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PID-Structured Controller Design for Interval Systems: Application to Piezoelectric Microactuators

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***Abstract*— This paper addresses the modeling and robust PID controller design for piezoelectric microsystems. Piezoelectric cantilevers, used as microactuators in micromanipulation and microassembly contexts, are particularly concerned. Due to their small sizes, these systems are very sensitive to environment (temperature, vibration, etc.) and to usury during functioning. Their behaviors often change because of the parameters variation. For that, linear modeling with uncertainty has been used to account the uncertainties, then classical H_∞ and μ-synthesis approaches were applied. These techniques were efficiency but they were of high order which is not suitable for embedded microsystems. Furthermore, when the number of uncertain parameters increases, the modeling of microsystems became delicate and difficult.**

In this paper, we propose to model the uncertain parameters by bounding them with intervals. Afterwards, we propose to design a robust PID controller by using interval arithmetic and related tools in order to ensure the specified performances. In addition to thesimplicity of the uncertainties modeling, the derived controller is of low order. The controller synthesis is formulated as a set-inversion problem. An application to the control of piezoelectric microactuators proves the efficiency of the proposed method.

I. Introduction

The development of high performances miniaturized systems - which is of great interest for various applications - presents technological and scientifical challenges. These miniaturized systems called microsystems are very sensitive to the environmental conditions (temperature, vibrations, etc.) and to the interaction and contact with surrounding systems (objects, other microsystems). The models of their behaviors are therefore subject to change and uncertainties that should be taken into acocunt during the controller design. Among these microsystems, piezoelectric microgrippers are used to manipulate and assembly or characterize artificial micro-objects and biological cells with sizes ranging between $10\mu m$ to $1mm$. A microgripper is composed of two piezoelectric cantilevered microactuators (piezocantilevers) [1] [2]. In fact, on the one hand piezoelectric materials are widely used because of their high resolution, large bandwidth and high force density [3], but on the other hand their high sensitivity makes the developed piezoelectric microactuators lose the accuracy.

To achieve the required performances in micromanipulation and microassembly tasks, linear modeling with Δ-matrix uncertainties have been used and classical robust control laws (H_2, H_∞ and μ-synthesis) were applied for each piezocantilever [4]–[7]. The efficiency of these advanced methods was proved in several applications (SISO and MIMO microsystems). However their major disadvantage is the derivation of high-order controllers which are time consuming and which limit their embedding possibilities, as required for real packaged microsystems.

A possible alternative to classical robust control laws is the use of interval analysis which is a suitable tool dealing with parametric uncertain models. These parametric uncertain models have known structures with unknown parameters, but their values are assumed constant within given intervals. Their possible values are usually bounded by intervals. The principle of the controller design is therefore based on the combination of the interval arithmetic with a linear control theory. In addition to its principle simplicity to model the uncertain parameters, the main advantage is the derivation of low-order controllers.

The first idea on interval arithmetic has been proposed in 1924 by Burkill and 1931 by Young, then later in 1966 with R.E. Moore's works [8]. Since, several applications on interval analysis have been raised. Since then, several applications appeared on the subject. Some of them relates to guaranteed estimation, robust stability and controllers design. The works in [9] [10] deal with guaranteed parameters estimation based on the SIVIA algorithm (Set Inversion Via Interval Analysis). In [11] [12] [13], the stability analysis of the closed-loop with a given controller was proposed using the Routh's criteria and/or the Kharitonov's theorem. Concerning the design of controller, *Chen and Wang* [14] proposed a systematic computational technique to design of robust stabilizing controller for interval systems basing on the transformation of the robust controller design problem into an equivalently non-linearly constrained optimization problem. In [15] an approach of state feedback control combined with the intervals for the parameters model was proposed to synthesize a controller that ensures the stability. In [16], a PID controller that ensures robust performances was proposed by using a set-inversion problem. The method suffers from the computational complexity, particularly when using high-order interval systems. *Chen and Wang* [17] also proposed

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a robust method to control interval systems. In their work, two controllers were necessary: a robust controller stabilizing the feedback first, and then a pre-filter must be computed to ensure the wanted performances. *Li et al.* [18] proposed a control algorithm prediction-based interval model that was efficiently applied to a welding process. In our previous work [19], a robust controller for interval systems with zero-order numerator was proposed. Its main advantage relative to the other existing works is that the order of the system's denominator is not limited and the derived controller has a low-order. This later work also proved that interval analysis and related controller design could be very promising for modeling and control microsystems.

