theta C_\theta)\hat{x}_k$ are LPV stable for all $\theta \in \Theta$. All such plants stabilised by K_θ can be parametrised as $G_{S,\theta} = \mathcal{F}_u(G_{0,\theta}, S_\theta)$, where

$$G_{0,\theta} = \left[\begin{array}{c|cc} A_\theta & -L_\theta & B_\theta \\ \hline -F_\theta & 0 & I \\ C_\theta & I & 0 \end{array} \right]$$

and $S_\theta = \left[\begin{array}{c|c} A_{S,\theta} & B_{S,\theta} \\ \hline C_{S,\theta} & 0 \end{array} \right]$ is any proper LPV stable system. S_θ is denoted the dual Youla-Kucera parameter.

Proof: We first show that under the given assumptions, K_θ stabilises $G_{S,\theta}$. The upper loop in the right part of Figure 1 is closed, yielding $G_{S,\theta}$ in the left part of the figure:

$$\begin{aligned} G_{S,\theta} &= \mathcal{F}_u(G_{0,\theta}, S_\theta) \\ &= \left[\begin{array}{cc|c} A_{S,\theta} & -B_{S,\theta}F_\theta & B_{S,\theta} \\ \hline -L_\theta C_{S,\theta} & A_\theta & B_\theta \\ C_{S,\theta} & C_\theta & 0 \end{array} \right] \end{aligned} \quad (11)$$

and when connecting K_θ as shown to this system, we obtain the autonomous LPV system

$$\begin{bmatrix} \xi_{k+1} \\ \eta_{k+1} \\ \chi_{k+1} \end{bmatrix} = \begin{bmatrix} A_{S,\theta} & -B_{S,\theta}F_\theta & 0 \\ 0 & A_\theta + L_\theta C_\theta & 0 \\ -L_\theta C_{S,\theta} & -L_\theta C_\theta & A_\theta + B_\theta F_\theta \end{bmatrix} \begin{bmatrix} \xi_k \\ \eta_k \\ \chi_k \end{bmatrix}$$

where ξ is the state vector of S_θ , χ is the controller state vector and $\eta = x - \chi$ is the difference between the state vector of $G_{0,\theta}$ and K_θ . Since $A_{S,\theta}$, $A_\theta + L_\theta C_\theta$ and $A_\theta + B_\theta F_\theta$ are all LPV stable, and $B_{S,\theta}F_\theta$, $L_\theta C_{S,\theta}$ and $L_\theta C_\theta$ are bounded for bounded θ , we can then conclude that the closed-loop system is LPV stable by applying Lemma 2 twice in succession.

We then show that, given a $K_\theta = U_\theta V_\theta^{-1}$, a nominal $G_\theta = N_\theta M_\theta^{-1}$ stabilised by K_θ and a $G_{S,\theta}$ also stabilised by K_θ , there exists an S_θ (connected as shown in Fig. 1) such that the interconnection of $G_{0,\theta}$ and S_θ is identical to $G_{S,\theta}$.

We construct the dual Youla-Kucera parameter as $S_\theta = \mathcal{F}_u(\bar{G}_\theta, G_{S,\theta})$, where

$$\bar{G}_\theta = \left[\begin{array}{c|cc} A_\theta + B_\theta F_\theta + L_\theta C_\theta & -L_\theta & B_\theta \\ \hline F_\theta & 0 & I \\ -C_\theta & I & 0 \end{array} \right]$$

First, we note that the (1,1)-block subsystem of \bar{G}_θ is identical to K_θ (cf. (5)); thus, since $\mathcal{F}_u(K_\theta, G_\theta)$ is LPV stable, $S_\theta = \mathcal{F}_u(\bar{G}_\theta, G_{S,\theta})$ is also LPV stable. Secondly, it is fairly easy to see that

$$\mathcal{F}_u(G_{0,\theta}, \bar{G}_\theta) = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}$$

which is the upper fractional transformation identity. Thus,

$$\begin{aligned}\mathcal{F}_u(G_{0,\theta}, S_\theta) &= \mathcal{F}_u(G_{0,\theta}, S_\theta) \\ &= \mathcal{F}_u(G_{0,\theta}, \mathcal{F}_u(\bar{G}_\theta, G_{S,\theta})) \\ &= \mathcal{F}_u(\mathcal{F}_u(G_{0,\theta}, \bar{G}_\theta), G_{S,\theta}) \\ &= G_{S,\theta}.\end{aligned}$$

which completes the proof. \blacksquare

Note that knowledge of a specific polytopic Lyapunov function is not required in the proof; we simply require the state transformations to be independent of the system states.