This paper deals with the interval modeling and robust control design for piezoelectric microactuators. While we bound the uncertain parameters with intervals, a PID structure controller is proposed. Contrary to the previous work [19], the proposed approach is extended to general transfer function, i.e. no limitation on the degrees of both numerator and denominator. Despite the limitation to SISO systems and the account of only parametric uncertainties, the proposed approach proposes a low-order cotroller (PID) that is suitable for real-time embedded microsystems.

The paper is organized as follows. In section-II, preliminaries related to interval analysis and systems are provided. Section-III is dedicated to the tuning of the robust PID controller. In section-IV is concerned with the application of the proposed method to control piezoelectric microactuators. Finally, the experimental results end the paper.

II. Mathematical preliminaries

A. Basic Terms and Concepts on intervals

More details on the preliminaries given here can be found in [8] or [11].

A closed interval number denoted by $[x]$ corresponds to a range of real values, it can be represented by the left and right endpoints x^- and x^+ respectively:

$$[x] = [x^-, x^+] = \{x \in R \backslash x^- \leq x \leq x^+\} \tag{1}$$

An ordinary real number x can be represented by a degenerate interval $[x, x]$ where $x^- = x^+$.

A vector (box) of n interval parameters is denoted by:

$$[\mathbf{x}] = [[x_1], [x_2], ..., [x_n]] \tag{2}$$

The width of an interval $[x]$ is given by:

$$w([x]) = x^+ - x^- \tag{3}$$

The midpoint of [x] is given by:

$$mid([x]) = \frac{x^+ + x^-}{2} \tag{4}$$

The radius of $[x]$ is defined by:

$$rad([x]) = \frac{x^+ - x^-}{2} \tag{5}$$

1) Operations on intervals: The elementary mathematical operations are also extended to intervals, the operation result between two intervals is an interval containing all the operations results of all pairs of numbers in the two intervals. So, if we have two intervals $[x] = [x^-, x^+]$ and $[y] = [y^-, y^+]$ and a law $\otimes \in \{+, -, *, /\}$, we can write:

$$[x] \otimes [y] = \{x \otimes y \,|x \in [x], y \in [y]\} \tag{6}$$

B. Interval system

Definition 2.1: A linear system under parametric uncertainties is often modeled by interval system. A SISO interval system $[G](s, [\mathbf{a}], [\mathbf{b}])$ is a family of systems:

$$[G](s, [\mathbf{a}], [\mathbf{b}]) = \left\{ \frac{\sum_{j=0}^{m} b_j s^j}{\sum_{i=0}^{n} a_i s^i} \,\Big|\, b_j \in [b_j^-, b_j^+], a_i \in [a_i^-, a_i^+] \right\} \tag{7}$$

such as: $[\mathbf{b}] = [[b_0], ..., [b_m]]$ and $[\mathbf{a}] = [[a_0], ..., [a_n]]$ are two boxes (interval vectors) and s the Laplace variable.

The following lemma which is a result for interval functions is due to [8].

Lemma 2.1: **(Containment Theorem)** Given $[F]([\mathbf{x}])$ a rational expression in the interval variables $[\mathbf{x}] = [[x_1], ..., [x_n]]$. Let $[\mathbf{y}] = [[y_1], ..., [y_n]]$ be a box of interval variables, if $[\mathbf{y}] \subseteq [\mathbf{x}]$, i.e. $[y_1] \subseteq [x_1], ..., [y_n] \subseteq [x_n]$, then $[F]([\mathbf{y}]) \subseteq [F]([\mathbf{x}])$.

Proof: see [8] ■

The following theorem is a straightforward consequence of Lemma 2.1.

Theorem 2.1: Given two SISO, linear and stable interval transfers $[G_1](s, [\boldsymbol{\alpha}], [\boldsymbol{\beta}])$ and $[G_2](s, [\boldsymbol{\gamma}], [\boldsymbol{\lambda}])$ defined as in Definition 2.1. The two systems have the same structure (same degree for their numerators, idem for their denominators). If $[\boldsymbol{\alpha}] \subseteq [\boldsymbol{\gamma}]$ and $[\boldsymbol{\beta}] \subseteq [\boldsymbol{\lambda}]$, then $[G_1](s, [\boldsymbol{\alpha}], [\boldsymbol{\beta}]) \subseteq [G_2](s, [\boldsymbol{\gamma}], [\boldsymbol{\lambda}])$.

Proof: Noting that $s = [s, s] = [s]$ and $[s] \subseteq [s]$, and applying Lemma 2.1 with $[F]([\mathbf{x}]) = [G_2](s, [\boldsymbol{\gamma}], [\boldsymbol{\lambda}])$ and $[F]([\mathbf{y}]) = [G_1](s, [\boldsymbol{\alpha}], [\boldsymbol{\beta}])$, where $[\mathbf{x}] = [[s], [\boldsymbol{\gamma}], [\boldsymbol{\lambda}]]$ and $[\mathbf{y}] = [[s], [\boldsymbol{\alpha}], [\boldsymbol{\beta}]]$, we obtain:

$$[\mathbf{y}] \subseteq [\mathbf{x}] \Rightarrow [F]([\mathbf{y}]) \subseteq [F]([\mathbf{x}])$$

which leads for any s to:

$$\begin{cases} [\boldsymbol{\alpha}] \subseteq [\boldsymbol{\gamma}] \\ [\boldsymbol{\beta}] \subseteq [\boldsymbol{\lambda}] \end{cases} \Rightarrow [G_1](s, [\boldsymbol{\alpha}], [\boldsymbol{\beta}]) \subseteq [G_2](s, [\boldsymbol{\gamma}], [\boldsymbol{\lambda}])$$ ■

C. Performances of interval systems

The following theorem [20] defines the inclusion of the time/frequency domains performances of interval systems.

Theorem 2.2: Let two SISO, linear and stable interval transfers $[G_1](s, [\boldsymbol{\alpha}], [\boldsymbol{\beta}])$ and $[G_2](s, [\boldsymbol{\gamma}], [\boldsymbol{\lambda}])$ with the same structure. If $[\boldsymbol{\alpha}] \subseteq [\boldsymbol{\gamma}]$ and $[\boldsymbol{\beta}] \subseteq [\boldsymbol{\lambda}]$, then the time and the frequency domains responses of $[G_1](s, [\boldsymbol{\alpha}], [\boldsymbol{\beta}])$ are bounded by those of $[G_2](s, [\boldsymbol{\gamma}], [\boldsymbol{\lambda}])$. These responses define the time and frequency performances respectively.

Proof: see [20] ■

III. Computation of the controller

This section aims to design PID controllers ensuring performances for a parametric uncertain system. The interval arithmetic and related tools are used for that.

A. Problem statement

Consider the closed-loop with an interval system $[G](s,[\mathbf{a}],[\mathbf{b}])$ as shown on (Fig. 1). The controller must ensure some given performances for the closed-loop whatever the parameters a_i and b_j ranging in $[a_i]$ and $[b_j]$ respectively. $[H_{cl}](s,[\mathbf{p}],[\mathbf{q}])$ denotes the closed-loop transfer.

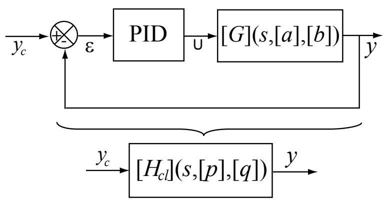

Fig. 1. Closed-loop transfer H_{cl}.