By Theorem 1, all LPV systems stabilized by K_θ can be written as $G_{S,\theta} = \mathcal{F}_u(G_{0,\theta}, S_\theta)$, with $G_{0,\theta}$ given in the theorem. By inspection, it is seen that

$$\begin{aligned}G_{0,\theta} &= \left[\begin{array}{c|cc} A_\theta & -L_\theta & B_\theta \\ \hline -F_\theta & 0 & I \\ C_\theta & I & 0 \end{array} \right] \\ &= \begin{bmatrix} -M_\theta^{-1}U_\theta & M_\theta^{-1} \\ \tilde{M}_\theta^{-1} & G_\theta \end{bmatrix} \\ &= \begin{bmatrix} -M_\theta^{-1}U_\theta & M_\theta^{-1} \\ V_\theta - N_\theta M_\theta^{-1}U_\theta & N_\theta M_\theta^{-1} \end{bmatrix}\end{aligned}$$

where the last equality is obtained by the Bezout identity. Then, it can be checked that

$$\begin{aligned}\mathcal{F}_u(G_{0,\theta}, S_\theta) &= (N_\theta + V_\theta S_\theta)(M_\theta + U_\theta S_\theta)^{-1} \\ &= \left(\tilde{M}_\theta + S_\theta \tilde{U}_\theta \right)^{-1} \left(\tilde{N}_\theta + S_\theta \tilde{V}_\theta \right)\end{aligned}\quad (12)$$

This setup is depicted in Figure 2 and will be used in the following.

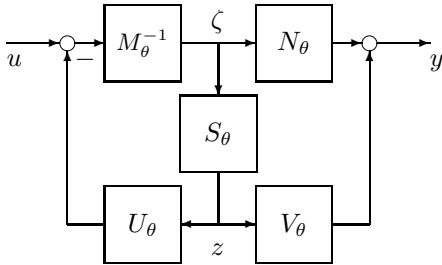


Fig. 2. Dual Youla-Kucera parametrization of all proper polytopic LPV plants stabilized by the LPV controller $K_\theta = U_\theta V_\theta^{-1}$.

IV. OPEN-LOOP-LIKE SYSTEM IDENTIFICATION

We assume that a nominal state space LPV model of an existing system, G_θ , has been found. The system takes control signals u as input, and yields corresponding output measurements y , which are affected by additive noise $n_y \in \mathbb{R}^p$. The parameter variation θ is measurable and satisfies the assumptions in the previous sections.

Based on this model, a stabilising observer-based LPV controller K_θ of the form (5) with stable observer and state feedback dynamics has been designed, for instance using the methods in [24]. However, for some reason, e.g., monitoring

of the plant during operation, it is suspected that there is additional un-modelled dynamics, which we wish to identify.

Since K_θ stabilises $G_{S,\theta}$ and (12) is a *full parametrisation* of all LPV systems stabilised by K_θ , Theorem 1 ensures that there exists an (LPV stable) parameter system S_θ such that $G_{S,\theta}$ can be written as in (12) (or, equivalently, as in (11)).

Consider now the setup shown in Figure 3, where K_θ and G_θ are shown in their factorised form as in (7) and (6), respectively. $n' = (\tilde{M}_\theta + S_\theta \tilde{U}_\theta)n_y$ is the measurement noise that would normally affect the measurements y , relocated in the block diagram to affect the output of the parameter system instead, and r_1 and r_2 are external excitation signals.

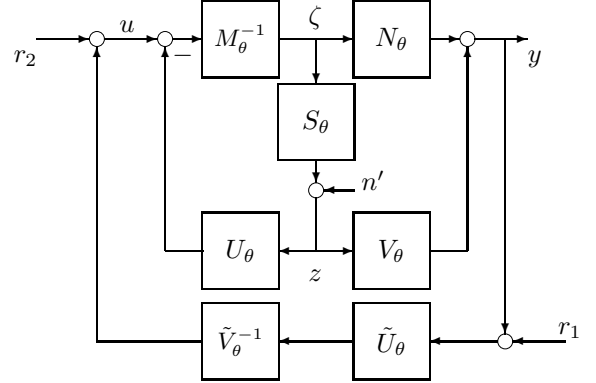


Fig. 3. ‘Hansen scheme’ setup for closed-loop system identification

From the block diagram, we find the following relations:

$$(N_\theta + V_\theta S_\theta)\zeta = y - V_\theta n' \quad (13)$$

and

$$\begin{aligned}(M_\theta + U_\theta S_\theta)\zeta &= u - U_\theta n' \\ &= r_2 + \tilde{V}_\theta^{-1} \tilde{U}_\theta (y + r_1) - U_\theta n'\end{aligned}\quad (14)$$