In the sequel, $[G](s,[\mathbf{a}],[\mathbf{b}])$ will be written as:

$$[G](s,[\mathbf{a}],[\mathbf{b}]) = \frac{\sum_{j=0}^{m}[b_j]s^j}{\sum_{i=0}^{n}[a_i]s^i} \tag{8}$$

with $[\mathbf{a}] = [[a_0],...,[a_n]]$, $[\mathbf{b}] = [[b_0],...,[b_m]]$ and $m \leq n$.

B. Computation of the closed-loop model

Let us define a fixed-order controller PID with adjustable parameters $[\boldsymbol{\theta}] = [[K_p],[K_i],[K_d]]$ as follows:

$$[C](s,[\boldsymbol{\theta}]) = [K_p] + [K_d]s + [K_i]\frac{1}{s} \tag{9}$$

The closed-loop model can be computed using the interval model (8) and the controller (9) as follows:

$$[H_{cl}](s,[\mathbf{a}],[\mathbf{b}],[\boldsymbol{\theta}]) = \frac{1}{\frac{1}{[C](s,[\boldsymbol{\theta}])[G](s,[\mathbf{a}],[\mathbf{b}])}+1} \tag{10}$$

After replacing $[G](s,[\mathbf{a}],[\mathbf{b}])$ and $[C](s,[\boldsymbol{\theta}])$, we get:

$$[H_{cl}](s) = \frac{\sum_{j=0}^{m+2}[\alpha_j]s^j}{\sum_{i=0}^{n}[a_i]s^{i+1} + \sum_{j=0}^{m+2}[\alpha_j]s^j} \tag{11}$$

Such as the coefficients of the box $[\alpha_j]$ for $j = 0,...,m+2$ are dependent and are function of the boxes $[\mathbf{b}]$ and $[\boldsymbol{\theta}]$.

After developing (11) and factorizing the last coefficient of the numerator (or the denominator), we obtain:

$$[H_{cl}](s,[\mathbf{p}],[\mathbf{q}]) = \frac{1+\sum_{j=1}^{e}[q_j]s^j}{\sum_{i=0}^{r}[p_i]s^i} \tag{12}$$

where $e = m+2$ and $r = max(n+1, m+2)$ and where the boxes of interval parameters $[\mathbf{q}]$ and $[\mathbf{p}]$ are function of the boxes $[\mathbf{a}]$, $[\mathbf{b}]$ and of the controller parameters $[\boldsymbol{\theta}]$.

C. Feasible controller parameters

The objective consists to compute the set Θ of controller parameters (9) for which the set of all possible closed-loop behaviors (12) is included inside the set of all wanted behaviors defined by an interval reference model $[H]$:

$$\Theta = \{\theta \in [\boldsymbol{\theta}] \,|\, [H_{cl}](s,[\mathbf{p}],[\mathbf{q}]) \subseteq [H](s)\} \tag{13}$$

The condition $[H_{cl}](s,[\mathbf{p}],[\mathbf{q}]) \subseteq [H](s)$ can be checked by applying the parameter by parameter inclusion as given in Theorem 2.1. For that, the interval reference model $[H]$ must have the same structure than $[H_{cl}]$ (12). Therefore, we use as reference model:

$$[H](s,[\mathbf{w}],[\mathbf{x}]) = \frac{1+\sum_{j=1}^{e}[x_j]s^j}{\sum_{i=0}^{r}[w_i]s^i} \tag{14}$$

Remark 3.1: $[\mathbf{w}]$ and $[\mathbf{x}]$ are two interval boxes chosen by the user from the specifications.

Based on (12), (14) and Theorem 2.2, the problem (13) can be reduced to the problem of finding Θ such as:

$$\Theta = \left\{\theta \in [\boldsymbol{\theta}] \;\middle|\; \begin{array}{ll} [q_j] \subseteq [x_j], & \text{for} \quad j = 1,...,e \\ [p_i] \subseteq [w_i], & \text{for} \quad i = 0,...,r \end{array}\right\} \tag{15}$$

Remark 3.2: The number of unknown parameters in (9) is 3 while the number of inclusions is $r+e+1$ (see (15)). Since $e = m+2$ and $r = max(n+1, m+2)$, we can write $r+e+1 > 3$. Therefore, there are more inclusions than unknown variables.