Applying the LPV operators \tilde{V}_θ and \tilde{U}_θ to (13) and (14), respectively, then yields

$$\begin{aligned}\tilde{V}_\theta(M_\theta + U_\theta S_\theta)\zeta &= \tilde{U}_\theta(r_1 + y) + \tilde{V}_\theta r_2 - \tilde{V}_\theta U_\theta n' \\ \tilde{U}_\theta(N_\theta + V_\theta S_\theta)\zeta &= \tilde{U}_\theta y - \tilde{U}_\theta V_\theta n'\end{aligned}$$

Subtracting the bottom equation from the top equation and using the Bezout identity then results in

$$\zeta = \tilde{U}_\theta r_1 + \tilde{V}_\theta r_2 \quad (15)$$

In a similar vein, from the block diagram, we have the relations

$$\begin{aligned}M_\theta \zeta &= u - U_\theta z \\ N_\theta \zeta &= y - V_\theta z\end{aligned}$$

Applying the LPV stable filters \tilde{N}_θ to the top expression and \tilde{M}_θ to the bottom one, subtracting one from the other and using the Bezout identity then results in

$$z = \tilde{M}_\theta y - \tilde{N}_\theta u \quad (16)$$

Thus, ζ and z can be obtained by filtering measurements through known, stable LPV filters. Furthermore, assuming

n_y is independent of r_1 and r_2 , then ζ is independent of n' as well.

As a consequence, although u and y are measured in closed-loop, the identification of S_θ using the signals θ , z and ζ becomes equivalent to an open-loop LPV identification problem. S_θ can in principle be identified using any of the methods mentioned in the Introduction. When the identification is complete, $G_{S,\theta}$ may then be recovered by inserting S_θ in (12), or, more conveniently, in (11).

V. SIMULATION EXAMPLE

We consider the following unstable system with a single time varying parameter $0 \leq \theta \leq 1$:

$$\begin{aligned} x_{k+1} &= A_\theta x_k + B u_k + K v_k \\ y_k &= C x_k + v_k, \end{aligned}$$

$$A_\theta = \begin{bmatrix} 0.9 & 0.05 & 0.1 & -0.3 & 0.4 \\ -0.2 - 0.7\theta & 0.9 & 0 & 0 & 0 \\ 0 & 0.1 & 0.9 & 0.1 & -0.1 \\ 0.3 + \theta & 0 & 0 & 0 & 0.3 + \kappa \\ 0 & 0.3 & -0.3 & 0.3 & 0.92 + 0.05\theta \end{bmatrix},$$

$$B = \begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \quad K = \begin{bmatrix} -0.8 \\ 0.3 \\ 0 \\ 0 \\ -0.7 \end{bmatrix},$$

$$C = [0 \quad 1 \quad 2 \quad 1 \quad -1],$$

with $\kappa = 0.3$ and $E\{v_k v_k^T\} = 10^{-6}$. We assume that we already have a reasonably accurate nominal model $(A_{m,\theta}, B_m, C_m)$ of the deterministic part. $A_{m,\theta}$ is equal to A_θ , except that the model assumes $\kappa = 0$, while the input and output matrices are correctly identified, i.e., $B_m = B$, $C_m = C$.

The system is open loop unstable and only barely detectable and stabilisable; in fact, although the model error may seem small, even a slightly larger error can in fact easily cause an unstable closed loop.

A stabilising LPV controller

$$\begin{aligned} x_{c,k+1} &= (A_{m\theta} + B_m F_\theta + L_\theta C_m) x_{c,k} - L_\theta y_k \\ u_k &= F_\theta x_{c,k} \end{aligned}$$

with

$$F_\theta = [0.11 - 0.27\theta \quad 0.42 \quad -0.43 \quad 0.12 + 0.05\theta \quad 0.7]$$

$$L_\theta = \begin{bmatrix} 0.87 - 0.37\theta \\ -0.26 - 0.77\theta \\ -0.19 \\ 0.47 + 0.4\theta \\ 0.87 \end{bmatrix}$$

has been designed for the system. It satisfies the requirements given in Theorem 1 for all $\theta \in [0; 1]$.

In closed loop operation, excitation in the form of white noise with variance 1 is added to the input (r_2 in Figure 3). The full output measurement sequence is shown in Figure 4 and a zoom of the signals along with the auxiliary signals used in the Hansen scheme is shown in Figure 5.

In all the identifications, models on the form $\hat{x}_{k+1} = \hat{A}_\theta \hat{x}_k + \hat{B}_\theta u_k$, $\hat{y}_k = \hat{C}_\theta \hat{x}_k$ are assumed, with \hat{A}_θ and \hat{B}_θ depending linearly on θ .