The problem of finding the set Θ so that (15) holds, is known as a set-inversion problem which can be solved using set inversion algorithms. An algorithm that can be used to solve such problem is the SIVIA algorithm [9].

IV. Control of piezocantilevers

This section is focused on the application of the proposed method to control piezoelectric microactuators (piezocantilevers) used in microgrippers. We particularly use unimorph piezocantilevers due to their ease of fabrication relative to multimorph ones. A unimorph piezocantilever is made up of one piezoelectric layer (often Lead Zirconate Titanate: PZT) and one passive layer (Copper). Indeed, the piezoelectric layer is used to actuate or produce energy while the non-piezoelectric layer (passive) is used to add stiffness as well as make the beam more durable. When a voltage U is applied to the piezoelectric layer, the cantilever expands/contracts which causes a global deflection δ (Fig. 2). Besides, a force F applied at the tip of piezocantilever may also cause a charge between the electrodes of the piezoelectric layer. In this situation, energy can be produced from the electrodes.

Piezocantilevers can be modeled by a transfer function with varying parameters. Unfortunately, such models with interval parameters are difficult to obtain. In addition, it is known that small differences in dimensions (somes microns) of similar piezocantilevers due to the imprecision of the microfabrication process, generate non-negligible difference on their model parameters. So, instead of having one piezocantilever with varying parameters during the experiment, we use two (or more) similar piezocantilevers. Thereafter, the derivation of the interval model $[G](s,[\mathbf{a}],[\mathbf{b}])$ is based on the two models of the used piezocantilevers. This interval model is then used to design controller that ensures performances not only for the both piezocantilevers but also for a set of piezocantilevers.

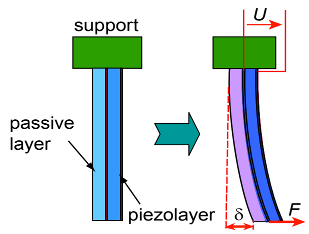

Fig. 2. Piezocantilever principle.

A. Presentation of the setup

Fig. 3 presents the experimental setup. It is composed of:

- two piezocantilevers each one having a total thickness of $0.3mm$, a width of $2mm$ and a length of $15mm$,
- a computer-DSpace hardware and the Matlab-Simulink software used for the data-acquisition and control,
- a Keyence optical sensor with $10nm$ of resolution used to measure the deflections of the piezocantilevers,
- and a high-voltage (HV) amplifier.

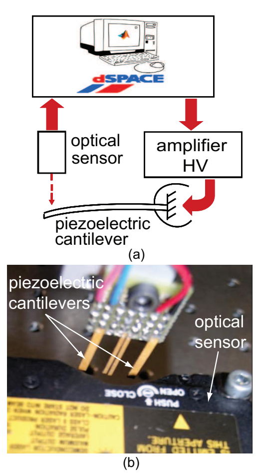

Fig. 3. The experimental setup.

B. Modeling and identification

The linear relation that relates the output deflection to the input voltage U applied to piezocantilevers [6]:

$$\delta = G(s)U \tag{16}$$

where, for us, the transfer functions $G_1(s)$ and $G_2(s)$ that model the two piezocantilevers must be identified.