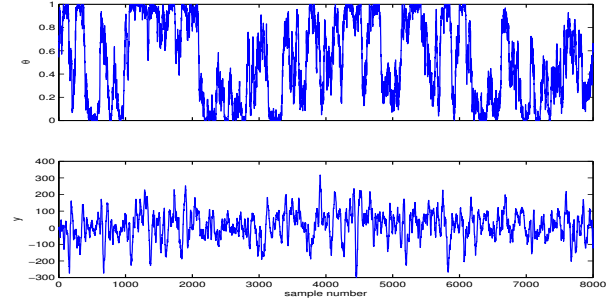


Fig. 4. Measurement data for system identification. Top: $\theta(k)$; bottom: y_k

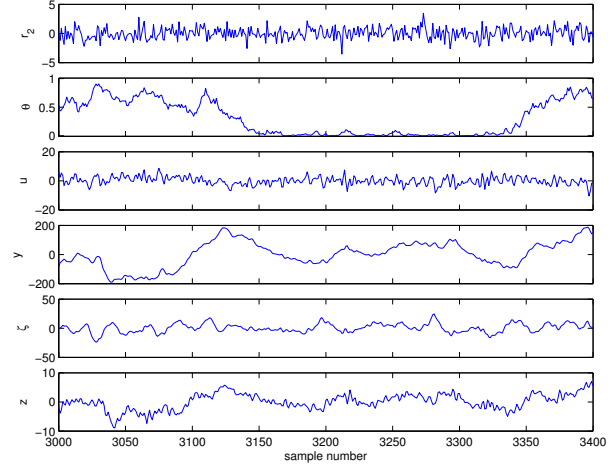


Fig. 5. Zoom of measurement data, including auxiliary signals. From top to bottom: $r_{2,k}$; $\theta(k)$; u_k ; y_k ; ζ_k ; z_k

In order to evaluate the obtained models, the ν -gap between the model and the real system is computed. The ν -gap is a value between 0 and 1 that expresses the difference between two transfer functions in terms of their similarity with respect to closed loop operation; that is, if the ν -gap between two plant models is small, then a good controller designed for one transfer function will also work well with the other [25]. The ν -gap is only defined for LTI systems, so the comparisons strictly speaking only hold for fixed values of θ . However, to the best of the authors' knowledge, no other meaningful tools for comparison of closed-loop LPV model fitness are known. Here, the ν -gap is evaluated for θ frozen at 0, 0.5 and 1.

The identifications are performed using an increasing number of samples, in order to evaluate how much excitation is needed.

Two identification methods, ARX and PBSIDopt, are tested, both in a direct form and using the Hansen scheme. The state space matrices are found by minimising the prediction error using least squares methods. Note that we do not assume any explicit knowledge of which entries in A_m are erroneous, so a direct grey box approach is not possible.

The first identification method examined is the LPV ARX method found in e.g. [7] and [13]. Here, the state estimate

simply consists of delayed outputs and inputs. In the direct application, the method is simply fed measured input and output data and model with 5 delayed outputs and 5 delayed inputs is identified. We assume a zero-order polynomial dependence on θ in the identification. The dash-dot line in Figure 6 shows the ν -gap as a function of the number of samples used. For $\theta = 1$ the model is acceptable, but for $\theta = 0$ and $\theta = 0.5$, even large numbers of samples do not yield acceptable models. Making delayed values of θ available to the identification algorithm did not improve the model, either.

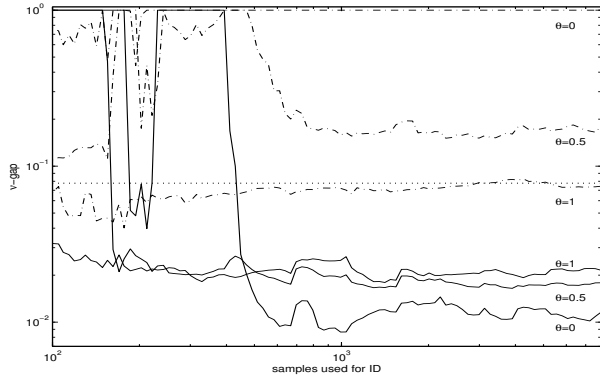


Fig. 6. ν -gap for different models identified using ARX methods, with frozen values of θ .

Next, the ARX method is used to identify a dual Youla parameter in a Hansen scheme. First the data is filtered as discussed in Section IV. Then the ARX method is used to identify S_θ , again with 5 delayed outputs and 5 delayed inputs, which is then combined with the nominal model as in Eqn. (11). The resulting model error is shown by the solid lines in Figure 6. The dotted lines show the ν -gap for the nominal model (which is approximately 0.08 for all frozen θ), indicating that a significant improvement is achieved with a reasonably small number of samples.

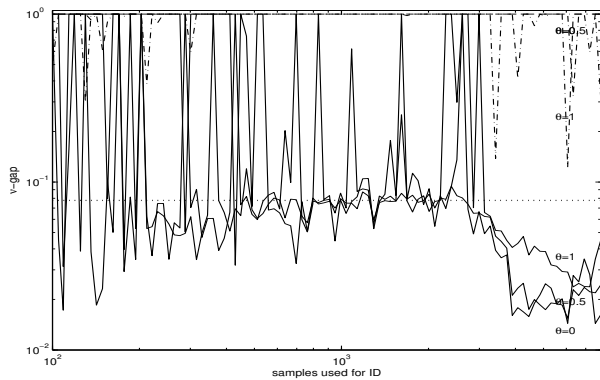


Fig. 7. ν -gap for different models identified using PBSIDOpt, with frozen values of θ .

The second method examined is PBSIDopt, which is presented in an LPV version in [16]. In this approach, a

subspace method is used to construct the state estimates, and consequently requires a lot of computational power.

First PBSIDopt (with a window length of 9) is applied directly to the measurements to obtain a 5th order LPV model, and the result, shown by the dash-dot lines in Figure 7, is quite poor. Changing the window length did not improve the identification noticeably.

Next, PBSIDopt (again with a window length of 9) is applied to obtain a 7th order LPV model of S_θ in the Hansen scheme. The ν -gaps of the resulting model is shown with solid lines in Figure 7; as can be seen, the ν -gap drops below those of the nominal model when more than 3000 samples are used. The result is not as good as for the Hansen ARX method, but it is a definite improvement over using PBSIDopt directly.

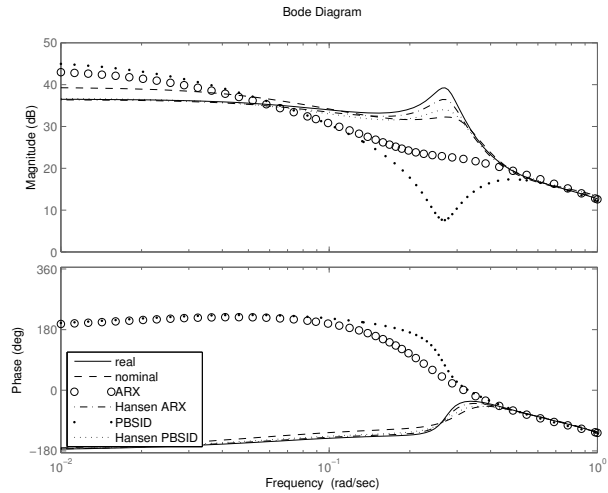


Fig. 8. Bode plots for different models identified using PBSIDOpt, with $\theta = 0.9$.

Figure 8 shows Bode plots for all the models obtained with the maximum number of samples, with θ frozen at 0.9. The picture is similar for all other values of θ ; the Hansen scheme is able to capture the spike, whereas the direct methods are not.

The reason that the Hansen scheme improves on the identification is likely different for the two different identification methods. For the ARX case, the closed-loop nature of the data affects the direct ARX method, and the Hansen scheme helps to decouple these effects. In PBSIDopt, the main approximation lies in assuming that the state transition is zero beyond the window length; in this example this is not the case. The Hansen scheme, on the other hand, focuses on the identification of a subsystem, where this assumption is closer to being satisfied. Finally, it should be noted that n' is the output noise filtered through a combination of known factors and the unknown S_θ . As pointed out in [26], this may be exploited in a grey box setup to further improve the results with the Hansen scheme.

VI. DISCUSSION

In this paper we considered incremental system identification of LPV systems that are modified during online operation, for instance due to replacement and/or addition of system components (so-called *plug-and-play control*). We used the notion of polyhedral Lyapunov functions to prove the existence of a dual Youla-Kucera parameter system for proper polytopic LPV systems in a non-conservative manner. Then we showed how the Hansen scheme can be used for incremental system identification of such LPV systems, taking the starting point in a nominal system model and identifying the unknown dynamics by means of identification of said dual Youla-Kucera parameter in an open-loop-like setting. The method is an extension of the Hansen scheme for LTI systems. This particular approach is suited for plug-and-play control, where system dynamics is changed during online operation e.g. due to replacement or introduction of new sensors, actuators or other components; only the changed dynamics need to be identified, while nominal plant and controller information may be retained.

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