For the identification, a step voltage $U = 20V$ is applied to each piezocantilever. A second order was chosen for each model because of its sufficiency to account (the first) resonance and its simplicity (low order). Using the output error method and the matlab software, we obtain:

$$\begin{aligned} G_1(s) &= \frac{8.0313\times10^{-8}s^2+1.808\times10^{-4}s+1}{9.794\times10^{-8}s^2+5.24\times10^{-6}s+1.44} \\ G_2(s) &= \frac{7.042\times10^{-8}s^2+1.809\times10^{-4}s+1}{8.802\times10^{-8}s^2+5.364\times10^{-6}s+1.291} \end{aligned} \tag{17}$$

C. Derivation of the interval model

Let us rewrite each model $G_i(s)$ $(i = 1, 2)$ as follows:

$$G_i(s) = \frac{b_{2i}s^2 + b_{1i}s + b_{0i}}{a_{2i}s^2 + a_{1i}s + a_{0i}} \tag{18}$$

The interval model $[G](s,[\mathbf{a}],[\mathbf{b}])$ which represents a family of piezocantilever models is derived using the two point models $G_i(s)$. Considering each parameter of $G_1(s)$ and the corresponding parameter in $G_2(s)$ as an endpoint of the interval parameter in $[G](s,[\mathbf{a}],[\mathbf{b}])$, we have:

$$[G](s,[\mathbf{a}],[\mathbf{b}]) = \frac{[b_2]s^2 + [b_1]s + [b_0]}{[a_2]s^2 + [a_1]s + [a_0]} \tag{19}$$

such as:

$$\begin{aligned} [b_2] &= [min(b_{21}, b_{22}), max(b_{21}, b_{22})] \\ [b_1] &= [min(b_{11}, b_{12}), max(b_{11}, b_{12})] \\ [b_0] &= [min(b_{01}, b_{02}), max(b_{01}, b_{02})] \\ [a_2] &= [min(a_{21}, a_{22}), max(a_{21}, a_{22})] \\ [a_1] &= [min(a_{11}, a_{12}), max(a_{11}, a_{12})] \\ [a_0] &= [min(a_{01}, a_{02}), max(a_{01}, a_{02})] \end{aligned}$$

After the numerical application, we obtain:

$$\begin{aligned} [b_2] &= [7.042, 8.0313] \times 10^{-8} \\ [b_1] &= [1.808, 1.809] \times 10^{-4} \\ [b_0] &= 1 \\ [a_2] &= [8.802, 9.794] \times 10^{-8} \\ [a_1] &= [5.24, 5.364] \times 10^{-6} \\ [a_0] &= [1.291, 1.44] \end{aligned}$$

In order to increase the stability margin of the closed-loop system and to ensure that the interval model really contains the models (17), we propose to expand the intervals of the model (19). However, it is noticed that when the interval width of the parameters in the model is too large, it is difficult to find a controller that respects both the stability and performances of the closed-loop. After some trials of controller design, we choose to expand the interval width of each parameter of (19) by 10%. 10% represents the maximal value allowed in this application. Finally, the extended parameters of the interval model which will be used for the controller design are given by:

$$\begin{aligned} [b_2] &= [6.992, 8.08] \times 10^{-8} \\ [b_1] &= [1.807, 1.809] \times 10^{-4} \\ [a_2] &= [8.753, 9.844] \times 10^{-8} \\ [a_1] &= [5.234, 5.37] \times 10^{-6} \\ [a_0] &= [1.283, 1.448] \end{aligned} \tag{20}$$

D. Definition of the specifications

Consider the following specifications for the closed-loop. These specifications correspond to the requirement in micropositioning tasks for microassembly and micromanipulation that use piezoelectric microgrippers.

- no overshoot,
- settling time $1ms \leq tr_{5\%} \leq 30ms$,
- static error $|\varepsilon| \leq 1\%$.

As the desired system behavior is without overshoot, we can use two first-order systems to create an interval reference model. The used time constants for both systems are $\frac{1}{3}ms$ and $10ms$, while -0.01 and 0.01 for their statical errors.

E. Controller structure

In this application, we consider a PI (Proportional-Integral) structure because of its low-order (two parameters). It is a particular case of PID controllers where the derivative action K_d is set to zero. Though PID structure can be easily used as presented in Section III.

$$[C](s,[K_p],[K_i]) = \frac{[K_p]s + [K_i]}{s} \tag{21}$$

such as K_p and K_i are the parameters to be adjusted and representing the proportional and the integral gains repsectively.

F. The closed-loop and the reference models computation

The general model of the closed-loop is given by (10). In our case, the closed-loop transfer is obtained using the model with interval parameters in (20) and the controller (21):

$$[H_{cl}](s,[\mathbf{p}],[\mathbf{q}]) = \frac{[q_3]s^3 + [q_2]s^2 + [q_1]s + 1}{[p_3]s^3 + [p_2]s^2 + [p_1]s + [p_0]} \tag{22}$$

such as:

$$\begin{aligned}
[q_3] &= \tfrac{[K_p][b_2]}{[K_i]} \\
[q_2] &= \tfrac{[K_p][b_1]}{[K_i]} + [b_2] \\
[q_1] &= \tfrac{[K_p]}{[K_i]} + [b_1] \\
[p_3] &= \tfrac{[a_2]+[K_p][b_2]}{[K_i]} \\
[p_2] &= \tfrac{[a_1]+[K_p][b_1]}{[K_i]} + [b_2] \\
[p_1] &= \tfrac{[a_0]+[K_p]}{[K_i]} + [b_1] \\
[p_0] &= 1
\end{aligned}$$

The computation of the interval reference model is based on the required specifications and the structure of the closed-loop (22). As we said before, a first order interval model would be considered (see Section IV-D):

$$[H](s,[K],[\tau]) = \frac{[K]}{[\tau]s + 1} \tag{23}$$

where the parameters $[K]$ and $[\tau]$ define the static error and settling time respectively and are deduced from specifications as follows:

- $[K] = 1 + \varepsilon = [0.99, 1.01]$,
- $[\tau] = \frac{[tr_{5\%}]}{3} = [0.33ms, 10ms]$.

However, the application of the parameter by parameter inclusion (15) requires that the reference model has the same structure than the closed-loop (22): the same degrees for the numerator, also for the denominators. $[H_{cl}](s,[\mathbf{p}],[\mathbf{q}])$ has a degree of 3 for both numerator and denominator. Thus we add some poles and zeros far from the imaginary axis to (23)

$$[H](s,[K],[\tau]) = \frac{[K](\frac{[\tau]}{10}s + 1)^3}{([\tau]s + 1).(\frac{[\tau]}{10}s + 1)^2} \tag{24}$$

which can also be rewritten as follows:

$$[H](s,[\mathbf{w}],[\mathbf{x}]) = \frac{[x_3]s^3 + [x_2]s^2 + [x_1]s + 1}{[w_3]s^3 + [w_2]s^2 + [w_1]s + [w_0]} \tag{25}$$

such as:

$$\begin{aligned}
[x_3] &= 0.001[\tau]^3 \\
[x_2] &= 0.03[\tau]^2 \\
[x_1] &= 0.3[\tau] \\
[w_3] &= \tfrac{0.01[\tau]^3}{[K]} \\
[w_2] &= \tfrac{0.21[\tau]^2}{[K]} \\
[w_1] &= \tfrac{1.2[\tau]}{[K]} \\
[w_0] &= \tfrac{1}{[K]}
\end{aligned}$$

G. Acheivement of robust performances

The controller defined in (21) ensures the required specifications in Section IV-D if its parameters $[K_p]$ and $[K_i]$ meet the following inclusions:

$$\begin{aligned}
&\tfrac{[K_p][b_2]}{[K_i]} \subseteq 0.001[\tau]^3 \\
&\tfrac{[K_p][b_1]}{[K_i]} + [b_2] \subseteq 0.03[\tau]^2 \\
&\tfrac{[K_p]}{[K_i]} + [b_1] \subseteq 0.3[\tau] \\
&\tfrac{[a_2]+[K_p][b_2]}{[K_i]} \subseteq \tfrac{0.01[\tau]^3}{[K]} \\
&\tfrac{[a_1]+[K_p][b_1]}{[K_i]} + [b_2] \subseteq \tfrac{0.21[\tau]^2}{[K]} \\
&\tfrac{[a_0]+[K_p]}{[K_i]} + [b_1] \subseteq \tfrac{1.2[\tau]}{[K]} \\
&1 \subseteq \tfrac{1}{[K]}
\end{aligned} \tag{26}$$

H. Derivation of the controller

Let us now compute the controller parameters. The problem given in (26) is known as a set-inversion problem which can be solved using SIVIA algorithm [9] if an initial box is provided. We denote S_c the set parameters of the controller that satisfy these conditions. After the application of the SIVIA algorithm implemented in the Matlab-Software, with an initial box $[K_{p0}] \times [K_{i0}] = [0.1, 0.6] \times [0.1, 500]$, we obtain the subpaving given in Fig. 4. The dark colored subpaving (S_c) corresponds to the set parameters $[K_p]$ and $[K_i]$ that define a family of controllers ensuring performances for the interval model.

Remark 4.1: Any choice of the parameters $[K_p]$ and $[K_i]$ in the dark colored subpaving S_c (see Fig. 4) satisfies the inclusions (26) and consequently ensures specifications in Section IV-D.

Remark 4.2: If the set-inversion problem is not feasible, i.e. $S_c = \emptyset$, the initial box of the parameters must be changed and/or one must modify the specifications.

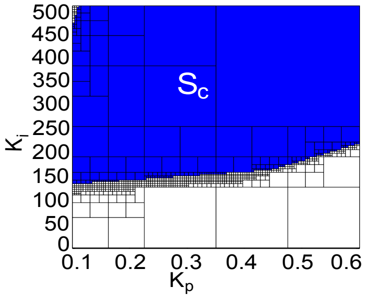

Fig. 4. Set parameters $[K_p] \times [K_i]$ ensuring performances.

I. Experimental results

For the implementation of the controller, (point) parameters K_p and K_i must be taken from the set solution S_c. In this application, we test two point controllers:

$$\begin{array}{l} C_1(s) = \frac{0.1s+200}{s} \\ C_2(s) = \frac{0.2s+400}{s} \end{array} \qquad (27)$$

These two controllers are implemented for the two piezocantilevers. Fig. 5 shows the experimental results when a step reference of $20\mu m$ is applied. In this figure, we also have the temporal envelope of the reference model $[H](s, [K], [\tau])$. We mean by the envelope of $[H](s, [K], [\tau])$ the step responses of two tranfer functions $H_1(s)$ and $H_2(s)$ such as: 1) $H_1(s)$ has the minimal constant time $\tau = 0.33ms$ and the maximal static gain $K = 1.01$, 2) and $H_2(s)$ has the maximal constant time $\tau = 10ms$ and the minimal static gain $K = 0.99$. As shown in Fig. 5, the controllers have played their roles and ensure the specifications. Indeed, experimental settling times are $tr_1 = 17.7ms$ and $tr_2 = 7ms$ with $C_1(s)$ and $C_2(s)$ respectively, and the static errors are neglected and belong to the required interval $|\varepsilon| \leq 1\%$.

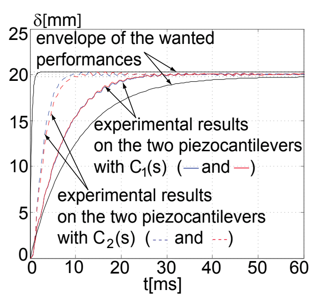

Fig. 5. Step response of the interval reference model and experimental results on the two piezocantilevers using $C_1(s)$ and $C_2(s)$.

V. Conclusion

The main contribution of this paper was the interval modeling and robust PID controller design for piezoelectric microsystems. These microsystems are known to be sensitive to usury functioning and to environmental disturbances making their models uncertain during micromanipulation or microassembly tasks. We therefore introduced interval techniques to model the uncertain parameters and to compute robust control law. The main advantage to use intervals is the ease and natural way to bound the parametric uncertainties. The proposed controller design is advantageous for deriving low-order robust controllers which are necessary to develop real packaged microsystems. Finally, the experimental results demonstrated the efficiency of the proposed method.

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